

Spike-layered Solutions of Singularly Perturbed Elliptic Problems in a Degenerate Setting

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In this paper we consider a class of nonlinear singularly perturbed elliptic problems of the form $-\Delta u + f(u) = 0$ in Ω , for a superlinear, subcritical function f . Here Ω a smooth bounded domain, $\epsilon > 0$ is a small parameter and we are interested in positive solutions to this equation satisfying zero Dirichlet or Neumann boundary conditions on $\partial\Omega$. In the study of the asymptotic behavior of the solutions, when the parameter ϵ approaches zero, a key role is played by a uniqueness-nondegeneracy assumption on the limiting equation. Our main purpose is to show that the least energy solutions can be studied without this delicate technical assumption, thus generalizing the work of Ni & Takagi and Ni & Wei, by enlarging considerably the class of nonlinearities. In fact, not even differentiability of the nonlinearity is needed. On the other hand, the proofs we present here are relatively short and elementary, simplifying the main step of estimating the energy from below.