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Information within Coalitions: Risk and Ambiguity

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INFORMATION WITHIN COALITIONS: RISK AND AMBIGUITY

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ABSTRACT. We address economies with asymmetric information where agents are not perfectly aware about the informational structure for coalitions. Thus, we introduce solutions that we refer to as *risky core* and *ambiguous core*. We provide existence results and a variety of properties of these cooperative solutions.

KEYWORDS: Differential Information - Risky core - Ambiguous core
JEL CLASSIFICICATION: D82, D51, C71.

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1. Introduction

The core is a cooperative solution that defines situations presenting coalitional stability, in the sense that groups of individuals have no incentive to deviate. That is, there is no coalition that is able to attain an outcome by itself improving all their members. Under the presence of asymmetric information, the set of outcomes that a coalition can block depends upon the initial information and the communication opportunities of its members. Thus, a variety of notions of core can be stated.

The different core concepts that we find in the literature consider only three possible scenarios regarding the information that members of a coalition are able to use: they are restricted to the common information, they keep their private information, or the information is shared. Moreover, the criteria that specify the information for coalitions do not involve uncertainty (see Wilson, 1978, Yannelis, 1991, Allen, 2006 and Hervés-Beloso, Meo and Moreno-García, 2014, among others).

Our aim is to provide and analyze alternative definitions of the core where, in contrast to the aforementioned notions, the information associated to coalitions is not uniquely determined. To be precise, when a coalition is formed, agents have beliefs about the possible informational profiles that will be followed by the group. When these beliefs are given by a probability distribution and the blocking mechanism considers expected utility functions over the possible informational scenarios, we obtain the *risky core*. More general, when beliefs are given by a set of probability distributions and agents are α-maxmin expected utility maximizers *a la* Ghirardato, Maccheroni, and Marinacci (2004), we get the *ambiguous core*.

Our results include topological properties of the risky core (Proposition 1) and show that it shrinks when agents either give larger probability to more informative scenarios (Proposition 2) or become less risk averse (Proposition 5). The risky core may be non-empty even when the fine core is empty and agents have access to more than their private information (Proposition 3). In addition, we prove existence of the risky core when agents cannot obtain more than their private information (Proposition 4) and when they are maxmin expected utility maximizers (Proposition 7).

Regarding the ambiguous core, the blocking power of coalitions depends on the parameter α, which determines individuals' degree of ambiguity aversion. We show that larger the ambiguity aversion more difficult to block allocations and then larger the ambiguous core. Moreover, when α increases the ambiguous core ranges between the intersection and the union of the associated risky cores (Proposition 6). These results jointly with our findings for the risky core allow us to obtain several properties for the ambiguous core (Corollaries 6.1-6.6 and Proposition 7).

The construction of the paper is as follows. Section 2 describes the economy. Section 3 focuses on the informational profiles that coalitions may have and on the veto mechanism. Section 4 defines the risky core and states its properties. Section 5 is devoted to the ambiguous core. Section 6 shows a result on the relationship between no trade and the non-emptiness of risky and ambiguous cores. An auxiliary Lemma is stated and proved in a final Appendix.

2. The economy

Let us consider an exchange economy $\mathcal{E}$ with differential information and with a finite set $N = \{1, \ldots, n\}$ of consumers. The economy extends over two time periods. The exogenous uncertainty is described by a finite set of states of nature Ω that can be realized at the second period, where consumption takes place. At the first period agents have access to a complete set of contingent contracts.

There is a finite number ℓ of commodities in each state and $(\mathbb{R}^{\ell}_{+})^k$ is the consumption set, where k denotes the number of elements of Ω. Each agent $i \in N$ is characterized by a continuous utility function $U_i : (\mathbb{R}^{\ell}_{+})^k \to \mathbb{R}_+$ and by her endowments $e_i = (e_i(\omega), \omega \in \Omega) \in (\mathbb{R}^{\ell}_{+})^k$. In addition, i is partially and privately informed about the states of nature in the economy: she only knows a partition P_i of Ω, in the sense that she is not able to distinguish those states of nature that are in the same element of P_i. Thus, we assume that e_i is P_i-measurable.[1]

An allocation x assigns a commodity bundle $x_i(\omega)$ to each consumer i in each state ω. An allocation is *feasible* if it is both physically feasible and informationally feasible. The allocation x is *physically feasible* if $\sum_{i \in N} x_i(\omega) \leq \sum_{i \in N} e_i(\omega)$, $\forall \omega \in \Omega$. The allocation x is *informationally feasible* if it is compatible with the private information of each consumer, i.e., $x_i := (x_i(\omega); \omega \in \Omega)$ is P_i-measurable for every agent $i \in N$. The set of feasible allocations is denoted by $\mathcal{F}$.

Let $\mathbb{P}$ be the set of all partitions of Ω. An information P' is finer than P (equivalently, P is coarser than P'), and we write $P \leq P'$, if $P'(\omega) \subseteq P(\omega)$ for every $\omega \in \Omega$. This binary relation is reflexive, transitive and antisymmetric. Thus, $(\mathbb{P}, \leq)$ is a partially ordered set. The join of $\{P_i : i \in S\}$, denoted by $P^{\vee}_S = \bigvee_{i \in S} P_i$, is the coarsest information that is finer than P_i for every $i \in S$. The meet of $\{P_i : i \in S\}$, denoted by $P^{\wedge}_S = \bigwedge_{i \in S} P_i$, is the finest information that is coarser than P_i for every $i \in S$.

[1] Given a partition P of Ω, $x = (x(w))_{w \in \Omega} \in (\mathbb{R}^{\ell}_{+})^k$ is said to be P-measurable when it is constant on the elements of the partition P. That is, $x(\omega) = x(\omega')$ for all states ω and ω' belonging to the same element of P.

3. Coalitions and information

In general terms, an allocation belongs to the core if it is feasible and it is not blocked by any coalition. Addressing differential information economies, to propose a cooperative solution concept like the core, one has to specify the information that coalitions have. In this work, we consider that individuals are not perfectly aware about the information that is going to be obtained when they join a coalition. Each member of a coalition may keep her private information but this information may also change and become finer or coarser. The finest information is attained when information is totally shared by the individuals and the coarsest information is given when they are restricted to the common information. Any other information that arises when they partially restrict or partially share their private information is also considered.

Consider that an agent i in a coalition S may get either a finer or coarser information than her private information P_i. Thus, the information structure that agent i may have belongs to $\Gamma^i(S) = F^i(S) \cup C^i(S)$, where $F^i(S) = \{P \in \mathbb{P} : P_i \leq P \leq P_S^\vee\}$ and $C^i(S) = \{P \in \mathbb{P} : P_S^\wedge \leq P \leq P_i\}$.[2] When the coalition S is formed a *profile of information structures* $\mathcal{P} = (P_i, i \in S)$ is an element of $\Gamma(S) = \prod_{i\in S} \Gamma^i(S) \subseteq \mathbb{P}^{|S|}$, where $|S|$ is the cardinality of S. The effects that the process of coalition formation has on the available information is described by a correspondence γ that associates to each coalition S the set of possible profiles of information structures $\gamma(S) \subseteq \Gamma(S)$ that are assigned to its members.

Within this framework, and in order to block an allocation, members of a coalition S consider net trades for every possible profile of information structures determined by $\gamma(S)$.

Definition (*Family of net trades attainable for coalitions*)
Given a coalition S, a vector $z = (z^{\mathcal{P}}, \mathcal{P} \in \gamma(S))$, where $z^{\mathcal{P}} = (z_i^{\mathcal{P}}, i \in S) \in \mathbb{R}^{\ell k|S|}$ is a *family of net trades attainable for* S if the following conditions are satisfied for each $\mathcal{P} = (P_i', i \in S) \in \gamma(S)$:

(i) $z^{\mathcal{P}}$ is physically feasible for S, i.e., $\sum_{i\in S} z_i^{\mathcal{P}}(\omega) \leq 0$ for every $\omega \in \Omega$.

(ii) For each $i \in S$, $z_i^{\mathcal{P}}$ is P_i'-measurable.

Definition (*Allocations attainable for coalitions*)
A vector $y = (y^{\mathcal{P}}, \mathcal{P} \in \gamma(S))$ is an *attainable allocation for* S if there exists a family of net trades z attainable for S such that, $y^{\mathcal{P}} = (e_i + z_i^{\mathcal{P}}, i \in S)$, for every $\mathcal{P} \in \gamma(S)$. Let $\mathcal{F}_\gamma(S)$ denote the set of

[2]Note that $\Gamma^i(S)$ is different to $\{P \in \mathbb{P} : P_S^\wedge \leq P \leq P_S^\vee\}$. This is basically due to the fact that the order $\leq$ is not complete in $\mathbb{P}$. However, $\leq$ is a complete order restricted to $\Gamma^i(S)$.

attainable allocations for S.

We assume that each consumer takes into account the coalition she joins and the set of bundles that coalitions are able to attain in each possible scenario. Therefore, when agent i is a member of a coalition S the preference relation over the set of allocations that S could obtain is represented by a utility function $V_i^S : \left(\mathbb{R}_+^{\ell k}\right)^{m_S} \to \mathbb{R}_+$, where m_S is the cardinality of $\gamma(S)$. To be consistent, we assume that for any $a \in \mathbb{R}_+^{\ell k}$ we have $V_i^S(a, \ldots, a) = U_i(a)$ for every S and every $i \in S$.

A feasible allocation is in the core if it is not blocked by any coalition. A coalition S blocks $x = (x_i, i \in N)$ if there exists an allocation $y = (y^{\mathcal{P}}, \mathcal{P} \in \gamma(S))$ attainable for S such that $V_i^S(y_i) > U_i(x_i)$ for every $i \in S$, where $y_i = (y_i^{\mathcal{P}}, \mathcal{P} \in \gamma(S))$.[3]

Remark 1. It is worth noting that well-known notions of core for asymmetric information economies can be obtained as particular cases (see, for instance, Wilson, 1978, Yannelis, 1991, Allen, 2006 and Hervés-Beloso, Meo and Moreno-García, 2014). To be precise, if $\gamma(S) = \{(P_S^{\wedge}, \ldots, P_S^{\wedge})\}$ we obtain the coarse core $\mathcal{C}^{\wedge}(\mathcal{E})$; when $\gamma(S) = \{(P_i, i \in S)\}$ we obtain the private core $\mathcal{C}^{\circ}(\mathcal{E})$; and when $\gamma(S) = \{(P_S^{\vee}, \ldots, P_S^{\vee})\}$ we obtain the fine core $\mathcal{C}^{\vee}(\mathcal{E})$.

Remark 2. An additional particular case is when only three possible scenarios may occur regarding the information that members of a coalition are able to use: they are restricted to the common information, they keep their private information, or the information is shared, i.e., $\gamma(S) = \{(P_S^{\wedge}, \ldots, P_S^{\wedge}), (P_i, i \in S), (P_S^{\vee}, \ldots, P_S^{\vee})\}, \forall S \subseteq N$. Within this framework a family of net trades for a coalition S will be denoted by $z = (z_i^{\wedge}, z_i^{\circ}, z_i^{\vee}; i \in S)$

4. The risky core

In this section, we analyze situations in which each agent has beliefs, which are determined by probability distributions, regarding the profile of information structures that coalitions will follow.

[3]Following the related literature, we consider the strong veto condition requiring that every member in a blocking coalition becomes better off. Even with continuous and monotone utility functions, in differential information economies the weak and strong veto are not equivalent. To show this, consider an economy with three consumers, three states and one commodity. Private information structures are $P_1 = \{\{a, b\}, \{c\}\}$, $P_2 = \{\{a, c\}, \{b\}\}$ and $P_3 = \{\{a\}, \{b, c\}\}$. Endowments are $e_1 = (1, 1, 0)$ $e_2 = (1, 0, 1)$, and $e_3 = (0, 0, 0)$. The expected utility functions are $U_1(x_a, x_b, x_c) = (x_a + x_b)/4 + x_c/2$, $U_2(x_a, x_b, x_c) = (x_a + x_c)/4 + x_b/2$ and $U_3(x_a, x_b, x_c) = (x_b + x_c)/4 + x_a/2$. The endowment allocation is blocked in the weak sense by the big coalition via the allocation that assigns $(0, 0, 1)$ to agent 1, $(0, 1, 0)$ to agent 2 and $(1, 0, 0)$ to agent 3. However, there is no coalition that blocks the endowments in the strong sense.

Given a coalition $S \subseteq N$, let $\mathcal{R}(S)$ be the set of vectors $(r_i; i \in S)$, where $r_i = (r_i^{\mathcal{P}}, \mathcal{P} \in \gamma(S))$ is a probability distribution on $\gamma(S)$ and hence $r_i^{\mathcal{P}}$ can be interpreted as the probability that agent i gives to the implementation of the profile of information $\mathcal{P}$ when S is formed. Let $\mathcal{R}^*(S)$ be the set of vectors $r \in \mathcal{R}(S)$ that induce distributions of probability that only depend on the coalition and not on the identity of each agent. Let $\mathcal{R} = \prod_{S \subseteq N} \mathcal{R}(S)$ and $\mathcal{R}^* = \prod_{S \subseteq N} \mathcal{R}^*(S)$.

DEFINITION (*Risky core*)
Given $r = (r(S); S \subseteq N) \in \mathcal{R}$, the *risky core* $\mathcal{C}_r(\mathcal{E})$ is the set of feasible allocations that are not blocked by any coalition S when for every agent $i \in S$

$$V_i^S(y) = \sum_{\mathcal{P} \in \gamma(S)} r_i^{\mathcal{P}}(S)\ U_i(y_i^{\mathcal{P}}), \qquad \forall i \in S, \forall S \subseteq N,$$

where $r(S) = (r_i(S); i \in S)$ and $r_i(S) = (r_i^{\mathcal{P}}(S); \mathcal{P} \in \gamma(S))$.

It is well known that $\mathcal{C}^{\vee}(\mathcal{E}) \subseteq \mathcal{C}^{\circ}(\mathcal{E}) \subseteq \mathcal{C}^{\wedge}(\mathcal{E})$. Moreover, as a direct consequence of the definition, $\mathcal{C}_r(\mathcal{E}) \subseteq \mathcal{C}^{\wedge}(\mathcal{E})$, $\forall r \in \mathcal{R}$. The following example shows not only that these inclusions are strict but also allows us to conclude that the risky core concept differs from the coarse, private, and fine core.

EXAMPLE 1. Consider an economy with two states of nature and one commodity. There are two individuals, A and B, that differ only in their private information: agent A has complete information, while B has no information about the realization of the state of nature. They have the same original utility function, $U(a,b) = ab$, and the same endowments, $e_A = e_B = (1,1)$. It is not difficult to verify that $\mathcal{C}^{\wedge}(\mathcal{E}) = \mathcal{C}^{\circ}(\mathcal{E}) = \mathcal{C}^{\vee}(\mathcal{E}) = \{((1,1),(1,1))\}$

Consider that $\gamma(\{A,B\})$ is the set given by the three informational profiles that are obtained when both agents use the sharing information, or the private information, or the common information. Let $r \in \mathcal{R}$ be such that the probability α associated to share information belongs to $(1/4, 3/4)$. Following the notation stated in Remark 2, let $(y_A, y_B) = (y_i^{\vee}, y_i^{\circ}, y_i^{\wedge}; i \in \{A,B\})$ be the attainable allocation given by $y_A^{\wedge} = y_A^{\circ} = y_B^{\vee} = (0,0)$ and $y_B^{\wedge} = y_B^{\circ} = y_A^{\vee} = (2,2)$. Then, $V_A(y_A) = 4\alpha$ and $V_B(y_B) = 4(1-\alpha)$. It follows that the coalition formed by both agents A and B blocks the initial allocation and therefore it does not belong to the risky core $\mathcal{C}_r(\mathcal{E})$. Actually, in this economy the risky core is empty. □

To show topological properties of the risky core, notice that for any $r \in \mathcal{R}$ the risky core can be written as $\mathcal{C}_r(\mathcal{E}) = \{x \in \mathcal{F} : \Omega(r, x) \leq 0\}$, where

$$\Omega(r,x) = \max_{S\subseteq N} \max_{y\in\mathcal{F}_\gamma(S)} \min_{i\in S} \left(\sum_{\mathcal{P}\in\gamma(S)} r_i^{\mathcal{P}}(S)\; U_i(y_i^{\mathcal{P}}) - U_i(x_i) \right).$$

PROPOSITION 1

If $\{U_i\}_{i\in N}$ are continuous functions, then the following properties hold:

(a) For any $(x, r) \in \mathcal{F} \times \mathcal{R}$, the risky core $\mathcal{C}_r(\mathcal{E})$ and $\{\mu \in \mathcal{R} : x \in \mathcal{C}_\mu(\mathcal{E})\}$ are closed sets.

(b) The set $\{r \in \mathcal{R} : \mathcal{C}_r(\mathcal{E}) \neq \emptyset\}$ is closed.

(c) For any $x \in \mathcal{F}$ and $S \subseteq N$, $\{r \in \mathcal{R} : x$ is blocked by $S\}$ is an open set.

Furthermore, if $\{U_i\}_{i\in N}$ are concave, $\{r \in \mathcal{R} : x$ is blocked by $S\}$ is convex.

Proof. (a) Under continuity of utility functions, $\Omega : \mathcal{R} \times \mathcal{F} \to \mathbb{R}$ is continuous. Therefore, the correspondence $r \in \mathcal{R} \twoheadrightarrow \mathcal{C}_r(\mathcal{E})$ has closed graph, which ensures the statement (a).

(b) Let $\{r_m\}_{m\in\mathbb{N}} \subseteq \mathcal{R}$ be a sequence converging to $\overline{r}$ and satisfying $\mathcal{C}_{r_m}(\mathcal{E}) \neq \emptyset$, $\forall m \in \mathbb{N}$. We want to prove that $\mathcal{C}_{\overline{r}}(\mathcal{E}) \neq \emptyset$. For it, take $x^m \in \mathcal{C}_{r_m}(\mathcal{E})$. Notice that, the compactness of the set $\mathcal{F}$ of feasible allocations implies that, taking a subsequence if it is necessary, $\{x^m\}_{m\in\mathbb{N}}$ converges to some $\overline{x} \in \mathcal{F}$. If $\overline{x} \notin \mathcal{C}_{\overline{r}}(\mathcal{E})$, then there exists a coalition S and an attainable allocation $y \in \mathcal{F}_\gamma(S)$ such that, for every $i \in S$ one has $\sum_{\mathcal{P}\in\gamma(S)} \overline{r}_i^{\mathcal{P}}(S)\; U_i(y_i^{\mathcal{P}}) > U_i(\overline{x}_i)$. Thus, the continuity of utility functions implies that, for m large enough, $x^m \notin \mathcal{C}_{r_m}(\mathcal{E})$, a contradiction. Therefore, $\mathcal{C}_{\overline{r}}(\mathcal{E})$ is non-empty.

(c) An attainable allocation x is blocked by a coalition S under a risk r if and only if there exists $y \in \mathcal{F}_\gamma(S)$ such that $F_i(r, y_i, x_i) := \sum_{\mathcal{P}\in\gamma(S)} r_i^{\mathcal{P}}(S)\; U_i(y_i^{\mathcal{P}}) - U_i(x_i) > 0$, $\forall i \in S$. The continuity of the functions $\{F_i\}_{i\in S}$ on r ensures that there exists $\epsilon > 0$ such that $F_i(\tilde{r}, y_i, x_i) > 0$, $\forall i \in S$, $\forall \tilde{r} \in \mathcal{R} : \|r - \tilde{r}\| < \epsilon$. That is, $\{r \in \mathcal{R} : x$ is blocked by $S\}$ is an open set.

Finally, suppose that $x \notin \mathcal{C}_r(\mathcal{E}) \cup \mathcal{C}_{\tilde{r}}(\mathcal{E})$ can be blocked by a same coalition S under r and $\tilde{r}$. Then, there are attainable allocations $y, \tilde{y} \in \mathcal{F}_\gamma(S)$ such that, for every $i \in S$ one has $\sum_{\mathcal{P}\in\gamma(S)} r_i^{\mathcal{P}}(S)\, U_i(y_i^{\mathcal{P}}) > U_i(x_i)$ and $\sum_{\mathcal{P}\in\gamma(S)} \tilde{r}_i^{\mathcal{P}}(S)\; U_i(\tilde{y}_i^{\mathcal{P}}) > U_i(x_i)$. Given $\gamma \in (0, 1)$, let $r_\gamma = \gamma r + (1 - \gamma)\tilde{r}$. It follows from the concavity of utility functions that for each $i \in S$ we have that,

$$\begin{aligned} U_i(x_i) &< \gamma \sum_{\mathcal{P}\in\gamma(S)} r_i^{\mathcal{P}}(S)\; U_i(y_i^{\mathcal{P}}) + (1-\gamma) \sum_{\mathcal{P}\in\gamma(S)} \tilde{r}_i^{\mathcal{P}}(S)\; U_i(\tilde{y}_i^{\mathcal{P}}) \\ &\leq \sum_{\mathcal{P}\in\gamma(S)} r_{\gamma,i}^{\mathcal{P}}(S)\; U_i\left(\frac{\gamma r_i^{\mathcal{P}}(S)}{r_{\gamma,i}^{\mathcal{P}}(S)} y_i^{\mathcal{P}} + \frac{(1-\gamma)\tilde{r}_i^{\mathcal{P}}(S)}{r_{\gamma,i}^{\mathcal{P}}(S)} \tilde{y}_i^{\mathcal{P}} \right). \end{aligned}$$

Therefore, $x \notin \mathcal{C}_{r_\gamma}(\mathcal{E})$ for any $\gamma \in (0,1)$. □

DEFINITION (*Totally ordered informational structure*)
The correspondence γ determines a *totally ordered informational structure* when $\gamma(S)$ is a totally ordered subset of $\Gamma(S)$, for any coalition $S \subseteq N$.

DEFINITION (*First-order stochastic dominance*)
Given a totally ordered informational structure and $\hat{r}, r \in \mathcal{R}$, $\hat{r}$ *first-order stochastically dominates* r if the probability distribution $\hat{r}_i(S)$ first-order stochastically dominates $r_i(S)$, for every coalition S and for each agent $i \in S$.[4]

The following result shows that the risky core becomes smaller when agents assign larger probabilities to more informative informational profiles.

PROPOSITION 2
Let γ be a totally ordered informational structure and $\hat{r}, r \in \mathcal{R}^*$. If $\{U_i\}_{i\in N}$ are concave functions and $\hat{r}$ first-order stochastically dominates r, then $\mathcal{C}_{\hat{r}}(\mathcal{E}) \subseteq \mathcal{C}_r(\mathcal{E})$.

Proof. Let $S \subseteq N$ be a coalition that blocks $x \notin \mathcal{C}_r(\mathcal{E})$. That is, there exists $y \in \mathcal{F}_\gamma(S)$ such that $\sum_{\mathcal{P}\in\gamma(S)} r^{\mathcal{P}}(S)\ \ U_i(y_i^{\mathcal{P}}) > U_i(x_i)$, $\forall i \in S$. Since γ is totally ordered, we can write $\gamma(S) = \{\mathcal{P}^1, \ldots, \mathcal{P}^{m_S}\}$, where $\mathcal{P}^h = (P_i^h, i \in S)$ and $P_i^h \leq P_i^{\tilde{h}}$, for every $h \leq \tilde{h}$ and every $i \in S$. Furthermore, as $\hat{r}$ first-order stochastically dominates r, it follows from the Lemma in the Appendix that, for each $k \in \{1, \ldots, m(S)\}$ and $h \in \{1, \ldots, k\}$ there exists $a_{k,h} \geq 0$ such that,

$$\sum_{h=k}^{m(S)} a_{h,k} = r^{\mathcal{P}^k}(S), \qquad \sum_{h=1}^{k} a_{k,h} = \hat{r}^{\mathcal{P}^k}(S), \qquad \forall k \in \{1, \ldots, m(S)\}.$$

Therefore, the fact that $y \in \mathcal{F}_\gamma(S)$ ensures that the allocation $\hat{y}$ characterized by

$$\hat{y}_i^{\mathcal{P}^k} = \sum_{h=1}^{k} \frac{a_{k,h}}{\hat{r}^{\mathcal{P}^k}(S)} y_i^{\mathcal{P}^h}, \qquad \forall i \in S,\ \forall k \in \{1, \ldots, m(S)\},$$

[4]Remember that the first-order stochastic dominance notion requires the support of probability distribution to be totally ordered, which is equivalent to require that γ is totally ordered.

is attainable for S. Furthermore, the concavity of utility functions implies that

$$\sum_{k=1}^{m(S)} \hat{r}^{\mathcal{P}^k}(S)\ U_i(\hat{y}_i^{\mathcal{P}^k}) \geq \sum_{h=1}^{m(S)} \left(\sum_{k=h}^{m(S)} a_{k,h} \right) U_i(y_i^{\mathcal{P}^h}) \geq \sum_{h=1}^{m(S)} r^{\mathcal{P}^h}(S)\ U_i(y_i^{\mathcal{P}^h}) > U_i(x_i), \quad \forall i \in S.$$

We conclude that $x \notin \mathcal{C}_{\hat{r}}(\mathcal{E})$. □

The fine core of a differential information economy may be an empty set, because coalitions increase their veto power if blocking allocations are just required to be compatible with the shared information. The next result shows that the non-emptiness of $\mathcal{C}_r(\mathcal{E})$ requires that agents do not assign large probabilities to the profile given by sharing information.

PROPOSITION 3

(a) If $\{U_i\}_{i\in N}$ are concave functions, then $\mathcal{C}^{\vee}(\mathcal{E}) \subseteq \mathcal{C}_r(\mathcal{E})$, $\forall r \in \mathcal{R}^*$.[5]

(b) If $\mathcal{C}^{\vee}(\mathcal{E}) = \emptyset$ and $\mathcal{P}_S^{\vee} := (P_S^{\vee}, \dots, P_S^{\vee}) \in \gamma(S)$ for any $S \subseteq N$, then there exists $\underline{\kappa} \in (0,1)$ such that $\mathcal{C}_r(\mathcal{E}) = \emptyset$ for any risk $r \in \mathcal{R}$ such that $\min\limits_{S\subseteq N} \min\limits_{i\in S} r_i^{\mathcal{P}_S^{\vee}}(S) \geq \underline{\kappa}$.

Proof. (a) Let $S \subseteq N$ be a coalition that blocks $x \notin \mathcal{C}_r(\mathcal{E})$. That is, there exists $y \in \mathcal{F}_\gamma(S)$ such that $\sum_{\mathcal{P}\in\gamma(S)} r^{\mathcal{P}}(S)\ U_i(y_i^{\mathcal{P}}) > U_i(x_i)$, $\forall i \in S$. Notice that, for every $\mathcal{P} \in \gamma(S)$ and $i \in S$, $y_i^{\mathcal{P}}$ is $P_S^{\vee}$-measurable. Since the concavity of utility functions implies that $U_i\left(\sum_{\mathcal{P}\in\gamma(S)} r^{\mathcal{P}} y_i^{\mathcal{P}}\right) > U_i(x_i), \forall i \in S$, we conclude that $x \notin \mathcal{C}^{\vee}(\mathcal{E})$.

(b) Given $S \subseteq N$, let $\mathcal{F}_S^{\vee}$ be the set of allocation $(y_i, i \in S)$ such that $\sum_{i\in S} y_i \leq \sum_{i\in S} e_i$ and y_i is $\bigvee_{h\in S} P_h$ measurable for every $i \in S$. Suppose that $\mathcal{C}^{\vee}(\mathcal{E}) = \emptyset$. The continuity of utility functions, the compactness of sets $\{\mathcal{F}_S^{\vee}\}_{S\subseteq N}$, and the emptiness of the fine core ensure that the continuous mapping Φ characterized by $(x,\theta) \in \mathcal{F} \times [0,1] \longrightarrow \Phi(x,\theta) := \max\limits_{S\subseteq N} \max\limits_{y\in\mathcal{F}_S^{\vee}} \min\limits_{i\in S} (\theta U_i(y_i) - U_i(x_i))$ satisfies the following properties: (i) for any $\theta', \theta \in [0,1]$, if $\theta' > \theta$ and $\Phi(x,\theta) > 0$, then $\Phi(x,\theta') > \Phi(x,\theta)$; and (ii) there exists $a > 0$ such that $\Phi(x,1) \geq a, \forall x \in \mathcal{F}$.[6] Given $x \in \mathcal{F}$, let $\Theta(x) = \{\theta \in [0,1] : \Phi(x,\theta) \geq \bar{a}\}$, where $\bar{a} \in (0,a)$. It is not difficult to verify that Θ is a continuous correspondence with non-empty and compact values.[7] Therefore, the Berge's Maximum Theorem

[5] Example 1 shows that without the concavity of utility functions this result does not hold.

[6] Since $\mathcal{C}^{\vee}(\mathcal{E}) = \emptyset$, we have that $\Phi(x,1) > 0, \forall x \in \mathcal{F}$. Thus, (ii) is a direct consequence of the compactness of $\mathcal{F}$ and the continuity of Φ.

[7] For any $x \in \mathcal{F}$, $1 \in \Theta(x)$. Also, the continuity of Φ guarantees that $\Theta(x)$ is a closed subset of $[0,1]$. Therefore, Θ has non-empty and compact values. Θ is upper hemicontinuous, because has closed graph and compact codomain.

guarantees that $x \in \mathcal{F} \longrightarrow \min\{\theta : \theta \in \Theta(x)\}$ is a continuous function and it has values strictly lower than one. We conclude that there exists $\underline{\kappa} \in (0,1)$ such that $\Phi(x,\theta) > 0$, $\forall x \in \mathcal{F}$, $\forall \theta \in [\underline{\kappa}, 1]$.

Given $r \in \mathcal{R}$ and $x \in \mathcal{F}$, by definition $x \notin \mathcal{C}_r(\mathcal{E})$ if and only if $\Omega(r,x) > 0$. Since U_i takes non-negative values, $\Omega(r,x) \geq \Phi\left(x, \min_{S\subseteq N} \min_{i \in S} r_i^{\mathcal{P}_S^{\vee}}(S)\right)$. Therefore, for any $r \in \mathcal{R}$ with $\min_{S\subseteq N} \min_{i \in S} r_i^{\mathcal{P}_S^{\vee}}(S) \geq \underline{\kappa}$, we have that $\mathcal{C}_r(\mathcal{E}) = \emptyset$. □

In our exchange economy with differential information a *Walrasian expectation equilibrium* is given by a pair $(p,x) \in \mathbb{R}_+^{lk} \times \mathbb{R}_+^{lkn}$ such that $x = (x_i; i \in N)$ is physically feasible and $U_i(x_i) \geq U_i(y)$, for each agent i and for any P_i-measurable bundle $y \in \mathbb{R}_+^{lk}$ such that $p \cdot y \leq p \cdot e_i$. Let $\mathcal{W}(\mathcal{E})$ be the set of Walrasian expectation equilibrium allocations of $\mathcal{E}$. The following result determines conditions to ensure that the risky core is non-empty.

PROPOSITION 4

Assume that agents are unable to get more than their private information.[8] *If utility functions* $\{U_i\}_{i\in N}$ *are concave and locally non-satiated, then* $\mathcal{W}(\mathcal{E}) \subseteq \mathcal{C}^{\circ}(\mathcal{E}) \subseteq \mathcal{C}_r(\mathcal{E})$, $\forall r \in \mathcal{R}^*$.

Proof. Under continuity, concavity and locally non-satiability of utility functions, the Walrasian expectation equilibrium allocation exists and belongs to the private core. Furthermore, given $r \in \mathcal{R}^*$ and $S \subseteq N$ blocking $x \notin \mathcal{C}_r(\mathcal{E})$, there exists $y \in \mathcal{F}_{\gamma}(S)$ such that $\sum_{\mathcal{P}\in\gamma(S)} r^{\mathcal{P}}(S)\ \ U_i(y_i^{\mathcal{P}}) > U_i(x_i)$, $\forall i \in S$. Since agents are unable to get more than their private information, for every $\mathcal{P} \in \gamma(S)$ and $i \in S$, $y_i^{\mathcal{P}}$ is P_i-measurable. Since the concavity of utility functions implies that $U_i\left(\sum_{\mathcal{P}\in\gamma(S)} r^{\mathcal{P}}(S) y_i^{\mathcal{P}}\right) > U_i(x_i), \forall i \in S$, we conclude that $x \notin \mathcal{C}^{\circ}(\mathcal{E})$. □

In order to allow changes on utility functions, taking as given individuals' endowments and private informations, for any $r \in \mathcal{R}$ denote by $\mathcal{C}_r(\mathcal{U})$ the risky core of the economy where utility functions are given by $\mathcal{U} = \{U_i\}_{i\in N}$. In this context, consider the following partial ordering on the set of utility profiles: $\widetilde{\mathcal{U}} \succ \mathcal{U} \iff \forall i \in N, \exists\, f_i : \mathbb{R}_+ \to \mathbb{R}_+$ concave and increasing : $\ U_i = f_i \circ \widetilde{U}_i$.

Since $x \twoheadrightarrow \dot{\Theta}(x) := \{\theta \in [0,1] : \Phi(x,\theta) > \bar{a}\}$ has open graph, its closure is lower-hemicontinuous. In addition, given $x \in \mathcal{F}$ and $\theta \in \Theta(x) \cap [0,1)$, it follows from (i) that for any $n \in \mathbb{N}$ we have that $\Phi(x, n^{-1} + (1-n^{-1})\theta) > \Phi(x,\theta) \geq 0.5a$. Hence $\{n^{-1} + (1-n^{-1})\theta\}_{n\in\mathbb{N}} \subseteq \dot{\Theta}(x)$ and converges to θ. This ensures that $\Theta(x) \subseteq \overline{\dot{\Theta}}(x)$. We conclude that $\Theta = \overline{\dot{\Theta}}$, which implies in the lower hemicontinuity of Θ.

[8] That is, for any $S \subseteq N$ and $(P_i'; i \in S) \in \gamma(S)$ we have that $P_i' \leq P_i$, $i \in S$.

Proposition 5
If $\widetilde{\mathcal{U}} \succ \mathcal{U}$*, then* $\mathcal{C}_r(\widetilde{\mathcal{U}}) \subseteq \mathcal{C}_r(\mathcal{U})$ *for any* $r \in \mathcal{R}$*.*

Proof. Given $r \in \mathcal{R}$, let $S \subseteq N$ be a coalition that blocks $x \notin \mathcal{C}_r(\mathcal{U})$, i.e., there is $y \in \mathcal{F}_\gamma(S)$ such that $\sum_{\mathcal{P}\in\gamma(S)} r_i^{\mathcal{P}}(S)\; U_i(y_i^{\mathcal{P}}) > U_i(x_i)$, $\forall i \in S$. Since $\widetilde{\mathcal{U}} \succ \mathcal{U}$, for each agent i there is an increasing and concave function $f_i : \mathbb{R}_+ \to \mathbb{R}_+$ such that $\sum_{\mathcal{P}\in\gamma(S)} r_i^{\mathcal{P}}(S)\; f_i \circ \widetilde{U}_i(y_i^{\mathcal{P}}) > f_i \circ \widetilde{U}_i(x_i)$. The concavity of f_i and the monotonicity of f_i^{-1} imply that $x \notin \mathcal{C}_r(\widetilde{\mathcal{U}})$. □

The following example illustrates the implications of our previous results, characterizing the risky core in differential information economies with two agents and one commodity.

Example 2. Consider an economy $\mathcal{E}$ with two agents and one commodity. There are three states of nature a, b and c. Utility functions $U_1, U_2 : \mathbb{R}^3_+ \to \mathbb{R}_+$ are continuous, non-decreasing, concave, and strictly monotonic in at least one contingent commodity. Agent 1 is fully informed about the realized state of nature, while agent 2 does not have any information, i.e., $P_1 = \{\{a\}, \{b\}, \{c\}\}$ and $P_2 = \{a, b, c\}$. Endowments $(e_1, e_2) \gg 0$ are measurable with respect to the private information and determine an inefficient distribution of resources, in the sense that there is a physically feasible allocation that improves both agents with respect to the endowments. The possible informational profiles for the big coalition are $\gamma(\{1,2\}) = (\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3)$, where $\mathcal{P}_1 = (P_1, P_1)$, $\mathcal{P}_2 = (P_2, P_2)$ and $\mathcal{P}_3 = (\{\{a\}, \{b, c\}\}, P_2)$.

In this context, it is not difficult to verify that $\mathcal{C}^{\vee}(\mathcal{E}) = \emptyset$ and $\mathcal{C}^{\wedge}(\mathcal{E}) = \mathcal{C}^{\circ}(\mathcal{E}) = \{(e_1, e_2)\}$. Therefore, as the risky core is a subset of the coarse core, for any $r \in \mathcal{R}$ we have that either $\mathcal{C}_r(\mathcal{E}) = \emptyset$ or $\mathcal{C}_r(\mathcal{E}) = \{(e_1, e_2)\}$.

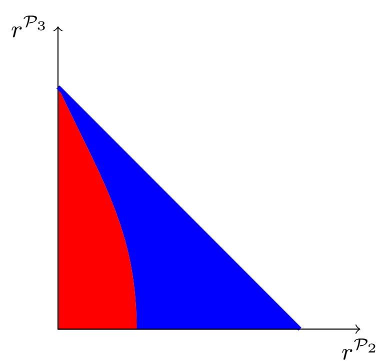

Figure 1

Consider a situation where the risk about the sharing rule following by the big coalition is captured by distributions of probability that do not depend on the identity of agents, i.e., admissible risks are elements of $\mathcal{R}^*$. Note that $\mathcal{R}^*$ can be identified with $\{(r^{\mathcal{P}_2}, r^{\mathcal{P}_3}) \in [0,1]\times[0,1] : r^{\mathcal{P}_2} + r^{\mathcal{P}_3} \leq 1\}$. This set is drawing in Figure 1 above, where two regions are shaded in order to apply our previous results. The blue zone represents the risks for which the risky core is non-empty whereas risks in the red zone leads to empty risky cores.

Properties of the blue zone. It follows from Proposition 1(b) that the risky core is non-empty in a closed subset of $\{(r^{\mathcal{P}_2}, r^{\mathcal{P}_3}) \in [0,1]\times[0,1] : r^{\mathcal{P}_2} + r^{\mathcal{P}_3} \leq 1\}$. Moreover, since under $\mathcal{P}_2$ and $\mathcal{P}_3$ agents get no more than their private information, Proposition 4 allows us to conclude that the risky core is non-empty in the line segment $\{(r^{\mathcal{P}_2}, r^{\mathcal{P}_3}) \in [0,1]\times[0,1] : r^{\mathcal{P}_2} + r^{\mathcal{P}_3} = 1\}$.

Properties of the red zone. It follows from Proposition 1 that $\{r \in \mathcal{R}^* : \mathcal{C}_r(\mathcal{E}) = \emptyset\}$ is convex. When the probability $r^{\mathcal{P}_2}$ to restrict the information to the common one is zero and the probability $r^{\mathcal{P}_1}$ to share information is positive, the concavity of the utility functions U_1, U_2 and the inefficiency of endowments ensure that (e_1, e_2) can be blocked by the grand coalition. For this reason the red zone includes the set $\{0\}\times[0,1)$. As a consequence of Propositions 2 and 3(b), the red zone is determined by the (relative) interior of the graph of a decreasing function and always includes a neighborhood the point $(0,0)$. Finally, Proposition 5 implies that, if utility functions became more concave, then the red zone may shrink in the direction of the origin. □

5. The ambiguous core

In this section, we provide a notion of core that reflects the circumstance that, when forming coalitions, individuals do not take decisions considering only one distribution of probability about the informational structures. That is, there is ambiguity about the information within coalitions and agents behave as α-maxmin expected utility maximizers *a la* Ghirardato, Maccheroni, and Marinacci (2004).

Given a coalition $S \subseteq N$, let $\mathcal{A}(S)$ be the set of vectors $(a_i; i \in S)$, where a_i is a *set of probability distributions* on $\gamma(S)$ that are the priors of agent i about the informational profiles that will be implemented when S is formed. Let $\mathcal{A}^*(S)$ be the elements of $\mathcal{A}(S)$ that induce distributions of probability that only depend on the coalition and not on the identity of each agent. Let $\mathcal{A} = \prod_{S\subseteq N} \mathcal{A}(S)$ and $\mathcal{A}^* = \prod_{S\subseteq N} \mathcal{A}^*(S)$.

DEFINITION (*Ambiguous core*)

Given $\alpha \in [0,1]$ and $a = (a(S); S \subseteq N) \in \mathcal{A}$ the *ambiguous core* $\mathcal{C}_a^\alpha(\mathcal{E})$ is the set of feasible allocations that are not blocked by any coalition S when for every agent $i \in S$

$$V_i^S(y) = \alpha \inf_{r_i \in a_i(S)} \sum_{\mathcal{P} \in \gamma(S)} r_i^{\mathcal{P}}(S)\ U_i(y_i^{\mathcal{P}}) + (1-\alpha) \sup_{r_i \in a_i(S)} \sum_{\mathcal{P} \in \gamma(S)} r_i^{\mathcal{P}}(S)\ U_i(y_i^{\mathcal{P}}),$$

where $a(S) = (a_i(S); i \in S)$ and $r_i = (r_i^{\mathcal{P}}(S); \mathcal{P} \in \gamma(S)) \in a_i(S)$.

Given $a \in \mathcal{A}$, we use the notation $r \in a$ to refer to any $r \in \mathcal{R}$ that is compatible with the priors contained in a, i.e., for every $S \subseteq N$ and $i \in S$ we have that $r_i(S) \in a_i(S)$. Furthermore, we say that $a \in \mathcal{A}$ is *closed* when $a_i(S)$ is a closed set for any agent $i \in S$ and for each coalition $S \subseteq N$.

PROPOSITION 6

For any $a \in \mathcal{A}$, we have that $\mathcal{C}_a^\alpha(\mathcal{E}) \subseteq \mathcal{C}_a^{\alpha'}(\mathcal{E})$ for any $\alpha \leq \alpha'$ and

$$\mathcal{C}_a^0(\mathcal{E}) \subseteq \bigcap_{r \in a} \mathcal{C}_r(\mathcal{E}) \subseteq \mathcal{C}_\mu(\mathcal{E}) \subseteq \bigcup_{r \in a} \mathcal{C}_r(\mathcal{E}) \subseteq \mathcal{C}_a^1(\mathcal{E}), \qquad \forall \mu \in a.$$

Furthermore, if a is closed, then

$$\mathcal{C}_a^0(\mathcal{E}) = \bigcap_{r \in a} \mathcal{C}_r(\mathcal{E}), \qquad \text{and} \qquad \bigcup_{r \in a} \mathcal{C}_r(\mathcal{E}) = \mathcal{C}_a^1(\mathcal{E}) \text{ when } n = 2.$$

Proof. Let $\alpha \leq \alpha'$ and $x \notin \mathcal{C}_a^{\alpha'}(\mathcal{E})$. Then, there is $S \subseteq N$ and $y \in \mathcal{F}_\gamma(S)$ such that, for every $i \in S$,

$$\alpha \inf_{\tilde{r}_i \in a_i(S)} \sum_{\mathcal{P} \in \gamma(S)} \tilde{r}_i^{\mathcal{P}}(S)\ U_i(y_i^{\mathcal{P}}) + (1-\alpha) \sup_{\tilde{r}_i \in a_i(S)} \sum_{\mathcal{P} \in \gamma(S)} \tilde{r}_i^{\mathcal{P}}(S)\ U_i(y_i^{\mathcal{P}})$$

$$\geq \alpha' \inf_{\tilde{r}_i \in a_i(S)} \sum_{\mathcal{P} \in \gamma(S)} \tilde{r}_i^{\mathcal{P}}(S)\ U_i(y_i^{\mathcal{P}}) + (1-\alpha') \sup_{\tilde{r}_i \in a_i(S)} \sum_{\mathcal{P} \in \gamma(S)} \tilde{r}_i^{\mathcal{P}}(S)\ U_i(y_i^{\mathcal{P}})\ >\ U_i(x_i).$$

This implies that $x \notin \mathcal{C}_a^\alpha(\mathcal{E})$. Analogously, if $x \notin \bigcap_{r \in a} \mathcal{C}_r(\mathcal{E})$, then there exists $r \in a$, $S \subseteq N$ and $y \in \mathcal{F}_\gamma(S)$ such that $\sup_{\tilde{r}_i \in a_i(S)} \sum_{\mathcal{P} \in \gamma(S)} \tilde{r}_i^{\mathcal{P}}(S)\ U_i(y_i^{\mathcal{P}})\ \geq\ \sum_{\mathcal{P} \in \gamma(S)} r_i^{\mathcal{P}}(S)\, U_i(y_i^{\mathcal{P}}) > U_i(x_i)$, $\forall i \in S$. Hence, $x \notin \mathcal{C}_a^0(\mathcal{E})$. This shows $\mathcal{C}_a^0(\mathcal{E}) \subseteq \bigcap_{r \in a} \mathcal{C}_r(\mathcal{E})$. Furthermore, if $x \notin \mathcal{C}_a^1(\mathcal{E})$, then there is a coalition S and $y \in \mathcal{F}_\gamma(S)$ such that, $\sum_{\mathcal{P} \in \gamma(S)} r_i^{\mathcal{P}}(S)\ U_i(y_i^{\mathcal{P}})\ \geq\ \inf_{\tilde{r}_i \in a_i(S)} \sum_{\mathcal{P} \in \gamma(S)} \tilde{r}_i^{\mathcal{P}}(S)\ U_i(y_i^{\mathcal{P}}) > U_i(x_i)$, $\forall r \in a$, $\forall i \in S$ and, therefore, $x \notin \bigcup_{r \in a} \mathcal{C}_r(\mathcal{E})$. This shows $\bigcup_{r \in a} \mathcal{C}_r(\mathcal{E}) \subseteq \mathcal{C}_a^1(\mathcal{E})$.

Suppose that a is closed. If $x \notin \mathcal{C}_a^0(\mathcal{E})$, then there is a coalition $S \subseteq N$ and $y \in \mathcal{F}_\gamma(S)$ such that $\sup_{\tilde{r}_i \in a_i(S)} \sum_{\mathcal{P} \in \gamma(S)} \tilde{r}_i^{\mathcal{P}}(S)\ U_i(y_i^{\mathcal{P}}) > U_i(x_i)$, $\forall i \in S$. Since $\sum_{\mathcal{P} \in \gamma(S)} \tilde{r}_i^{\mathcal{P}}(S)\ U_i(y_i^{\mathcal{P}})$ is a continuous function in $\tilde{r}_i(S)$ and $a_i(S)$ is compact, the supremum is attained. This implies that there exists

$r \in a$ such that $x \notin \mathcal{C}_r(\mathcal{E})$. Therefore, $\bigcap_{r\in a} \mathcal{C}_r(\mathcal{E}) \subseteq \mathcal{C}_a^0(\mathcal{E})$. The fact that $\mathcal{C}_a^0(\mathcal{E}) \subseteq \bigcap_{r\in a} \mathcal{C}_r(\mathcal{E})$ allows us to conclude that $\mathcal{C}_a^0(\mathcal{E}) = \bigcap_{r\in a} \mathcal{C}_r(\mathcal{E})$.

Assume that $N = \{1, 2\}$ and $x \notin \bigcup_{r\in a} \mathcal{C}_r(\mathcal{E})$. If x is not individually rational, i.e., there exists $i \in N$ such that $U_i(x_i) < U_i(e_i)$, then $x \notin \mathcal{C}_a^1(\mathcal{E})$. Otherwise x is individually rational and for any $r \in a$ there is $y \in \mathcal{F}_\gamma(N)$ such that $\sum_{\mathcal{P}\in\gamma(N)} r_i^{\mathcal{P}}(N)\ U_i(y_i^{\mathcal{P}}) > U_i(x_i)$, $\forall i \in N$.[9] Since a is closed and the functions on the left hand side of these inequalities are continuous in $r_i(N)$, we have $\inf_{\tilde{r}_i\in a_i(N)} \sum_{\mathcal{P}\in\gamma(N)} \tilde{r}_i^{\mathcal{P}}(N)\ \ U_i(y_i^{\mathcal{P}}) > U_i(x_i)$, $\forall i \in N$, implying that $x \notin \mathcal{C}_{\mathcal{A}}^1(\mathcal{E})$. Taking into account that $\bigcup_{r\in a} \mathcal{C}_r(\mathcal{E}) \subseteq \mathcal{C}_a^1(\mathcal{E})$, we conclude that $\mathcal{C}_a^1(\mathcal{E}) = \bigcup_{r\in a} \mathcal{C}_r(\mathcal{E})$. □

These results allow us to characterize several properties of the ambiguous core as a direct consequence of Propositions 2-5. The first of these properties, which is an immediate consequence of Propositions 2 and 6, determines conditions that ensure the coincidence of the ambiguous core and the risky core.

COROLLARY 6.1

Let γ be a totally ordered informational structure, $\{U_i\}_{i\in N}$ concave functions, and $a \in \mathcal{A}^*$ closed. If $\hat{r} \in a$ first-order stochastically dominates every $r \in a$, then $\mathcal{C}_a^0(\mathcal{E}) = \mathcal{C}_{\hat{r}}(\mathcal{E})$. Moreover, if $n = 2$ and $\tilde{r} \in a$ is first-order stochastically dominated by every $r \in a$, then $\mathcal{C}_a^1(\mathcal{E}) = \mathcal{C}_{\tilde{r}}(\mathcal{E})$.

Under concavity of utility functions, Propositions 3 and 6 guarantee that the fine core is always contained in the ambiguous core. Furthermore, the non-emptiness of $\mathcal{C}_a^\alpha(\mathcal{E})$ requires the existence of risk scenarios compatible with a where agents do not assign large probabilities to the profile given by sharing information.

COROLLARY 6.2

If utility functions $\{U_i\}_{i\in N}$ are concave, then for any $a \in \mathcal{A}^*$ we have that $\mathcal{C}^\vee(\mathcal{E}) \subseteq \bigcap_{r\in a} \mathcal{C}_r(\mathcal{E})$. Furthermore, if a is also closed, then $\mathcal{C}^\vee(\mathcal{E}) \subseteq \mathcal{C}_a^\alpha(\mathcal{E})$ for all $\alpha \in [0, 1]$.

[9]Notice that, when $n > 2$ we cannot ensure that for any risk $r \in a$ a same coalition blocks x.

COROLLARY 6.3

If $\mathcal{C}^{\vee}(\mathcal{E}) = \emptyset$ and $\mathcal{P}_S^{\vee} := (P_S^{\vee}, \ldots, P_S^{\vee}) \in \gamma(S)$ for any $S \subseteq N$, then there is $\underline{\kappa} \in (0,1)$ such that the following properties hold:

(a) $\mathcal{C}_a^0(\mathcal{E}) = \emptyset$ *for any* $a \in \mathcal{A}$ *such that* $a \cap \left\{ r \in \mathcal{R} : \min_{S \subseteq N} \min_{i \in S} r_i^{\mathcal{P}_S^{\vee}}(S) \geq \underline{\kappa} \right\} \neq \emptyset$

(b) $\mathcal{C}_a^1(\mathcal{E}) = \emptyset$ *for any* $a \in \mathcal{A}$ *such that* $a \subseteq \left\{ r \in \mathcal{R} : \min_{S \subseteq N} \min_{i \in S} r_i^{\mathcal{P}_S^{\vee}}(S) \geq \underline{\kappa} \right\}$ *and* $n = 2$.

When agents may be unable to get more than their private information, it is possible to ensure that the set of Walrasian expectation equilibrium allocations of $\mathcal{E}$ is contained in the ambiguous core and, therefore, it is a non-empty set.

COROLLARY 6.4

Suppose that the utility functions $\{U_i\}_{i \in N}$ are concave and locally non-satiated. Let $\mathcal{P}_S := (P_i, i \in S)$ and $a \in \mathcal{A}^*$ such that, for some $r \in a$, $\sum_{\substack{\mathcal{P} \in \gamma(S): \\ \mathcal{P} \leq \mathcal{P}_S}} r^{\mathcal{P}}(S) = 1, \forall S \subseteq N$. *Then,* $\mathcal{W}(\mathcal{E}) \subseteq \mathcal{C}^{\circ}(\mathcal{E}) \subseteq \mathcal{C}_a^1(\mathcal{E})$.

COROLLARY 6.5

Suppose that utility functions $\{U_i\}_{i \in N}$ are concave and locally non-satiated. If agents are unable to get more than their private information, then the following properties hold for any $a \in \mathcal{A}^*$:

(a) $\mathcal{W}(\mathcal{E}) \subseteq \mathcal{C}^{\circ}(\mathcal{E}) \subseteq \bigcap_{r \in a} \mathcal{C}_r(\mathcal{E})$.

(b) *If a is closed, then* $\mathcal{W}(\mathcal{E}) \subseteq \mathcal{C}^{\circ}(\mathcal{E}) \subseteq \mathcal{C}_a^{\alpha}(\mathcal{E})$ *for all* $\alpha \in [0,1]$.

As for the risky core, Propositions 5 and 6 ensure that the ambiguous core does not shrink when agents became more risk adverse.

COROLLARY 6.6

If $\widetilde{\mathcal{U}} \succ \mathcal{U}$, then for any $a \in \mathcal{A}$:

(a) $\bigcup_{r \in a} \mathcal{C}_r(\widetilde{\mathcal{U}}) \subseteq \bigcup_{r \in a} \mathcal{C}_r(\mathcal{U})$ *and* $\bigcap_{r \in a} \mathcal{C}_r(\widetilde{\mathcal{U}}) \subseteq \bigcap_{r \in a} \mathcal{C}_r(\mathcal{U})$.

(b) *If a is closed, then* $\mathcal{C}_a^0(\widetilde{\mathcal{U}}) \subseteq \mathcal{C}_a^0(\mathcal{U})$ *and, when* $n = 2$ *we have* $\mathcal{C}_a^1(\widetilde{\mathcal{U}}) \subseteq \mathcal{C}_a^1(\mathcal{U})$.

EXAMPLE 3. Following the economy in Example 2, the figure below illustrates different scenarios where there exists ambiguity about the information within coalition.

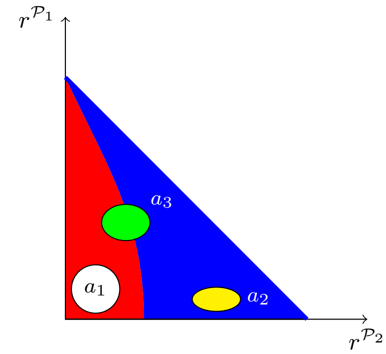

Figure 2

It follows from Proposition 6 that, independently of the parameter $\alpha \in [0,1]$, $\mathcal{C}^{\alpha}_{a_1}(\mathcal{E})$ is an empty set. Moreover, when the set of priors is a closed subset of the blue zone, as the yellow ellipse a_2, then Proposition 6 implies that $\mathcal{C}^{\alpha}_{a_2}(\mathcal{E}) = \{(e_1, e_2)\}$ for any $\alpha \in [0,1]$. Finally, if the ambiguity is captured by risks belonging to a_3, then the ambiguous core varies with the aversion to ambiguity α, as $\mathcal{C}^{0}_{a_3}(\mathcal{E}) = \emptyset$ and $\mathcal{C}^{1}_{a_3}(\mathcal{E}) = \{(e_1, e_2)\}$. Finally, from Corollary 6.6 we deduce that the parameter given by $\inf\{\alpha \in [0,1] : \mathcal{C}^{\alpha}_{a_3}(\mathcal{E}) \neq \emptyset\}$ does not increase when agents become more risk averse. □

6. ON THE NON-EMPTINESS OF RISKY AND AMBIGUOUS CORES

The existence results for the risky and ambiguous cores we have showed in the previous sections are based on the existence of Walrasian expectation equilibrium and assume that agents do not obtain more than their private information when forming coalitions. In this section, inspired in Billot, Chateauneuf, Gilboa, and Tallon (2000), we show the non-emptiness of our cooperative solutions without requiring any property on the informational profiles for coalitions.

Essentially, we prove that when agents are maxmin expected utility maximizers and endowments are riskless, the existence of a common prior about the realization of the state of nature ensures that non-trade is an stable outcome regarding to the blocking power of coalitions.

PROPOSITION 7

Consider an economy with one commodity such that, for every $i \in N$,

$$U_i(x_i) = \inf_{\pi \in \Delta_i} \sum_{\omega \in \Omega} \pi(\omega) u_i(x_i(\omega)),$$

where Δ_i is a non-empty set of priors,[10] *u_i is increasing and concave, and $e_i = \delta_i(1,\dots,1)$ for some $\delta_i > 0$. If $\bigcap_{i\in N}\Delta_i \neq \emptyset$, then $(e_i)_{i\in N} \in \bigcap_{r\in\mathcal{R}^*}\mathcal{C}_r(\mathcal{E})$.*

Proof. Suppose that $(e_i)_{i\in N} \notin \mathcal{C}_r(\mathcal{E})$ for some $r \in \mathcal{R}^*$. That is, there exist $S \subseteq N$ and $y \in \mathcal{F}_\gamma(S)$ such that $\sum_{\mathcal{P}\in\gamma(S)} r^{\mathcal{P}} U_i(y_i^{\mathcal{P}}) > U_i(e_i), \forall i \in S$. Given $\pi \in \bigcap_{i\in S}\Delta_i$, the concavity of functions $\{u_i\}_{i\in S}$ guarantees that $u_i\left(\sum_{\mathcal{P}\in\gamma(S)} r^{\mathcal{P}} \sum_{\omega\in\Omega}\pi(\omega)y_i^{\mathcal{P}}(\omega)\right) > u_i(\delta_i), \forall i \in S$. Therefore, $\sum_{i\in S}\sum_{\mathcal{P}\in\gamma(S)} r^{\mathcal{P}} \sum_{\omega\in\Omega}\pi(\omega)y_i^{\mathcal{P}}(\omega) > \sum_{i\in S}\delta_i$, which contradicts the fact that $y \in \mathcal{F}_\gamma(S)$. □.

A direct consequence of Propositions 6 and 7 is that, for every closed $a \in \mathcal{A}^*$ we have that the ambiguous core $\mathcal{C}_a^\alpha(\mathcal{E})$, with $\alpha \in [0,1]$, contains the initial allocation $(e_i)_{i\in N}$.

APPENDIX

LEMMA. *Suppose that $\hat{r} = (\hat{r}^1,\dots,\hat{r}^m)$ first-order stochastically dominates $r = (r^1,\dots,r^m)$. Then, for each $k \in \{1,\dots,m\}$ and $h \in \{1,\dots,k\}$ there exists $a_{k,h} \geq 0$ verifying*

$$\sum_{h=k}^{m} a_{h,k} = r^k, \qquad \sum_{h=1}^{k} a_{k,h} = \hat{r}^k, \qquad \forall k \in \{1,\dots,m\}.$$

Proof. We show it by induction. When $m = 1$ there is no uncertainty and the result trivially holds. Assume that the result is true for $m = t$ and let us prove that it is also true for $m = t+1$. Notice that $\hat{r}_* = (\hat{r}^1,\dots,\hat{r}^{t-1},\hat{r}^t+\hat{r}^{t+1})$ first-order stochastically dominates $r_* = (r^1,\dots,r^{t-1},r^t+r^{t+1})$. Therefore, it follows from the induction hypothesis that, for each $k \in \{1,\dots,t\}$ and $h \in \{1,\dots,k\}$ there exists $a^*_{k,h} \geq 0$ verifying $\sum_{h=k}^{t} a^*_{h,k} = r^k_*$ and $\sum_{h=1}^{k} a^*_{k,h} = \hat{r}^k_*$.

Given $k \in \{1,\dots,t+1\}$ and $h \in \{1,\dots,k\}$, define

$$a_{k,h} = \begin{cases} a^*_{k,h}, & h \leq k < t; \\ a^*_{k,h} - \alpha_h, & h < k = t; \\ a^*_{k,h} - r^{t+1} - \alpha_h, & h = k = t; \\ \alpha_h, & h < k = t+1; \\ r^{t+1}, & h = k = t+1, \end{cases}$$

where $(\alpha_h)_{1\leq h\leq t} \geq 0$ satisfies $\sum_{h=1}^{t}\alpha_h = \hat{r}^{t+1} - r^{t+1}$.

[10]That is, $\Delta_i \subseteq \{\pi \in \mathbb{R}^\Omega_+ : \sum_{\omega\in\Omega}\pi(\omega) = 1\}$.

It follows that $\sum_{h=k}^{t+1} a_{h,k} = r^k$ and $\sum_{h=1}^{k} a_{k,h} = \hat{r}^k$, for all $k \in \{1, \ldots, t+1\}$. □

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