Short-Run and Long-Run Welfare Implications of Free Trade
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Short-run and long-run welfare implications of free trade

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Abstract. We consider a two-factor (capital and labour), two-good (consumption and investment goods), one-country, overlapping-generations model. For the case in which the closed economy follows an efficient path we prove that if trade lowers (raises) the relative price of the capital-intensive good, the current old people, who only own capital, lose (gain) from the opening of the economy, while all subsequent generations, whose only endowment is labour, benefit (lose) from it. It is also shown that the country gains from trade, in the sense that the generations made better off by trade can compensate those that lose from the opening of the economy.

Les implications du libre-échange en termes de bien-être à court et à long terme. L'auteur propose un modèle d'économie avec deux types de biens (biens de consommation et biens d'investissement) et deux types de facteurs de production (capital et travail) pour un pays dans lequel il y a chevauchement de générations. Partant d'une économie qui est fermée et suit un sentier efficient, on montre que si le commerce international réduit (augmente) le prix relatif du bien à forte intensité de capital, les vieilles gens de la génération présente, qui possèdent le capital, perdent (gagnent) en conséquence de l'ouverture de l'économie, alors que toutes les générations subséquentes, dont la seule dotation est le travail, gagnent (perdent) à cause de cette ouverture de l'économie. On montre aussi que le pays dans son entier gagne grâce à ce commerce international, en ce sens que les générations qui gagnent grâce au commerce international sont en mesure de compenser celles qui perdent à cause de l'ouverture de l'économie.

I. INTRODUCTION

This paper deals with the short-run and long-run welfare implications of opening an economy to trade. There is a vast literature discussing this subject, most of

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which, however, uses ad hoc saving functions or infinitely lived agents. We believe that dynamic models that derive savings from finitely lived agents' behaviour have more intuitive appeal and perhaps more predictive power than those that fail to do so.

Kareken and Wallace (1977), Fried (1980), and Buiter (1981) pioneered the use of overlapping generations to represent finitely lived agents' saving behaviour in open-economy models. Kareken and Wallace (1977), and Fried (1980) employed a two-good, two-factor model where land and labour are the factors, to analyse the welfare effects of portfolio and commercial openness, respectively. Buiter (1981) used a one-commodity, two-factor, two-country model, where the factors are labour and reproducible capital, to compare welfare under financial autarky and openness.

More recent authors using overlapping generations models include Dornbusch (1985), who investigates the gains from intertemporal trade in an economy with one good and a single asset; Serra (1985) and Galor (1988), who consider the optimality of a free trade regime for a small open economy; and Eaton (1987), who employs a specific-factors model to discuss how factor prices and output levels are affected by changes in the terms of trade and in the endowments of factors in fixed supply.

We also use an overlapping-generations model. A two-factor (capital and labour), two-good (consumption and investment good) small economy is considered. Goods are produced from capital and labour according to constant returns to scale and diminishing marginal productivities. The useful life of investment goods is one period. At the beginning of each period a given number of individuals are born. They live for two periods but work in the first only. They have rational expectations but do not anticipate policy shocks. All markets are perfectly competitive.

In what follows we summarize the main results of this paper. Our analysis considers a small economy, which ensures that the free-trade stationary path is attained in just one period, so that the generation that is old when trade opens is the sole transition generation. Let us assume first that the country, which is in stationary equilibrium, does not overinvest relative to the Golden Rule. We prove that if the domestic relative price of the capital-intensive good is above its international level, then trade lowers the welfare of the sole transition generation but raises the stationary level of utility. If the domestic price is below the international price, then trade has opposite effects; that is, it raises the welfare of the sole transition generation but lowers the stationary level of utility. Furthermore, we prove that individuals made better off by trade can compensate those made worse off by trade.[1]

From the perspective of the first period of free trade, members of the transition generation are capital owners and the only endowment of members of all subsequent generations is labour. Thus if the world price of the capital-intensive good is below (above) the domestic price, then trade hurts (benefits) capital owners and benefits (hurts) workers. The distributional consequences of the Stolper-Samuelson theorem for labourers and capitalists thus extend to our model. In this sense, the above result

1 To the best of our knowledge Dornbusch (1985) is the first to consider intergenerational compensations in this kind of model.

may be seen as the analogue of the consequences of the Stolper-Samuelson theorem in an overlapping-generations model.[2]

We also examine the effects induced by trade when the economy, which is in closed-economy equilibrium, overinvests relative to the Golden Rule. We first consider the case in which the world relative price of the capital-intensive good is above the domestic price. We show that trade raises both the welfare of the transition generation and the stationary level of utility. This non-standard result, namely, that trade benefits both capital-owners and workers, is due to the fact that trade, in addition to creating the standard gains from trade, reduces the inefficiency of the economy.

Next, we consider the case when (1) the economy, which is in stationary equilibrium, overinvests relative to the Golden Rule path, and (2) the world relative price of the capital-intensive good is below the domestic price. We show that trade lowers both the welfare of the transition generation and the stationary level of utility. Consequently, in this case trade is Pareto-inferior to autarky. Though it is well known that trade might reduce welfare when the economy is inefficient, our result has theoretical interest because the inefficiencies considered here, namely, capital overaccumulation and misallocation of intergenerational consumption, are barely discussed in the trade literature.

In a recent paper, Galor (1988) shows that the imposition of a tariff, in a small overlapping-generations economy engaged in free trade, improves steady-state welfare if either (1) tariff revenues are rebated to young individuals and the steady-state free-trade equilibrium is characterized by underinvestment relative to the Golden Rule, or (2) tariff revenues are rebated to old individuals and the steady-state free-trade equilibrium is characterized by overinvestment relative to the Golden Rule. His steady-state comparisons agree with ours. In addition, Galor (1988) holds that in case (2), departure from free trade may result in Pareto improvement. This result should cause no surprise. It is well known that if the economy follows an inefficient path, an active commercial policy may be welfare improving. However, a tariff is not the best policy; in case (2) the optimal policy is an intergenerational transfer.

The rest of the paper is organized as follows: Part II formally describes the basic structure of the model. The closed-economy stationary equilibrium is analysed in part III. The welfare impact of free trade is discussed in part IV.

II. THE MODEL

We study a two-factor, two-good economy. The factors are capital and labour, the commodities are consumption goods and investment goods. Both goods are produced from labour and capital with constant returns to scale. We assume that the useful life of the investment goods is one period and that it is the only store

2 It is quite easy to prove that the formal Stolper-Samuelson theorem extends to our model; that is, if the world relative price of the capital-intensive good is below (above) the domestic price, then trade benefits (hurts) workers and hurts (benefits) capitalists.

of value; that is, investment goods last as long as a person's working life. The analysis, however, is perfectly applicable to any level of capital depreciation.

Labour supply in each period is normalized to equal one. Given the previous set of assumptions, the production side of the economy is summarized in the following equations:

$$i_t = (1 - l_t) \quad g(k_{it}) \tag{1}$$

$$c_t = l_t \quad f(k_{ct}) \tag{2}$$

and

$$l_t k_{ct} + (1 - l_t) k_{it} = k_t, \tag{3}$$

where, for period t, i_t denotes the production of investment goods; c_t, the output of consumption goods; l_t, the share of total labour absorbed by the consumption goods industry; k_{ct}, the capital-labour ratio in the consumption goods industry; k_{it}, the investment goods industry's capital-labour ratio; k_t, the stock of capital in the economy; and f and g are increasing and concave functions. In the remainder of the paper we preclude factor-intensity reversals; that is, one of the two goods is capital intensive at any wage-rental ratio.

Throughout this paper, subscript j denotes the partial derivate with respect to the jth argument of a function. Let the investment good be the numeraire, and let w_t, r_t, and p_t denote the wage, the return on capital and the relative price of the consumption good, respectively. Then, assuming that both goods are produced, profit-maximizing behaviour implies that the factor prices satisfy the following relations:

$$w_t = p_t(f(k_{ct}) - k_{ct} f_1(k_{ct})) = g(k_{it}) - k_{it} g_1(k_{it}) \tag{4}$$

and

$$r_t = p_t f_1(k_{ct}) = g_1(k_{it}). \tag{5}$$

The production side of our model is standard Heckscher-Ohlin-Samuelson. Thus, all the well-known relationships holding in the Heckscher-Ohlin-Samuelson model extend to ours. Because factor-intensity reversals are precluded and assuming that the economy is incompletely specialized in production, functions $r(p)$ and $w(p)$, respectively representing the equilibrium rental on capital and wage for a given price ratio p, are well defined. In what follows, n denotes the supply of investment goods, that is

$$i_t = n(p_t, \ k_t). \tag{6}$$

Having thus introduced the production side of the model, we turn to consumption. The country has a constant population. At the beginning of each period a given number of individuals are born. They live for two periods but work in the first only. Since they work only during the first period of their lives, part of their salary is saved to pay for consumption during the second period. As consumption goods are non-storeable, it follows that savings are invested in physical capital, and consequently an individual born in period t faces the following budget constraints:

$$p_t c_t^y + x_t \leqq w_t \tag{7}$$

$$p_{t+1} c_t^o \leqq x_t r_{t+1}, \tag{8}$$

where c_t^y denotes consumption while young; c_t^o, consumption while old; and x_t, savings.

People's intertemporal preferences are assumed to be the same within and across generations and do not include a bequest motive. Individuals have rational expectations but do not anticipate policy shocks. Utility-maximizing savings are represented by $s(w_t, p_t, p_{t+1}^e / r_{t+1}^e)$, where p_{t+1}^e and r_{t+1}^e denote the expected values of p_{t+1} and r_{t+1}, respectively. It is assumed that function s is differentiable. Throughout the paper we assume normality in consumption, that is $0 \leqq s_1 \leqq 1$. In the absence of a policy shock p_{t+1}^e and r_{t+1}^e are equal to p_{t+1} and r_{t+1}, respectively.

Capital goods to be used in period $t + 1$ must be produced in period t. This, in conjunction with the assumption that the rate of depreciation is equal to one, implies that the dynamic equations of the model are

$$k_{t+1} = n(p_t,\ k_t) \tag{9}$$

$$k_{t+1} = s(w_t,\ p_t,\ p_{t+1}/r_{t+1}). \tag{10}$$

The rational-expectation, single-period equilibrium of the economy is characterized by equations (4), (5), (9), and (10).

III. CLOSED ECONOMY

A stationary equilibrium is characterized by a price p^0 and a capital stock k^0 that satisfy.

$$k^0 = n(p^0,\ k^0) \tag{11}$$

$$k^0 = s(w(p^0),\ p^0,\ p^0/r(p^0)). \tag{12}$$

We introduce functions $\Phi(p)$, representing the solution for k to $n(p, k) = k$, and $S(p)$, defined by $S(p) = s(w(p), p, p/r(p))$. Functions Φ and S might be seen, respectively, as the stationary demand for, and supply of, investment goods. Given these definitions, the stationary equilibrium can also be characterized by the price p^0 that solves the equation $S(p^0) = \Phi(p^0)$.

The existence of equilibrium and the stability of the model in the neighbourhood of the equilibrium is examined in appendix A. In the remainder of the paper we assume that there is one and only one stationary equilibrium. We also assume that the model is stable.

Now we turn to the problem of finding the Golden Rule path. In stationary equilibrium the utility level is $\mu = v(w, p, p/r)$, where μ denotes an individual's utility level and v his or her indirect utility function. Consequently, the effect on the level of utility caused by a marginal price change is given by

$$\frac{d\mu}{dp} = \left(\frac{dw}{dp} - \frac{w}{p} + \frac{S(p)}{r} \frac{dr}{dp} \right) v_1. \tag{13}$$

In appendix B we show that equation (13) can be rewritten as follows:

$$\frac{d\mu}{dp} = \left[\left[\frac{S(p)}{r} - \Phi(p) \right] \frac{g(k_i)}{(k_c - k_i)} + [S(p) - \Phi(p)] \right] \frac{v_1}{p}. \tag{14}$$

In a closed-economy equilibrium $S(p)$ equals $\Phi(p)$. Consequently, the Golden Rule price p^* and the golden stock of capital k^* (where $d\mu/dp = 0$) require $r(p^*) = 1$. If the economy's stock of capital exceeded k^*, the country would be following an inefficient path in that it could, by discarding capital, raise the utility level of all current and future generations.

In this model the population growth rate is zero and the rate of capital depreciation is one. Thus we obtain the standard results: (1) the optimal capital stock is attained when the return to capital equals the sum of the rates of population growth and capital depreciation; and (2) if consumption allocation decisions are individually made, then a necessary and sufficient condition for the optimal intertemporal consumption allocation is, again, a rate of interest equal to the sum of the rates of population growth and capital depreciation.[3]

IV. TRADE

This section examines the welfare effects of commercial openness. Throughout this part we assume that the economy is initially in closed-economy stationary equilibrium. We also assume that in an open-economy régime the country remains incompletely specialized in production, or alternatively we assume that (1) there is international lending and borrowing, and (2) home and the rest of the world share the same technology. Given these assumptions, the free-trade stationary equilibrium is attained in one period.

We discuss first the pattern of trade. Let p^w denote the world relative price of consumption goods. This paper assumes a unique autarky stationary equilibrium, which ensures that if the world relative price of consumption goods, p^w, is lower (higher) than the domestic price, p^0, then in a free-trade regime the country becomes a net exporter (importer) of investment goods.

3 Note that in order to allow for overinvestment relative to the Golden Rule either the rate of capital depreciation or the rate of population growth should be positive.

Results are derived assuming that consumption goods are capital intensive. However, they do not depend on this assumption, as is shown at the end of this section. Let $k_c^w(k_i^w)$ represent the consumption (investment) goods industry's capital-labour ratio. Consider first the case when both $S(p^w)$ and $S(p^0)$ are contained between k_c^w and k_i^w. In this case in an open-economy régime the country remains incompletely specialized in production. In fact, if $p^w < p^0$, it exports $n(p^w, k^0) - S(p^w)$ units of investment goods in the transition period and exports $n(p^w, S(p^w)) - S(p^w)$ units in all subsequent periods. Now if $p^w > p^0$, then the country imports $S(p^w) - n(p^w, k^0)$ units of investment goods in the transition period, and imports $S(p^w) - n(p^w, S(p^w))$ units in all subsequent periods.

Let us examine now the case where both $S(p^0)$ and $S(p^w)$ are greater than k_c^w, which implies that the economy becomes specialized in the production of (capital-intensive) consumption goods.[4] Assume that factor movements are the smallest that equalize domestic and international factor prices. At the beginning of the transition period the home country's old lend $(S(p^0) - k_c^w)$ units of investment goods in the international market. In the transition period and in all future periods the home country's young buy $S(p^w)$ units of investment goods in foreign markets but bring home only k_c^w. It follows that the home imports of investment goods are $(2k_c^w - S(p^0))$ in the transition period and rise to k_c^w units for all future periods. On the other hand, home exports $[S(p^w) + r(p^w)(k^w - S(p^\circ)]$ in consumption goods in the transition period and $[S(p^w) + r(p^w)(k^w - S(p^w)]$ in all subsequent periods.

Let us now consider the welfare implications of free trade. Given that the free-trade stationary equilibrium is attained in one period, the generation that is old when trade opens is the sole transition generation. Therefore we need to assess only the trade-induced changes on the transition generation's welfare and on the stationary level of utility.

It is simple to ascertain the effect of opening the economy on the sole transition generation. If p^0 is higher than p^w, then trade hurts the transition generation, but if p^0 is lower than p^w, the opposite result holds. It is somewhat more complicated to examine the trade-induced changes on the stationary level of utility. Equation (14) can be rewritten as follows

$$\frac{d\mu}{dp} = \left[\left(\frac{1}{r} - 1\right) \frac{S(p)g(k_i)}{(k_c - k_i)} - pm\right] \frac{v_1}{p}, \tag{15}$$

where

$$m = [\Phi(p) - S(p)]\left(1 + \frac{g(k_i)}{(k_c - k_i)}\right)/p. \tag{16}$$

Note that if $p^0 > p^w$, then $m > 0$. Otherwise $m < 0$.[5]

4 For the sake of space the trade patterns for other cases are not examined.

5 From equation A10 it follows that $pm = n(p, S(p)) - S(p)$. Thus if the economy remains incompletely specialized in production, then m represents the per capita imports of consumption goods.

PROPOSITION 1. *If $p^0 > p \geqq p^*$, then $d\mu/dp$ evaluated at p is negative.*

Proof. From $p^0 > p$ it follows that $m > 0$. This, in conjunction with $r(p) \geqq 1$ and equation (15), implies that $d\mu/dp < 0$.

Due to proposition 1, if $p^0 > p^w \geqq p^*$, then trade raises the stationary level of utility and lowers the welfare of the sole transition generation. $p^0 > p^w \geqq p^*$ is not a necessary condition for proving that trade raises the stationary level of utility. We can specify conditions under which $d\mu/dp < 0$ for any p.

CONDITION I. *A price p is said to satisfy condition I if and only if*

$$S(p)\left[1 + \frac{g(k_i)}{r(k_c - k_i)}\right] \leqq \Phi(p)\left[1 + \frac{g(k_i)}{r(k_c - k_i)}\right]. \tag{17}$$

Note that if $r^0 > 1$, then condition I is satisfied in the vicinity of p^0.

PROPOSITION 2. *If $p^0 > p^*$ and p satisfies condition I, then $d\mu/dp$ is negative.*

Proof. Immediate from (14) and condition I.

We have extended the Stolper-Samuelson theorem to this context, at the cost of accepting some minor technical assumptions (namely, that condition I is satisfied by any p in the interval $[p^0, p^w]$). At the time of the trade opening members of the transition generation possess only capital, while the only endowment of members of subsequent generations is current or future labour. Therefore the former can be deemed to be capital owners and the latter workers. We have shown that if (1) the economy is on an efficient path, and (2) trade lowers the relative price of the capital-intensive good, then trade hurts capital owners and benefits workers. If trade raises the relative price of the capital-intensive good, the opposite results hold.

Having thus discussed the case in which the closed economy is on an efficient stationary path, let us now turn to the inefficient case.

PROPOSITION 3. *If $p^0 < p < p^*$, then $d\mu/dp$ evaluated at p is positive.*

Proof. $p^0 < p < p^*$ implies that $r(p) < 1$ and $m < 0$. Recalling equation (15), the proposition follows.

Therefore, if $p^0 < p^W < p^*$, trade raises the welfare of both the transition and all subsequent generations. This unfamiliar result – trade raising the welfare of both workers and capital owners – is due to the fact that, in addition to the standard gains from trade, commercial openness reduces the economy's inefficiency.

Assume now that $p^0 > p^w$. In order to get unambiguous results an additional assumption is required.

PROPOSITION 4. *If $p^0 < p^*$ and p does not satisfy condition* I, *then $d\mu/dp$ evaluated at p is positive.*

Proof. Immediate from (14) and condition I.

If $r^0 < 1$, then condition I is not met by any p in the neighbourhood of p^0. It follows that if p^w is contained in a neighbourhood of p^0 and $p^w < p^0$, then free trade lowers both the welfare of the transition generation and the stationary level of utility. Therefore, free trade is Pareto inferior to autarky in this case. Though it is well known that in the presence of inefficiencies trade could reduce welfare, this last result is of theoretical interest because the inefficiencies encountered in this case, namely, an intertemporal consumption misallocation and capital overaccumulation, are barely discussed in the trade literature.

Consider now the issue of compensating the transition generation. Let τ denote the lump-sum transfer required to restore the welfare of the transition generation to its autarky level, that is,

$$\tau = k^0 \left(\frac{r^0}{p^0} - \frac{r}{p} \right) p. \tag{18}$$

Imagine that a permanent intergenerational transfer equal to τ is instituted. Individuals pay a tax τ while young and receive a subsidy τ while old. Let $T(p, \theta)$ denote the value of an individual's savings in stationary equilibrium when a transfer equal to θ is in force. If the individual's desired working-period consumption exceeds $1 - \theta$, then $T(p, \theta) = 0$; otherwise

$$T(p,\ \theta) = s\left(w - \theta \left(1 - \frac{1}{r} \right),\ p,\ \frac{p}{r} \right) - \frac{\theta}{r} \tag{19}$$

and

$$T_2(p,\ \theta) = -s_1 \left(1 - \frac{1}{r} \right) - \frac{1}{r}. \tag{20}$$

Normality in consumption – that is, $0 \leqq s_1 \leqq 1$ – implies $T_2 \leqq 0$. Therefore, a transfer $\theta > 0$ moves the savings curve downwards; a transfer $\theta < 0$ has the opposite effect.

Assuming that $T(p, \tau) > 0$, the stationary level of utility is given by

$$\mu = v\left(w - \tau \left(1 - \frac{1}{r} \right),\ p,\ \frac{p}{r} \right). \tag{21}$$

Differentiating equation (21) with respect to p results in

$$\frac{d\mu}{dp} = \left[\frac{dw}{dp} - \frac{w}{p} + \frac{T}{r}\frac{dr}{dp} + \left(1 - \frac{1}{r} \right) \left(\frac{dr}{dp} - \frac{r}{p} \right) k^{\circ} \right] v_1 \tag{22}$$

We show in appendix B that (22) becomes

$$\frac{d\mu}{dp} = \left[\left(1 - \frac{1}{r}\right) (k^0 - \Phi) + (T - \Phi) \frac{pf}{gr} \right] \frac{v_1 g}{(k_c - k_i)p} \tag{23}$$

At $p = p^0, \tau = 0$ and $k^0 = S(p) = T(p, \tau) = \Phi(p)$; hence $d\mu/dp$ evaluated at p^0 is equal to zero.

PROPOSITION 5. *Given a permanent intergenerational transfer equal to* τ, *if* $T(p, \tau) > 0$ *and* $p^0 > p > p^*$, *then* $d\mu/dp < 0$.

Proof. If $T(p, \tau) > 0$, then $d\mu/dp$ is given by equation (23). $p^0 > p > p^*$ implies that $\tau > 0, \Phi(p) > k^0, T(p, \tau) < S(p) < \Phi(p)$, and $r(p) > 1$. Consequently $d\mu/dp < 0$.

PROPOSITION 6. *Assume that* $p > p^0 > p^*$ *and that the government establishes a permanent intergenerational transfer equal to* τ. *Then* $d\mu/dp$ *evaluated at* p *is positive.*

Proof. The condition $p > p^0 > p^*$ implies that (1) $r(p) > 1$; (2) $\tau < 0$; (3) $T(p, \tau) > S(p) > \Phi(p)$; and (4) $\Phi(p) < k^0$. From equation (23) it follows that $d\mu/dp > 0$.

Thus we have extended a familiar result in trade theory to our model. We have shown that if the economy is on an efficient stationary path (i.e., $p^0 \geqq p^*$) and the international price ratio is less than, or equal to, the home country's Golden Rule price ratio (i.e., $p^w \geqq p^*$), then the country gains from trade in the sense that trade winners can compensate trade losers. Therefore, we conclude that free trade is superior to autarky.

All conclusions obtained assuming that consumption goods are capital-intensive extend to the case in which consumption goods are labour-intensive. This is done by modifying propositions 1 through 6 to take into account the factor-intensity change. We illustrate it for proposition 1. If consumption goods are labour intensive, then the economy follows a closed economy efficient path; that is, $r^0 > 1$, when $p^0 < p^*$.

PROPOSITION 1′. *If* $p^0 < p \leqq p^*$, *then* $d\mu/dp$ *evaluated at* p *is positive.*

Proof. From $p^0 < p$ it follows that $m < 0$. This, in conjunction with $r > 1$ and equation (15), implies that $d\mu/dp \geqq 0$.

APPENDIX A

Additional assumptions are required to ensure the existence of a stationary equilibrium. We assume that there is a capital stock k' and a commodity price ratio p', which satisfy $g(k') = k'$ and $n(p', k') = k'$ (the existence proof can easily be modified to consider the case in which this condition does not hold). Let us consider first the case in which consumption goods are capital intensive. Then Φ is defined only in the interval $[p', \infty]$, and Φ_1 is negative in the domain of definition. At p' $k' = g(k')$, while $w' = g(k') - k'g_1(k')$, consequently $S(p') < \Phi(p')$. It follows that the sufficient but not necessary condition for the existence of a stationary closed economy equilibrium is

$$\lim_{p \to 0} S(p)/\Phi(p) = 1 + \epsilon \tag{A1}$$

where ϵ is a positive number. In addition, condition (A1) implies that function S intersects function Φ from below at least once; that is, there is at least one steady state characterized by a price p^0 that satisfies $S_1(p^0) > \Phi_1(p^0)$.

When investment goods are capital intensive, function Φ is only defined in the interval $[0, p']$ and Φ_1 is positive on its domain of definition. In this case, a sufficient but not necessary condition for the existence of a stationary closed-economy equilibrium is

$$\lim_{p \to 0} S(p)/\Phi(p) = 1 + \epsilon. \tag{A2}$$

In addition, condition (A2) implies that function S intersects function Φ from above at least once; that is, there is at least one stationary equilibrium such that $S_1(p^0) < \Phi_1(p^0)$.

The stability of the model in the neighbourhood of the stationary equilibrium is analysed next. In what follows we assume that there is one and only one stationary equilibrium. This assumption implies that $S_1(p^0)/\Phi_1(p^0) < 1$. From the definitions of S and Φ it follows that $S_1 = \beta - \alpha^{-1}$ and that $\Phi_1 = n_1/(1 - n_2)$, where $\alpha = [s_3 d(p_{t+1}/r_{t+1})/dp]^{-1}$ and $\beta = s_2 + s_1 dw_t/dp$. Hence the condition $S_1(p^0) > \Phi_1(p^0)$ becomes

$$\beta - \alpha^{-1} > n_1/(1 - n_2). \tag{A3}$$

Consider the stability of the model in the neighbourhood of the stationary equilibrium. Equations (9) and (10) can be linearized around the steady state as follows:

$$\left(s_1 \frac{dw_t}{dp} + s_2 \right) \mathbf{p}_t - s_3 \frac{d}{dp} \left(\frac{p_{t+1}}{r_{t+1}} \right) \mathbf{p}_{t+1} = \mathbf{k}_{t+1} \tag{A4}$$

$$n_1 \mathbf{p}_t + n_2 \mathbf{k}_t = \mathbf{k}_{t+1}, \tag{A5}$$

where $s_1, s_2, s_3, n_1, n_2, dw_t/dp$, and $d(p_{t+1}/r_{t+1})/dp$ assume their stationary values, and $\mathbf{p}_t = p_t - p^0$ and $\mathbf{k}_t = k_t - k^\circ$. These two equations constitute a second-order system of difference equations that can be expressed as

$$\begin{bmatrix} \mathbf{k}_{t+1} \\ \mathbf{p}_{t+1} \end{bmatrix} = \begin{bmatrix} n_2 & n_1 \\ -n_2\beta & (\beta - n_1)\alpha \end{bmatrix} \begin{bmatrix} \mathbf{k}_t \\ \mathbf{p}_t \end{bmatrix} \tag{A6}$$

Function s is linear homogeneous; thus $s_1 w_t + s_2 p_t + s_3(p_{t+1}/r_{t+1}) = 0$; and consequently $\beta = s_1[dw/dp - (w/p)_t] - (s_3/p_t)(p_{t+1}/r_{t+1})$. Finally as $dw/dp = w/p + n_2 k_c/p$, it follows that β is equal to $s_1 n_2 k_c/p - (s_3/p_t)(p_{t+1}/r_{t+1})$.

The characteristic roots of the matrix in (A6), λ_1 and λ_2 satisfy

$$\lambda_1 + \lambda_2 = n_2 + (\beta - n_1)\alpha \tag{A7}$$

$$\lambda_1 \lambda_2 = \alpha\beta n_2 \tag{A8}$$

Therefore, a necessary and sufficient condition for the existence of a unique stable convergence path is

$$|n_2 + (\beta - n_1)\alpha| > |1 + \beta\alpha n_2|. \tag{A9}$$

Let us consider the case in which (i) consumption goods are capital-intensive, and (ii) $s_3 \leqq 0$ – the latter assumption implies a low degree of intertemporal substitution in consumption: for the current young an increase in p_t lowers consumption while old. Then n_1, n_2, and β are negative, and α is positive. These assumptions, in conjunction with equation (A3), imply condition (A9). We have described a set of assumptions that ensure the saddle point stability of the system. Alternative stability conditions do exist; they are not examined here, however, owing to space limitations.

APPENDIX B

In the model $n_2(p, k) = -g(k_i)/(k_c - k_i)$; consequently,

$$n(p,\ k) - n(p,\ \Phi(p)) = [\Phi(p) - k]g(k_i)/(k_c - k_i). \tag{B10}$$

Now in stationary equilibrium,

$$n(p,\ k) = (1 - l)g(k_i). \tag{B11}$$

Also, by definition,

$$n(p,\ \Phi(p)) = \Phi(p). \tag{B12}$$

Thus,

$$(1 - l)g(k_i) - \Phi(p) = [\Phi(p) - k]g(k_i)/(k_c - k_i). \tag{B13}$$

Rearranging terms results in

$$\left[\frac{g}{k_c - k_i} + 1\right]\Phi = \frac{gk_c}{k_c - k_i}. \tag{B14}$$

Now

$$\frac{dr}{dp} - \frac{r}{p} = \frac{g}{(k_c - k_i)p} \tag{B15}$$

$$\frac{dw}{dp} - \frac{w}{p} = -\frac{k_c g}{(k_c - k_i)p}. \tag{B16}$$

From (B14) and (B16) it follows

$$\frac{dw}{dp} - \frac{w}{p} = -\left[\frac{g}{k_c - k_i} + 1\right] \frac{\Phi}{p}. \tag{B17}$$

Replacing (B15) and (B17) in (13) we obtain

$$\frac{d\mu}{dp} = \left[-\left(\frac{g}{k_c - k_i} + 1\right) \frac{\Phi}{p} + \left(\frac{g}{k_c - k_i} + \frac{r}{p}\right) \frac{S}{r}\right] v_1. \tag{B18}$$

From (B18) it follows (14). Now replacing (B15) and (B17) in (22) we obtain

$$\frac{d\mu}{dp} = \left[-\left(\frac{g}{k_c - k_i} + 1\right) \frac{\Phi}{p} + \left(\frac{g}{k_c - k_i} + \frac{r}{p}\right) \frac{T}{r} + \left(1 - \frac{1}{r}\right) \frac{gk^0}{(k_c - k_i)p}\right] v_1. \tag{B19}$$

Rearranging terms results in

$$\frac{d\mu}{dp} = \left[\left(1 - \frac{1}{r}\right)(k^0 - \Phi) + (T - \Phi)\left(\frac{1}{r} + \frac{k_c - k_i}{g}\right)\right] \frac{gv_1}{(k_c - k_i)p}. \tag{B20}$$

From (B20) it follows (23).

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