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Estimation of travel time variability for cars, buses, metro and door-to-door public transport trips in Santiago, Chile

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The analysis of travel time variability (TTV) is attracting attention among policy makers due to the increasing awareness that users assign a high value to level-of-service attributes. In this paper, the TTV of cars and public transport trips is analysed. We estimate the effect of each trip stage on the TTV for complete door-to-door public transport trips, including access, waiting, transfer and in-vehicle time. We employ data from Santiago, Chile, in which surveyors performed predetermined trips and recorded each stage on several days between 2007 and 2011, which were complemented by recorded bus GPS data. We found that (i) bus waiting and in-vehicle times are highly significant in explaining total (door-to-door) TTV relative to metro (subway) travel times, whereas walking time is not significant; (ii) metro travel time is generally more stable but may be more skewed compared with the travel time of buses on a segregated right-of-way; and (iii) buses that travel in mixed traffic have not only a larger mean travel time but also a larger variability than buses that travel in bus lanes and segregated busways. Formal cost-benefit analysis should consider the effect of (total or partial) segregation of public transport operation on reducing travel time variability.

1. Introduction: the relevance of characterizing travel time variability

Travel time variability reflects the degree of variation in the travel time of a trip that is repeated in similar conditions over several days. Travel time variability is a key factor that travellers consider when making basic travel decisions, such as decisions regarding mode, route and departure time. Numerous studies have attempted to quantify how much people value reductions in travel time variability (e.g., Bates, Polak, Jones, & Cook, 2001; Jackson & Jucker, 1982; Lam & Small, 2001; Li, Hensher, & Rose, 2010; Noland & Small, 1995; Senna, 1994). Basically, a reduction in travel time variability enables more predictable travel times and better activity scheduling decisions for all users of a transport network, including car drivers, public transport riders, cyclists and cargo operators.

To monetise the value of the reductions in travel time variability, two modelling approaches are usually proposed: the scheduling model and the mean–variance model. The scheduling model introduces a scheduling delay that penalises modal utility when a user is either early or late to a destination relative to a preferred arrival time (PAT). In the mean–variance model, a PAT is unknown or undefined, and travel time variability is assumed to be a cost by itself, regardless if travellers arrive early or late, as both the mean travel time and the standard deviation of the travel time enter a modal utility function. Empirical evidence suggests that the value of the travel time variability reductions from a scheduling model may be smaller than the value of the travel time variability reductions from a mean-variance model (Börjesson, Eliasson, & Franklin, 2012), as uncertainty for some travellers is a source of disutility regardless if the final outcome is arriving early, on time or late to a destination.

In the literature, the concept of a reliability ratio (RR) has been defined as the ratio between the value of reducing the standard deviation of travel time and the value of reducing the mean travel time. A review by Li et al. (2010) shows an extensive range of empirical estimates of the RR, from 0.5 to 3.3 (see Carrion & Levinson, 2012 for a meta-analysis of empirical evidence). De Jong, Kouwenhoven, Kroes, Rietveld, and Warffemius (2009)

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suggests a value of 0.8 for cars and a value of 1.4 for public transport, and Bates et al. (2001) suggest an RR of 1.3 for cars and a maximum RR of two for public transport. All in all, despite the empirical differences in the value of the RR, the literature clearly indicates a value to reduce travel time variability; therefore, it is relevant to identify and analyse the variables that affect TTV.

Although evidence exists regarding the importance of travel time variability to travellers, there is no agreement on which is the best way to measure it. Several constructs have been employed to analyse the level of variability in travel times for different modes and travel conditions, and a number of studies have compared different measures of network reliability or travel time variability, either for particular roads or specific modes (Cambridge Systematics et al., 2013; Lomax, Schrank, Turner, & Margiotta, 2003; van Lint, van Zuylen, & Tu, 2008). The measures of TTV that have been proposed and analysed in the literature can be approximately classified into two groups (Pu, 2011): (i) performance reliability measures, which quantify the performance of transport systems; and (ii) measures to estimate travellers' responses to unreliability, which improve travel behaviour models (such as the standard deviation of travel time and the probability of arriving early or late at a destination, which are employed in mean–variance or scheduling models, respectively). The advantages of the standard deviation of travel time are its simplicity and the fact that it can be readily introduced in a mean–variance model to analyse users' responses to travel time variability. A summary of selected travel time reliability measures that have been proposed in the literature is presented in Table 1.

Recently, Tirachini, Hensher, and Bliemer (2014) introduced a mean–variance modal utility form in a social welfare maximisation model that obtains optimal values of bus supply (frequency and size), bus fare and car congestion toll in a transport corridor. The results indicate that the optimal toll linearly increases as the reliability ratio increases, whereas the optimal bus fare remains almost constant. A linear relationship between the mean travel time and the standard deviation of travel time, which was empirically obtained in Sydney, Australia, was employed to relate traffic congestion to travel time variability. This relation provides a simple link to use a mean–variance model to set optimal prices and public transport supply levels in multimodal networks.

Given the relevance of travel time reliability for traveller satisfaction and network performance assessment, the relationship between a TTV measure and a measure of mean travel times is useful for policy analysis as the latter is easier to estimate either with empirical, analytic or simulation methods. Several authors have estimated functions to link average travel time with a measure of travel time variability—usually the standard deviation—as estimated for cars by May, Bonsall, and Marler (1989), Mahmassani, Hou, and Dong (2012), Peer, Koopmans, and Verhoef (2012), Cambridge Systematics et al. (2013) and Tirachini et al. (2014) and for buses by Mazloumi, Currie, and Rose (2010) and Moghaddam, Noroozi, Casello, and Hellinga (2011). First, we employ the standard deviation as a measure of the TTV even though it is a symmetric measure that hides attributes of skew and width of travel time distributions, which are important aspects of the lack of reliability (van Lint et al., 2008). In this paper, the standard deviation is investigated due to its simplicity and its direct application in a mean–variance model to analyse travel behaviour, which makes it one of the most commonly employed TTV measures in the literature. Second, the skew and width parameters as introduced by van Lint et al. (2008) are computed and analysed for all study modes. The difference between symmetrical measures and asymmetrical measures of travel time variability is observed when comparing trips by metro (subway) against road-based modes (bus and car).

An analysis of travel time variability in public transport is more complicated than an analysis of travel time variability of car traffic due to, at least, three factors (Tirachini et al., 2014): (i) buses and trains stop for the boarding and alighting of passengers, which is a process that involves other sources of variability (speed and number of passengers who board and alight, choice of fare payment method, and number of buses that stop), (ii) unreliable travel times have a negative effect on waiting times at bus stops and train stations, and (iii) the uncertainty of travel times in public transport induces a cost to service providers, who may introduce larger recovery times in the schedule if travel times are less reliable. Ad-hoc measures that are proposed to analyse the reliability of a public transport service surpass the standard deviation of travel time to include constructs such as the probability of on-time performance, the travel time ratio (observed travel time/scheduled travel time) and several measures of the variability of headways, which increase

Table 1
Selected measures of travel time variability.

TTV measure	Source
Standard deviation of travel time	May et al. (1989) Eliasson (2007) Mahmassani et al. (2012) Peer et al. (2012) Tirachini et al. (2014)
Difference between 90th and 10th percentile of travel time	Eliasson (2007) Tu, van Lint, and van Zuylen (2007) van Lint and van Zuylen (2005)
Coefficient of variation	May et al. (1989) Eliasson (2006)
Standard deviation of delay (delay: difference between actual travel time and free flow travel time)	Mott MacDonald (2008b, 2008a)
Variance of delay	Mott MacDonald (2008b, 2008a)
Travel time index (TTI): ratio of actual travel time to free-flow travel time	Cambridge Systematics et al. (2013)
80% percentile TTI	Cambridge Systematics et al. (2013)
Buffer time index: difference between 95th percentile travel time per km and average travel time per km, divided by travel time per km.	Lomax et al. (2003) van Lint et al. (2008)
Misery index: average of the highest 5% or 20% of travel times, divided by free-flow travel time	van Lint et al. (2008) Kim et al. (2013)
Planning time index: 95th percentile travel time divided by free-flow travel time.	Lomax et al. (2003) Kim et al. (2013)
Skew: distance between the 90th and 50th travel time percentiles, divided by the distance between the 50th and 10th percentiles.	van Lint et al. (2008)
Width: Distance between the 90th and 50th travel time percentiles, divided by the median travel time.	van Lint et al. (2008)

waiting times (Abkowitz & Engelstein, 1983; El-Geneidy, Horning, & Krizek, 2008; Strathman & Hopper, 1993; Strathman et al., 1999).

In this paper, we characterise the reliability of both cars and public transport trips using data from Santiago, Chile. We employ three databases of repeated observations of trips in different areas of the city by car and public transport. The contributions of this paper to the literature on travel time variability are three-fold. First, we compare the travel time variability of three modes—car, bus and metro trains—using a single distance-free measure: the mean and standard deviation of travel time in minutes per kilometre (min/km). Second, in the case of public transport trips, one of our databases encompasses door-to-door trips that are repeated over several days, in which surveyors record times for walking (access and egress), waiting, in-vehicle and transferring between vehicles (bus–bus, bus–metro or metro–metro). Therefore, we can surpass previous public transport studies that focus on in-vehicle travel time or headway reliability, to separately analyse each stage of a trip (walking, waiting, in-vehicle bus, in-vehicle metro and transfer time) and how each of these stages influences the total (door-to-door) travel time variability. In particular, we are able to estimate which stages, and to what extent, are statistically significant in explaining total travel time variability. To the best of our knowledge, this study is the first study that includes walking and waiting to calculate the total travel time variability in public transport. Finally, a comparison of different bus priority measures (buses in mixed traffic, bus lanes and segregated median busways) in terms of travel time variability is performed.

The remainder of the paper is organised as follows: Section 2 describes the data used in this paper. In Section 3 we analyse the probability distributions of travel times for car and multimodal public transport trips. In Section 4, the variability of travel time is analysed per mode and trip stage (in the case of public transport). Section 5 analyses the TTV in segregated busways versus the TTV in bus lanes and mixed traffic operations. In Section 6, other reliability measures (width and skew) are introduced and calculated. Section 7 presents the study of door-to-door travel time variability in public transport. Section 8 presents the conclusion.

2. Data description

Three datasets are employed to investigate the characteristics of travel time variability in Santiago. The analysed modes are car, bus and metro; currently, car trips account for 25.7% of trips and public transport accounts for 25% of trips (Muñoz, Thomas, Navarrete, & Contreras, 2015). The first database includes travel times by car provided by *Unidad Operativa de Control de Tránsito* (UOCT), which is a public agency that controls traffic signals for the Santiago Metropolitan Area. These data consist of travel times by car for 25 road stretches for different time periods. Trips in the morning peak period (8:00–9:00) and afternoon (18:00–20:00) peak period were recorded. Data are recorded on one working day every three months, and the total database contains 2616 travel time measurements between 2010 and 2014. The floating car method is used to measure travel time. Between three and six repetitions of the same trip are recorded during the morning peak period, and between six and ten repetitions are recorded in the afternoon on a single day.

The second database is provided by the Metropolitan Public Transport Agency (*Directorio de Transporte Público Metropolitano*, DPTM), which is the institution in charge of the planning and regulation of Santiago's public transport system, Transantiago, which integrates bus routes and the metro network using a single fare system with tickets that are validated using a contactless smartcard. The database is derived from a large project that recorded travel times for multimodal trips in Santiago from 2007 to 2012. This project was requested by DTPM and managed by a private consultant, who hired surveyors to make specific trips daily and record the travel time for each stage of a trip. The main difference between other databases that are employed to analyse public transport reliability and ours is that our database record door-to-door trips, i.e., including access, waiting, in-vehicle time (by bus and/or metro), transfer and egress times for 66 different origin–destination pairs in the metropolitan area. Trips were made in one, two, three and four vehicles during peak and off-peak periods. We have a total of 35,340 observations for different stages of trips. Table 2 summarises relevant information about the three databases. The peak periods differ among the databases as the periods were defined by the authorities in charge of each survey.

The third database was provided by DTPM and contains travel times for each trip recorded in Santiago's public transport system between May 25th and May 31st, 2014. Trips are recorded using smartcard transactions by passengers when boarding buses. Alighting is not recorded; however, a methodology has been developed to estimate the number of alightings using the full day record of boardings and bus travel times from GPS pulses (Munizaga & Palma, 2012). A total of 23.6 million trips were recorded during the entire week; however, a subsample with 42,125 observations is chosen to compare in-vehicle travel times by bus for three alternative right-of-way configurations: mixed traffic operation, bus lanes and segregated median busways.

3. Probability distribution of travel times

Travel time variability (TTV) is the result of random variations in travel time that are caused by a number of variables whose impact cannot be anticipated by travellers (Tu, 2008). Amongst the most common causes of TTV are temporal demand differences (peak/off-peak and weekday/weekend), driving attitude, weather, roadwork, accidents, special events, network effects (effect of traffic in one road on travel times for adjacent roads) and differences in traffic signal programming and other traffic control devices (Cambridge Systematics et al., 2013; Kim, Mahmassani, Vovsha, Stogios, & Dong, 2013; Tu, 2008). These factors cause travel times to vary both within one day and between two days. By recording repeated observations of a trip on the same route at the same time (or time period) every working day, the daily travel time variability, which users consider in their commuting decisions, can be analysed.

In this section, we estimate the probability distributions for travel time by car, bus and metro. The accurate calculation of any parametric distribution in modelling travel time observations for a particular route is a useful tool when performing analytical comparisons between different reliability measures (e.g., standard deviation, buffer index, planning time index), as noted by Pu (2011) who assumed a lognormal distribution for travel times.

Some articles have estimated continuous probability distributions for car traffic in cities such as San Antonio (Rakha, El-Shawarby, & Arafeh, 2010), Adelaide (Susilawati, Taylor, & Somenahalli, 2013; Taylor & Susilawati, 2012), Paris (Aaron, Bhori, & Guessous, 2014) and Stockholm (Eliasson, 2007), whereas some studies of public transport selected bus routes in Melbourne (Mazloumi et al., 2010) and Brisbane (Kieu, Bhaskar, & Chung, 2014). Distributions such as lognormal, gamma, Burr and Weibull are the most commonly proposed functions to fit repeated travel time observations. A common finding is that travel times are skewed with long right tails (Cambridge Systematics et al., 2013; Susilawati et al., 2013; van Lint and van Zuylen, 2005). In theory, therefore, asymmetrical distributions are more suitable than symmetrical distributions to model travel time variability; however, symmetrical distributions also exist (Eliasson, 2007). Even the bimodality of the travel time distribution has been observed in specific cases (Susilawati et al., 2013).

We identified probability distributions that fit in-vehicle travel times of the three motorised modes in our study: car, bus and

Table 2
Description travel time databases.

Variable	DB1: Car database	DB2: Public transport – travel time survey	DB3: Public transport – Smartcard + GPS
Trip stage recorded	Car travel time	Access time Waiting time In-vehicle time (bus and metro) Transfer time Total (door-to-door) travel time	Bus in-vehicle time
Observation period	March 2010–June 2014	May 2007–December 2012	25–31 May 2014
Time periods	Morning peak: 08:00–09:00 Afternoon peak: 18:00–20:00	Morning peak: 6:30–9:30 Off-peak: 9:30–12:30 Afternoon: 14:30–16:30 Afternoon peak: 17:30–20:30 Night: 20:30–01:00	Morning peak: 06:30–08:30 Morning peak transition: 08:30–09:30 Morning off-peak: 09:30–12:30 Noon peak: 12:30–14:00 Afternoon off-peak: 14:00–17:30 Afternoon peak: 17:30–20:30 Afternoon peak transition: 20:30–21:30 Off-peak (night): 21:30–23:00
Observations	2616 trips in 25 O–D pairs.	35,340 trips stages in 66 O–D pairs.	42,125 trip stages in 130 O–D pairs.
Average speed (km/h)	Car morning peak: 24.1 Car afternoon peak: 20.7	Bus morning peak: 17.6 Bus off-peak: 18.2 Metro morning peak: 28.2 Metro off-peak: 31.0	Bus morning peak: • Mixed traffic: 10.5 • Bus lane: 14.2 • Seg busway: 21.6 Bus morning off-peak: • Mixed traffic: 12.3 • Bus lane: 16.9 • Seg busway: 24.2
Average trip length (km)	2.4	Bus: 5.6 Metro: 9.7	2.01

metro, using databases DB1 and DB2, based on tests of goodness-of-fit, such as a chi-square test and the Kolmogorov–Smirnov test. The software *Statgraphics* was employed for this task.

As a summary, it is found that in the majority of cases (84% for car trips, 66% for bus trips and 54% for metro trips), asymmetrical distributions such as the lognormal, loglogistic and triangular, provide a better fit for the measured travel times than the symmetrical distributions. For the remaining cases, symmetrical distributions such as the normal, logistic and Laplace distributions are useful to model the variability of travel times. In the particular case of car driving, the lognormal distribution fit the observed travel times for 80% of the routes. The lognormal distribution has been previously proposed in the literature of travel time variability (e.g., Pu, 2011; Rakha et al., 2010; Susilawati, Taylor, & Somenahalli, 2010). For bus and metro, the loglogistic distribution adequately fits several observed travel times (45% of observations of buses and 46% of observations of metro). The shape of lognormal and loglogistic distributions are similar; they are employed for reliability analyses of the lifetimes of components and systems.[1] Equations (1) and (2) show the probability density functions of a lognormal distribution and a loglogistic distribution, respectively,

$$f\left(x;\mu_y,\sigma_y\right)=\frac{1}{x\sigma_y\sqrt{2\pi}}e^{-\frac{\left(\ln(x)-\mu_y\right)^2}{2\sigma_y^2}} \tag{1}$$

where μ_y and σ_y are the mean and the standard deviation, respectively, of $y=ln(x)$.

$$f(x;\alpha,\beta)=\frac{\left(\frac{\beta}{\alpha}\right)\left(\frac{x}{\alpha}\right)^{\beta-1}}{\left(1+\left(\frac{x}{\alpha}\right)^{\beta}\right)^2} \tag{2}$$

[1] Wolfram Documentation Center, http://reference.wolfram.com/language/guide/DistributionsUsedInReliabilityAnalysis.html, accessed 30 Oct 2015.

where $\alpha>0$ is the median (parameter of scale), and $\beta>0$ is the parameter of shape.

In Figs. 1–3 and in Table 3, we provide examples of probability distributions for travel times by car, metro and bus for specific routes.

4. Travel time variability: modal differences

4.1. The identification of incidents

Traffic congestion as a source of travel time variability should be analysed by distinguishing recurrent congestion (e.g., the daily increase in traffic during the morning peak on working days) and nonrecurring congestion, which is caused by infrequent incidents such as accidents and extreme weather that may cause very long travel times (Tu, 2008). The infrequent existence of very long travel times generally skews the travel time distribution (van Lint et al., 2008). Based on examples in Figs. 1–3, the existence of outliers in these cases is limited as the estimated (asymmetrical) distributions do not exhibit a large skewness (refer to Table 3).

The influence of incidents is discernible in our data. For illustration purposes, we focus on car trips. Fig. 4 shows the travel time observations of three different car trips. Fig. 4a depicts the travel time along route 10 for 113 observations of morning peak trips (8:00–9:00), where sets of bars (either green (in the web version) or blue) represent observations on the same day. In Fig. 4a, no distinct outliers are observed, and all randomness seems to stem from recurrent congestion. Fig. 4b shows the travel time observations for Route 9 in the morning peak period; two observations (red bars 54 and 73) stand out. These are trips that likely occur during an incident that caused an increase in travel time, which cannot be explained by recurrent congestion. The identification of incidents is performed using a simple test for outliers (a value of approximately three standard deviations from the mean is candidate to be an outlier).

Fig. 4c shows another interesting case, in which two observations are identified as outliers (red bars). However, based on the two subsequent trips (orange bars), the travel times for both trips

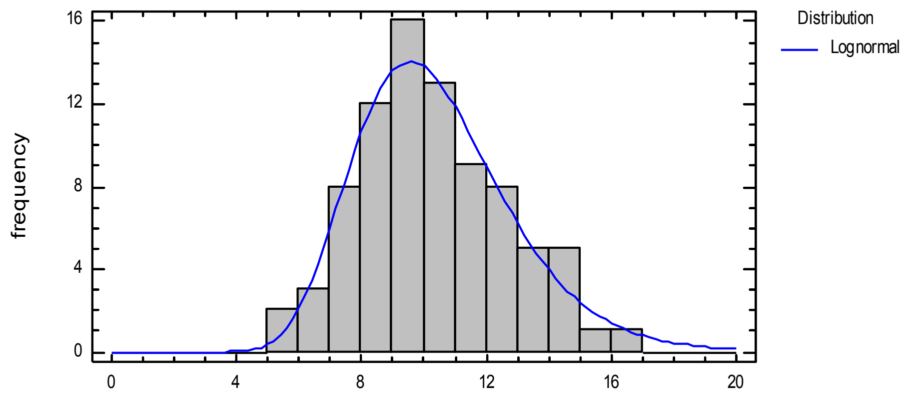


Fig. 1. Histogram and lognormal distribution for car travel time (minutes): Eliodoro Yáñez Avenue, between Américo Vespucio and Los Leones.

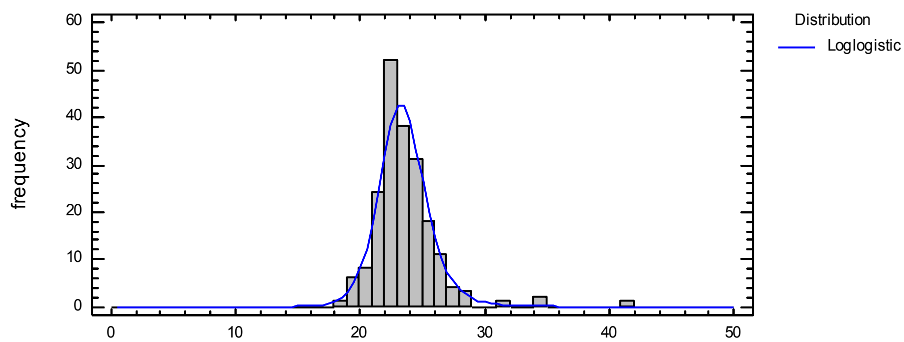


Fig. 2. Histogram and loglogistic distribution for metro in-vehicle time (minutes): Trip between stations Plaza de Armas and Vicente Valdés.

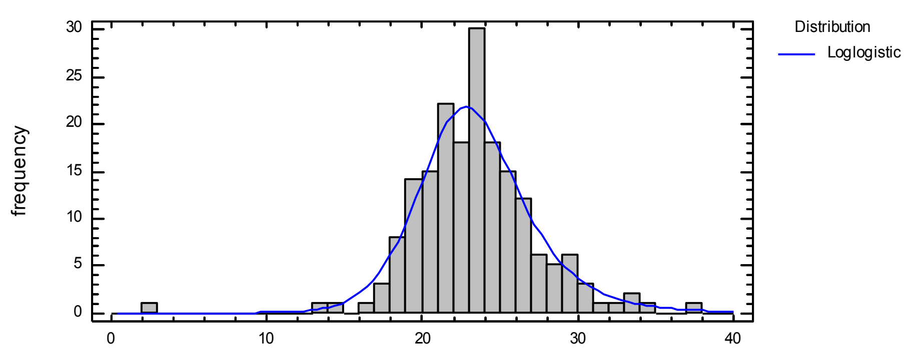


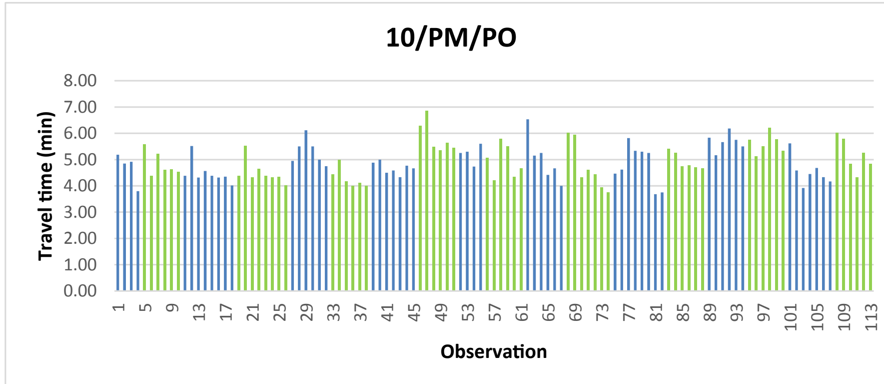
Fig. 3. Histogram and loglogistic distribution for bus in-vehicle time (minutes): Trip made on bus service 105, from bus stop Cardenal Raúl Silva H. and Pegaso to bus stop No. 1 Metro San Alberto Hurtado.

Table 3
Estimated parameters for distributions in Figs. 1–3.

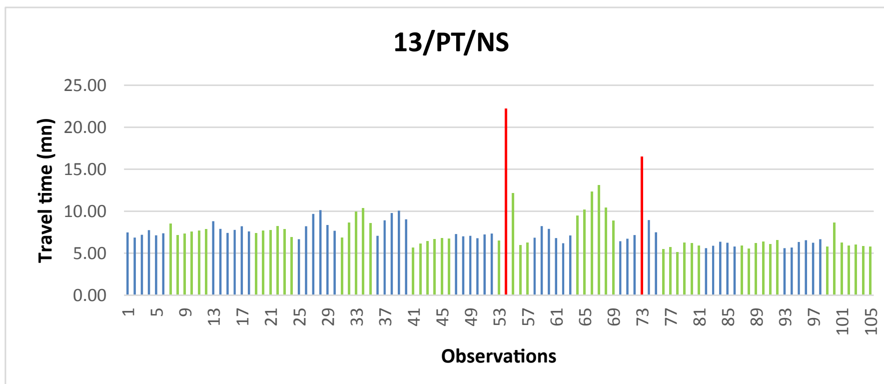
Mode	Car	Bus	Metro
Distribution	Lognormal	Loglogistic	Loglogistic
μ (Mean)	10.42	23.53	23.65
σ (Standard deviation)	2.53	3.64	2.58
μ_y	2.32		
σ_y	0.24		
α (median)		23.18	23.38
β (shape)		0.09	0.05
Skewness	0.27	0.73	2.64
Kurtosis	−0.28	1.49	14.33
Chi-squared test P-Value	0.78	0.97	0.57
Kolmogorov–Smirnov test P-Value	0.98	0.91	0.82

are longer than usual due to a previous incident (red bars) that negatively affected the travel time. Even though the orange bars are not be statistical outliers, they are considered to be the result of incidents as well. Of a total of 2616 car travel time observations for 25 routes, only 13 trips were detected as outliers; an additional six trips were assumed to be incidents, two of which are shown in Fig. 4c. In Table 4, we show the number of outliers in DB1 for the travel times in cars and the number of outliers in DB2 for travel times in bus and metro. The number of trips that are affected by incidents is between 0.7 and 0.8% of all travel time observations.

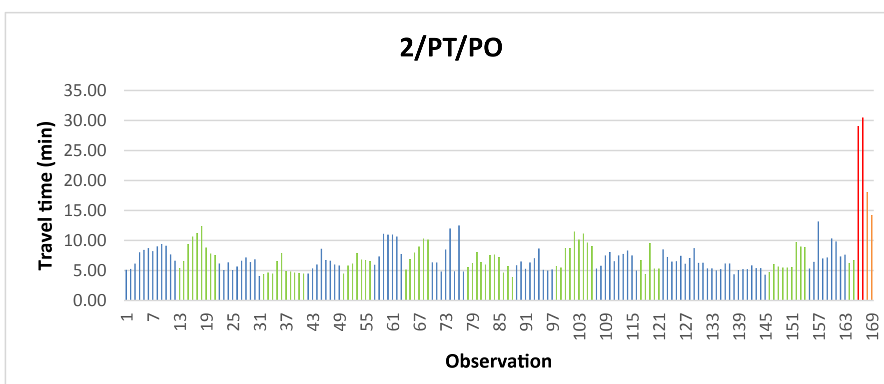
The effect of including or removing incidents based on the characterisation of travel time variability is analysed in the next section.



(a) Travel time Route 10, morning peak



(b) Travel time Route 9, morning peak



(c) Travel time Route 2, afternoon peak

Fig. 4. Travel time variability: cases with and without incidents.

Table 4
Number of incidents in DB1 and DB2.

Mode	No. OD pairs	Observations	Incidents	% Observations with incidents	No. outliers
Car	25	2.616	19	0,7%	13
Bus	209	35.164	286	0,8%	307
Metro	51	9.659	78	0,8%	188

4.2. Travel time variability: differences by mode and stages of a public transport trip

4.2.1. Car travel time

Fig. 5 depicts the relationship between the mean travel time and the standard deviation of travel time in minutes per kilometre. Linear regression parameters for Fig. 5 and the following figures are presented in Table 5. The estimated regressions have the form of Equation (3).

$$\sigma = m^{*}\mu + n + \varepsilon \quad (3)$$

where σ is the standard deviation of travel time, μ is the mean travel time, and ε is a random error.

The effect of the incidents in Fig. 5 is illustrated via a comparison of both plots: incidents increase the variability of a few observations as shown in Fig. 5a. Removing incidents (0.7% of observations) produces the plot in Fig. 5b. Both scatterplots can be regressed with linear relationships. Interestingly, removing incidents impacts the goodness-of-fit of the relationships but the effect on the slope of the linear relationship is only 10%, as its values reduces from 0.32 (all observations) to 0.29 (only recurrent congestion). Therefore, our data suggests that an average increase of 1 min per kilometre in mean travel time is associated with an increase between 17 s and 19 s of the standard deviation of travel time, which match the results obtained by Tirachini et al. (2014), using data from 423 roads in Sydney (a regression between the mean travel times and the standard deviation of the travel times, with a slope of 0.32).

A linear relationship between the standard deviation (SD) and the mean of travel times is a very simple method for applying a mean–variance model that only employs estimations of mean travel time, provided that the reliability ratio (ratio between the parameter of the mean travel time and the parameter of the SD of the travel time) is known. The data for Sydney and Santiago indicate a slope of 0.3 for the relationship between the SD and mean when the unit is min/km; data from other cities is necessary to assess the generalizability of these results. Mahmassani et al. (2012) employed simulated travel time data from different cities in the United States and estimated linear regressions with slopes between 0.25 and 0.53, which corresponds with our results.

For public transport, we characterise the time variability for each stage in subsequent sections.

4.2.2. Walking time (access and transfers)

Although walking is the predominant method for accessing bus stops and metro stations in cities and to transfer between vehicles on trips with more than one motorised stage, previous studies of public transport reliability do not characterise walking time variability. In our dataset (DB2), surveyors were required to walk from a given corner to a specific bus stop or metro station and record their walking time over several days. The relationship between the mean and variability of time is depicted in Fig. 6a, where a positive relationship is observed. This relationship shows that, on average, travel time variability for walking increases with travel distance due to the likely influence of traffic signals and other elements. The tendency is less distinct when analysing the walking time variability for transferring between vehicles due to high variability points for both short average walking times and long average walking times (Fig. 6b). A caveat of this analysis is that different surveyors perform trips for a given O–D pair in the database in different days; therefore, a systematic source of variation due to personal characteristics may be embedded in the scatter plot of Fig. 6.

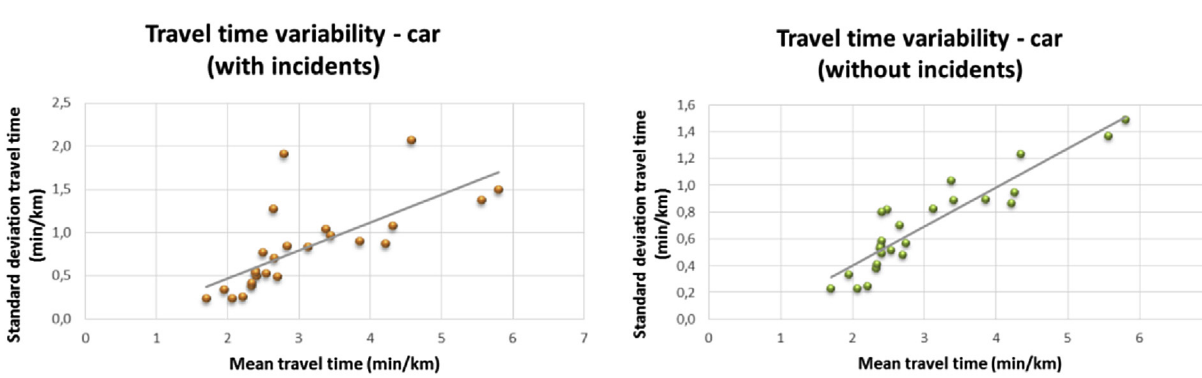
Table 5
Linear regression parameters for Figs. 5–8.

Regression	m	n	Adj. R2
Fig. 5 a	0.32 (0.19; 0.46)	−0.17* (−0.63; 0.28)	0.48
Fig. 5 b	0.29 (0.24; 0.35)	−0.19 (−0.36; −0.01)	0.83
Fig. 6 a	0.14 (0.09; 0.19)	0.56 (0.32; 0.79)	0.33
Fig. 6 b	0.11 (0.03; 0.19)	1 (0.67; 1.32)	0.12
Fig. 7 a	0.84 (0.64; 1.05)	0.09* (−0.33; 0.51)	0.57
Fig. 7 b	1 (0.66; 1.33)	−0.19* (−0.80; 0.43)	0.42
Fig. 7 c	1.01 (0.891; 1.13)	0.29* (−0.31; 0.88)	0.71
Fig. 7 d	0.85 (0.75; 0.95)	0.77 (0.25: 1.29)	0.76
Fig. 8 a	0.25 (0.13; 0.36)	−0.18* (−0.41; 0.05)	0.27
Fig. 8 c	0.49 (0.44; 0.55)	−0.83 (−1.02; −0.63)	0.58

Note: The 95% confidence interval for each parameter is shown in parenthesis. * indicate parameters that are not statistically significant.

4.2.3. Waiting time

Several authors analyse the stability of bus headways (e.g., Byon, Cortés, Martinez, Munizaga, & Zúñiga, 2011; Chen, Yu, Zhang, & Guo, 2009; Strathman et al., 1999) as headway variability has been demonstrated to increase mean waiting times (Osuna & Newell, 1972). Therefore, strategies such as bus holding have been investigated and implemented in both frequency-based and schedule-based public transport services to maintain intervals as even as possible. Although the link between headway variability and mean travel time has been established, the extension to understanding waiting time variability has not received much attention in the literature, as bus headways are easily recorded with automatic vehicle location devices (e.g., GPS devices), but obtaining repeated observations of actual waiting times for several routes over several days is a cumbersome task that usually requires field observations and/or video recording and processing. We are able to



(a) All data **(b) Removing incidents**

Fig. 5. Relationship between SD and mean travel time: cars.

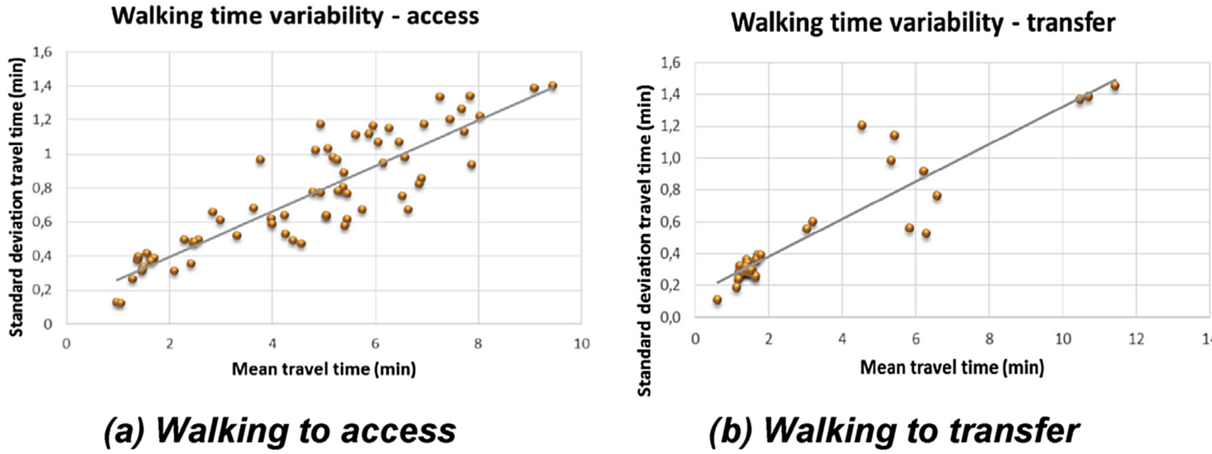


Fig. 6. Relationship between SD and mean of walking times.

characterise the daily variation in waiting times due to the repeated surveys for estimating travel times of different trips by public transport in Santiago, as shown in Fig. 7.

For the surveyed Metro trips, the majority of mean waiting times are less than 3 min (75 of 77 metro trip stages). When analysing all observations, Fig. 7a shows that the standard deviation of the waiting times increases for trips with a mean waiting time less than 3 min. For the two observations of the mean waiting times greater than 3 min, the SD is between 3.3 min and 4.2 min. This finding may indicate a concave relationship between the mean and SD of the waiting times (such as the logarithmic regression in Fig. 7a). However, the low number of observations with mean waiting times greater than 3 min prevents us from concluding these results with certainty. By focussing on the 75 cases with mean waiting times below 3 min, Fig. 7b is obtained, in which a slope of 0.997 is obtained for the linear relationship between the mean and the SD of the waiting times, which is similar to the slope (1.01) obtained for buses (Fig. 7c). The fact that the slope is near unity in both cases indicates an exponential distribution in the interarrival time between vehicles and users, which is characteristic of a Poisson process. Fig. 7d presents the analysis of waiting time variability when transferring between buses or between bus and metro. A fairly linear relationship is also observed, with a slope that is slightly lower than the case of the waiting times at first boarding (0.85 vs 1.01). Given that transfers between vehicles in Santiago are not coordinated, a significant difference between both regressions was not expected (Fig. 7c and d).

4.2.4. In-vehicle time: bus and metro

Next, we study the travel time variability for in-vehicle times of public transport trips. As for the case of cars, we normalise the travel times by distance (min/km) to determine the relationship between travel time and congestion.

Fig. 8a depicts the standard deviation of the travel times by metro. The data supports a positive relationship between the mean and SD of the travel times; however, the data shows high dispersion, which indicates that the travel time variability is weakly related to the mean travel time, in apparent opposition to the relationships observed for the other two modes according to Figs. 5 and 8c. However, note that mean in-vehicle times by metro range between 1.4 and 3.0 min/km in Fig. 8a, which is significantly lower than the range for cars (between 1.6 and 6.0 min/km in Fig. 5) and buses (between 1.5 and 8.2 in Fig. 8c).

The bus travel time plot (Fig. 8b) reveals some interesting issues. First, 207 of the 209 trip stages have a mean travel time less than 8.2 min/km (commercial speed greater than 7.4 km/h), and only 28 trip stages (13%) have a mean travel time greater than 4 km/min (speed less than 15 km/h). A positive relationship is observed between the SD and the mean of the travel times in Fig. 8b, which tends to stabilise if the cases with extreme congestion in Fig. 8b are included (speed less than 4 km/h). Removing the two cases of extreme congestion, we obtain Fig. 8c, in which a strong linear relationship is observed.

5. Comparison of bus TTV in mixed traffic versus sections with bus lanes and segregated busways

In this section, we analyse the effect of alternative right-of-way configurations for buses on TTV. Database DB2 is not useful due to an inadequate number of observations for bus lanes and segregated busways. Therefore, we resort to DT3, which contains over 23 million records of bus (in-vehicle) travel times in Santiago, which were obtained from smartcard transactions and GPS data. From this database, we selected 197 origin–destination (OD) pairs of trips on workdays in mixed traffic, 71 OD pairs that fully employ one bus lane (Alameda-Providencia-Apoquindo Avenue) during the in-vehicle stage, and 48 OD pairs in four segregated busways: Vicuña Mackenna Avenue, Las Rejas Avenue, Santa Rosa Avenue and Grecia Avenue. Each origin–destination pair was also disaggregated according to time period.

Fig. 9 shows the relationship between mean travel time and the standard deviation of travel times in minutes per kilometre for the three types of right-of-way under study. The linear regression parameters for Fig. 9 are presented in Table 6. Considering of all observations in mixed traffic (blue dots), the relationship between MTT and TTV is positive and can be fitted by a linear relationship. The maximum travel time in our sample is 15 min/km (4 km/h), which indicates a level of extreme congestion; however, we obtained no evidence of reductions in travel time variability, as shown in Fig. 8b, in which two bus stages with extreme congestion have a relatively low standard deviation of travel time. Additional observations under extreme congestion are needed to assess the trends of TTV in these conditions.

Fig. 9 also depicts the relationship between mean travel time and standard deviation of bus travel times in bus lanes (green (in the web version) dots) and segregated busways (red dots). As shown in Table 6, we note that the slope of the TTV curve is significantly larger for mixed traffic (0.61) than for bus lane and segregated busways (0.23–0.25). We also note that the confidence interval of the slope of the bus lane regression is within the confidence interval of the slope of the segregated busway; therefore, no significant difference between both regression lines is observed.

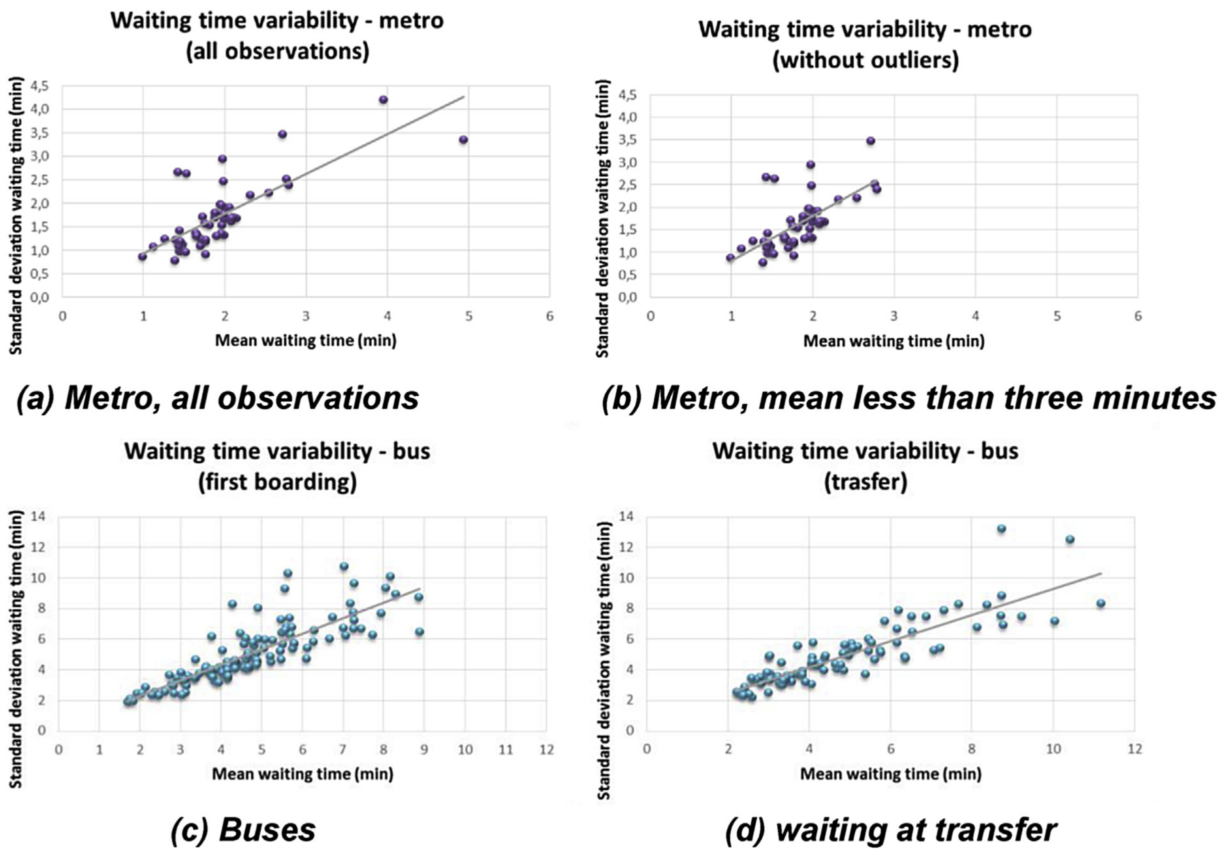


Fig. 7. Relationship between SD of waiting times and mean waiting time: bus and metro.

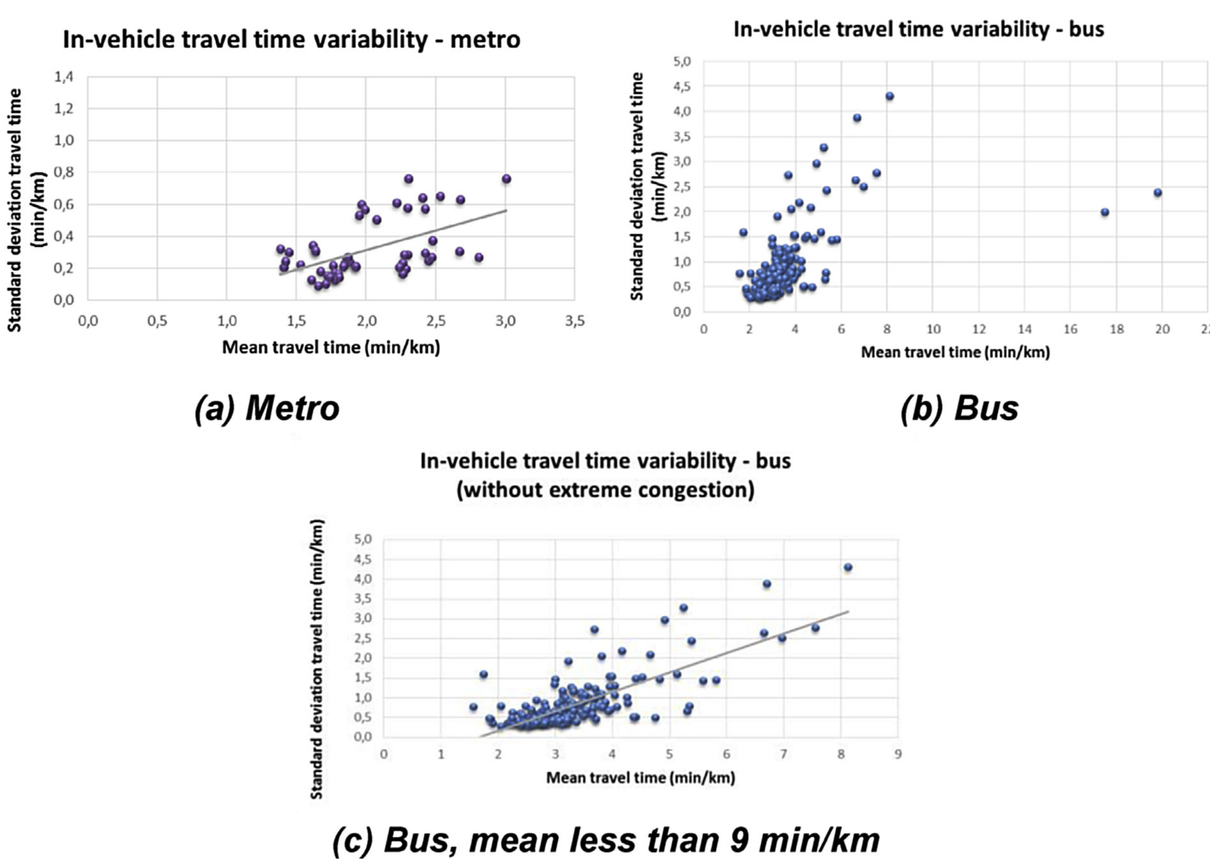


Fig. 8. Relationship between SD and mean of in-vehicle times: bus and metro.

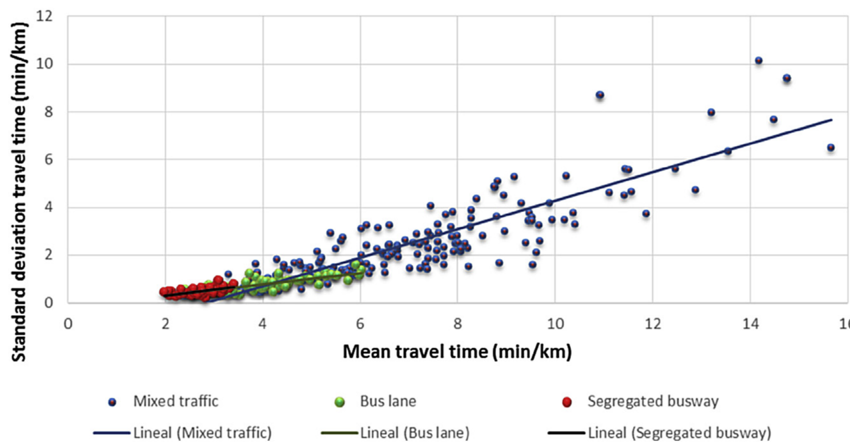


Fig. 9. TTV in mixed traffic, bus lanes and segregated busways: all data.

Table 6
Linear regression parameters for Figs. 9 and 10.

Regression	m	n	Adj. R2
Mixed traffic	0.61	−1.78	0.75
	(0.56; 0.66)	(−2.15; −1.41)	
Bus lane	0.23	−0.16*	0.61
	(0.19; 0.28)	(−0.35; 0.02)	
Segregated busway	0.25	−0.19*	0.31
	(0.14; 0.37)	(−0.5; 0.11)	
Mixed traffic	0.39	−0.64	0.37
(travel time shorter than 6 min/km)	(0.28; 0.51)	(−1.21; −0.07)	
All data – linear regression	0.55	−1.36	0.81
	(0.52; 0.58)	(−1.54; −1.17)	

Note: The 95% confidence interval for each parameter is shown in parenthesis. * indicate parameters that are not statistically significant.

Fig. 9 shows that bus lanes and segregated busways have similar range of TTV even though segregated busways have shorter mean travel times, which is also revealed when analysing the coefficient of variation (CV) of all trips, as presented in Fig. 10. We conclude that buses that travel in mixed traffic not only have a larger mean travel time but also a larger travel time variability relative to buses that travel in bus lanes and segregated busways.

6. Other reliability measures

To complement the results obtained from the SD as a measure of TTV, we apply two other measures, as suggested by van Lint et al. (2008):

- Skew of the travel time distribution (λ_{skew}): the ratio of the distance between the difference between the 90th percentile and the 50th percentile and the distance between the 50th percentile and the 10th percentile. For large values of λ_{skew}, the distribution is strongly skewed and the travel time reliability is low.
- Width of the travel time distribution (λ_{var}): When λ_{skew} is one, the distribution is symmetric and the width of the distribution should be considered. λ_{var} is defined as the ratio of the difference between the 90th percentile and the 10th percentile and the median travel time. The wider the distribution is relative to the median, the larger is the range of travel times that may occur and the lower is the travel time reliability.

Fig. 11 shows λ_{skew} and λ_{var} for the travel times by car and metro, using DB1 and DB2, respectively. Fig. 12 depicts λ_{skew} and λ_{var} for the bus travel times, distinguishing between mixed traffic, bus lanes and segregated busways (DB3).

A large value of λ_{skew} is observed for Metro trips compared with buses that travel in bus lanes and segregated busways. For the metro, 94% of the analysed trips have a λ_{skew} greater than 1 compared with 58% of the trips in the bus lanes, which have a λ_{skew} greater than 1. This finding is attributed to the fact that the travel time variability for Metro is primarily based on a few observations with long travel times, relative to a larger proportion of observations that are close to the mean, as shown in Fig. 2. Therefore, the skew parameter indicates that metro has a different type of variability, which is not evident when focussing on the standard deviation as the measure of TTV.

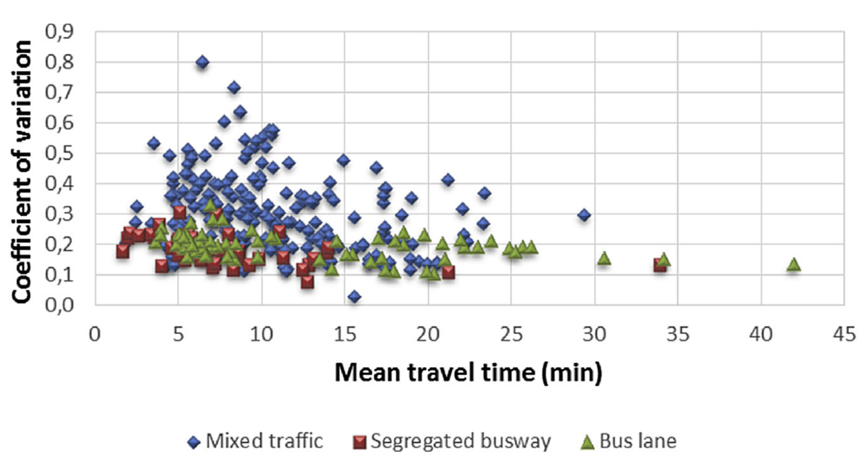


Fig. 10. Coefficient of variation in mixed traffic, bus lanes and segregated busways.

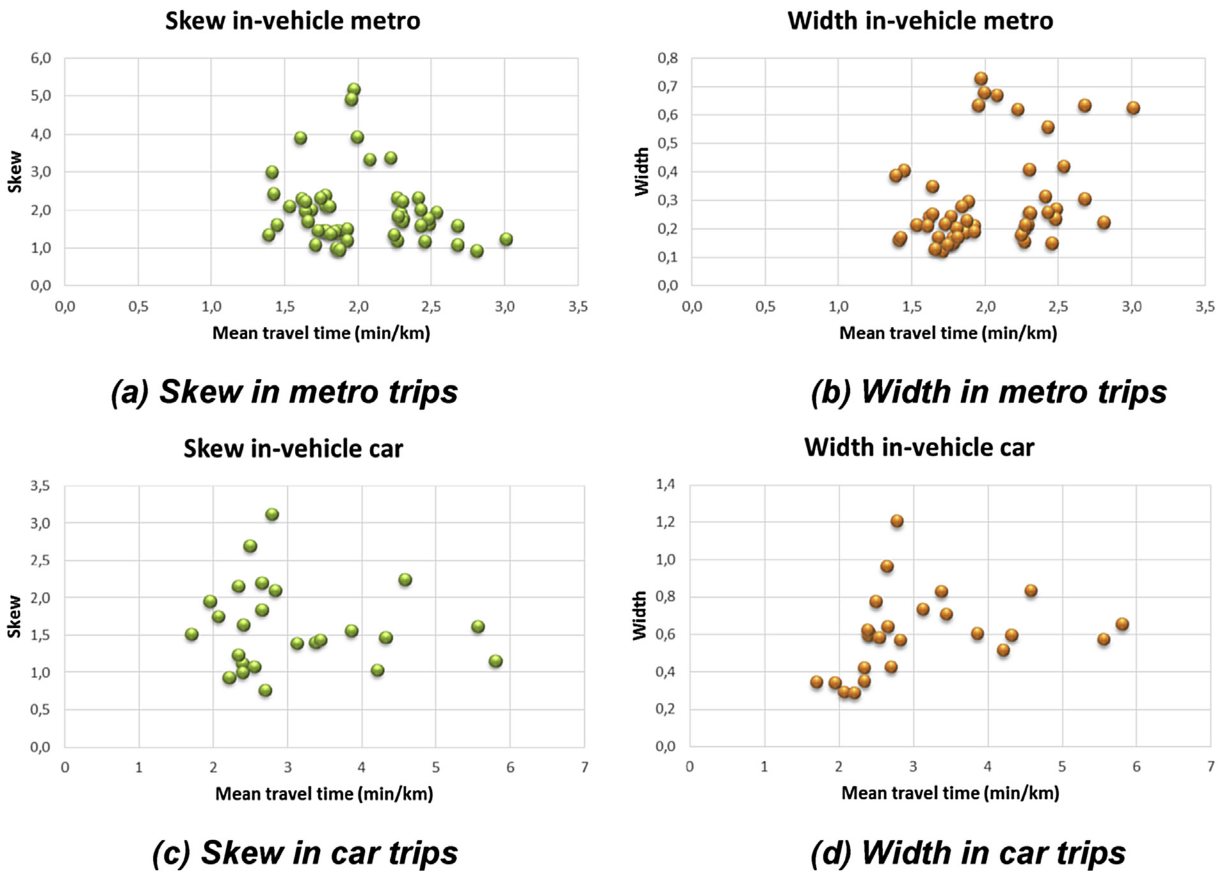


Fig. 11. Other reliability measures – in-vehicle travel time for metro and car.

This finding reinforces the remarks of van Lint et al. (2008), who compared different TTV measures, particularly measures that include or disregard the skewness of the distributions. The analysis of the width parameter in Figs. 11 and 12 also indicate that the travel time distribution of the modes that are subject to congestion (cars and buses on mixed traffic), is greater than the travel time distribution for Metro and buses with preferential right-of-way.

7. Travel time variability: door-to-door public transport trips

The previous analysis was independently performed for each trip stage to observe differences in travel time variability for walking, waiting and in-vehicle times. Given that we have repeated observations of door-to-door trips in database DB2, we can surpass the analysis of individual stages to identify which trip stages, and to what extent, are statistically significant in explaining total (door-to-door) travel time variability. In this section, we estimate a regression model for the standard deviation of total travel time as a function of the mean access, waiting, transfer and in-vehicle times per mode, as shown in Equation (4). All variables are expressed in minutes:

$$\sigma = b_0 + b_1 t_{walk-access} + b_2 t_{wait-bus} + b_3 t_{wait-metro} + b_4 t_{veh-bus} + b_5 t_{veh-metro} + b_6 t_{walk-trans} \quad (4)$$

Table 7 shows the number of origin–destination (OD) pairs of which travel time data was registered between 2007 and 2012 in DB2 and the mean number of observations in each OD pair.

Year 2007 has the largest number of measured OD pairs (209 pairs), which decreases to 189 from 2008 to 2010. In 2012, only 64 OD pairs were surveyed. The number of observations per OD pair is significantly reduced in 2010, 2011 and 2012 as reliable data about travel times was obtained from GPS and smartcard data since 2010. Furthermore, in 2007, the Transantiago public transport system was launched and severe operational problems arose (Muñoz & Gschwender, 2008). Therefore, travel time measurements in 2007 are not representative of subsequent years. Thus, data from 2008 to 2011 will be considered for additional scrutiny. A statistical analysis for each of these four years is separately performed for the entire 2008–2011 period and for the 2008–2009 period given that 2008 and 2009 have a larger number of observations per OD pair than 2010 and 2011.

Table 8 presents the estimated parameters for the models estimated with yearly data; 2008 to 2011 are grouped, and 2008 and 2009 are grouped. Assessing the goodness-of-fit, the best models are the models that include data from 2008 to 2009 (first four columns), in which the adjusted R-squared value is between 0.62 and 0.66. The models from 2010 to 2011 have too few observations per OD pair (Table 7) to calculate reliable TTV models, which is reflected in a lower goodness-of-fit and non-significance of variables in the case of the model estimated using only data from 2010.

Based on the variables and their significance, in the models based on the most reliable data (2008, 2009 and 2008–2009), both bus waiting times and bus in-vehicle times are statistically significant in explaining total TTV at the 0.1 percent confidence level. Second, metro in-vehicle travel time is statistically significant at the 5 percent confidence level in the models estimated with data from 2009 and from 2008 to 2009 (and it is significant at 6% confidence for the 2008–2011 model), but it is not significant in years 2008

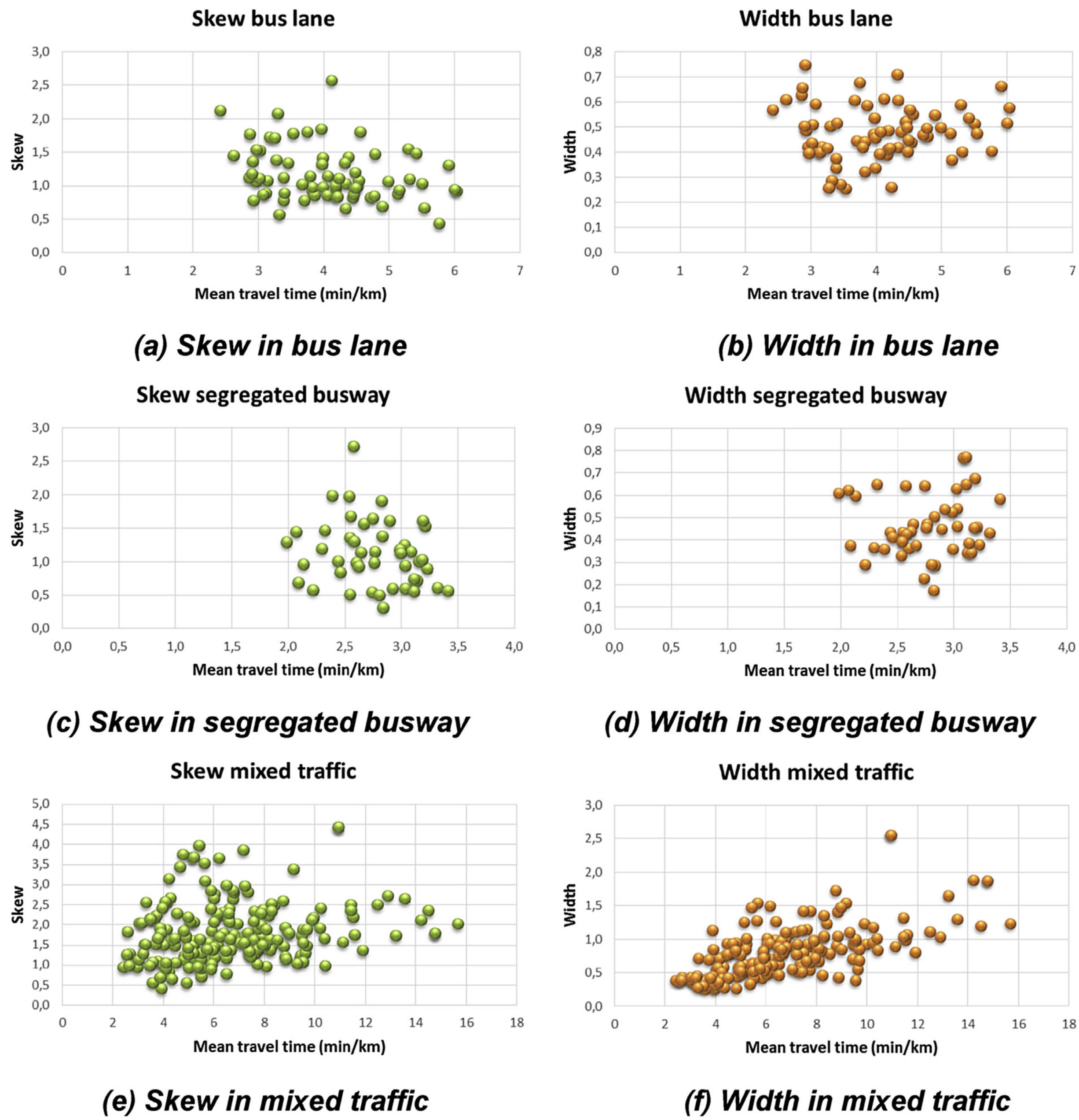


Fig. 12. Other reliability measures – in-vehicle travel time for bus in mixed traffic, bus lane and segregated busway.

Table 7
Data characterization by year.

	2007	2008	2009	2010	2011	2012
Number of OD pairs	209	189	189	189	171	64
Mean number of observations in each OD pair	134	172	136	21	16	3

and 2011 alone. Finally, mean walking (access and transfer) and metro waiting times are not significant in explaining total travel time variability.

Table 8 is the first effort to determine if specific trip stages by public transport are significant in explaining total door-to-door travel time variability. The results confirm the results of Section 4, which indicate that bus waiting and in-vehicle time are the main drivers of total TTV, followed by metro in-vehicle times. Note that waiting times at metro stations are not significant due to the stability of metro intervals in Santiago's metro and low mean waiting time. Although walking time is subject to variability, it does not significantly influence the results.[2] Therefore, our results suggest that efforts to reduce public transport TTV should be aimed at reducing mean travel times for waiting and in-vehicle bus times and to control the stability of metro travel times. We conclude that measures such as the segregation of buses from cars (bus lanes, exclusive streets, and segregated busways) and a fleet management system to reduce bus bunching (and reduce mean waiting times)

[2] This result is obtained despite the fact that data in DB2 are collected by different people who may have different walking speeds, which increased walking time variability as recorded in the survey.

Table 8
Regression models for total TTV, aggregated and by year.

Model	2008–2011	2008–2009	2008	2009	2010	2011
Constant	2,03	1,83	1,68*	2,04	6,29	1,72*
	(2,21)	(1,97)	(1,60)	(2,05)	(3,38)	(0,79)
Mean access time	−0,01*	0,04*	0,05*	0,01*	−0,27*	−0,12*
	(−0,05)	(0,36)	(0,34)	(0,09)	(−1,42)	(−0,63)
Mean waiting time (bus)	**0,51**	**0,53**	**0,57**	**0,45**	−0,01*	**0,40**
	(5,78)	**(6,21)**	**(6,50)**	**(4,21)**	(−0,02)	**(1,91)**
Mean waiting time (metro)	−0,50*	−0,56*	−0,21*	−0,72*	−1,27*	1,73
	(−0,60)	(−0,63)	(−0,19)	(−0,88)	(−1,22)	(2,63)
Mean in-vehicle time (bus)	**0,07**	**0,07**	**0,07**	**0,07**	0,02*	**0,13**
	(5,61)	**(5,30)**	**(4,40)**	**(5,48)**	(0,61)	**(3,84)**
Mean in-vehicle time (metro)	**0,15**	**0,16**	0,14*	**0,16**	0,12*	−0,13*
	(1,94)	**(1,95)**	(1,48)	**(2,07)**	(1,31)	(−1,67)
Mean transfer time	−0,02*	−0,10*	−0,20*	−0,04*	0,54*	0,01*
	(−0,13)	(−0,54)	(−0,93)	(−0,21)	(1,30)	(0,03)
R squared	0,78	0,78	0,76	0,73	0,28	0,57
Adjusted R squared	0,72	0,72	0,69	0,66	0,08	0,44

Note: The t-test is shown in brackets bellow the parameter value. * indicate variables that are not statistically significant. Bold values for variables that are significant.

are significant in increasing the reliability of door-to-door travel times by public transport.

8. Concluding remarks

We investigated the travel time variability of cars and public transport trips in the city of Santiago, Chile. Three databases were employed: one database for cars trips that were obtained with the floating car method for different routes and two databases for public transport trips. One of our databases considers door-to-door trips by bus and/or metro (subway), which were performed over several days by surveyors.

The main results are summarised. First, a distinct and strong relationship between the standard deviation of the travel time and the mean travel time is observed for car, bus waiting times and bus in-vehicle times, whereas walking times and waiting and travel times by metro are subject to variability but to a lesser extent than the other modes and trip stages. Metro in-vehicle times are more stable when analysing the standard deviation of travel times (symmetrical variability measure); however, metro travel time variability is mainly driven by a few observations with large travel times relative to the mean, which is obtained by analysing the skew parameter of the travel time observations (asymmetrical variability measure).

Second, when analysing the car travel time variability, a linear relationship between the mean and SD of the travel times has a slope between 0.30 and 0.32, which corresponds to the results obtained for Sydney (Tirachini et al., 2014), i.e., an average increase of 1 min per kilometre in mean travel time is associated with an increase between 18 s and 19 s of standard deviation. Similar analyses from other cities should be performed to assess the generalisability of this finding.

Third, for door-to-door public transport trips, we discovered that the total travel time variability is significantly explained by bus waiting and in-vehicle times and explained at a lower level by metro in-vehicle times, whereas walking and metro waiting times were not statistically significant. This finding has relevant policy implications on the interventions that should be preferred to reduce total travel time variability, such as increasing bus frequency and introducing bus priority measures. For example, the relationship between the mean and the standard deviation of the bus waiting times can be used to assess the value of reducing bus bunching, not only for reducing average waiting times but also for decreasing its variability.

Finally, we analysed the effect of mixed traffic, bus lanes and segregated busways on TTV in bus. We found that buses that travel in mixed traffic have not only a larger mean travel time but also a larger variability compared with the buses in bus lanes and segregated busways. This link between preferential right-of-way configurations and travel time variability highlights the hidden benefit of bus lanes and segregated corridors for reducing travel time variability, which should be monetised and incorporated in a formal cost-benefit analysis of public transport priority interventions.

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