LIQUIDITY SHRINKAGES AND INCOMPLETE FINANCIAL PARTICIPATION IN DYNAMIC ECONOMIES WITH ASSET-BACKED SECURITIES

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ABSTRACT. We address an infinite horizon economy with collateralized debt and asset-backed securities, allowing for general prepayment specifications, liquidity contractions, and incomplete financial participation. We guarantee equilibrium existence without requiring uniform impatience on preferences or imposing debt constraints other than those endogenously determined by collateral requirements.

KEYWORDS. Asset-Backed Securities - Equilibrium - Rational Asset Pricing Bubbles

JEL CLASSIFICATION. D50, D52.

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1. Introduction

The analysis of sequential economies with incomplete markets has been a central topic of research in theoretical economics since the pioneering articles of Radner (1972) and Hart (1975). These works led to a vast literature which gave account of the different imperfections that may appear when financial markets are incomplete (for a review of the principal results see Geanakoplos (1990)). This framework also offered the possibility to study asset-pricing in a general equilibrium context, providing macroeconomics with a useful widely-applicable tool based on microeconomic analysis.

Despite the development achieved by this line of investigation, the question of existence of equilibrium in infinite horizon economies supposes a complex issue. One of the principal problems in this context is that, when agents are allowed to sell short, they can make use of Ponzi schemes. That is, they can postpone the commitment of its financial obligations by acquiring new debt at each successive period, undetermining the problem of a utility maximizing individual. The classical solution to this problem—applied in the macroeconomic equilibrium literature with complete markets—was the imposition of debt constraints or transversality conditions to the individuals' choice sets, eliminating exogenously the possibility of agents committing into these non-solvent plans.

The application of such techniques to the incomplete markets framework was non-trivial, since there is not necessarily a unique vector of present values of future resources to define the transversality conditions. To solve this issue, Magill and Quinzii (1994) proposed to use personalized deflators given by the Kuhn-Tucker multipliers induced by the budget constraints. Using this approach, they show that when agents are uniformly impatient—a joint requirement in preferences and endowments—and assets are short-lived, an equilibrium with transversality condition always exists. Also, there is a one-to-one correspondence between equilibria with transversality conditions and equilibria with implicit debt constraints. This last property allows to determine explicit debt constraints that are never binding in equilibrium. Similarly, Hernández and Santos (1996) show the existence of equilibrium for uniform impatient agents when assets are short-lived and transversality conditions are determined by using the most punishing deflator among those compatible with non-arbitrage. For long-lived assets, equilibrium does not necessarily exists, because the dependence of deliveries in prices may induce discontinuities on individual demands (cf., Magill and Quinzii (1996), Hernández and Santos (1996), Levine and Zame (1996)).

Debt constraints and transversality conditions are financial frictions included on budget sets that do not have an economic *rationale*. As an improvement to this respect, Araujo, Páscoa and Torres-Martínez (2002) introduced credit risk to the infinite-horizon analysis of incomplete markets, by allowing agents to default on collateral-backed debt. With the collateral structure they were able to endogenously rule out Ponzi schemes and prove equilibrium existence without the need of assuming that agents are uniformly impatient, or including either debt constraints or transversality conditions on budget sets. Moreover, these results can be extended to economies with long-lived assets (Araujo, Páscoa and Torres-Martínez (2005, 2011)) without requiring further assumptions.

In the present project, we follow these previous works and address an infinite-horizon general equilibrium model with default, where assets are backed by collateral. When agents decide to default, the physical guarantees—that they were burdened to constitute when selling the debt contract—are seized, without any additional credit recovery mechanism or utility punishment.

We add three principal features to this framework: (i) the possibility of *prepayment of debt*, which was introduced to the general equilibrium literature in a recent paper by Iraola and Torres-Martínez (2013); (ii) the existence of *incomplete financial participation*, developed by Angeloni and Cornet (2006), Aouani and Cornet (2009), and Seghir and Torres-Martínez (2011), among others; and (iii) the possibility of individuals to attain *leasing transactions*.

Our model extends that of Iraola and Torres-Martínez (2013) not only to a multi-period setting (allowing infinite horizon analysis), but also adds rental markets and incomplete financial participation on investment opportunities. We allow the durability of commodities to depend on wether they are consumed by its owner or by a lessee. Thus, in our economy the aggregated physical resources available in any state of nature is determined endogenously, as it can vary with individual decisions about demand and consumption.

Under this framework, we are able to prove existence of equilibrium with analogous hypotheses to those described on Iraola and Torres-Martínez (2013), for the finite horizon case. However, when the relevant horizon is infinite, an additional condition—not required on the previous literature with collateral-backed securities—is included to guarantee the existence of equilibrium. This hypothesis is related to individual preferences, and can be described as a particular kind of impatience, although it is not the uniform patience used on default-free models.

To build the asset structure of our economy we follow closely the model by Iraola and Torres-Martínez (2013), where debt contracts are pooled in passthrough securities that serve as the investment instruments. Thus, each credit contract is characterized by its emission node, coupon payments, prepayment rule, and collateral requirements; while passthrough securities are completely defined by its price and payments delivered to investors through time. Since each credit contract is uniquely associated to one passthrough security, payments delivered from debtors of the former are proportionally distributed among creditors of the latter.

The possibility of prepayment in a general equilibrium context is particularly interesting since it allows the occurrence of loans with *negative equity* as an equilibrium phenomena. That is, agents can rationally decide to honor the coupons of a debt, even when the associated collateral has a lower market value than the prepayment cost. In order to observe this kind of behavior, it is fundamental to incorporate liquidity contractions or incomplete financial participation, since payment decisions are determined by the availability of alternative credit opportunities. In other words, if credit opportunities are not subject to any class of shrinkage, individuals optimally decide to apply strategic default on their debt (cf., Araujo, Páscoa and Torres-Martínez (2005, 2011)).

In this framework it is also possible to observe heterogeneous decisions among agents when it comes to the fulfillment of their financial commitments. That is, different individuals with the same obligation can make different decisions —either to pay, prepay or default on their debt— at the same state of nature, depending on their respective preferences and financial opportunities. We illustrate these possibilities in a numerical example.

The rest of the paper is organized as follows. Section 2 describes the model, notation, and equilibrium definition; Section 3 displays our principal results on equilibrium existence for the finite and infinite horizon cases; Section 4 develops the numerical example where loans with negative equity occurs in equilibrium. Finally, Section 5 contains some concluding remarks. Proofs are left to appendices.

2. An Economy with Securitization and Liquidity Contractions

Information structure. We consider a discrete time economy with time horizon $T \in \mathbb{N} \cup \{+\infty\}$ and where periods are denoted by $t \in \{0, 1, \ldots, T\}$. Uncertainty is characterized by a set S of states of nature. At each period t the available information is homogeneous across agents and is given by a finite partition $\mathbb{F}_t$ of S. There is no information at $t = 0$, i.e., $\mathbb{F}_0 = \{S\}$. Information is revealed through, as $\mathbb{F}_s$ is at least as fine as $\mathbb{F}_t$, for any $s > t$. When $T < \infty$, we assume that $\#S < \infty$ and $\mathbb{F}_T = \{\{s\} : s \in S\}$.

A node is a pair $\xi = (t, \sigma)$, where $t \in \{0, 1 \ldots, T\}$ and $\sigma \in \mathbb{F}_t$. The only initial node is denoted by ξ_0. Let D be the event-tree composed of all nodes in the economy. Given $\xi \in D$, t_ξ and σ_ξ are respectively the date and the information set associated with ξ. Let $D_s := \{\xi \in D : t_\xi = s\}$ be the set of nodes at period s, where the set of terminal nodes D_T is assumed to be empty when the economy has infinite horizon. The set of intermediate nodes is denoted by $\mathring{D} := D \setminus (\{\xi_0\} \cup D_T)$.

If both $t_\mu > t_\xi$ and $\sigma_\mu \subseteq \sigma_\xi$, then μ is a successor node of ξ, denoted as $\mu > \xi$. As customary, $\mu \geq \xi$ means that either $\mu = \xi$ or $\mu > \xi$. Let ξ^- be the only immediate predecessor of ξ (i.e., the only node that satisfies $\xi > \xi^-$ and $t_\xi = t_{\xi^-} + 1$), and $\xi^+ := \{\mu \in D : \mu > \xi \wedge t_\mu = t_\xi + 1\}$ the set of immediate successor nodes of $\xi \in D$.

Physical markets. There is a finite an ordered set L of commodities which can be traded, consumed and leased. At every $\xi \in D$, spot markets for each commodity are available, and characterized by a vector of spot prices $p_\xi = (p_{\xi,l})_{l \in L} \in \mathbb{R}^L_+$. The plan of prices along the event-tree is denoted by $p = (p_\xi)_{\xi \in D}$.

Commodities are durable and suffers transformations between periods, which are characterized at each $\xi \in D$ by two $L \times L$ matrices with non-negative entries, (Y^c_ξ, Y^r_ξ). Thus, if a bundle $x \in \mathbb{R}^L_+$ is consumed at node ξ by its owner, then it is transformed into $Y^c_\mu x$ at each $\mu \in \xi^+$. However, if the same bundle is consumed by a lessee, it is transformed into the bundle $Y^r_\mu x$ at each $\mu \in \xi^+$. On the one hand, this structure captures perfectly durable, perishable or depreciable commodities, and also allows for the transformation of some commodities into others. On the other, we allow owners and lessees to have a different treatment on the commodities they consume.

Rental markets. Since commodities are durable, we allow individuals to attain lease transactions. At each $\xi \in D$, there is a set $R(\xi)$ of rental contracts available. Each rental contract $a \in R(\xi)$

is characterized by a bundle $M_{\xi,a} \in \mathbb{R}_+^L$. The bundle $M_{\xi,a}$ is bought by a lessor at node ξ, who delivers it to a lessee for a price $r_{\xi,a}$. At each immediate successor $\mu \in \xi^+$ the contract is finished, and the lessor receives back the transformed bundle $Y_\mu^r M_{\xi,a}$. Let $r = (r_\xi)_{\xi \in D}$ be the plan of rental prices, where $r_\xi = (r_{\xi,a})_{a \in R(\xi)} \in \mathbb{R}_+^{R(\xi)}$ denotes the vector of rental prices at node ξ.

Financial contracts. Borrowing possibilities are characterized by finite and ordered sets of credit contracts $(J(\xi); \xi \in D)$. Each credit contract $j \in J(\xi)$ has an associated passthrough security, which distributes borrowers' payments and is denoted with the subscript j. The issuing node of a debt contract j is denoted by ξ_j (i.e., $j \in J(\xi)$ if and only if $\xi = \xi_j$). Each debt contract is only traded at its issuing node, while passthrough securities can be negotiated along the event-tree.

Let $K(\xi) := \bigcup_{\mu \in D : \xi \geq \mu} J(\mu)$ be the set of investment opportunities available at a non-terminal node $\xi \in D$. Without loss of generality, assume that at the issuing node ξ_j the price of the credit contract j is the same as the price of the associated passthrough security. Let $q = (q_\xi)_{\xi \in D}$ be the financial prices, where $q_\xi = (q_{\xi,j})_{j \in K(\xi)} \in \mathbb{R}_+^{K(\xi)}$.

The issuer of one unit of credit contract $j \in J(\xi)$ has the obligation to pledge a physical collateral $C_{\xi,j} \in \mathbb{R}_+^L \setminus \{0\}$ and promises to pay coupons $(A_{\mu,j}(\pi))_{\mu > \xi}$ at the successor nodes of ξ, where $\pi = (p, q, r)$ denotes the prices in the economy. The collateral is consumed by the issuer of the debt as long as the short position is maintained open.

At each intermediate node $\mu \in \mathring{D}$ such that $\mu > \xi_j$, the issuer of a debt j may deliver the coupon, prepay, or default. The prepayment of one unit of credit contract j at μ has a cost $B_{\mu,j}(\pi) \geq A_{\mu,j}(\pi)$. If a borrower decides to default, collateral guarantees are seized. Let $C_{\mu,j}$ be the bundle obtained at $\mu > \xi_j$ as a consequence of the transformation generated over $C_{\xi_j,j}$ by its consumption through time, recursively defined by $C_{\mu,j} = Y_\mu^c C_{\mu^-,j}$. Therefore, the cost of defaulting on one unit of j at $\mu > \xi_j$ equals $p_\mu C_{\mu,j}$. Notice that, at terminal nodes, borrowers may deliver the coupon or default, because the prepayment cost implicitly equals the coupon value.

The buyer of one unit of passthrough security $j \in J(\xi)$ receives, at each node $\mu > \xi$, a unitary payment $N_{\mu,j}$ such that borrowers' deliveries are fully distributed among security investors. For convenience of notation, let $N = (N_\xi)_{\xi > \xi_0}$ be the vector of security payments, where $N_\xi := (N_{\xi,j})_{j \in K(\xi^-)}$, $\forall \xi > \xi_0$.

Households. A finite set H of agents demand commodities and trade financial instruments through the whole event-tree. Each agent $h \in H$ is characterized by a utility function $U^h : \mathbb{R}_+^{D\times L} \to \mathbb{R}_+ \cup \{+\infty\}$ and an endowment process $w^h = (w_\xi^h)_{\xi\in D} \in \mathbb{R}_+^{D\times L}$.

Individuals may face restricted access to financial instruments. Thus, $J^h(\xi) \subseteq J(\xi)$ and $K^h(\xi) \subseteq K(\xi)$ denote, respectively, the set of credit contracts and the set of securities that agent h can trade at ξ. We denote by $\widehat{J}^h(\xi)$ the set of credit contracts issued before ξ that were available to agent h. We assume that agents do not loss the access to investment opportunities along the event-tree. That is, for every $h \in H$, $K^h(\xi^-) \subseteq K^h(\xi)$ at any non-terminal node $\xi \in D$.

At the initial node ξ_0 every agent $h \in H$ chooses an allocation $z_{\xi_0}^h = (x_{\xi_0}^h, \theta_{\xi_0}^h, \varphi_{\xi_0}^h, \phi_{\xi_0}^h, \psi_{\xi_0}^h)$ belonging to $\mathbb{E}^h(\xi_0) := \mathbb{R}_+^L \times \mathbb{R}_+^{K^h(\xi)} \times \mathbb{R}_+^{J^h(\xi)} \times \mathbb{R}_+^{R(\xi)} \times \mathbb{R}_+^{R(\xi)}$, where

$x_{\xi_0}^h = (x_{\xi_0,l}^h)_{l\in L}$ is the autonomous consumption bundle;[1]

$\theta_{\xi_0}^h = (\theta_{\xi_0,j}^h)_{j\in K^h(\xi_0)}$ is the investment portfolio;

$\varphi_{\xi_0}^h = (\varphi_{\xi_0,j}^h)_{j\in J^h(\xi_0)}$ is the debt position;

$\phi_{\xi_0}^h = (\phi_{\xi_0,a}^h)_{a\in R(\xi_0)}$ is the position as a lessor;

$\psi_{\xi_0}^h = (\psi_{\xi_0,a}^h)_{a\in R(\xi_0)}$ is the position as a lessee.

At every intermediate node $\xi \in \mathring{D}$, agent h chooses $z_\xi^h = (x_\xi^h, \theta_\xi^h, \varphi_\xi^h, \varphi_\xi^{\alpha,h}, \varphi_\xi^{\beta,h}, \phi_\xi^h, \psi_\xi^h)$ in the set

$$\mathbb{E}^h(\xi) := \mathbb{R}_+^L \times \mathbb{R}_+^{K^h(\xi)} \times \mathbb{R}_+^{J^h(\xi)} \times \mathbb{R}_+^{\widehat{J}^h(\xi)} \times \mathbb{R}_+^{\widehat{J}^h(\xi)} \times \mathbb{R}_+^{R(\xi)} \times \mathbb{R}_+^{R(\xi)},$$

where $\varphi_{\xi,j}^{\alpha,h}$ are the coupons of debt j honored at ξ, and $\varphi_{\xi,j}^{\beta,h}$ are the units of j prepaid at ξ. At every terminal node $\xi \in D_T$, agent h chooses an allocation $z_\xi^h = (x_\xi^h) \in \mathbb{E}^h(\xi) := \mathbb{R}_+^L$. It follows from the description above that agent h choses strategies $z^h = (z_\xi^h)_{\xi\in D}$ in the space $\mathbb{E}^h := \prod_{\xi\in D} \mathbb{E}^h(\xi)$.

For convenience of notation, at every node $\mu \in D$ we write $\varphi_{\mu,j}^{\alpha,h} = \varphi_{\mu,j}^h$ whenever $j \in J(\mu)$. Thus, at any $\mu \in \xi^+$, agent h defaults on $\varphi_{\mu,j}^{\gamma,h} := (\varphi_{\xi,j}^{\alpha,h} - \varphi_{\mu,j}^{\alpha,h} - \varphi_{\mu,j}^{\beta,h})$ units of credit contract $j \in K(\xi)$.

Individuals choice constraints. The existence of collateralized contracts and rental markets impose patrimonial obligations on borrowers and lessors. While borrowers are required to constitute collateral guarantees, lessors only can rent a bundle when they have its property. Thus, associated to an allocation $z_\xi^h \in \mathbb{E}^h(\xi)$ at node ξ, the total demand for commodities of agent $h \in H$ at this node is equal to the sum of her autonomous consumption, the bundles leased, and the

[1]Autonomous consumption is one that is not related to collateral guarantees nor to lease transactions.

collateral requirements associated to debt positions, i.e.,

$$d_\xi^h(z_\xi^h) := \begin{cases} x_\xi^h + \sum_{a\in R(\xi)} M_{\xi,a}\,\phi_{\xi,a}^h + \sum_{j\in J^h(\xi)} C_{\xi,j}\varphi_{\xi,j}^h & \text{if } \xi=\xi_0; \\ x_\xi^h + \sum_{a\in R(\xi)} M_{\xi,a}\,\phi_{\xi,a}^h + \sum_{j\in J^h(\xi)} C_{\xi,j}\varphi_{\xi,j}^h + \sum_{j\in \widehat{J}^h(\xi)} C_{\xi,j}\varphi_{\xi,j}^{\alpha,h} & \text{if } \xi\in \mathring{D}\,; \\ x_\xi^h & \text{if } \xi\in D_T. \end{cases}$$

Similarly, to determine consumption allocations we need to consider that lessees consume the bundles that they rent and borrowers hold collateral guarantees. Thus, associated to an allocation $z_\xi^h \in \mathbb{E}^h(\xi)$ at node ξ, the individual consumption at this node is given by

$$c_\xi^h(z_\xi^h) := \begin{cases} x_\xi^h + \sum_{a\in R(\xi)} M_{\xi,a}\,\psi_{\xi,a}^h + \sum_{j\in J^h(\xi)} C_{\xi,j}\varphi_{\xi,j}^h & \text{if } \xi=\xi_0; \\ x_\xi^h + \sum_{a\in R(\xi)} M_{\xi,a}\psi_{\xi,a}^h + \sum_{j\in J^h(\xi)} C_{\xi,j}\varphi_{\xi,j}^h + \sum_{j\in \widehat{J}^h(\xi)} C_{\xi,j}\varphi_{\xi,j}^{\alpha,h} & \text{if } \xi\in \mathring{D}\,; \\ x_\xi^h & \text{if } \xi\in D_T. \end{cases}$$

A key property of our model is that durability of commodities may depend on whether an agent consumes or leases them and, therefore, the availability of physical resources is endogenously determined in equilibrium. In particular, when agent h chooses an allocation $z_\xi^h \in \mathbb{E}^h(\xi)$ at node ξ, the physical resources available at immediate successor node $\mu\in\xi^+$ are given by

$$W_\mu^h(z_\xi^h) := w_\mu^h + Y_\mu^c d_\xi^h(z_\xi^h) - \sum_{a\in R(\xi)} (Y_\mu^c - Y_\mu^r) M_{\xi,a}\,\phi_{\xi,a}^h.$$

Given prices $\pi=(p,q,r)\in\mathbb{P}$ and security payments $N=(N_\xi)_{\xi>\xi_0}\in\mathcal{N}$, where

$$\mathbb{P} := \mathbb{R}_+^{D\times L} \times \prod_{\xi\in D\setminus D_T} \mathbb{R}_+^{K(\xi)} \times \prod_{\xi\in D\setminus D_T} \mathbb{R}_+^{R(\xi)}, \qquad \mathcal{N} := \prod_{\xi\in D\setminus\{\xi_0\}} \mathbb{R}_+^{K(\xi^-)},$$

the objective of each agent $h\in H$ is to choose a plan $z^h\in\mathbb{E}^h$ to maximize the utility derived from consumption, subject to the following constraints,

$$g_{\xi_0}^h(z_{\xi_0}^h;(\pi,N)) := p_{\xi_0}\left(d_{\xi_0}^h(z_{\xi_0}^h) - w_{\xi_0}^h\right) + \sum_{j\in K^h(\xi_0)} q_{\xi_0,j}\theta_{\xi_0,j}^h - \sum_{j\in J^h(\xi_0)} q_{\xi_0,j}\varphi_{\xi_0,j}^h - r_{\xi_0}(\phi_{\xi_0}^h - \psi_{\xi_0}^h) \le 0;$$

for every intermediate node $\xi\in\mathring{D}$,

$$\begin{aligned} g_\xi^h(z_\xi^h, z_{\xi^-}^h;(\pi,N)) := {} & p_\xi\left(d_\xi^h(z_\xi^h) - W_\xi^h(z_{\xi^-}^h)\right) + \sum_{j\in K^h(\xi)} q_{\xi,j}\theta_{\xi,j}^h - \sum_{j\in J^h(\xi)} q_{\xi,j}\varphi_{\xi,j}^h \\ & - \sum_{j\in K^h(\xi^-)} (q_{\xi,j}+N_{\xi,j})\,\theta_{\xi^-,j}^h + \sum_{j\in\widehat{J}^h(\xi)} \Phi_{\xi,j}^h(z_\xi^h, z_{\xi^-}^h;\pi) - r_\xi(\phi_\xi^h - \psi_\xi^h) \le 0\,; \end{aligned}$$

$$\varphi_{\xi,j}^{\alpha,h} + \varphi_{\xi,j}^{\beta,h} \le \varphi_{\xi^-,j}^{\alpha,h}, \quad \forall j\in\widehat{J}^h(\xi)\,;$$

where $\Phi^h_{\xi,j}(z^h_\xi, z^h_{\xi^-};\pi)$ is the j-debt payment delivered at node ξ, i.e.,

$$\Phi^h_{\xi,j}(z^h_\xi, z^h_{\xi^-};\pi) := A_{\xi,j}(\pi)\varphi^{\alpha,h}_{\xi,j} + B_{\xi,j}(\pi)\varphi^{\beta,h}_{\xi,j} + p_\xi C_{\xi,j}\varphi^{\gamma,h}_{\xi,j}.$$

For every terminal node $\xi \in D_T$,

$$g^h_\xi(z^h_\xi, z^h_{\xi^-};(\pi,N)) := p_\xi\left(d^h_\xi(z^h_\xi) - W^h_\xi(z^h_{\xi^-})\right) - \sum_{j\in K^h(\xi^-)} N_{\xi,j}\theta^h_{\xi^-,j}$$
$$+ \sum_{j\in \widehat{J}^h(\xi)} \min\{A_{\xi,j}(\pi), p_\xi C_{\xi,j}\}\varphi^{\alpha,h}_{\xi^-,j} \leq 0.$$

This last set of restrictions makes explicit the fact that, for finite horizon economies, utility maximizing agents apply strategic default at the last period. That is, at terminal nodes they will pay the minimum between the coupon and the current value of the collateral.

Given prices and security payments $(\pi, N) \in \mathbb{P}\times\mathcal{N}$, the choice set of agent h, denoted by $\Gamma^h(\pi,N)$, is the collection of plans $z^h \in \mathbb{E}^h$ satisfying restrictions above.

Definition 1. An equilibrium for this economy is given by prices, unitary payments and allocations

$$\left(\overline{\pi}, \overline{N}, (\overline{z}^h)_{h\in H}\right) \in \mathbb{P}\times\mathcal{N}\times\prod_{h\in H}\mathbb{E}^h,$$

where $\overline{\pi} = (\overline{p}, \overline{q}, \overline{r})$ and $\overline{z}^h = (\overline{x}^h, \overline{\theta}^h, \overline{\varphi}^h, \overline{\varphi}^{\alpha,h}, \overline{\varphi}^{\beta,h}, \overline{\phi}^h, \overline{\psi}^h)$, such that:

(1) For each $h\in H$, $\overline{z}^h \in \underset{z\in\Gamma^h(\overline{\pi},\overline{N})}{\operatorname{argmax}}\ U^h(c^h(z))$.

(2) Physical markets are cleared,

$$\sum_{h\in H} d^h_{\xi_0}(\overline{z}^h_{\xi_0}) = \sum_{h\in H} w^h_{\xi_0}; \qquad \sum_{h\in H} d^h_\xi(\overline{z}^h_\xi) = \sum_{h\in H} W^h_\xi(\overline{z}^h_{\xi^-}), \qquad \forall \xi > \xi_0.$$

(3) Rental markets are cleared,

$$\sum_{h\in H}\overline{\phi}^h_\xi = \sum_{h\in H}\overline{\psi}^h_\xi, \qquad \forall\xi\in D\setminus D_T.$$

(4) Security markets are cleared,

$$\sum_{h\in H^+_j(\xi)}\overline{\theta}^h_{\xi,j} = \sum_{h\in H^+_j(\xi_j)}\overline{\theta}^h_{\xi_j,j} = \sum_{h\in H^-_j}\overline{\varphi}^h_{\xi_j,j}, \qquad \forall\xi\in D\setminus D_T,\ \forall j\in K(\xi);$$

where $H^+_j(\xi) := \{h\in H : j\in K^h(\xi)\}$ are the agents with access to security j at node ξ, and $H^-_j := \{h\in H : j\in J^h(\xi_j)\}$ are the potential borrowers of j.[2]

[2]Notice that, as investors does not loss access to financial opportunities (i.e., $K^h(\xi^-)\subseteq K^h(\xi)$, for any non-terminal node ξ), it follows that $H^+_j(\xi^-)\subseteq H^+_j(\xi)$ at any non-terminal node.

(5) For any security $j \in K(\xi)$, payments $(\overline{N}_{\mu,j})_{\mu\in\xi^+}$ are determined in such form that resources delivered by borrowers are proportionally distributed among lenders, i.e.,

$$\overline{N}_{\mu,j} \sum_{h\in H_j^+(\xi)} \overline{\theta}^h_{\xi,j} = \sum_{h\in H_j^-} \Phi^h_{\mu,j}(\overline{z}^h_\mu, \overline{z}^h_\xi; \overline{\pi}), \qquad \forall \mu \in \xi^+,\, \forall j \in K(\xi);$$

$$\overline{N}_{\mu,j} \sum_{h\in H_j^+(\xi)} \overline{\theta}^h_{\xi,j} = \sum_{h\in H_j^-} \min\{A_{\mu,j}(\overline{\pi}), \overline{p}_\mu C_{\mu,j}\}\overline{\varphi}^{\alpha,h}_{\xi,j}, \qquad \forall \mu \in \xi^+ \cap D_T,\, \forall j \in K(\xi).$$

3. Existence of Equilibrium

This section establishes the existence of equilibrium. For the finite horizon case, the result is an extension of that obtained by Iraola and Torres-Martínez (2013) to a multi-period setting, including investment constraints and rental markets. The hypotheses required to guarantee the existence of equilibrium are completely analogous to those described on that previous work.

Theorem 1. Suppose that a finite horizon economy satisfies the following hypotheses:

(H1) For each $h \in H$, U^h is continuous, strictly increasing, and strongly quasi-concave.[3]

(H2) For each $h \in H$, $(W_\xi^h)_{\xi \in D} \gg 0$ where $W_{\xi_0}^h = w_{\xi_0}^h$ and $W_\xi^h = w_\xi^h + Y_\xi^c W_{\xi^-}^h$, $\forall \xi > \xi_0$.

(H3) For each $\xi \in D$ and $j \in K(\xi^-)$, functions $A_{\xi,j}$ and $B_{\xi,j}$ are continuous.

(H4) For each $p \in \mathbb{R}_{++}^{D\times L}$, there exists $\xi \in D$ and $j \in J(\xi)$ such that, for some $\mu \in \xi^+$ we have

$$\min\{A_{\mu,j}(p,\cdot,\cdot), \|Y_\mu C_{\mu,j}\|_\Sigma\} > 0.$$

Then, there is an equilibrium where both $\overline{N} \neq 0$ and $\overline{q} \neq 0$.

Assumption (H1) imposes classical requirements on utility functions, which are satisfied for any individual with rational, continuous, strictly monotonic, and strictly convex preferences. Essentially, Assumptions (H1)-(H3) ensure the existence of well behaved demand correspondences. Assumption (H4) precludes the existence of trivial security markets. Indeed, it implies that there is at least one debt contract with non-trivial coupons and collateral requirements and, therefore, there is at least one security with positive price and payments in equilibrium.

The incompleteness of financial markets or the presence of long-lived assets may compromise equilibrium existence in sequential economies with infinite horizon. In the absence of credit risk, uniform impatience and exogenous transversality conditions are required to guarantee equilibrium existence in economies with short-lived assets (cf., Hernández and Santos (1996), Magill and Quinzii (1994), Levine and Zame (1996)). In addition, when assets live for more than one period, generic existence of equilibrium has been shown by Hernández and Santos (1996) and Magill and Quinzii (1996).

In a seminal work Araujo, Páscoa and Torres-Martínez (2002) prove that collateralized asset markets endogenously avoid Ponzi schemes and, therefore, uniform impatience and exogenous

[3]A function $f : \mathbb{R}_+^n \to \mathbb{R}$ is *strongly quasi-concave* if $f(\lambda x + (1-\lambda)y) > \min\{f(x), f(y)\}$, for any $\lambda \in (0,1)$ and $(x,y) \in \mathbb{R}_+^n \times \mathbb{R}_+^n$ such that $f(x) \neq f(y)$.

transversality conditions are not required to guarantee equilibrium existence. Furthermore, in collateralized security markets with long-lived loans it is possible to show that equilibrium always exists, as the scarcity of physical collateral induces endogenous upper bounds on short-sales (see Araujo, Páscoa and Torres-Martínez (2005, 2011)).

In our framework, with incomplete financial participation, endogenous liquidity contractions and general prepayment mechanisms, we are also able to show equilibrium existence under mild conditions on preferences and endowments.

THEOREM 2. Under Assumptions (H2)-(H4), suppose that an infinite horizon economy $\mathcal{E}$ satisfies:

(H5) For any $h \in H$, the utility function satisfies $U^h((c_\xi)_{\xi \in D}) = \sum_{\xi \in D} u^h_\xi(c_\xi)$, where kernels $u^h_\xi : \mathbb{R}^L_+ \to \mathbb{R}_+$ are continuous, strictly concave, and strictly increasing.

(H6) For any $h \in H$, $U^h((\widehat{W}_\xi)_{\xi \in D})$ is finite, where $\widehat{W}_\xi := (Y^c_\xi + Y^r_\xi)\widehat{W}_{\xi^-} + \sum_{h \in H} w^h_\xi$ and $\widehat{W}_{\xi_0} := \sum_{h \in H} w^h_{\xi_0}$.

(H7) For each $\xi \in D$ and $j \in K(\xi^-)$, $A_{\xi,j}$ and $B_{\xi,j}$ depend only on prices until period t_ξ.[4]

(H8) For any $\xi \in D$ and $j \in J(\xi)$ there exists $h \in H^+_j(\xi_j)$ such that,

$$\lim_{c_\mu \gg 0,\ \|c_\mu\| \to +\infty} u^h_\mu(c_\mu) = +\infty, \qquad \forall \mu > \xi.$$

Then, there is an equilibrium where both $\overline{N} \neq 0$ and $\overline{q} \neq 0$.

Assumption (H5) and (H6) are common in the infinite horizon general equilibrium literature. The separability of the utility function allows to approximate the economy with finite horizon truncations. The second assumption is crucial to obtain an equilibrium allocation as a limit of equilibria in finite horizon economies. Assumption (H7) complements (H3), avoiding the dependence of coupons and prepayment costs on infinite streams of prices.

Assumption (H8) is not considered in previous equilibrium existence results with collateralized asset markets (see Araujo, Páscoa, and Torres-Martínez (2002, 2005, 2011)). Its objective is to guarantee the existence of upper bounds on security prices.[5]

[4] That is, their domain is given by the set $\mathbb{P}_\xi := \prod_{\mu \in D : \xi \geq \mu} \left(\mathbb{R}^L_+ \times \mathbb{R}^{K(\mu)}_+ \times \mathbb{R}^{R(\mu)}_+\right)$. The continuity property is relative to the Euclidean topology on $\mathbb{P}_\xi$.

[5] Essentially, investors could liquidate long positions at nodes where security prices were high enough, obtaining resources to buy huge commodity bundles. However, this strategy should not induce utility levels over those compatible with the availability of commodities. For these reasons, when (i) the utility level can increase without

To deepen our understanding of (H8), let us consider a particular case. Given a continuous, strictly concave and strictly increasing function $u : \mathbb{R}^L_+ \to \mathbb{R}_+$ define $U : \mathbb{R}^{D\times L}_+ \to \mathbb{R}_+ \cup \{+\infty\}$ by

$$U((c_\xi)_{\xi\in D}) = \sum_{\xi\in D} \beta_{t_\xi}\rho(\xi)u(c_\xi),$$

where $(\beta_t)_{t\geq 0}$ are strictly positive discount factors and $\rho(\xi) > 0$ is the probability to reach node $\xi \in D$, which satisfies $\rho(\xi) = \sum_{\mu\in\xi^+}\rho(\mu)$ and $\rho(\xi_0) = 1$. Hence, we can specify a wide variety of functional forms for u satisfying (H5) and (H8), for instance quasi-linear, Coob-Douglas, and CES functions.

Furthermore, if we fix a consumption plan $\tilde{c} := (\tilde{c}_\xi)_{\xi\in D} \in \mathbb{R}^{D\times L}_+$ such that $U(\tilde{c})$ is finite, then for any $\delta \in (0,1)$ and $\xi \in D$, Assumption (H8) implies that there exists a bundle $\zeta(\xi,\delta) \in \mathbb{R}^L_+$ such that,

$$\sum_{\eta\in D\setminus D(\xi)} \beta_{t_\eta}\rho(\eta)u(\tilde{c}_\eta) + \beta_{t_\xi}\rho(\xi)u(\tilde{c}_\xi + \zeta(\xi,\delta)) + \sum_{\eta>\xi}\beta_{t_\eta}\rho(\eta)u(\delta\tilde{c}_\eta) > U(\tilde{c}),$$

where $D(\xi)$ denotes the sub-tree with root ξ. In other words, (H8) ensures that at any node a reduction of future consumption can be compensated with an increase of the current consumption, which is a type of impatience. However, the bundle required to compensate a reduction of future consumption is not necessarily uniform across nodes, or bounded by a fixed multiple of the aggregated endowment. Hence, (H8) differs from the *uniform impatience* conditions, which are joint requirements on preferences and endowments imposed in general equilibrium models with sequential trade to avoid Ponzi schemes (cf. Hernandez and Santos (1996, Assumption C.3), Magill and Quinzii (1996, Assumptions B2 and B4), or Páscoa, Petrassi and Torres-Martínez (2010, Assumption 2)).

We illustrate the difference between uniform impatience and (H8) with an example. Assume that individual endowments are uniformly bounded from above and away from zero, i.e., there exists $(\underline{e},\overline{e}) \in \mathbb{R}^2_{++}$ such that, for any $(h,\xi) \in H \times D$ we have, $\underline{e} \leq w^h_{\xi,l} \leq W^h_{\xi,l} \leq \overline{e}$, $\forall l \in L$. In addition, following the notation above, suppose that for any $t \geq 0$, $\beta_t = \frac{1}{(1+t_\xi)^2}$. Then, Páscoa, Petrassi, and Torres-Martínez (2010, Proposition 1) and the example given by Moreno-García and Torres-Martínez (2012, page 133) guarantee that uniform impatience is not satisfied by U, although it satisfies (H5), (H6) and (H8).

an upper bound (Assumption (H8)); (ii) preferences are continuous (Assumption (H5)); and (iii) agents are restricted to demand consumption in compact sets—as in our truncated economies—it follows that security prices are bounded (see Lemma 5 for further details).

4. An Example of the Prevalence of Negative Equity Loans

The objective of this section is to show an example of the occurrence of negative equity loans in equilibrium. It is a simple setting, where there is no uncertainty. However, it shows how the generality of our model can be applied in simple cases to study particular issues.

Consider a three period economy without uncertainty. There is only one commodity that depreciates 50% between two successive periods, i.e., $Y_1^c = Y_2^c = 0.5$. There is a unique credit contract at the first period, which is characterized by

$$(C, A_1, A_2, B_1) = \left(\frac{5}{2}, \frac{1}{2}, \frac{1}{4}, \frac{11}{8}\right).$$

The credit contract is securitized into an asset that delivers to lenders the payments made by borrowers. There are two agents, a and b. Agent b is the only that has access to credit, while agent a is the only that can invest in the security.

Individuals are characterized by,

$$U^a(x_0, x_1, x_2) = 8\sqrt{x_0} + 3\sqrt{x_1} + 6\sqrt{x_2}; \qquad (w_0^a, w_1^a, w_2^a) = \left(\frac{31}{22}, 0, \frac{1}{4}\right);$$

$$U^b(x_0, x_1, x_2) = \sqrt{2816\, x_0} + \sqrt{198\, x_1} + \sqrt{252\, x_2}; \qquad (w_0^b, w_1^b, w_2^b) = \left(\frac{103}{44}, \frac{1}{2}, 0\right).$$

Taking the commodity price as numeraire, the following set of prices, security payments and consumption-financial allocations constitutes an equilibrium:

$$\begin{aligned}
(\overline{q}_0, \overline{q}_1, \overline{N}_1) &= \left(\frac{9}{22}, \frac{1}{4}, \frac{1}{2}\right); \\
(\overline{x}_0^a, \overline{x}_1^a, \overline{x}_2^a, \overline{\theta}_0^a, \overline{\theta}_1^a) &= (1, 1, 1, 1, 1); \\
(\overline{x}_0^b, \overline{x}_1^b, \overline{x}_2^b, \overline{\varphi}_0^b) &= \left(\frac{11}{4}, \frac{11}{8}, \frac{7}{16}, 1\right).
\end{aligned}$$

In this equilibrium agent b pays the coupon of his debt at the second period, although the loan has negative equity, because the prepayment cost $B_1 = 1.375$ is greater than the depreciated collateral value $Y_1 C = 1.250$.[6]

Suppose that commodities can be rented at the second period by a unitary price r. Also, tenants depreciates the commodity more than owners, i.e., a lessor of one unit of commodity at $t = 1$ receives $Y_2^r < 0.5$ units at the last period. Hence, if the rent r and the depreciation rate $(1 - Y_2^r)$ are relatively high, then agents are not interested in change their financial positions—reducing investments or closing underwater loans—to enter into the rental market. In fact, under prices and payments defined above, there is no trade in the rental market when $0.5 < r < (1 - Y_2^r)$.

[6] Kuhh-Tucker multipliers are given by $(\lambda_0^a, \lambda_1^a, \lambda_2^a) = (5.5, 3, 3)$ and $(\lambda_0^b, \lambda_1^b, \lambda_2^b) = (22, 12, 12)$.

5. Concluding Remarks

We have developed a dynamic general equilibrium model with asset-backed securities, where agents can make multiple decisions with respect to their financial obligations: paying, prepaying or defaulting.

Although our model is very general—in the sense of allowing different payment and prepayment rules, incomplete financial participation, and rental markets—we were able to prove existence of equilibrium under mild conditions when trade occurs during a finite number of periods. However, we have found that in an infinite horizon economy, conditions previously described by the literature are not sufficient to guarantee existence of equilibrium. Thus, even though collateral still avoid Ponzi schemes and we do not use debt constrains or transversality conditions to prove equilibrium, we require a particular kind of impatience in our more general setting to guarantee existence. It is worth noting that this additional requirement is not related to the incomplete financial participation present in our model. Indeed, as long as there exist liquidity contractions in the economy, we would still need Assumption (H8) to guarantee existence of equilibria in the infinite horizon case.

It has been shown in a numerical example that loans with negative equity is a phenomena compatible with equilibrium. This example complements the one developed by Iraola and Torres-Martínez (2013), since we allowed individuals to attain leasing transactions. Thus, even in the presence of rental markets, borrowers may find optimal to keep short positions.

As a matter of future research, it is relevant to analyze the prevalence of price bubbles in asset markets. Since bubbles existence results are often conditional on assets being on zero or positive net supply, this question is interesting in our model because securities emitted in previous periods appear as assets with endogenously determined positive net supply.

Appendix A: Equilibrium Existence in Finite Horizon Economies

Let $\mathcal{E}$ be an exchange economy with $T \in \mathbb{N}$ periods. We prove the existence of an equilibrium for $\mathcal{E}$ following a generalized game approach.

For any $\xi \in D \setminus D_T$, let $(\tau_\xi, \nu_\xi) := \left((T - t_\xi) \max_{j \in K(\xi)} \|C_{\xi,j}\|_\Sigma, \ \max_{a \in R(\xi)} \|M_{\xi,a}\|_\Sigma\right)$. We restrict prices to belong to $\Delta := \prod_{\xi \in D} \Delta_\xi$, where for every $\xi \in D \setminus D_T$

$$\Delta_\xi := \left\{(p_\xi, q_\xi, r_\xi) \in \mathbb{R}^L_+ \times \mathbb{R}^{K(\xi)}_+ \times \mathbb{R}^{R(\xi)}_+ : \|p_\xi\|_\Sigma = 1, \, q_\xi \leq 2\tau_\xi(1, \ldots, 1), \, r_\xi \leq 2\nu_\xi(1, \ldots, 1)\right\},$$

and for every $\xi \in D_T$, $\Delta_\xi := \{p_\xi \in \mathbb{R}^L_+ : \|p_\xi\|_\Sigma = 1\}$.

The equilibrium definition guarantees that there exists an upper bound $\Omega := (\Omega_\xi)_{\xi \in D}$ for market feasible plans such that, for any $\xi \in D$ the vector $\Omega_\xi := (\Omega_{\xi,x}, \Omega_{\xi,\theta}, \Omega_{\xi,\varphi}, \Omega_{\xi,\varphi^\alpha}, \Omega_{\xi,\varphi^\beta}, \Omega_{\xi,\phi}, \Omega_{\xi,\psi})$ satisfies

$$(\Omega_{\xi,x}, \Omega_{\xi,\varphi}, \Omega_{\xi,\phi}) \geq 2\left(\|\widehat{W}_\xi\|_\Sigma, \ \frac{\|\widehat{W}_\xi\|_\Sigma}{\min_{j \in J(\xi)} \|C_{\xi,j}\|_\Sigma}, \ \frac{\|\widehat{W}_\xi\|_\Sigma}{\min_{a \in R(\xi)} \|M_{\xi,a}\|_\Sigma}\right);[7]$$

$$(\Omega_{\xi,\theta}, \Omega_{\xi,\psi}) = 2\#H\,(\Omega_{\xi,\varphi}, \Omega_{\xi,\phi}); \qquad (\Omega_{\xi,\varphi^\alpha}, \Omega_{\xi,\varphi^\beta}) = 2 \max_{\mu < \xi} \Omega_{\mu,\varphi}\ (1,1).$$

For any agent $h \in H$, let $\mathbb{E}^h(\Omega)$ be the set of plans $(x, \theta, \varphi, \varphi^\alpha, \varphi^\beta, \phi, \psi) \in \mathbb{E}^h$ such that, for every $\xi \in D$

$$(x_{\xi,l}, \theta_{\xi,k}, \varphi_{\xi,j}, \varphi^\alpha_{\xi,k}, \varphi^\beta_{\xi,k}, \phi_{\xi,a}, \psi_{\xi,a}) \leq \Omega_\xi, \quad \forall (l,k,j,a) \in L \times K^h(\xi) \times J^h(\xi) \times R(\xi).$$

Let $\overline{\mathcal{N}} := \{N \in \mathcal{N} : N_{\xi,j} \leq \|C_{\xi,j}\|_\Sigma, \ \forall \xi > \xi_0, \forall j \in K(\xi^-)\}$.

The generalized game $\mathcal{G}(\Omega)$ is defined by:

(P1) Given prices and payments $(\pi, N) \in \Delta \times \overline{\mathcal{N}}$, each agent $h \in H$ chooses $z^h \in \Gamma^h(\pi, N) \cap \mathbb{E}^h(\Omega)$ in order to maximize the utility function U^h.

(P2) Given $(z^h)_{h \in H} \in \prod_{h \in H} \mathbb{E}^h(\Omega)$, a player chooses $(p_{\xi_0}, q_{\xi_0}, r_{\xi_0}) \in \Delta_{\xi_0}$ to maximize

$$p_{\xi_0} \sum_{h \in H} \left(d^h_{\xi_0}(z^h_{\xi_0}) - w^h_{\xi_0}\right) + \sum_{j \in J(\xi_0)} q_{\xi_0,j} \left(\sum_{h \in H^+_j(\xi_0)} \theta^h_{\xi_0,j} - \sum_{h \in H^-_j} \varphi^h_{\xi_0,j}\right) + r_{\xi_0} \sum_{h \in H} (\phi^h_{\xi_0} - \psi^h_{\xi_0}).$$

(P3) For every intermediate node $\xi \in \mathring{D}$, given $(z^h)_{h \in H} \in \prod_{h \in H} \mathbb{E}^h(\Omega)$ a player chooses a vector of prices $(p_\xi, q_\xi, r_\xi) \in \Delta_\xi$ to maximize

$$p_\xi \sum_{h \in H} \left(d^h_\xi(z^h_\xi) - W^h_\xi(z^h_{\xi^-})\right) + \sum_{j \in J(\xi)} q_{\xi,j} \left(\sum_{h \in H^+_j(\xi)} \theta^h_{\xi,j} - \sum_{h \in H^-_j} \varphi^h_{\xi,j}\right)$$

$$+ \sum_{j \in K(\xi^-)} q_{\xi,j} \left(\sum_{h \in H^+_j(\xi)} \theta^h_{\xi,j} - \sum_{h \in H^+_j(\xi^-)} \theta^h_{\xi^-,j}\right) + r_\xi \sum_{h \in H} (\phi^h_\xi - \psi^h_\xi).$$

[7] Recall that, $\widehat{W}_{\xi_0} := \sum_{h \in H} w^h_{\xi_0}$ and, for each $\xi > \xi_0$, $\widehat{W}_\xi := (Y^c_\xi + Y^r_\xi)\widehat{W}_{\xi^-} + \sum_{h \in H} w^h_\xi$.

(P4) For every terminal $\xi \in D_T$, given $(z^h)_{h\in H} \in \prod_{h\in H} \mathbb{E}^h(\Omega)$ a player chooses $p_\xi \in \Delta_\xi$ to maximize

$$p_\xi \sum_{h\in H} \left(d^h_\xi(z^h_\xi) - W^h_\xi(z^h_{\xi^-}) \right).$$

(P5) For each $\xi \in \mathring{D}$ and $j \in K(\xi^-)$, given $\left(\pi, (z^h)_{h\in H}\right) \in \Delta \times \prod_{h\in H} \mathbb{E}^h(\Omega)$, a player chooses $N_{\xi,j} \in [\min\{A_{\xi,j}(\pi), p_\xi C_{\xi,j}\} \max\{t_{\xi_j} - t_\xi + 2, 0\}, p_\xi C_{\xi,j}]$ to maximize

$$-\left(N_{\xi,j} \sum_{h\in H_j^-} \varphi^h_{\xi_j,j} - \sum_{h\in H_j^-} \Phi^h_{\xi,j}(z^h_\xi, z^h_{\xi^-}; \pi) \right)^2,$$

where ξ_j is the issuing node of contract j.[8]

(P6) For each $\xi \in D_T$ and $j \in K(\xi^-)$, given $\left(\pi, (z^h)_{h\in H}\right) \in \Delta \times \prod_{h\in H} \mathbb{E}^h(\Omega)$, a player chooses $N_{\xi,j} \in [\min\{A_{\xi,j}(\pi), p_\xi C_{\xi,j}\} \max\{t_{\xi_j} - t_\xi + 2, 0\}, p_\xi C_{\xi,j}]$ to maximize

$$-\left(N_{\xi,j} - \min\{A_{\xi,j}(\pi), p_\xi C_{\xi,j}\}\right)^2.$$

A Cournot-Nash equilibrium of $\mathcal{G}(\Omega)$ is an allocation $\left(\overline{\pi}, \overline{N}, (\overline{z}^h)_{h\in H}\right) \in \Delta \times \overline{\mathcal{N}} \times \prod_{h\in H} \mathbb{E}^h(\Omega)$, which is individually optimal given the actions of other players.

Lemma 1. *Under Assumptions (H1)-(H3), the generalized game $\mathcal{G}(\Omega)$ has a Cournot-Nash equilibrium.*

Proof. From (H1), the objective function U^h is continuous and quasi-concave for all $h \in H$. The upper hemi-continuity of the correspondences of admissible strategies $(\pi, N) \twoheadrightarrow \Gamma^h(\pi, N) \cap \mathbb{E}^h(\Omega)$ follows from the continuity of functions $(g^h_\xi; \xi \in D)$, while the lower hemi-continuity is a consequence of the strict positiveness of the plan $(W^h_\xi, \xi \in D)$ (see Assumption (H2)).[9] Therefore, for any player $h \in H$, the correspondence of admissible strategies is continuous, with compact, convex and non-empty values.

The objective functions of the players defined in items (P2)-(P6) are continuous and quasi-concave on their respective strategies. Furthermore, the correspondences of admissible strategies for these players are continuous and have non-empty, compact and convex values.

It follows from Berge's Maximum Theorem that players' best response correspondences are closed with non-empty, compact and convex values. Since $\Delta \times \overline{\mathcal{N}} \times \prod_{h\in H} \mathbb{E}^h(\Omega)$ is compact, convex and non-empty, we can apply Kakutani's Fixed Point Theorem to the cartesian product of best response correspondences to find a Cournot-Nash equilibrium for $\mathcal{G}(\Omega)$. □

[8]Notice that, the lower bound of $N_{\xi,j}$ could be positive only at the immediate successor nodes of ξ_j.

[9]Given Assumption (H2), for any $(\pi, N) \in \Delta \times \overline{\mathcal{N}}$ the plan $\left(W^h_\xi/2^{t_\xi+1}, 0, 0, 0, 0, 0, 0\right)_{\xi\in D} \in \mathbb{E}^h(\Omega)$ is an interior point of $\Gamma^h(\pi, N)$. Thus, the correspondence that associates to any $(\pi, N) \in \Delta \times \overline{\mathcal{N}}$ the interior of $\Gamma^h(\pi, N) \cap \mathbb{E}^h(\Omega)$ relative to $\mathbb{E}^h(\Omega)$ has non-empty values. Since this correspondence also has an open graph, it is lower-hemicontinuous. Therefore, $(\pi, N) \twoheadrightarrow \Gamma^h(\pi, N) \cap \mathbb{E}^h(\Omega)$ is lower hemi-continuous because it is the closure of a lower hemi-continuous correspondence.

Lemma 2. *Under Assumption (H1), let $(\overline{\pi}, \overline{N}, (\overline{z}^h)_{h\in H})$ be a Cournot-Nash equilibrium of $\mathcal{G}(\Omega)$. Then, for each $\xi \in D \setminus D_T$, we have that*

$$\sum_{h\in H} d^h_\xi(\overline{z}^h_\xi) \leq \sum_{h\in H} W^h_\xi(\overline{z}^h_{\xi^-}) \implies \overline{q}_{\xi,j} < \tau_\xi, \quad \forall j \in J(\xi);$$

$$\sum_{h\in H} d^h_\xi(\overline{z}^h_\xi) \leq \sum_{h\in H} W^h_\xi(\overline{z}^h_{\xi^-}), \quad \sum_{h\in H^+_j(\xi^-)} \overline{\theta}^h_{\xi^-,j} < \sum_{h\in H^+_j(\xi)} \overline{\theta}^h_{\xi,j} \implies \overline{q}_{\xi,j} < \tau_\xi, \quad \forall j \in K(\xi^-);$$

$$\sum_{h\in H} d^h_\xi(\overline{z}^h_\xi) \leq \sum_{h\in H} W^h_\xi(\overline{z}^h_{\xi^-}), \quad \sum_{h\in H} \overline{\phi}^h_{\xi,a} < \sum_{h\in H} \overline{\psi}^h_{\xi,a} \implies \overline{r}_{\xi,a} < \nu_\xi, \quad \forall a \in R(\xi).$$

Proof. Fix $\xi \in D \setminus D_T$ such that $\sum_{h\in H} d^h_\xi(\overline{z}^h_\xi) \leq \sum_{h\in H} W^h_\xi(\overline{z}^h_{\xi^-})$ and assume that, for some $j \in J(\xi)$, $\overline{p}_\xi C_{\xi,j} \leq \overline{q}_{\xi,j}$. Assumption (H1) implies that, for any player $h \in H^-_j$ we have $\overline{\varphi}^h_{\xi,j} = \Omega_{\xi,\varphi}$. Otherwise, player h could increase her utility without any additional cost, by increasing her debt j at node ξ, consuming the associated collateral, and defaulting on this additional short-position in the successor nodes.[10] Therefore, $\Omega_{\xi,\varphi} \|C_{\xi,j}\|_\Sigma \leq \|C_{\xi,j}\|_\Sigma \sum_{h\in H^-_j} \overline{\varphi}^h_{\xi,j} \leq \left\|\sum_{h\in H} d^h_\xi(\overline{z}^h_\xi)\right\|_\Sigma \leq \sum_{h\in H} \|W^h_\xi(\overline{z}^h_{\xi^-})\|_\Sigma$. Since $\sum\limits_{h\in H} W^h_\xi(\overline{z}^h_{\xi^-}) \leq \widehat{W}_\xi$, we contradict the definition of $\Omega_{\xi,\varphi}$. Hence, $\sum\limits_{h\in H} d^h_\xi(\overline{z}^h_\xi) \leq \sum\limits_{h\in H} W^h_\xi(\overline{z}^h_{\xi^-})$ guarantees that $\overline{q}_{\xi,j} < \overline{p}_\xi C_{\xi,j} \leq \tau_\xi$.

To ensure the second property, notice that for every $\xi \in D \setminus \{\xi_0\}$, unitary security payments satisfies $\overline{N}_{\xi,j} \leq \overline{p}_\xi C_{\xi,j}$, $\forall j \in K(\xi^-)$. Fix a node $\xi \in D_{T-1}$ and $j \in K(\xi^-)$ such that $\mathcal{X}_{\xi,j} := \sum_{h\in H^+_j(\xi)} \overline{\theta}^h_{\xi,j} - \sum_{h\in H^+_j(\xi^-)} \overline{\theta}^h_{\xi^-,j} > 0$. Then, we have that $\overline{q}_{\xi,j} < \overline{p}_\xi C_{\xi,j}$. Otherwise, investors on security j at ξ could increase their utility by reducing this position in $\varepsilon > 0$ units, in order to implement the consumption of the bundle $\varepsilon C_{\xi,j}$, where $\varepsilon > 0$ is chosen small enough to ensure that the new consumption is admissible. Indeed, with this strategy they increase the consumption allocation and receive at any $\mu \in \xi^+$ the amount $\overline{p}_\mu C_{\mu,j} \geq \overline{N}_{\mu,j}$.

Analogously, fix $\xi \in D_{T-2}$ and $j \in K(\xi^-)$ such that $\mathcal{X}_{\xi,j} > 0$. Then $\overline{q}_{\xi,j} < 2\overline{p}_\xi C_{\xi,j}$, since otherwise the consumption of the bundle $2C_{\xi,j}$ would be utility-improving with respect to investing on one unit of security j at ξ,[11] a contradiction with the optimality of $(\overline{z}^h)_{h\in H}$. Repeating this argument recursively, we get that for each $\xi \in \mathring{D}$ and $j \in K(\xi^-)$ we have that $\overline{q}_{\xi,j} < (T - t_\xi)\overline{p}_\xi C_{\xi,j}$ whenever $\mathcal{X}_{\xi,j} > 0$. Therefore, $\overline{q}_{\xi,j} < \tau_\xi$, $\forall \xi \in D \setminus D_T$, $\forall j \in K(\xi^-)$.

Finally, fix $\xi \in D \setminus D_T$ and $a \in R(\xi)$ such that $\sum_{h\in H}(\overline{\phi}^h_{\xi,a} - \overline{\psi}^h_{\xi,a}) < 0$, and suppose that $\overline{p}_\xi M_{\xi,a} \leq \overline{r}_{\xi,a}$. Then, every agent h with $\overline{\psi}^h_{\xi,a} > 0$ might be better off by reducing the lessee position in $\varepsilon > 0$ units and using those resources to buy the bundle $\varepsilon M_{\xi,a}$, where ε is small enough to make the new consumption allocation admissible. This contradicts the optimality of $\overline{\psi}^h_{\xi,a}$. □

[10] Every player can increase the consumption at ξ, because when $\sum_{h\in H} d^h_\xi(\overline{z}^h_\xi) \leq \sum_{h\in H} W^h_\xi(\overline{z}^h_{\xi^-})$ upper bounds on consumption allocations are non-binding, i.e., $\overline{x}^h_\xi < \Omega_{\xi,x}$.

[11] Remember that, $\overline{N}_{\mu,j} \leq \overline{p}_\mu C_{\mu,j}$, $\forall \mu \in \xi^+$.

LEMMA 3. *Under Assumptions (H1) and (H4), any Cournot-Nash equilibrium of $\mathcal{G}(\Omega)$ induces an equilibrium for the economy $\mathcal{E}$ where both $\overline{N} \neq 0$ and $\overline{q} \neq 0$.*

PROOF. Let $(\overline{\pi}, \overline{N}, (\overline{z}^h)_{h\in H}) \in \Delta \times \overline{\mathcal{N}} \times \prod_{h\in H} \mathbb{E}^h(\Omega)$ be a Cournot-Nash equilibrium of t$\mathcal{G}(\Omega)$. Since for each $h \in H$ we have $g^h_{\xi_0}(\overline{z}^h_{\xi_0}; (\overline{\pi}, \overline{N})) \leq 0$, it follows that

$$(1) \qquad \overline{p}_{\xi_0} \sum_{h\in H} \left(d^h_{\xi_0}(\overline{z}^h_{\xi_0}) - w^h_{\xi_0}\right) + \sum_{j\in J(\xi_0)} \overline{q}_{\xi_0,j} \left(\sum_{h\in H^+_j(\xi_0)} \overline{\theta}^h_{\xi_0,j} - \sum_{h\in H^-_j} \overline{\varphi}^h_{\xi_0,j} \right) + \overline{r}_{\xi_0} \sum_{h\in H} (\overline{\phi}^h_{\xi_0} - \overline{\psi}^h_{\xi_0}) \leq 0.$$

Thus, the optimal value of player (P2) objective function is non-positive. Given that $\overline{\pi}_{\xi_0} = (\overline{p}_{\xi_0}, \overline{q}_{\xi_0}, \overline{r}_{\xi_0}) \in \Delta_{\xi_0}$, we have that $\sum_{h\in H} (d^h_{\xi_0}(\overline{z}^h_{\xi_0}) - w^h_{\xi_0}) \leq 0$, otherwise the player who chooses prices at ξ_0 could make positive her objective function assigning a non-zero price only to those commodities with a positive excess of demand, and making asset and rental prices equal to zero. Therefore, given $j \in J(\xi_0)$ we have $\sum_{h\in H^+_j(\xi_0)} \overline{\theta}^h_{\xi_0,j} - \sum_{h\in H^-_j} \overline{\varphi}^h_{\xi_0,j} \leq 0$. Otherwise, player (P2) would choose $\overline{q}_{\xi_0,j} = 2\tau_{\xi_0}$, which contradicts Lemma 2. Similarly, given $a \in R(\xi_0)$ we have $\sum_{h\in H}(\overline{\phi}^h_{\xi_0,a} - \overline{\psi}^h_{\xi_0,a}) \leq 0$, because in other case it would be optimal for (P2) to choose $\overline{r}_{\xi_0,a} = 2\nu_{\xi_0}$, which contradicts Lemma 2. Hence, there is no excess of demand on commodity, financial, and rental markets at the initial node.

This implies that, for any $h \in H$, $(\overline{x}^h_{\xi_0,l}, \overline{\psi}^h_{\xi_0,a}) < (\Omega_{\xi_0,x}, \Omega_{\xi_0,\psi})$, $\forall (l,a) \in L \times R(\xi_0)$. Thus, the strict monotonicity of preferences guarantees that $(\overline{p}_{\xi_0}, \overline{r}_{\xi_0}) \gg 0$ and $g^h_{\xi_0}(\overline{z}^h_{\xi_0}; \overline{\pi}) = 0$, $\forall h \in H$. Therefore, we conclude that

$$(2) \qquad \sum_{h\in H} (d^h_{\xi_0}(\overline{z}^h_{\xi_0}) - w^h_{\xi_0}) = 0;$$

$$(3) \qquad \sum_{h\in H^+_j(\xi_0)} \overline{\theta}^h_{\xi_0,j} - \sum_{h\in H^-_j} \overline{\varphi}^h_{\xi_0,j} \leq 0, \quad \overline{q}_{\xi_0,j} \left(\sum_{h\in H^+_j(\xi_0)} \overline{\theta}^h_{\xi_0,j} - \sum_{h\in H^-_j} \overline{\varphi}^h_{\xi_0,j} \right) = 0, \ \forall j \in J(\xi_0);$$

$$(4) \qquad \sum_{h\in H} (\overline{\phi}^h_{\xi_0} - \overline{\psi}^h_{\xi_0}) = 0.$$

Conditions (2)-(4) imply that upper bounds on individual plans determined by Ω are non-binding. Since $\sum_{h\in H^+_j(\xi_0)} \overline{\theta}^h_{\xi_0,j} - \sum_{h\in H^-_j} \overline{\varphi}^h_{\xi_0,j} < 0 \implies \overline{q}_{\xi_0,j} = 0$, the monotonity of preferences guarantees that $(\overline{N}_{\mu,j}, \overline{q}_{\mu,j})_{\mu>\xi_0} = 0$ when there is excess of demand for credit contract $j \in J(\xi_0)$ at the initial node.

For any $h \in H$ and $j \in J(\xi_0)$ define

$$\widehat{\theta}^h_{\xi_0,j} = \begin{cases} \overline{\varphi}^h_{\xi_0,j} & \text{if } (\overline{N}_{\mu,j})_{\mu>\xi_0} = 0 \wedge (\overline{q}_{\mu,j})_{\mu\geq\xi_0} = 0; \\ \overline{\theta}^h_{\xi_0,j} & \text{otherwise.} \end{cases}$$

Since $\Omega_{\xi_0,\theta} > \Omega_{\xi_0,\varphi}$, the modified portfolios $(\widehat{\theta}^h_{\xi_0}, \overline{\varphi}^h_{\xi_0})_{h\in H}$ are still feasible, optimal, and satisfy

$$(5) \qquad \sum_{h\in H^+_j(\xi_0)} \overline{\theta}^h_{\xi_0,j} - \sum_{h\in H^-_j} \overline{\varphi}^h_{\xi_0,j} = 0, \ \forall j \in J(\xi_0).$$

Furthermore, for any $\xi \in \xi_0^+$, payments $(\overline{N}_{\xi,j})_{j\in J(\xi_0)}$ chosen by players in (P5) satisfy,

$$(6) \qquad \overline{N}_{\xi,j} \sum_{h\in H_j^-} \overline{\varphi}^h_{\xi_0,j} = \sum_{h\in H_j^-} \Phi^h_{\xi,j}\left(\overline{z}^h_\xi, \widehat{z}^h_{\xi_0}; \overline{\pi}\right), \quad \forall j \in J(\xi_0).$$

where for each $h \in H$, $\widehat{z}^h_{\xi_0}$ is obtained from $\overline{z}^h_\xi$ by replacing $\overline{\theta}^h_{\xi_0}$ with $\widehat{\theta}^h_{\xi_0}$. Since for every node $\xi \in \xi_0^+$ we have $g^h_\xi(\overline{z}^h_\xi, \widehat{z}^h_{\xi_0}; (\overline{\pi}, \overline{N})) = g^h_\xi(\overline{z}^h_\xi, \overline{z}^h_{\xi_0}; (\overline{\pi}, \overline{N})) \leq 0$, it follows from (5)-(6) that,

$$\overline{p}_\xi \sum_{h\in H}\left(d^h_\xi(\overline{z}^h_\xi) - W^h_\xi(\widehat{z}^h_{\xi_0})\right) + \sum_{j\in J(\xi)} \overline{q}_{\xi,j}\left(\sum_{h\in H_j^+(\xi)} \overline{\theta}^h_{\xi,j} - \sum_{h\in H_j^-} \overline{\varphi}^h_{\xi,j}\right)$$
$$+ \sum_{j\in K(\xi^-)} \overline{q}_{\xi,j}\left(\sum_{h\in H_j^+(\xi)} \overline{\theta}^h_{\xi,j} - \sum_{h\in H_j^+(\xi^-)} \widehat{\theta}^h_{\xi_0,j}\right) + \overline{r}_\xi \sum_{h\in H}(\overline{\phi}^h_\xi - \overline{\psi}^h_\xi) \leq 0.$$

Hence, the objective function of (P3) is non-positive in equilibrium. This implies, by analogous arguments to those made above, that for every node $\xi \in \xi_0^+$ we have $(\overline{p}_\xi, \overline{r}_\xi) \gg 0$ and

$$(7) \qquad (\overline{x}^h_{\xi,l}, \overline{\theta}^h_{\xi,k}, \overline{\varphi}^h_{\xi,j}, \overline{\varphi}^{\alpha,h}_{\xi,j}, \overline{\varphi}^{\beta,h}_{\xi,j}, \overline{\phi}^h_{\xi,a}, \overline{\psi}^h_{\xi,a}) < \Omega_\xi, \ \forall h \in H, \ \forall (l,j,k,a) \in L \times J^h(\xi) \times K^h(\xi) \times R(\xi);$$

$$(8) \qquad \sum_{h\in H}\left(d^h_\xi(\overline{z}^h_\xi) - W^h_\xi(\widehat{z}^h_{\xi_0})\right) = 0;$$

$$(9) \qquad \sum_{h\in H_j^+(\xi)} \overline{\theta}^h_{\xi,j} - \sum_{h\in H_j^-} \overline{\varphi}^h_{\xi,j} \leq 0, \quad \overline{q}_{\xi,j}\left(\sum_{h\in H_j^+(\xi)} \overline{\theta}^h_{\xi,j} - \sum_{h\in H_j^-} \overline{\varphi}^h_{\xi,j}\right) = 0, \forall j \in J(\xi);$$

$$(10) \qquad \sum_{h\in H}(\overline{\phi}^h_\xi - \overline{\psi}^h_\xi) = 0.$$

We also conclude that for each $j \in K(\xi_0)$, $\sum_{h\in H_j^+(\xi)} \overline{\theta}^h_{\xi,j} - \sum_{h\in H_j^+(\xi_0)} \widehat{\theta}^h_{\xi_0,j} \leq 0$. Otherwise, (P3) would choose $q_{\xi,j} = 2\tau_\xi$, a contradiction with Lemma 2. Hence,

$$(11) \qquad \sum_{h\in H_j^+(\xi)} \overline{\theta}^h_{\xi,j} - \sum_{h\in H_j^+(\xi_0)} \widehat{\theta}^h_{\xi_0,j} \leq 0, \quad \overline{q}_{\xi,j}\left(\sum_{h\in H_j^+(\xi)} \overline{\theta}^h_{\xi,j} - \sum_{h\in H_j^+(\xi_0)} \widehat{\theta}^h_{\xi_0,j}\right) = 0, \ \forall j \in K(\xi_0).$$

Given $\xi \in \xi_0^+$, if there exists $j \in J(\xi)$ such that $\sum\limits_{h\in H_j^+(\xi)} \overline{\theta}^h_{\xi,j} - \sum\limits_{h\in H_j^-} \overline{\varphi}^h_{\xi,j} < 0$, then (7) and (9) imply that $\overline{q}_{\xi,j} = 0$ and, therefore, the monotonicity of preferences ensures that $(\overline{N}_{\mu,j}, \overline{q}_{\mu,j})_{\mu>\xi} = 0$. Also, if for some $k \in K(\xi_0)$ we have $\sum\limits_{h\in H_j^+(\xi)} \overline{\theta}^h_{\xi,j} - \sum\limits_{h\in H_j^+(\xi_0)} \widehat{\theta}^h_{\xi_0,j} < 0$, it follows from (7) and (11) that $\overline{q}_{\xi,j} = 0$, which in turn implies that $(\overline{N}_{\mu,j}, \overline{q}_{\mu,j})_{\mu>\xi} = 0$.

For any $h \in H$ and $j \in K(\xi)$ define

$$\widehat{\theta}^h_{\xi,j} = \begin{cases} \overline{\varphi}^h_{\xi_j,j} & \text{if } (\overline{N}_{\mu,j})_{\mu>\xi} = 0 \wedge (\overline{q}_{\mu,j})_{\mu\geq\xi} = 0; \\ \overline{\theta}^h_{\xi,j} & \text{otherwise.} \end{cases}$$

Since $\Omega_{\xi,\theta} > \Omega_{\xi,\varphi}$, the financial position $(\widehat{\theta}^h_\xi, \overline{\varphi}^h_\xi)_{h\in H}$ is feasible, optimal, and satisfy

$$\sum_{h\in H_j^+(\xi)} \widehat{\theta}^h_{\xi,j} - \sum_{h\in H_j^-} \overline{\varphi}^h_{\xi,j} = 0, \forall j \in J(\xi); \qquad \sum_{h\in H_j^+(\xi)} \widehat{\theta}^h_{\xi,j} - \sum_{h\in H_j^+(\xi_0)} \widehat{\theta}^h_{\xi_0,j} = 0, \;\; \forall j \in K(\xi_0).$$

Following the same argument through the event-tree, we can ensure that $(\overline{p}, \overline{r}) \gg 0$ and we can construct modified plans $(\widehat{z}^h)_{h\in H}$ that are optimal choices for households in $\mathcal{G}(\Omega)$ and satisfy market feasibility conditions along the event-tree D.[12]

Given that commodity prices are strictly positive, both the characteristics of players in (P5)-(P6) and (H4) imply that for some $\xi \in D$, $j \in J(\xi)$ and $\mu \in \xi^+$ we have $\overline{N}_{\mu,j} \geq \min\{A_{\mu,j}(\overline{\pi}), \overline{p}_\mu C_{\mu,j}\} > 0$. Moreover, since upper bounds on individuals' admissible plans are non-binding, it follows that $\overline{q}_{\xi,j} > 0$.

To ensure that $(\overline{\pi}, \overline{N}, (\widehat{z}^h)_{h\in H})$ is an equilibrium of $\mathcal{E}$ it remains prove that, for every $h \in H$ the allocation $\widehat{z}^h \in \mathbb{E}^h(\Omega)$ is an optimal choice for agent h in $\Gamma^h\left(\overline{\pi}, \overline{N}\right)$. This property is a direct consequence of the strong quasi-concavity of functions U^h. Indeed, if there exists $\tilde{z}^h \in \Gamma^h\left(\overline{\pi}, \overline{N}\right)$ strictly preferred to $\widehat{z}^h$, then any convex combination of these plans is also strictly preferred to $\widehat{z}^h$. Since $\widehat{z}^h$ belongs to $\Gamma^h\left(\overline{\pi}, \overline{N}\right) \cap \text{interior}(\mathbb{E}^h(\Omega))$, some of these convex combinations are in $\Gamma^h\left(\overline{\pi}, \overline{N}\right) \cap \mathbb{E}^h(\Omega)$, a contradiction with the optimality of $\widehat{z}^h$ in the generalized game. Thus, $(\overline{\pi}, \overline{N}, (\widehat{z}^h)_{h\in H})$ is an equilibrium of $\mathcal{E}$. $\square$

[12]Notice that, as financial market feasibility holds, we have that

$$\overline{N}_{\xi,j} \sum_{h\in H_j^+(\xi_j)} \widehat{\theta}^h_{\xi_j,j} = \sum_{h\in H_j^-} \Phi^h_{\xi,j}\left(\widehat{z}^h_\xi, \widehat{z}^h_{\xi^-}; \overline{\pi}\right), \forall \xi \in \mathring{D}, \forall j \in K(\xi^-).$$

Appendix B: Equilibrium Existence in Infinite Horizon Economies

We construct an equilibrium of the infinite horizon economy $\mathcal{E}$ as a limit of equilibria in finite horizon economies. These truncated economies will have equilibria as a consequence of Theorem 1.

Truncated Economies.

Fix $T \in \mathbb{N}$ and consider a truncated economy $\mathcal{E}^T$ where households can consume and trade at nodes in $D^T := \{\xi \in D : t_\xi \leq T\}$. Individual plan spaces $(\mathbb{E}^{h,T})_{h\in H}$, the space of prices $\mathbb{P}^T$, and the space of security payments $\mathcal{N}^T$ are defined as in any finite horizon economy by considering D^T as the full event-tree. In particular, for each $h \in H$, $\mathbb{E}^{h,T}$ is such that assets and rental portfolios are equal to zero at each terminal node $\xi \in D_T$ (even though in the original economy $\mathcal{E}$ it is not necessarily true).

Given prices $(\pi, N) \in \mathbb{P}^T \times \mathcal{N}^T$, the optimization problem of $h \in H$ is defined as:

$$\max_{z\in\Gamma^{h,T}(\pi,N)} \quad \sum_{\xi\in D^T} u_\xi^h\left(c_\xi^h(z_\xi)\right),$$

where $\Gamma^{h,T}(\pi, N)$ is the set of plans $z^h = (x^h, \theta^h, \varphi^h, \varphi^{\alpha,h}, \varphi^{\beta,h}, \phi^h, \psi^h) \in \mathbb{E}^{h,T}$ such that

$$\begin{aligned}
&g_{\xi_0}^h(z_{\xi_0}^h; (\pi, N)) \leq 0;\\
&g_\xi^h(z_\xi^h, z_{\xi^-}^h; (\pi, N)) \leq 0, \ \forall \xi \in D^T \setminus \{\xi_0\};\\
&\varphi_{\xi,j}^{\alpha,h} + \varphi_{\xi,j}^{\beta,j} \leq \varphi_{\xi^-,j}^{\alpha,h}, \ \forall \xi \in D^T \setminus \{\xi_0\}, \forall j \in K^h(\xi^-).
\end{aligned}$$

An equilibrium of $\mathcal{E}^T$ is a tuple $(\overline{\pi}, \overline{N}, (\overline{z}^h)_{h\in H}) \in \mathbb{P}^T \times \mathcal{N}^T \times \prod_{h\in H} \mathbb{E}^{h,T}$, satisfying individual optimality and market clearing conditions of Definition 1.

Lemma 4. *Under Assumptions (H2)-(H5) and (H7), $\mathcal{E}^T$ has an equilibrium for any $T \in \mathbb{N}$.*

Proof. Under Assumption (H5), utility functions for the truncated economy are well defined and satisfy (H1). From (H7), functions $A_{\xi,j}$ and $B_{\xi,j}$ are continuous on $\mathbb{P}^T$ for each $\xi \in D^T(\xi_0)$. Thus, when (H2)-(H5) and (H7) hold, the economy $\mathcal{E}^T$ satisfies the hypotheses of Theorem 1. $\square$

Asymptotic Equilibria.

In order to find an equilibrium as a limit of truncated economies' equilibria we need to count with uniform bounds for plans and prices. Notice that, for any $\xi > \xi_0$, the bounds found on Lemma 2 for the prices of securities in $K(\xi^-)$ are dependent on the time horizon T. For this reason, Assumption (H8) is key to find appropriate bounds for these prices in the infinite horizon case, as the next lemma shows.

Lemma 5. *Suppose that Assumptions (H2), (H5), (H6) and (H8) hold. Then, there exists $(\Upsilon_\mu)_{\mu\in D} \gg 0$ such that, for any $T\in\mathbb{N}$ and for every equilibrium $(\overline{\pi}^T, \overline{N}^T, (\overline{z}^{h,T})_{h\in H})$ of $\mathcal{E}^T$ we have that*

$$\overline{p}^T_{\xi_j} C_{\xi_j,j} \geq \overline{q}^T_{\xi_j,j} \quad \Longrightarrow \quad \overline{q}^T_{\mu,j} < \Upsilon_\mu, \quad \forall \mu \in D^T : \mu > \xi_j,$$

where $\xi_j \in D^{T-1}$ is the emission node of credit contract j.

Proof. Given $\xi \in D^{T-1}$ and $j \in J(\xi)$, assume that $\overline{p}^T_\xi C_{\xi,j} \geq \overline{q}^T_{\xi,j}$.

Let $\widehat{H}^+_j \subseteq H^+_j(\xi_j)$ be the non-empty set of agents whose preferences satisfy Assumption (H8). Then, hypotheses (H5)-(H6) imply that

$$\max_{h\in \widehat{H}^+_j} \sum_{\eta\in D^T} u^h_\eta\left(c^h_\eta(\overline{z}^{h,T}_\eta)\right) \leq \Xi := \max_{h\in H} \sum_{\eta\in D} u^h_\eta(\widehat{W}_\eta) < +\infty.$$

On the other hand, Assumption (H8) guarantees that, for any successor node $\mu > \xi_j$ there exists $\Theta_{\mu,j} \in \mathbb{R}^L_+$ such that, $\min\limits_{h\in\widehat{H}^+_j} u^h_\mu\left(\frac{1}{2^{t_\mu+1}} W^h_\mu + \Theta_{\mu,j}\right) > \Xi$.

Fix $\mu \in D^T$ such that $\mu > \xi_j$. Any agent $h \in \widehat{H}^+_j$ can implement the following plan:

(i) At any $\eta \in D^T$, consume $\frac{1}{2^{t_\eta+1}} W^h_\eta$;

(ii) At node ξ_j, invest on $\dfrac{\min_{l\in L} W^h_{\xi_j,l}}{2^{t_{\xi_j}+1} \max_{l\in L} C_{\xi_j,j,l}}$ units of security j;[13]

(iii) At node μ, sell the position on security j.

Since this strategy is admissible, the resources that it collects at μ must be lower than the cost of the bundle $\Theta_{\mu,j}$, implying

$$\overline{q}^T_{\mu,j} < \overline{m}_{\mu,j} := \max_{h\in\widehat{H}^-_j} \left(2^{t_{\xi_j}+1} \frac{\max\limits_{l\in L} C_{\xi_j,j,l}}{\min\limits_{l\in L} W^h_{\xi_j,l}} \|\Theta_{\mu,j}\|_\Sigma \right).$$

Defining $\Upsilon_\mu = \max\limits_{k\in K(\mu^-)} \overline{m}_{\mu,k}$, we obtain the result. □

Consider a sequence $(\overline{\pi}^T, \overline{N}^T, (\overline{z}^{h,T})_{h\in H})_{T>0}$ of equilibria for truncated economies obtained by the application of Theorem 1. This sequence is uniformly bounded node by node. Indeed, given $\xi \in D$, market feasibility conditions (2)-(4) in the equilibrium definition guarantee that individual plans $((\overline{z}^{h,T}_\xi)_{h\in H})_{T>t_\xi}$ are uniformly bounded from above by the upper bound Ω_ξ that was defined in the proof of Theorem 1. Commodity prices are bounded by construction and, hence, security payments are bounded too. Financial and rental prices are bounded as a consequence of Lemmas 2 and 5.

For any $h \in H$ and $T > 0$ define

$$\mathbb{G}^{h,T} := \left\{(x,\theta,\varphi,\varphi^\alpha,\varphi^\beta,\phi,\psi) \in \mathbb{E}^{h,T} : \varphi^\alpha_{\mu,j} + \varphi^\beta_{\mu,j} \leq \varphi^\alpha_{\mu^-,j}, \ \forall \mu \in D^T \setminus \{\xi_0\}, \forall j \in K(\mu^-)\right\}.$$

[13]Given that $\overline{p}^T_\xi C_{\xi,j} \geq \overline{q}^T_{\xi,j}$, this investment can be financed by the available resources after consumption, which are equal to $\left(1 - \frac{1}{2^{t_{\xi_j}+1}}\right) \overline{p}_{\xi_j} w^h_{\xi_j} + \frac{1}{2^{t_{\xi_j}+1}} \overline{p}_{\xi_j} Y^c_{\xi_j} W^h_{\xi_j^-}$.

Kuhn-Tucker conditions guarantee that, for every h there are multipliers $(\overline{\gamma}_\xi^{h,T})_{\xi\in D}\geq 0$ such that,

$$\overline{\gamma}_{\xi_0}^{h,T} g_{\xi_0}^h(\overline{z}_{\xi_0}^{h,T};(\overline{\pi}^T,\overline{N}^T))=0, \tag{12}$$

$$\overline{\gamma}_{\xi}^{h,T} g_{\xi}^h(\overline{z}_{\xi}^{h,T},\overline{z}_{\xi^-}^{h,T};(\overline{\pi}^T,\overline{N}^T))=0,\ \forall\xi\in D^T\setminus\{\xi_0\}, \tag{13}$$

and, for any $z=(z_\xi)_{\xi\in D^T}\in\mathbb{G}^{h,T}$, where $z_\xi=(x_\xi,\theta_\xi,\varphi_\xi,\varphi_\xi^\alpha,\varphi_\xi^\beta,\phi_\xi,\psi_\xi)$, we have that

$$\sum_{\xi\in D^T}\left(u_\xi^h(c_\xi^h(z_\xi))-\overline{\gamma}_\xi^{h,T} g_\xi^h(z_\xi,z_{\xi^-};(\overline{\pi}^T,\overline{N}^T))\right)\leq\sum_{\xi\in D^T}u_\xi^h(c_\xi^h(\overline{z}_\xi^{h,T})). \tag{14}$$

Therefore, for any $z=(z_\xi)_{\xi\in D^T}\in\mathbb{G}^{h,T}$

$$\sum_{\xi\in D^T}u_\xi^h(c_\xi^h(z_\xi))-\sum_{\xi\in D^T}\overline{\gamma}_\xi^{h,T} g_\xi^h(z_\xi,z_{\xi^-};(\overline{\pi}^T,\overline{N}^T))\leq\sum_{\xi\in D}u_\xi^h\left(\widehat{W}_\xi\right). \tag{15}$$

Fix $\eta\in D^T$ and consider the plan $\tilde{z}=(\tilde{x}_\xi,0,0,0,0,0,0)_{\xi\in D^T}\in\mathbb{G}^{h,T}$ such that

$$\tilde{x}_\xi=\begin{cases} W_\xi^h, & \text{when } t_\xi<t_\eta;\\ 0, & \text{otherwise.}\end{cases}$$

Evaluating inequality (15) in $\tilde{z}$, it follows that

$$0\leq\overline{\gamma}_\eta^{h,T}\min_{l\in L}W_{\eta,l}^h\leq\sum_{\xi\in D^T:t_\xi=t_\eta}\overline{\gamma}_\xi^{h,T}\overline{p}_\xi^T W_\xi^h\leq U^h\left(\widehat{W}\right).$$

Hence, we can define the following uniform upper bounds for the Kuhn-Tucker multipliers,

$$0\leq\overline{\gamma}_\eta^{h,T}\leq\frac{\max_{k\in H}U^k\left(\widehat{W}\right)}{\min_{(k,l)\in H\times L}W_{\eta,l}^k},\quad\forall T\in\mathbb{N},\,\forall h\in H,\,\forall\eta\in D^T.$$

Therefore, for each $\xi\in D$ the sequence $(\overline{\pi}_\xi^T,\overline{N}_\xi^T,(\overline{z}_\xi^{h,T},\overline{\gamma}_\xi^{h,T})_{h\in H})_{T>t_\xi}$ is uniformly bounded. It follows from Tychonoff's Theorem that there exists a subsequence $(T_k)_{k>0}\subseteq\mathbb{N}$ such that, for any node $\xi\in D$, $(\overline{\pi}_\xi^{T_k},\overline{N}_\xi^{T_k},(\overline{z}_\xi^{h,T_k},\overline{\gamma}_\xi^{h,T_k})_{h\in H})_{T_k>t_\xi}$ converges to some vector $\left(\overline{\pi}_\xi,\overline{N}_\xi,(\overline{z}_\xi^h,\overline{\gamma}_\xi^h)_{h\in H}\right)$ as k goes to infinity, where $\overline{z}_\xi^h:=(\overline{x}_\xi^h,\overline{\theta}_\xi^h,\overline{\varphi}_\xi^h,\overline{\varphi}_\xi^{\alpha,h},\overline{\varphi}_\xi^{\beta,h},\overline{\phi}_\xi^h,\overline{\psi}_\xi^h)$.

Lemma 6. *For any $h\in H$ and $z=(x,\theta,\varphi,\varphi^\alpha,\varphi^\beta,\phi,\psi)\in\Gamma^h(\overline{\pi},\overline{N})$ we have that,*

$$\sum_{\xi\in D^M}\left(u_\xi^h(c_\xi^h(z_\xi))-u_\xi^h(c_\xi^h(\overline{z}_\xi^h))\right)\leq\sum_{\xi\in D\setminus D^M}u_\xi^h\left(\widehat{W}_\xi\right),\quad\forall M\in\mathbb{N}.$$

Proof. Fix $M\in\mathbb{N}$ and $z\in\Gamma^h(\overline{\pi},\overline{N})$. Then, there exists $k(M)\in\mathbb{N}$ such that $T_k>M$ for any $k>k(M)$. Therefore, for any $\xi\in D^M$ and $k>k(M)$, if we evaluate the inequality (14) in the plan

$$\begin{cases}\overline{z}_\mu^{h,T_k}, & \text{if } \mu\in D^{T_k}\setminus\{\xi\};\\ z_\xi, & \text{otherwise;}\end{cases}$$

we have that

$$u_\xi^h(c_\xi^h(z_\xi)) - u_\xi^h(c_\xi^h(\overline{z}_\xi^{h,T_k})) \leq \overline{\gamma}_\xi^{h,T_k} g_\xi^h(z_\xi, \overline{z}_{\xi^-}^{h,T_k}; (\overline{\pi}^{T_k}, \overline{N}^{T_k})) + \sum_{\mu\in\xi^+} \overline{\gamma}_\mu^{h,T_k} g_\mu^h(\overline{z}_\mu^{h,T_k}, z_\xi; (\overline{\pi}^{T_k}, \overline{N}^{T_k})).$$

Adding across nodes on D^M we obtain that,

$$\begin{aligned}
(16)\qquad & \sum_{\xi\in D^M} \left(u_\xi^h(c_\xi^h(z_\xi)) - u_\xi^h(c_\xi^h(\overline{z}_\xi^{h,T_k}))\right) \\
\leq\ & \sum_{\xi\in D^M} \left(\overline{\gamma}_\xi^{h,T_k} g_\xi^h(z_\xi, \overline{z}_{\xi^-}^{h,T_k}; (\overline{\pi}^{T_k}, \overline{N}^{T_k})) + \sum_{\mu\in\xi^+} \overline{\gamma}_\mu^{h,T_k} g_\mu^h(\overline{z}_\mu^{h,T_k}, z_\xi; (\overline{\pi}^{T_k}, \overline{N}^{T_k}))\right), \\
\leq\ & \sum_{\mu\in D: t_\mu = M+1} \overline{\gamma}_\mu^{h,T_k} g_\mu^h(\overline{z}_\mu^{h,T_k}, z_{\mu^-}; (\overline{\pi}^{T_k}, \overline{N}^{T_k})) \\
\leq\ & \Lambda_{M+1}^{h,T_k} := \sum_{\mu\in D: t_\mu = M+1} \overline{\gamma}_\mu^{h,T_k} g_\mu^h(\overline{z}_\mu^{h,T_k}, 0; (\overline{\pi}^{T_k}, \overline{N}^{T_k})) + \sum_{\mu\in D: t_\mu = M+1} \overline{\gamma}_\mu^{h,T_k} \overline{p}_\mu^{T_k} w_\mu^h,
\end{aligned}$$

where the last two inequalities follow from equations (12)-(13), the budget feasibility of the allocation $z = (x, \theta, \varphi, \varphi^\alpha, \varphi^\beta, \phi, \psi)$, and the fact that $(\overline{p}_\mu^{T_k}, \overline{p}_\mu^{T_k} C_{\mu,j} - \overline{q}_{\mu,j}^{T_k}, \overline{q}_\mu^{T_k}, \overline{r}_\mu^{T_k}) \geq 0$ for any $\mu \in D^{T_k}$, $j \in J(\mu)$.

On the other hand, given $s \leq T_k$, if we evaluate inequality (14) in the plan

$$\begin{cases} \overline{z}_\mu^{h,T_k}, & \text{if } \mu \in D^{T_k}, t_\mu \neq s; \\ 0, & \text{otherwise;} \end{cases}$$

the non-negativity of the utility function implies that,

$$\begin{aligned}
& - \sum_{\mu\in D^{T_k}(\xi_0): t_\mu = s} \overline{\gamma}_\mu^{h,T_k} g_\mu^h(0, \overline{z}_{\mu^-}^{h,T_k}; (\overline{\pi}^{T_k}, \overline{N}^{T_k})) - \sum_{\mu\in D^{T_k}(\xi_0): t_\mu = s+1} \overline{\gamma}_\mu^{h,T_k} g_\mu^h(\overline{z}_\mu^{h,T_k}, 0; (\overline{\pi}^{T_k}, \overline{N}^{T_k})) \\
& \qquad\qquad \leq \sum_{\mu\in D^{T_k}(\xi_0): t_\mu = s} u_\mu^h(c_\mu^h(\overline{z}_\mu^{h,T_k})), \quad \forall s < T_k;
\end{aligned}$$

$$- \sum_{\mu\in D^{T_k}(\xi_0): t_\mu = s} \overline{\gamma}_\mu^{h,T_k} g_\mu^h(0, \overline{z}_{\mu^-}^{h,T_k}; (\overline{\pi}^{T_k}, \overline{N}^{T_k})) \leq \sum_{\mu\in D^{T_k}(\xi_0): t_\mu = s} u_\mu^h(c_\mu^h(\overline{z}_\mu^{h,T_k})), \quad \forall s = T_k.$$

Therefore, as equations (12) and (13) guarantee that

$$-\overline{\gamma}_\mu^{h,T_k} g_\mu^h(0, \overline{z}_{\mu^-}^{h,T_k}; (\overline{\pi}^{T_k}, \overline{N}^{T_k})) = \overline{\gamma}_\mu^{h,T_k} g_\mu^h(\overline{z}_\mu^{h,T_k}, 0; (\overline{\pi}^{T_k}, \overline{N}^{T_k})) + \overline{\gamma}_\mu^{h,T_k} \overline{p}_\mu^{T_k} w_\mu^h,$$

we have

$$\Lambda_s^{h,T_k} \leq \sum_{\mu\in D^{T_k}(\xi_0): t_\mu \geq s} u_\mu^h(c_\mu^h(\overline{z}_\mu^{h,T_k})) \leq \sum_{\mu\in D: t_\mu \geq s} u_\mu^h\left(\widehat{W}_\mu\right), \quad \forall s \leq T_k.$$

This last result and inequality (16) imply that,

$$\sum_{\xi\in D^M} \left(u_\xi^h(c_\xi^h(z_\xi)) - u_\xi^h(c_\xi^h(\overline{z}_\xi^{h,T_k}))\right) \leq \sum_{\mu\in D: t_\mu \geq M+1} u_\mu^h\left(\widehat{W}_\mu\right).$$

Taking the limit in the inequality above when T_k goes to infinity, we conclude that

$$\sum_{\xi\in D^M} \left(u_\xi^h(c_\xi^h(z_\xi)) - u_\xi^h(c_\xi^h(\overline{z}_\xi^h))\right) \leq \sum_{\mu\in D: t_\mu \geq M+1} u_\mu^h\left(\widehat{W}_\mu\right). \qquad \square$$

Since for each $T \in \mathbb{N}$, $\left(\overline{\pi}_\xi^T, \overline{N}_\xi^T, (\overline{z}_\xi^{h,T})_{h\in H}\right)_{\xi\in D^T}$ satisfies market clearing and payment compatibility conditions (2)-(5) in the equilibrium definition, the limit $\left(\overline{\pi}_\xi, \overline{N}_\xi, (\overline{z}_\xi^h)_{h\in H}\right)_{\xi\in D}$ also satisfies market clearing conditions. Furthermore, since the functions defining individual choice sets $(g_\xi^h; (h,\xi) \in H \times D)$ are continuous, it follows that, for any $h \in H$ the plan $\overline{z}^h := (\overline{z}_\xi^h; \xi \in D)$ belongs to $\Gamma^h(\overline{\pi}, \overline{N})$.

To prove the optimality of individual limit plans suppose, by contradiction, that for some $h \in H$ there is a plan $z \in \Gamma^h(\overline{\pi}, \overline{N})$ which is strictly preferred to $\overline{z}^h$. Then, there exists $\delta > 0$ such that,

$$\sum_{\xi\in D} u_\xi^h(c_\xi^h(z_\xi)) - \sum_{\xi\in D} u_\xi^h(c_\xi^h(\overline{z}_\xi^h)) \geq \delta.$$

Thus, there exists $N^* \in \mathbb{N}$ such that for any $N > N^*$,

$$\sum_{\xi\in D^N} u_\xi^h(c_\xi^h(z_\xi)) - \sum_{\xi\in D^N} u_\xi^h(c_\xi^h(\overline{z}_\xi^h)) \geq \frac{\delta}{2}.$$

It follows from Lemma 6 that,

$$\frac{\delta}{2} \leq \sum_{\xi\in D\setminus D^N} u_\xi^h\left(\widehat{W}_\xi\right), \forall N > N^*.$$

Since the utility evaluated in the aggregated consumption is finite (Assumption (H6)), taking the limit as N goes to infinity, we obtain a contradiction. Thus, for any $h \in H$, the plan $\overline{z}^h$ is an optimal choice in the set $\Gamma^h(\overline{\pi}, \overline{N})$. The strict monotonicity of preferences ensures that $(\overline{p}, \overline{r}) \gg 0$.

Fix $\xi \in D$, $\mu \in \xi^+$, and $j \in J(\xi)$. The proof of Theorem 1 implies that $\overline{N}_{\mu,j}^T \geq \min\{A_{\mu,j}(\overline{\pi}^T), \overline{p}_\mu^T C_{\mu,j}\}$ for any $T \in \mathbb{N}$. Since $\left(\overline{\pi}_\xi, \overline{N}_\xi, (\overline{z}_\xi^h)_{h\in H}\right)_{\xi\in D}$ is obtained as a node by node limit of equilibria in truncated economies, the continuity of coupons ensures that $\overline{N}_{\mu,j} \geq \min\{A_{\mu,j}(\overline{\pi}), \overline{p}_\mu C_{\mu,j}\}$.

Finally, it follows from arguments above, the fact that $\overline{p} \gg 0$, and Assumption (H4) that there exists at least one node $\xi \in D$ such that, for some credit contract $j \in J(\xi)$ and some immediate successor node $\mu \in \xi^+$ we have that $\overline{N}_{\mu,j} > 0$. This ensures that $\overline{q}_{\xi,j} > 0$ as a consequence of Assumption (H5).

Therefore, $\left(\overline{\pi}_\xi, \overline{N}_\xi, (\overline{z}_\xi^h)_{h\in H}\right)_{\xi\in D}$ is an equilibrium for $\mathcal{E}$ where both $\overline{q} \neq 0$ and $\overline{N} \neq 0$.

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