Multi-Lagrangians, hereditary operators and Lax pairs for the Korteweg - De Vries positive and negative hierarchies

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We present an approach to the construction of action principles (the inverse problem of the calculus of variations), for first order (in time derivatives) differential equations, and generalize it to field theory in order to construct systematically, for integrable equations which are based on the existence of a Nijenhuis (or hereditary) operator, a (multi-Lagrangian) ladder of action principles which is complementary to the well-known multi-Hamiltonian formulation. We work out results for the Korteweg - de Vries (KdV) equation, which is a member of the positive hierarchy related to a hereditary operator. Three negative hierarchies of (negative) evolution equations are defined naturally from the hereditary operator as well, in a concise way, suitable for field theory. The Euler - Lagrange equations arising from the action principles are equivalent to deformations of the original evolution equation, and the deformations are obtained explicitly in terms of the positive and negative evolu