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A market game approach to differential information economies

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Abstract In this paper we recast a differential information economy as a strategic game in which players propose net trades and prices. Pure strategy Nash equilibria are strong and determine both consumption plans and commodity prices that coincide with the Walrasian Expectations equilibria of the underlying economy.

Keywords Differential information · Walrasian Expectations equilibrium · market games

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1 Introduction

The formation of prices plays a central role in any discussion of the market process, and this has given rise to a growing literature on market games. In particular, Schmeidler (1980) presented a market game in which the exchange mechanism that characterizes the economic institutions of trade is given by strategic outcome functions, with players proposing consumption bundles and prices. In this way, he explained the price formation mechanism and proved that Nash equilibria of this market game are strong and coincide with the Walrasian equilibria of the underlying Arrow–Debreu pure exchange economy.

In this paper we provide a strategic approach to economies with differential information. Our starting point is essentially the model of Radner (1968). Each agent has a private and incomplete information structure about the future states of nature that describes the events she can observe. It is supposed that a consumer can only carry out trades that are compatible with her private information, that is, she does not trade differently on states she is not able to distinguish. The noncooperative solution, here called Walrasian Expectations equilibrium, presumes that decisions are made in an ex-ante stage, that information constraints are explicitly considered, and that agents do not infer any additional information from the prevailing prices.

As long as in a differential information context different agents can differ in their degrees of knowledge about uncertainty, it is not surprising that a trade mechanism only based on a Schmeidler-type outcome function is not enough to characterize the equilibrium solutions. In fact, in a strategic approach to Walrasian Expectations equilibria, the main difficulty to overcome is that the outcomes that an agent receives have to be compatible with her own private information.

For this reason, we propose a market game mechanism that links Schmeidler-type outcome functions and a delegation rule, as well as it allows agents to inform anonymous players about their objective functions (who, by themselves, incorporate the information constraints). These players, who are perfectly informed, propose profiles involving both prices and net trades. As in Schmeidler (1980), the outcome function maps players' simultaneous selections of strategies into allocations.

Our main result guarantees that (1) every Walrasian Expectations equilibrium can be implemented as a strong Nash equilibrium of the market game described above, and (2) each Nash equilibrium is strong and determines bundles and prices that constitute a Walrasian Expectations equilibrium. We state an example which shows that, without the delegation rule, it is no longer possible to obtain the result. In addition, we provide an axiomatic characterization of the outcome functions introduced by Schmeidler (1980) and that are used here.

In a previous work, Hahn and Yannelis (2001) demonstrate that the private core is implementable as a strong (coalitional) Nash equilibrium. Thus, each Walrasian Expectations equilibrium can be obtained as a Bayesian Nash equilibrium. However, as long as the private core strictly contains as a subset the set of equilibrium allocations, a Bayesian Nash equilibrium is not necessarily a Walrasian Expectations equilibrium. Therefore, in this context, our main result compliments Hahn and Yannelis (2001) contribution.

The rest of the paper is organized as follows. In Sect. 2 we set the basic formal model of a differential information economy and discuss both the non-free disposal condition and the relationship between the concepts of Walrasian Expectations equilibria and Arrow–Debreu equilibria. In Sect. 3 we recast the economy as a market game and we present our main result. Section 4 provides some examples that justify both our market game structure and our assumptions. Finally, the last section lays the axiomatic characterization of the natural outcome function.

2 Model

Consider an economy $\mathcal{E}$ with two dates, $t \in \{0, 1\}$, in which there is no uncertainty at the first period, and there is a finite set of states of nature, Ω, that can be revealed at $t = 1$. There is a finite number of commodities, ℓ. Trading contingent contracts at the first period, a finite set of agents, N, makes consumption plans for each state $\omega \in \Omega$.

Given a partition P of Ω, a commodity bundle $x = (x(w))_{w\in\Omega} \in (\mathbb{R}^{\ell}_{+})^k$, where k denotes the number of elements of Ω, is said to be P-measurable when it is constant on the elements of the partition P.[1]

Each agent $i \in N$ is partially and privately informed about the states of nature in the economy: she only knows a partition P_i of Ω. Thus, she does not distinguish, after the realization of the uncertainty, those states of nature that are in the same element of P_i. Also, she does not necessarily know the set of possible states of nature Ω.

Utility functions are given by $U_i : (\mathbb{R}^{\ell}_{+})^k \to \mathbb{R}_+$ and are defined over the consumption set $(\mathbb{R}^{\ell}_{+})^k$. Moreover, by denoting $\mathbb{P}_i = \left\{x \in (\mathbb{R}^{\ell}_{+})^k \mid x \text{ is } P_i\text{-measurable}\right\}$ as the set of consumption bundles that are compatible with the information structure of agent i, we suppose that initial endowments vector $e_i \in (\mathbb{R}^{\ell}_{++})^k$ belongs to $\mathbb{P}_i$. We assume that

(A1) *Utilities are strictly monotone in* $(\mathbb{R}^{\ell}_{++})^k$*, strictly quasi-concave, and differentiable. Moreover, agents prefer an interior commodity bundle to any consumption bundle in the frontier of* $(\mathbb{R}^{\ell}_{+})^k$.

We refer to an allocation $(x_i)_{i\in N}$ as *physically feasible* if $\sum_{i\in N}(x_i - e_i) \leq 0$, and as *informationally feasible* if $x_i \in \mathbb{P}_i$, for every i. A *feasible allocation* is both physically and informationally feasible.

A price system is a vector $p = (p(\omega))_{\omega\in\Omega}$, that specifies a commodity price $p(\omega) \in \mathbb{R}^{\ell}_{+}$ at each state $\omega \in \Omega$. Without loss of generality, we suppose that $p \in \Delta := \left\{q \in (\mathbb{R}^{\ell}_{+})^k \mid \sum_{h=1}^{\ell\times k} q_h = 1\right\}$.

Each agent i is a price taker individual who maximizes her utility functions restricted to the allocations in her budget set:

$$B_i(p) = \left\{x_i \in \mathbb{P}_i \mid \sum_{\omega\in\Omega} p(\omega)\cdot(x_i(\omega) - e_i(\omega)) \leq 0\right\}.$$

We stress that though commodity prices, that agents take as given, can be different across the states of nature that are indistinguishable for them, the market cannot communicate any information through the price system.[2]

[1] That is, $x(\omega) = x(\omega')$, for all $\{\omega, \omega'\} \subseteq S$, for some $S \in P$.

[2] Following Maus (2004), agents do not infer any new information from prices. They observe prices according to their action possibilities, which are determined by their private information. Agent i perceives a price system p under her information P_i as $(p(S_i))_{S_i\in P_i}$, with $p(S_i)$ representing the same observed price in each state of S_i, given by the average price $\frac{1}{\#S_i}\sum_{w\in S_i} p(\omega)$, where $\#S_i$ denotes the cardinality of S_i.

Definition *A Walrasian Expectations equilibrium for the economy $\mathcal{E}$ is a pair (x, p), where $x = (x_i)_{i \in N}$ is a feasible allocation and p is a price system, such that, x_i maximizes U_i on $B_i(p)$ for all $i \in N$.*

Typically, a differential information economy is recast as an Arrow–Debreu economy in which the information constraint is built into the consumption set of each agent. However, in this paper, we will set up our Walrasian Expectations economy as an Arrow–Debreu economy in which agents have the same consumption sets, but information structures are incorporated directly into the utility functions.

More formally, given the economy $\mathcal{E}$ we can construct a *complete information* economy in which the consumption set of agent i is $(\mathbb{R}^\ell)^k$ and her utility is

$$\tilde{U}_i(x_i) = \begin{cases} U_i(x_i) & \text{if } x_i \in \mathbb{P}_i, \\ 0 & \text{otherwise.} \end{cases}$$

It is easy to check that both economies are equivalent with regard to the equilibria solutions. In fact, Walrasian Expectations equilibria of the differential information economy $\mathcal{E}$ are precisely competitive equilibria in the Arrow–Debreu economy above described. Though preferences are known to be no continuous, Assumption (A1) guarantees the existence of Walrasian Expectations equilibria and, therefore, there exist equilibria for this Arrow-Debreu economy.

We remark that equilibria of this economy can present free disposal. As it was shown in Glycopantis et al. (2003), the Radner equilibrium need not to be Bayesian incentive compatible because of the free disposal requirement [see also Hervés-Beloso et al. (2005)]. Moreover, Glycopantis et al. (2003) provide examples of economies with differential information without any Radner equilibria with positive prices in the case of non-free disposal. It should be stressed that in these examples free disposal occurs only at those states of nature that no agent can discern. Despite this, it is not difficult to prove that, if each state of nature is distinguished by at least one agent then any Walrasian Expectations equilibrium is a non-free disposal equilibrium and prices are strictly positive. To formalize this idea, we suppose that

(A2) *Given any state $\omega \in \Omega$, there exists an agent $i \in N$ such that, $\{w\} \in P_i$.*

Note that, whenever there exists an agent who is completely informed about Ω, the assumption above holds. Moreover, if the number of agents is much bigger than the set of states of nature, the hypothesis seems to be not very restrictive.

Proposition *Let $\mathcal{E}$ be an information economy satisfying hypothesis* (A2). *If preferences are strongly monotone then any Walrasian Expectations equilibrium is a non-free disposal equilibrium.*

Proof Let $((x_i)_{i \in N}, p)$ be a Walrasian Expectations equilibrium for the economy $\mathcal{E}$. Suppose that $\sum_{i \in N} x_i^m(\omega) < \sum_{i \in N} e_i^m(\omega)$ for a state of nature ω and for a physical commodity m. Then, strictly monotonicity of preferences implies $p^m(\omega) = 0$.

By Assumption (A2), there exists an agent j who distinguishes ω. Consider the consumption bundle y which coincides with x_j except for the commodity m and the state ω, where $y^m(\omega) = x_j^m(\omega) + \left(\sum_{i=1}^n e_i^m(\omega) - \sum_{i=1}^n x_i^m(\omega)\right)$. Observe that y is P_j-measurable and since $p^m(\omega) = 0$, we have $p \cdot y = p \cdot x_j$. Therefore,

y belongs to $B_j(p)$ and, by strong monotonicity of preferences, $U_j(y) > U_j(x_j)$, which is a contradiction. □

3 A market game approach to differential information economies

The aim of this section is to recover Walrasian Expectations equilibria as Nash equilibria of a game. For it, given the economy $\mathcal{E}$ described in Sect. 2, we construct a game where each consumer is represented by a player with no informational restriction.

Actually, in our game we suppose that agent i delegates to another individual, identified as *player* i, the duty to find an informationally compatible outcome that is optimal given the behavior of the other market participants. In fact, agent i realizes that a fully informed representant will not have problems in understanding strategy profiles, which may involve bundles and prices that are not measurable regarding her private information. With this mechanism, agents know that they can obtain the best response to the allocations chosen by the others.

Of course, we also suppose that (1) there is no economic incentive which allows agent i to obtain more information directly from player i, and (2) even in the case that this player is altruistic, she only knows the objective function of the agent, $\tilde{U}_i$, that internalizes the information restriction and, therefore, she does not know whether a null utility level is a consequence of either preferences or the impossibility of agent i to understand the consumption bundle.

Therefore, players, although fully informed, are only interested in finding an optimal response that is *compatible* with the preferences of the agents.

As in our economy agents have incomplete information, it is not very surprising that we need a more sophisticated type of market game than the one in Schmeidler (1980). In fact, avoiding fully informed players, it is not possible to neutralize the diversity of agents' information structures because, if (partially informed) consumers are by themselves the players, Nash equilibria of the corresponding market game may not lead to Walrasian Expectations equilibria (see Example 1, in the next section).

Now, let $\Gamma = \{\Theta_i, \pi_i\}_{i \in N}$ be a game where Θ_i is the strategy set and π_i the payoff function of player i. A strategy θ_i for player i is a vector $z_i \in (\mathbb{R}^{\ell})^k$ and a price system $p_i \in \Delta$ such that $p_i \cdot z_i = 0$. Hence, $\Theta_i = \left\{(z_i, p_i) \in (\mathbb{R}^{\ell})^k \times \Delta \mid p_i \cdot z_i = 0\right\}$. We stress that the amount vector $z_i \in (\mathbb{R}^{\ell})^k$ that player i proposes is not required to be measurable with respect to her private information.

Let $\Theta = \prod_{i=1}^{n} \Theta_i$ be the set of strategy profiles. Given a strategy profile θ, each player i will trade only with those individuals that propose the same prices, $A_i(\theta) = \{j \in N \mid p_j = p_i\}$.

As exchange of commodities takes place among members that choose the same prices, their aggregated net outcome need to be zero. Therefore, as in Schmeidler (1980), each player receives the original net demand proposed adjusted by the average excess of demand of individuals that choose the same price as her. Formally, given a strategy profile θ, the agent i receives

$$f_i(\theta) := z_i - \frac{\sum_{j \in A_i(\theta)} z_j}{\# A_i(\theta)},$$

where $\#A_i(\theta)$ denotes the cardinality of the set $A_i(\theta)$. Hence, the i^{th} player payoff function $\pi_i : \Theta \to \mathbb{R}$ is defined by $\pi_i(\theta) = \tilde{U}_i(f_i(\theta) + e_i)$.

For a profile θ, let θ_{-S} denote a strategy selection for all players except those belonging to the coalition S. We write $\theta = (\theta_{-S}, \theta_S)$. A strategy profile $\theta^* = (\theta_i^*)_{i \in N}$ is a **Nash equilibrium** if, for each player $i \in N$, $\pi_i(\theta^*) \geq \pi_i(\theta^*_{-i}, \theta_i)$, for all $\theta_i \in \Theta_i$. In addition, a strategy profile θ^* is said to be a **strong Nash equilibrium** if it is not upset by any coalition of players. That is, if it does not exist a coalition S and a strategy profile θ such that, for every player $i \in S$, $\pi_i(\theta^*_{-S}, \theta_S) \geq \pi_i(\theta^*)$, with strict inequality holding for some player in the coalition S.

Theorem *Let $\mathcal{E}$ be an economy with private information satisfying assumptions* (A1)–(A2)*, with at least three agents. Let Γ be the associated game. Then,*

I. *If (x^*, p^*) is a Walrasian Expectations equilibrium of $\mathcal{E}$, then $\theta^* = ((x_i^* - e_i, p^*)_{i \in N})$ is a strong Nash equilibrium of Γ.*
II. *Reciprocally, if θ^* is a pure strategy Nash equilibrium of the game Γ, then all the players propose the same prices p^* and $((f_i(\theta^*) + e_i)_{i \in N}, p^*)$ is a Walrasian Expectations equilibrium of $\mathcal{E}$.*

Moreover, pure strategy Nash equilibria are strong.

Proof **I.** Let (x^*, p^*) be a Walrasian Expectations equilibrium of $\mathcal{E}$ and define $\theta_i^* = (x_i^* - e_i, p^*)$ for every i. By definition, it follows that $f_i(\theta^*) + e_i = x_i^* = d_i(p^*) := argmax_{x \in B_i(p^*)} U_i(x)$.
Let $\theta_i = (z_i, p)$. If $p \neq p^*$, then $f_i(\theta^*_{-i}, \theta_i) = 0$ and $\pi_i(\theta^*_{-i}, \theta_i) = U_i(e_i) \leq \pi_i(\theta^*) = U_i(d_i(p^*))$. If $p = p^*$ then $\pi_i(\theta^*_{-i}, \theta_i) = U_i(z_i + e_i) \leq \pi_i(\theta^*) = U_i(d_i(p^*))$.
Therefore, given the strategy profile θ^*, no agent i can improve her payoffs by choosing a strategy different from θ_i^*, while the other players choose θ^*_{-i}. Hence, θ^* is a Nash equilibrium of Γ.
Moreover, suppose that θ^* is not a strong Nash equilibrium. Then, there exists a coalition S and a strategy profile θ such that $\pi_i\left(\theta^*_{-S}, \theta_S\right) \geq \pi_i(\theta^*)$ with strict inequality holding for at least one $j \in S$. Then, $p_j \neq p^*$ and $\#A_j(\theta^*_{-S}, \theta_S) > 1$. Thus, the coalition $A_j(\theta^*_{-S}, \theta_S)$ privately blocks the allocation x^*, which is a contradiction with the fact that x^* belongs to the private core of $\mathcal{E}$.[3]

II. Let θ^* be a Nash equilibrium of Γ. To see that $(f_i(\theta^*) + e_i, p^*)$ is a Walrasian Expectations equilibrium of $\mathcal{E}$ let us show that $f_i(\theta^*) + e_i \in \mathbb{P}_i$ for all $i \in N$. Otherwise, there exists an agent i such that $\pi_i(\theta^*) = \tilde{U}_i(f_i(\theta^*) + e_i) = 0$. Consider that player i chooses $\theta_i = (0, p)$ with $p \neq p_j^*$ for any $j \neq i$. Note that in this case $A_i(\theta^*_{-i}, \theta_i) = \{i\}$ and $f_i(\theta^*_{-i}, \theta_i) = 0$. Then, by monotonicity of the preferences, it follows that $\tilde{U}_i(e_i) = U_i(e_i) > 0$, that is, $\pi_i(\theta^*_{-i}, \theta_i) > \pi_i(\theta^*) = 0$, which is a contradiction. Therefore, if θ^* is a Nash equilibrium of Γ, then $f_i(\theta^*) + e_i \in \mathbb{P}_i$ for all $i \in N$ and $\pi_i(\theta^*) = U_i(f_i(\theta^*) + e_i)$.

[3] The private core is the set of allocations that are not privately blocked. An allocation is privately blocked by a coalition S if there exists another feasible allocation for S such that every member becomes better off [see Yannelis (1991)].

In order to obtain the result, and following the proof stated in (Schmeidler, 1980, pp. 1588–1589) it is not difficult to guarantee, firstly, that for any different agents $i, j \in N$, $U_i(f_i(\theta^*) + e_i) \geq U_i(d_i(p_j^*))$. Secondly, that if θ^* is a Nash equilibrium, then all players propose the same prices and if $\#A_i(\theta^*) \geq 2$ then $\#A_i(\theta^*) = N$. Finally, we confirm that there exists an agent i such that $\#A_i(\theta^*) \geq 2$ and, therefore, we conclude that under any Nash equilibrium all players propose the same prices.

Finally, given a Nash equilibrium, θ^*, it follows from the items above that $\theta^\diamond = ((f_i(\theta^*), p^*)_{i\in N})$ is a strong Nash equilibrium of the game Γ. Indeed, as (i) both equilibria θ^* and $\theta^\diamond$ implement the same consumption allocations, $((f_i(\theta^*) + e_i)_{i\in N})$, and as (ii) for each $i \in N$, $f_i(\theta^*) + e_i = d_i(p^*)$, the same arguments used in the first item ensure that θ^* is also a strong Nash equilibrium. □

Note that there is an indetermination on the Nash equilibrium allocation that implements a given Walrasian Expectation equilibrium (x^*, p^*). In fact, if $\theta^* = ((z_i^*, p_i^*)_{i\in N})$ is a Nash equilibrium that implements (x^*, p^*) (in terms of the second item in Theorem), then:

- All agents propose an identical price, i.e., there is p^* such that $p^* = p_j^*$, for each $j \in N$;
- The vector $\theta_\alpha^* = ((z_i^* + \alpha, p^*)_{i\in N})$, where α satisfies $\alpha \cdot p^* = 0$, is also a Nash equilibrium that implements (x^*, p^*).

Thus, associated to each Walrasian Expectations equilibrium there is a continuum of Nash equilibria implementing them. However, only final outcomes are observed, as the information about the original profiles proposed by players is lost once the strategic outcome functions are applied. Therefore, this indetermination does not produce any real effects on the market mechanism running.

Finally, we note that Daher et al. (2006) have recently reinterpreted the traditional model of differential information: today, each agent $i \in N$ has a complete information structure about the set of possible states of nature, but she is not able to discern, tomorrow, among those states of nature that are contained in a same event $S \in P_i$. In this context, (1) agents decide to choose informational-compatible allocations, as they do not have the possibility of distinguishing those states of nature that are in a same event $S \in P_i$, and (2) prices of state-dependent contracts, that are observed today, do not reveal any additional information, as agents perfectly know the set of possible future contingencies.

With this interpretation of agents' informational structure it is possible to avoid the delegation rule in our market game. In fact, as players know the possible states of nature, they have the necessary information to understand the strategic profiles chosen by the other players.

4 Some Counterexamples

In this section we firstly present an example which enables us to show that if informational feasibility is required for the quantities proposed, i.e., agents are those who play the game, then Nash equilibria do not coincide with Walrasian Expectations equilibria of the economy.

Example 1 Consider a differential information economy with three types of agents and two consumers of each type. There are three states of nature $\{a, b, c\}$ and one commodity in each state. All agents have the same utility function $U(x, y, z) = xyz$, and the three types are characterized by the following private information and initial endowments,

$$\begin{aligned} P_1 &= \{\{a, b\}, \{c\}\}, \quad e_1 = (1, 1, 2), \\ P_2 &= \{\{a\}, \{b, c\}\}, \quad e_2 = (2, 1, 1), \\ P_3 &= \{\{a, c\}, \{b\}\}, \quad e_3 = (1, 2, 1). \end{aligned}$$

The unique Walrasian Expectations equilibrium is given by the price system $p = (1, 1, 1)$ and the equalitarian allocation $x_i = (\frac{4}{3}, \frac{4}{3}, \frac{4}{3})$ which, for all agent i, is informationally feasible, independently of the information structure.

Now, consider the profile θ given by an identical strategy θ_i for each player of type i,

$$\begin{aligned} \theta_1 &= \left(\left(-\frac{1}{2}, -\frac{1}{2}, 2\right), \left(1, 1, \frac{1}{2}\right)\right), \\ \theta_2 &= \left(\left(-1, \frac{1}{4}, \frac{1}{4}\right), (1, 2, 2)\right), \\ \theta_3 &= \left(\left(-\frac{1}{2}, 2, -\frac{1}{2}\right), \left(1, \frac{1}{2}, 1\right)\right). \end{aligned}$$

Note that, in this case, the net bundles and price vectors that each player proposes in her strategy set are measurable with respect to the type's information that she is reproducing.

It is not difficult to see that, when players are restricted to choose prices and bundles in accordance to the information of the agent that they are representing, θ is a Nash equilibrium in which there is no trade, and therefore it does not coincide with the Walrasian Expectations equilibrium of the underlying economy.

In the following two examples, we remark two essential elements in the Schmeidler (1980) contribution that remain valid in the differential information framework.

On one hand, trade mechanism is carried out among players who choose the same prices for all commodities. Thus, agents who announce different price systems do not trade at all, even if prices are equal for some goods. The next example shows that if we consider a mechanism in the game that enables agents to trade the commodity h whenever they announce the same price for it, Walrasian equilibria cannot be supported as Nash equilibria.

Example 2 Consider a pure exchange economy with three agents and two commodities. All the consumers have the same utility function $U(x, y) = xy$ and their initial endowments are $\omega_1 = (1, 2)$, $\omega_2 = (2, 1)$, and $\omega_3 = (1, 1)$. Then, the unique Walrasian equilibrium is given by the price system $(p_x, p_y) = (1, 1)$ and the allocation $(x_1, y_1) = (\frac{3}{2}, \frac{3}{2})$, $(x_2, y_2) = (\frac{3}{2}, \frac{3}{2})$, and $(x_3, y_3) = (1, 1)$. From our main result, it follows that the strategy profile given by $\theta_1 = ((\frac{1}{2}, -\frac{1}{2}), (1, 1))$, $\theta_2 = ((-\frac{1}{2}, \frac{1}{2}), (1, 1))$, and $\theta_3 = ((0, 0), (1, 1))$, is a Nash equilibrium of our market game.

Now, consider that it is enough for the trade mechanism to run that players propose the same price for only one commodity, and not the whole price vector.[4] Then $\theta = (\theta_1, \theta_2, \theta_3)$ is not longer a Nash equilibrium. For instance, player 1 has incentives to deviate and announce the strategy $\bar{\theta}_1 = ((\frac{1}{2}, -1), (1, \frac{1}{2}))$.

Finally, we give an example which shows that in our main result it is necessary to have more than two agents.

Example 3 Consider a pure exchange economy with two agents and two commodities. Both agents have the same utility function, $U(x, y) = xy$, and endowments given by $\omega_1 = (2, 2)$ and $\omega_2 = (2, 1)$. The unique equilibrium for this economy is given by the prices $(p_x, p_y) = (1, \frac{4}{3})$ and the allocations $(x_1, y_1) = (\frac{7}{3}, \frac{7}{4})$ and $(x_2, y_2) = (\frac{5}{3}, \frac{5}{4})$.

If we consider the profile $\theta_1 = ((0, 0), (1, 2))$ and $\theta_2 = ((0, 0), (1, 1))$ then $\pi_r(\theta_1, \theta_2) = w_r$, for each player r. It is not difficult to see that (θ_1, θ_2) is a Nash equilibrium which does not result in a Walrasian equilibrium.

5 An axiomatic approach to the outcome functions

The outcome function used to frame a differential information economy as a strategic market game is the same as that in Schmeidler (1980) paper. In this section, in spite of the intuition of this outcome function, we provide an axiomatic approach that exhibits this function as the unique solution.

Firstly, note that, it is natural to suppose that an arbitrary outcome function H_i for a player i is *anonymous* in the sense that: (1) gives the same treatment to player i as the outcome function H_j gives to j; and, (2) only takes into account the profiles chosen by the players, and not their identity. Moreover, given a profile, the outcome that i receives depends only on the strategies chosen by those players that propose an identical price, because in any other case trade is not possible. Mathematically, given two profiles θ^1, θ^2 such that $\theta^1_i = \theta^2_j$ and $\{\theta^1_h \mid h \in A_i(\theta^1)\} = \{\theta^2_h \mid h \in A_j(\theta^2)\}$, we suppose that $H_i(\theta^1) = H_j(\theta^2)$.

Indeed, we assume that not only function H_i is linear in the net demand chosen by the players, but also that both (a) the outcome of a commodity h that the ith player receives only depends on net demand profiles $(z_{j,h})_{j\in N}$, and (b) the final commodity h outcome that player i obtains only changes with the amounts of $(z_{j,h})_{j\in N}$. So, we have that

$$H_i(\theta) = \alpha(\theta) z_i + \beta(\theta) \sum_{j\in A_i(\theta): j\neq i} z_j,$$

for some real functions $(\alpha(\cdot), \beta(\cdot))$ and for each profile $\theta = (z_j, p_j)_{j\in N}$.

[4] Formally, outcome functions are given by $g_i(\theta) = (g_{i,h}(\theta))_{h\in\{1,2,\ldots,\ell\}}$, where

$$g_{i,h}(\theta) = z_{i,h} - \frac{\sum_{j\in A^h_i(\theta)} z_{j,h}}{\# A^h_i(\theta)},$$

and $A^h_i(\theta) = \{j \in N \mid p_{j,h} = p_{i,h}\}$ denotes the set of players proposing the same price for commodity $h \in \{1, 2, \ldots, \ell\}$.

Thus, requiring that (i) outcomes will be feasible across the families of players that choose the same prices, $\sum_{j \in A_i(\theta)} H_j(\theta) = 0$; and (ii) strategies that are originally physically feasible will not be affected by the outcome function (i.e., if $\sum_{j \in A_i(\theta)} z_j = 0$ then $H_i(\theta) = z_i(\theta)$), it follows that, for each profile θ, $\alpha(\theta) - \beta(\theta) = 1$ and $\alpha(\theta) + \beta(\theta)(\#A_i(\theta) - 1) = 0$.

Therefore, if $\#A_i(\theta) > 1$, $\alpha(\theta) = 1 - \frac{1}{\#A_i(\theta)}$ and $\beta(\theta) = -\frac{1}{\#A_i(\theta)}$. When $\#A_i(\theta) = 1$, equations above imply $\alpha(\theta) = 0$ and $\beta(\theta) = -1$. In any case, we have $H_i(\theta) = f_i(\theta)$, for all profile θ. This shows that the unique outcome function satisfying the conditions above is the one we are using in our market game.

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