POSTGRADO ECONOMÍA Y NEGOCIOS UNIVERSIDAD DE CHILE

Congelamiento del Mercado del Crédito y Redes Bancarias

Banking networks in credit freezes

Tesis para optar al grado de

Magister en Análisis Economico

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Santiago, Junio 2017

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by Camilo Poblete

Abstract

We develop a model to study the impact of banking networks on the probability of credit freezes. Under the defined framework, we untangle the question about how bad economic outcomes in one region can affect the willingness of the financial system to provide funding to operating firms. We also we use it to understand some of the policies implemented by the authority during a global crisis. The model presented consists of two independent regions connected by cross holding deposits; banks use these deposits to insure against regional shocks. In each region, we have interdependent operating firms, whose success depends on macroeconomic conditions and on the ability of other firms to find funding. In our economy, two types of credit freezes may arise, an efficient one and an inefficient one. The latter is caused by the inability of financial firms to coordinate with one another. The main result of this thesis, gives us conditions under which banking networks affect the probability of an inefficient credit freeze. We also propose two channels under which this relationship can operate.

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1 Introduction

The global financial crisis had ramifications and implications for many sectors of the economy. Credit market froze, and funds injected into banks by the authority were not completely transmitted to the real economy, therefore diminishing the impact of this policy. The lack of adequate information about the soundness of banks and the financial system was also accompanied by uncertainty about the ability of non-financial firms to produce real returns. Despite the constant efforts and non-conventional strategies followed by monetary authorities around the globe, most of the new liquidity ended up being parked as Central Banks Deposits. While some argue that this is justified by the absence of profitable projects due to the poor level of economic variables, others say that the drought of the loan market can be explained by a lack of supply due to a very conservative approach of the financial sector.

This work analyzes conditions under which inefficient credit market freezes arise and how banking networks could be amplifying them. An inefficient credit market freeze happens, when the economy is on a credit freeze equilibrium, even though there are profitable projects but they cannot be executed because of the lack of supply of credit by the financial firms. Under the model presented, we can understand that some banks rationally back off from the credit market because they fear others will do so. Also, we will determine conditions under which bad economic fundamentals in one region could impact another region, specifically by deteriorating the circumstances under which banks would lend to firms. This kind of failure originates from the lack of coordination between banks and therefore doubts regarding their course of action. The main contribution of this work will be to shed some light on when existing banking connections could have a negative impact in the credit market, and how the monetary authority in the affected region could react to this.

The model presented in this thesis is an extension of the one developed by Bebchuk and Goldstein (2011). Our main innovation is the extension of their model to one with two identical regions, with cross-regional deposits between banks. It has been widely studied, and there is extensive evidence, that banks hold cross regional deposits. As explained by Allen and Gale (1998), banks hold these deposits to insure themselves against regional shocks. The way we include interactions between regions in the model is similar to the one presented by Dasgupta (2004).

The main motivation to include banking connections in a credit freeze model, comes from the fact that ongoing globalization and interconnections in the financial system have presented new challenges to monetary policy. Georgiadis and Mehl (2016) state that globalization increases the relevance of the exchange rate channel, due to foreign currency exposure of external balance sheets. In the same line, Meier (2013) says that financial integration has weakened the interest rate channel (mainly because firms can find funds aboard), but monetary policy is more effective due to the exchange rate channel. Also, Goldberg (2013) mentions that global banking may, under certain circumstances, enhance the financial trilemma.

The relevance of banking networks became evident in 2008 when investment bank Lehman Brothers failed. Panic spread rapidly through the financial system and capital connections between banks generated an amplifying effect that was difficult to foresee. Despite the rapid, innovative and ongoing response from the authorities, the global uncertainty remained and balance sheets of central banks have kept growing, expecting that financial markets would eventually start fueling most of the new liquidity to the real economy.

The study of coordination failures using the Global Games framework, has been widely used to study bank runs and speculative attacks. In recent years, there has been a shift towards applications in other contexts. The model presented by Bebchuck and Goldstein uses global games to study an economy where risk neutral banks must decide whether to lend to firms or invest in a secure asset. Firms' returns roughly depend only on two variables, i.e. macroeconomic fundamentals and the number of firms in the economy that receive funding from the financial sector to execute their projects. The latter represents a source of interfirm complementarities that is based on the premise that a significant fraction of operating firms benefit from the success of other operating firms (Cooper and John, 1988). There are two main channels under which this condition can operate, a direct channel, i.e., selling their goods or services to other operating firms, and an indirect channel, where firms sell their goods or services to individuals, who obtain the money to buy them from other operating firms. When macroeconomics fundamentals are sufficiently good or sufficiently bad, the interfirm complementarities play no significant role. There is however an intermediate range of fundamentals where they can make the difference. In this range it would be socially optimal for all banks to lend to firms. However, this will not always happen, because banks are uncertain about the actions of other banks, and thus act conservatively and back off from lending money to firms. This coordination failure between different financial institutions is what causes that inefficient credit freezes may appear in the economy. One consequence of this, is that a financial institution will extend loans not only based on their assessment of the project, but also based on their expectations, on whether other firms will obtain or not funding.

Another innovation included in our model, is the fact that banks are risk averse instead of risk neutral. The impact of this change is two-fold, as it allows us to justify the inclusion of cross regional deposits, and also that the level of risk-averseness influences the contagion of outcomes between regions.

As we are interested in the possibility of strategic behavior by banks, the timing of the arrival of information will play an important role. To analyze this, we suppose that the information is revealed to one region first and banks there act on this information. Then, banks in the other region receive information about their own region, but can also check the outcome of the aforementioned one. This could lead to strategic behavior, as banks knowing the other region returns, will then decide to invest or not in firms' risky projects. For example, there could be cases where banks knowing that they will receive a low return on their deposits have less incentive to invest in risky projects.

Under this extension, the main results obtained by Goldstein and Bebchuck about credit freezes still hold. We find that a reduction of the risk free rate can help stimulate the lending from banks, and that an injection of capital, via banks or directly to operating firms, can also reduce the probability of a credit freeze. Our main result is that bad outcomes in one region, defined as low returns on cross region deposits, can increase the probability of a credit freeze in the other region.

Here, we can identify two opposing forces interacting. The first one is the income effect of deposits, which operates through the utility of banks and is, the ratio between the utility they get from investing in the risk free asset and the utility they get from lending to firms. When this is closer to 1, then banks will, more probably, back off from lending. The second force operates through risk aversion. If banks have a greater risk aversion when their consumption is low, then a bad realization of the deposit rate increases the probability of a credit freeze. So, in conclusion, to have contagion between regions, the risk aversion effect must be greater than the income effect.

To summarize, our model builds on existing work of credit freezes and expands it to include banking networks through cross-regional deposits. There deposits play a role in the utility of the banks, that are now risk averse, and their willingness to extend or not loans to operating firms. In the next section, we will review in detail some of the literature on global games and the study of coordination failures, then in section 3 we will define the model and the timing under which the game is resolved. Then in section 4, we will propose an equilibrium solution for the model and describe the conditions of its uniqueness. In section 5, we focus on the analysis of the equilibrium and obtain the principal results of this study and finally, in section 6 we will present some conclusions and limitations to this work.

2 Related Literature

This work uses the global games methodology developed first by Carlsson and Van Damme (1993). The mentioned framework allows us to obtain a unique solution to problems with positive complementarities between players and incomplete information. Under perfect information this kind of problems usually presents multiple equilibriums, but when we move to settings to where this is not the case, we can uniquely characterize the solution. This fact makes the methodology developed by Carlsson and Van Damme useful to study problems where coordination between actors plays a significant role, with some of these problems being: Bank runs, currency attacks or even social uprising. Specifically, this piece of work is built on the model of Self-Fulfilling Credit Freezes, by Bebchuck and Goldstein (2011). In their work, they use the framework developed by Morris and Shin (2004) and apply it to study when banks could back off from lending to firms, causing a credit freeze. Their model distinguishes two types of credit freezes, an efficient one, which corresponds to a freeze justified by a bad realization of economic fundamentals, and also a type of inefficient credit freeze, which is only justified by the lack of coordination between participants. This kind of equilibrium is self-fulfilling, as banks decide against lending to non-financial firms due to the fear that other financial institutions will also withdraw.

One of the main conclusions obtained from the work of Bebchuck and Goldstein, is that losses of capital by the banks will increase the probability of a credit freeze in the economy. The monetary authorities can try to fight this, and one way to do it, is by the reduction of the risk free rate. Other alternative that they evaluate, is the injection of capital. This can be done injecting capital to banks so they can lend it to firms or by direct participation in the credit market by the government. They conclude that, due to the expertise of financial institutions in screening good projects, the best alternative is to inject the capital through them. Finally, they conclude that despite the measures adopted by the authorities, there still could be inefficient credit freeze outcomes, i.e. there still exist a range of the economic fundamentals where the banks will not lend to firms despite that they will be better off if all of them did.

Other papers relevant to this work include Morris and Shin (2004). They develop the framework that is later adapted by Bebchuck and Goldstein. Their main results, in a game that moves away from common knowledge, is that after establishing conditions regarding the precision of the private information agents receive, the game has a unique equilibrium. With this equilibrium, they can analyze the role of public information in a crisis and how information transparency not always is beneficial. Finally, they amend the model by introducing defaultable debt and conclude that by neglecting the coordination effect among debt holders, the creditor will not properly determine the value at risk. This paper provides many of the key elements included in this thesis and in Bebchuck and Goldstein's model.

Bank runs and deposit contracts were first studied by Diamond and Dybvig (1983). They introduced demand deposit contracts, a kind of contract that lets depositors withdraw their money at any time. With this type of contract, banks improve the sharing of liquidity risk among depositors, but are also prone to the possibility of bank runs. The presented full information problem is associated with multiple equilibriums. Goldstein and Pauzner (2005) built on this model by moving away from the common knowledge and into the global games methodology. With this framework, they characterize the ex-ante possibility of a panic-based bank run.

Another important paper on this topic is Allan and Gale (1998). They introduce a model where banks are interconnected through demand deposit contracts. Their agents have different liquidity preferences and each region in the model could be subjected to independent liquidity shocks. Their banks use cross regional deposits to insure against regional shocks. The authors conclude that in their model when banks have deposits in all 4 regions of the economy, shocks can be absorbed and no distortions are caused. By having these deposits, banks can improve general wealth and therefore, it is efficient to hold them. The situation changes when banks are only partially connected between them, as in this case perturbations in the demand for liquidity in one region can spread from one to another and eventually spread bank failures through the system.

The model developed by Dasgupta (2004) introduces a case where contagion can arise even in the context of complete networks, and even when liquidity shocks are not that large. This paper studies capital connections between banks in two non-overlapping regions, where there is no global liquidity uncertainty, but regional shocks exist. Agents are offered demand deposit contracts to deposit their initial endowment in banks. The uncertainty about returns of deposits can cause patient agents to prefer early withdrawals and run on banks. All this happens when information about economic fundamentals is not good. In his model, the timing in which information is revealed to the players plays a significant role in the behavior of the agents. This is because contagion can flow only from the region that experiences high demand of liquidity to the other region, and will flow only in the case that agents in that region act first. Their main conclusion is that, when probability of bank failure is low, cross regional deposits should be maximized.

The study of coordination failures under the global games methodology, as stated by Morris and Shin (2000), enables us to obtain a unique equilibrium in games usually associated with a multiple equilibrium. In this type of setting, players choose after observing a private signal of the fundamental variable and forming their best belief about other players' actions. The fact that agents, in this framework, cannot coordinate, gives space for the existence of an inefficient equilibrium, where players decide against acting, even when acting would be the better choice. Modeling information in this way is considered more realistic than games assuming perfect information and gives us a framework to analyze what happens when private information becomes more and more precise. The latter can help to explain some reactions that market participants have when a new piece of information is revealed.

Finally, another relevant extension of the Global Games literature is the Dynamic Global Games methodology, as the game is played in different stages and ends when the action is successful. In each of this stages, agents receive private information and decide to act accordingly. Angeletos, Hellwig and Pavan (2007) develop a model where players in each iteration decide to attack or not a current regime. If the number of attacking players at a time exceeds the threshold, then the regime is overthrown and players get a payoff. Mathevet and Steiner (2013) present an invariant result for dynamic global games that lets us characterize the solutions. These results are applicable to different models such as investment or currency attacks.

3 The Model

We have an economy with two non-overlapping regions ($\{A, B\}$) and three periods of time ($\{0, 1, 2\}$). In each region, we find a continuum of $[0, K]$ identical weakly risk-averse banks with utility function $u(\cdot)(u'(\cdot) > 0, u''(\cdot) \leq 0)$. In order to ease the definitions of the model we will refer to i as the region where banks are located and $-i$ to the other region. Banks only care about the amount they consume at $T = 2$ (i.e. $u(\cdot) = u(C_2)$) and receive an initial endowment of 1 unit at $T = 0$. This endowment can be understood as a capital injection into the banks or as deposits by agents who will be entitled to them at a time that is beyond the scope of the model.

At T = 1, banks can invest their capital only in their own region. Each region presents two possibilities of investment. One is a risk-free asset, this could be thought as a central bank deposit, that returns $(1 + r_i)$. The other possibility is to lend capital to firms (non-financial).

In this economy, firms in each region have access to investment projects but do not have the capital to execute them, so they must rely on external sources of funding. The firms in each region thus must rely on bank lending to execute their projects. Each project cannot be funded by more than one bank and costs exactly 1 unit of capital. The projects of firms in region i pay a return, to banks, of $(1 + R_i)$ if they are successful (i.e. firms obtain a private return greater than $1 + R_i$). The success of the projects depends on the state of macroeconomic fundamentals (summarized by the random variable θ_i) and the premise that firms benefit from the success of other operating firms. The latter condition is formalized as the proportion of firms that receive funding from banks and can be understood by the fact that success of firms present an interdependence between each other (this conclusion can be related to works such as Coordinating Coordination Failures in Keynesian Models by Copper and John (1988)). Interdependence between firms can operate at different levels. We can think of one firm selling their output to other operating firms, or in order to be successful one firm needs essential input provided by other firms in the economy. Firms that are completely independent of others, and sell their output directly to individuals will still benefit from others firms success, as when there are more successful firms this will mean there is more wealth on individuals. In this model, to have a successful economy we need coordination at operational firms level and a financial sector that provides funding to them.

The condition for projects to be successful or not is given by $an_iK + \theta_i \geq b$, where n_i corresponds to the proportion of banks in region i that decide to lend to firms (determined endogenously in the model, $n_i \in [0, 1]$) and K is the amount of capital that is available in every region. The parameter a measures the importance of inter-firm complementarities to projects' profitability and b is the threshold that must be breached for projects to become profitable, both parameters are fixed and exogenously determined. Projects will always be successful under good realizations of the macroeconomic variable (i.e. $\theta_i \geq b$). If the realization is bad (i.e. $\theta_i \leq b - aK$) then projects will always fail. Inter-firms' complementarities thus play a role only when θ_i is between these two values.

Specifically, the returns that banks can get on their investments in firms is summarized by the following functional form:

$$\begin{cases} 1 + R_i & , if \quad an_iK + \theta_i \geq b \\ \rho_i & , if \quad an_iK + \theta_i < b \end{cases}$$

Here, ρ_i can correspond to the liquidation value of the project, or to the value of collateral seize, in case projects are not successful.

At time 0 and after receiving their initial endowments, each bank in region A deposits d_A in banks of region B, conversely banks in region B deposit d_B in banks of region A (with d_A,d_B<1). The quantity d_A is equal among all banks in region A and d_B is equal among all banks in region B. This means that the bank j in region i will deposit a fraction d_i of its initial endowment in region $-i$ and will also receive d_{-i} from banks in that region. This deposits settle at time $T = 2$ and they pay a stochastic return that depends on the results of the investments in the region where they were deposited. As a return, each bank will pay a fraction d_{-i} of what they obtain as result of their investments. The implication is that banks will pay $d_{-i}(1 + r_i)$ if they invest in the risk free asset, $d_{-i}(1 + R_i)$ if they fund firms projects and they are successful or $d_{-i}(\rho_i)$ if they fund firms projects and they fail. The payments of the deposits by banks in region $-i$ are centralized at regional level and then equally distributed among banks in region i. Consequently, each bank will receive a $d_i(1+\phi_{-i})$ for each d_i invested, where ϕ_{-i} will only depend on the results of the investments of region

$-i$ ($\rho_{-i} - 1 \leq \phi_{-i} \leq R_{-i}$). For simplicity in this model we will assume that the amount deposited by each region is equal, this means $d_A = d_B = d$. We will also suppose that banks cannot hide the results of their investments and interbank payments are completed using any resource available.

We assume that the macroeconomic fundamentals of each region, θ_i, are not publicly known until after banks make their investment decisions. However, the distribution of θ_i is public knowledge, which is an independent normal distribution with mean y and variance σ_θ, let denote $\tau_\theta = \frac{1}{\sigma_\theta^2}$ the precision of the distribution. At T=1, and before making their investment decisions, banks receive a private signal about the macroeconomic fundamentals in their own region. Bank j in region i receives a signal $x_{ij} = \theta_i + \varepsilon_{ij}$, where the ε_{ij} is independent and identically distributed normal with mean 0 and standard derivation σ_ε, again let denote $\tau_\theta = \frac{1}{\sigma_\varepsilon^2}$ the precision of the distribution. Each bank can only see its own private signal x_{ij} but the distribution of ε_{ij} is common knowledge.

To help illustrate the model, if a bank decided to invest in the risk free asset, its balance sheet will look like this:

Assets	Liabilities
Central Bank Deposits = 1	
Deposits in region $i = d$	Deposits from region $i = d$
	Capital = 1

Table 1: Sample Balance Sheet 1

In case that banks decided to lend funds to firms, the balance sheet will look like this:

Assets	Liabilities
Loan to operating firms = 1	
Deposits in region $i = d$	Deposits from region $i = d$
	Long-Term Liabilities = 1

Table 2: Sample Balance Sheet 2

3.1 Timing

At time 0 banks are created in each region and are given the endowment of 1, also cross-regional deposits are interchanged. At time 1 state variables are set, then nature selects (with equal probability) which region receives its private information first. Without loss of generality we will assume region A receives information first and that B receives it later. Then, bank j in region A receives its own private signal x_{Aj} and decides its investments. After every bank in region A has made its investments information in this region is revealed, which makes the realization of θ_A and the proportion of banks that decided to lend, (n_A) public information. With this, all participants in the economy can calculate ϕ_A. Then, banks in region B receive their private signal and make their investment decisions. Finally, at time 2, banks in both regions receive payments from their investments and then pay a fraction d_{-i} of it as return for deposits they received from the other region. The amount paid for the deposits is collected at regional level and equally distributed among banks of the region.

Then timing of the game can be summarized in:

- At time 0:
  - Banks in each region receive their endowments
  - Cross regional deposits are made, banks in region A deposit on region B and banks on region B deposit on region A
- At time 1:
  - The macroeconomic fundamentals are determined and nature selects, with equal probability, which region receives information first (without loss of generality we will assume region A receives information first)
  - Bank j in region A receives a private signal (x_{Aj}) about the fundamentals of the economy in its region. The signal is $x_{Aj} = \theta_A + \varepsilon_{Aj}$ with ε_{Aj} independently distributed normal with mean 0 and standard derivation σ_ε
  - Each bank in region A, considering its private information (x_{Aj}) , decides to lend or not to, to firms
  - After every bank in region A makes its investments, it becomes publicly known if firms' projects in region A were successful or not. Also the realization of θ_A and the

value of n_A becomes public information, then ϕ_A can be determined using public information

  - Banks j in region B receive a private signal (x_{Bj}) about the fundamentals of the economy in its region. The signal is $x_{Bj} = \theta_B + \varepsilon_{Bj}$ with ε_{Bj} independently distributed normal with mean 0 and standard derivation σ_ε.
  - Each bank in region B, considering all the public (ϕ_A) and private information (x_{Bj}), decide to lend or not to, to firms

- At time 2:
  - Banks in both regions receive the returns on their investments
  - Each bank in region i pays a fraction d_{-i} as return of the deposits from region i
  - The amounts paid by each bank to the other region are collected and equally divided among all banks of the region

3.2 Model Summary

- Two identical regions $i = \{A, B\}$
- Three period economy $\{0, 1, 2\}$
- Each region has a continuum of banks $[0, K]$ with an endowment of 1 at $T = 0$
- Banks are weakly risk-averse and consume only in $T = 2$ ($u(\cdot) = u(C_2)$, with $u'(\cdot) > 0$ and $u''(\cdot) \leq 0$)
- Banks can invest in a risk free asset with return $(1 + r_i)$
- Banks can also fund firms' projects, the return of this kind of investment is given by:

$$g_i(\theta_i) = \begin{cases} 1 + R_i & , if \quad an_iK + \theta_i \geq b \\ \rho_i & , if \quad an_iK + \theta_i < b \end{cases}$$

  We assume that $u(\rho_i) < u(1+r_i) \leq \mathbb{E}_{\theta_i}[u(g(\theta_i))], \forall i$, i.e. the risk free asset pays a higher return than liquidation value of the projects and projects pay a higher expected return than the risk free asset. Under these conditions, banks have incentives to fund firms' projects.

- At time 0, Banks in each region exchange deposits.
  - Banks in each region deposit d_i in the other region
  - At time $T = 2$ and after receiving the return on their investments, banks pay a fraction d_{-i} of their wealth as return for the deposits of region $-i$ (i.e. $d_{-i}(1 + r_i)$ if they invest in the risk free asset, $d_{-i}(1 + R_i)$ if they lend to firms and projects are successful, or $d_{-i}\rho_i$ if they lend and projects fail)
  - After this all banks in region i receive an equal return on their deposits on region $-i$, i.e. $d_i(1 + \phi_{-i})$, with $\rho_i - 1 < \phi_i < R_i$ for all i
- Definitions:
  - d_i: corresponds to the amount each bank in region i deposits in region i. It is defined exogenously and is identical for all banks in the region
  - ϕ_{-i}: corresponds to the return of deposits by banks of region i in region $-i$
  - r_i: corresponds to the risk free rate in region i
  - R_i: corresponds to the return of lending to fund firm projects conditional on being successful
  - ρ_i: corresponds to the liquidation value of firm projects conditional on being unsuccessful
  - θ_i: corresponds to a variable that determines the macroeconomic fundamentals in region i. It is not publicly known until after banks in the region make their investment decisions
  - n_i: corresponds to the proportion of banks in region i that decide to lend to firms, is endogenously determined in the model
  - a: corresponds to a fixed parameter that measures the importance of inter-firm complementarities to projects profitability
  - b: corresponds to a fixed parameter that determines the threshold that projects must be breached to become profitable
- Assumptions:

- No exchange rate is considered between regions or the exchange rate is considered invariant and equal to 1
- Banks can either invest in the risk free asset or fund firms, they cannot invest in a combination of both
- Banks follow a trigger strategy where they will only fund firms if the fundamentals are above a given threshold (conversely they will invest only if they private signal is above another threshold)
- Banks cannot hide the profits of their investments
- Interbank liabilities are paid on full value; banks must pay even if their counterpart does not
- For simplicity we will assume that $d_A = d_B = d$. Under this assumption, the total capital in each region will still be K (each bank will have $1 + d_{-i} - d_i = 1$ as $d_i = d_{-i}$)

4 Equilibrium Analysis

To study the equilibrium of this game, we will divide the task in three parts. First, we will start assuming the game presents a trigger equilibrium, where banks lend to operating firms only if their estimate of the underlying fundamentals is higher than a given threshold, and back off from the credit market if their estimate falls below this threshold. Using this assumption, we will begin by writing the payoff function of the game and the ex-ante expected utility for each region. Then, we will find the equilibrium for the game and finally, we will determine the conditions for its uniqueness.

4.1 Payoffs

Let us denote $\{\theta_i^*(\phi_{-i}), x_i^*(\phi_{-i})\}$ as the threshold strategy for region i, where $\theta_i^*(\phi_{-i})$ corresponds to the minimum fundamental level for projects to be successful, $x_i^*(\phi_{-i})$ corresponds to the minimum private signal a bank has to receive to lend to firms and ϕ_{-i} is the amount paid by region $-i$ for its deposits (this quantity is unknown to the region that plays first and known to the region that plays second). Then, the utility function for the bank j in region i

that receives a signal x_{ij} will be:

$$u(x_{xj},\phi_{-i}) = \begin{cases} u(1+r_i+d(1+\phi_{-i})-d(1+r_i)) & ,with \quad \mathbb{P}(x_{ij} < x_i^*(\phi_{-i})) \\ u(1+R_i+d(1+\phi_{-i})-d(1+R_i)) & ,with \quad \mathbb{P}(x_{ij} \geq x_i^*(\phi_{-i}), \theta_i \geq \theta_i^*(\phi_{-i})) \\ u(\rho_i+d(1+\phi_{-i})-d(\rho_i)) & ,with \quad \mathbb{P}(x_{ij} \geq x_i^*(\phi_{-i}), \theta_i < \theta_i^*(\phi_{-i})) \end{cases}$$

$$\Rightarrow u(x_{xj},\phi_{-i}) = \begin{cases} u(1+r_i+d(\phi_{-i}-r_i)) & ,with \quad \mathbb{P}(x_{ij} < x_i^*(\phi_{-i})) \\ u(1+R_i+d(\phi_{-i}-R_i)) & ,with \quad \mathbb{P}(x_{ij} \geq x_i^*(\phi_{-i}), \theta_i \geq \theta_i^*(\phi_{-i})) \\ u(\rho_i+d(1+\phi_{-i}-\rho_i)) & ,with \quad \mathbb{P}(x_{ij} \geq x_i^*(\phi_{-i}), \theta_i < \theta_i^*(\phi_{-i})) \end{cases}$$

Here $\mathbb{P}(\Theta)$ corresponds to the probability measure of the event Θ.

To ease notation, let us define:

$$u_{r_i}(\phi_{-i}) = u(1+r_i+d(\phi_{-i}-r_i))$$
$$u_{R_i}(\phi_{-i}) = u(1+R_i+d(\phi_{-i}-R_i))$$
$$u_{\rho_i}(\phi_{-i}) = u(\rho_i+d(1+\phi_{-i}-\rho_i))$$

We can write,

$$\Rightarrow u(x_{xj},\phi_{-i}) = \begin{cases} u_{r_i}(\phi_{-i}) & ,with \quad \mathbb{P}(x_{ij} < x_i^*(\phi_{-i})) \\ u_{R_i}(\phi_{-i}) & ,with \quad \mathbb{P}(x_{ij} \geq x_i^*(\phi_{-i}), \theta_i \geq \theta_i^*(\phi_{-i})) \\ u_{\rho_i}(\phi_{-i}) & ,with \quad \mathbb{P}(x_{ij} \geq x_i^*(\phi_{-i}), \theta_i < \theta_i^*(\phi_{-i})) \end{cases}$$

Then, we can write the ex-ante expected utility of a bank j in region i as:

$$\begin{aligned} \mathbb{E}[u_{ij}(x_{ij})] = {} & u(1+r_i+d(\phi_{-i}-r_i)\mathbb{P}(x_{ij} < x_i^*(\phi_{-i})) \\ & + u(1+R_i+d(\phi_{-i}-R_i)\mathbb{P}(x_{ij} \geq x_i^*(\phi_{-i}), \theta_i \geq \theta_i^*(\phi_{-i})) \\ & + u(\rho_i+d(1+\phi_{-i}-\rho_i)\mathbb{P}(x_{ij} \geq x_i^*(\phi_{-i}), \theta_i < \theta_i^*(\phi_{-i})) \end{aligned} \tag{1}$$

The main take up is that the expected utility function is well defined and continuous for every signal x_{ij}, and for every pair of thresholds $x^*_{ij}(\phi_{-i}), \theta^*_{ij}(\phi_{-i})$, if $u(\cdot)$ is well defined.

4.2 Triggers

In this section, we will continue to assume that region A acts first. As the scope of this work is to study the effects of the region that acts first on the region that acts second, so we will only find the thresholds for region B.

The first equation to find $\{\theta^*_B(\phi_A), x^*_B(\phi_A)\}$, is given by the fact that when bank j receives a signal $x_{Bj} = x^*_B(\phi_A)$, it must be indifferent to invest in the risk-free asset and earn $(1+r_B)$ or lend to firms. Then,

$$u\Big(1 + r_B + d(\phi_A - r_B)\Big) = u\Big(1 + R_B + d(\phi_A - R_B)\Big)\mathbb{P}(\theta_B \geq \theta^*_B(\phi_A)|x_{Bj} = x^*_B(\phi_A)) + u\Big(\rho_B + d(1 + \phi_A - \rho_B)\Big)\mathbb{P}(\theta_B < \theta^*_B(\phi_A)|x_{Bj} = x^*_B(\phi_A))$$

Or using the notation defined before:

$$u_{r_B}(\phi_A) = u_{R_B}(\phi_A)\mathbb{P}(\theta_B \geq \theta^*_B(\phi_A)|x^*_B(\phi_A)) + u_{\rho_B}(\phi_A)\mathbb{P}(\theta_B < \theta^*_B(\phi_A)|x^*_B(\phi_A)) \quad (2)$$

Note that after receiving a signal $x_{Bj} = x^*_B(\phi_A)$, the bank j has a posterior distribution of θ_B normal with mean $\frac{\tau_\theta y + \tau_\varepsilon x^*_b(\phi_A)}{\tau_\theta + \tau_\varepsilon}$ and precision $\tau_\theta + \tau_\varepsilon$ (DeGroot, 1970). So if we name $\theta^P_B(x_{Bj})$, as the distribution of θ_B after receiving a signal x_{Bj}, then

$$\begin{aligned}\mathbb{P}(\theta_B < \theta^*_B(\phi_A)|x_{Bj} = x^*_B(\phi_A)) &= \mathbb{P}(\theta^P_B(x^*_B(\phi_A)) < \theta^*_B(\phi_A)) \\ &= \Phi\Big[\sqrt{\tau_\theta + \tau_\varepsilon}\Big(\theta^*_B(\phi_A) - \frac{\tau_\theta y + \tau_\varepsilon x^*_B(\phi_A)}{\tau_\theta + \tau_\varepsilon}\Big)\Big] \\ &= \Phi\Big[\sqrt{\tau_\theta + \tau_\varepsilon}\Big(\frac{\tau_\theta(\theta^*_B(\phi_A) - y)}{\tau_\theta + \tau_\varepsilon} + \frac{\tau_\varepsilon(\theta^*_B(\phi_A) - x^*_B(\phi_A))}{\tau_\theta + \tau_\varepsilon}\Big)\Big]\end{aligned}$$

Where Φ is the cumulative distribution function for the standard normal.

We also know that:

$$\begin{aligned}\mathbb{P}(\theta_B \geq \theta^*_B(\phi_A)|x_{Bj} = x^*_B(\phi_A)) = 1 - \mathbb{P}(\theta_B < \theta^*_B(\phi_A)|x_{Bj} = x^*_B(\phi_A)) &= 1 - \mathbb{P}(\theta^P_B(x^*_B(\phi_A)) < \theta^*_B(\phi_A)) \\ &= 1 - \Phi\Big[\sqrt{\tau_\theta + \tau_\varepsilon}\Big(\frac{\tau_\theta(\theta^*_B(\phi_A) - y)}{\tau_\theta + \tau_\varepsilon} + \frac{\tau_\varepsilon(\theta^*_B(\phi_A) - x^*_B(\phi_A))}{\tau_\theta + \tau_\varepsilon}\Big)\Big]\end{aligned}$$

Plugging these two expressions into (2), we get:

$$u_{r_B}(\phi_A) = u_{R_B}(\phi_A)(1 - \mathbb{P}(\theta^P_B(x^*_B(\phi_A)) < \theta^*_B(\phi_A))) + u_{\rho_B}(\phi_A)\mathbb{P}(\theta^P_B(x^*_B(\phi_A)) < \theta^*_B(\phi_A))$$

Re-arranging terms

$$\Rightarrow \mathbb{P}(\theta_B^P(x_B^*(\phi_A)) < \theta_B^*(\phi_A))(u_{R_B}(\phi_A) - u_{\rho_B}(\phi_A)) = (u_{R_B}(\phi_A) - u_{r_B}(\phi_A))$$

$$\Rightarrow \mathbb{P}(\theta_B^P(x_B^*(\phi_A)) < \theta_B^*(\phi_A)) = \frac{u_{R_B}(\phi_A) - u_{r_B}(\phi_A)}{u_{R_B}(\phi_A) - u_{\rho_B}(\phi_A)}$$

$$\Rightarrow \phi\Big[\sqrt{\tau_\theta + \tau_\varepsilon}\Big(\frac{\tau_\theta(\theta_B^*(\phi_A) - y)}{\tau_\theta + \tau_\varepsilon} + \frac{\tau_\varepsilon(\theta_B^*(\phi_A) - x_B^*(\phi_A))}{\tau_\theta + \tau_\varepsilon}\Big)\Big] = \frac{u_{R_B}(\phi_A) - u_{r_B}(\phi_A)}{u_{R_B}(\phi_A) - u_{\rho_B}(\phi_A)}$$

$$\Rightarrow \Big(\frac{\tau_\theta(\theta_B^*(\phi_A) - y)}{\tau_\theta + \tau_\varepsilon} + \frac{\tau_\varepsilon(\theta_B^*(\phi_A) - x_B^*(\phi_A))}{\tau_\theta + \tau_\varepsilon}\Big) = \frac{1}{\sqrt{\tau_\theta + \tau_\varepsilon}}\Phi^{-1}\Big[\frac{u_{R_B}(\phi_A) - u_{r_B}(\phi_A)}{u_{R_B}(\phi_A) - u_{\rho_B}(\phi_A)}\Big]$$

After doing some algebra we obtain the following expression:

$$\theta_B^*(\phi_A) - x_B^*(\phi_A) = \frac{\tau_\theta}{\tau_\varepsilon}\Big(- (\theta_B^*(\phi_A) - y) + \frac{\sqrt{\tau_\theta + \tau_\varepsilon}}{\tau_\theta}\Phi^{-1}\Big[\frac{u_{R_B}(\phi_A) - u_{r_B}(\phi_A)}{u_{R_B}(\phi_A) - u_{\rho_B}(\phi_A)}\Big]\Big) \qquad (3)$$

The second equation to determine $\{\theta_B^*(\phi_A), x_B^*(\phi_A)\}$, comes from the fact that when θ_B is exactly $\theta_B^*(\phi_A)$, the conditions to finance firms must hold with equality. Then,

$$an_B K + \theta_B^*(\phi_A) = b$$

Here, n_B corresponds exactly to the proportion of agents that receive a private signal above $x_B^*(\phi_A)$ and this is given by:

$$n_B = \mathbb{P}(x_{Bj} > x_B^*(\phi_A)) = \mathbb{P}(\varepsilon_{Bj} > x_B^*(\phi_A) - \theta_B^*(\phi_A)) = 1 - \mathbb{P}(\varepsilon_{Bj} < x_B^*(\phi_A) - \theta_B^*(\phi_A))$$

$$\Rightarrow n_B = 1 - \Phi[\sqrt{\tau_\varepsilon}(x_B^*(\phi_A) - \theta_B^*(\phi_A))]$$

Using this, in the last equation and re-arranging terms:

$$\theta_B^*(\phi_A) = b - aK + aK\Phi[\sqrt{\tau_\varepsilon}(x_B^*(\phi_A) - \theta_B^*(\phi_A))] \qquad (4)$$

Now combining expressions (3) and (4), we can obtain an expression for $\theta_B^*(\phi_A)$

$$\theta_B^*(\phi_A) = b - aK + aK\Phi\Big[\frac{\tau_\theta}{\sqrt{\tau_\varepsilon}}\Big(\theta_B^*(\phi_A) - y - \frac{\sqrt{\tau_\theta + \tau_\varepsilon}}{\tau_\theta}\Phi^{-1}\Big[\frac{u_{R_B}(\phi_A) - u_{r_B}(\phi_A)}{u_{R_B}(\phi_A) - u_{\rho_B}(\phi_A)}\Big]\Big)\Big] \qquad (5)$$

Let us note that equation (4) can be re-organized considering the fact that $\Phi(\alpha) = -\Phi(1-\alpha)$, then

$$\theta_B^*(\phi_A) = b - aK + aK\Phi\Big[\frac{\tau_\theta}{\sqrt{\tau_\varepsilon}}\Big(\theta_B^*(\phi_A) - y + \frac{\sqrt{\tau_\theta + \tau_\varepsilon}}{\tau_\theta}\Phi^{-1}\Big[\frac{u_{r_B}(\phi_A) - u_{\rho_B}(\phi_A)}{u_{R_B}(\phi_A) - u_{\rho_B}(\phi_A)}\Big]\Big)\Big] \qquad (6)$$

The threshold in the region that acts second, is well defined if for every ϕ_A, as returns of successful projects are not equal to the return of projects in case of failure. This condition holds by definition of the model, so with this result we can see that the threshold depends on the return (ϕ_A) and amount (d) of cross regional deposits.

4.3 Uniqueness of equilibrium

Having defined θ_B, we can analyze the uniqueness of the solution. Let us look at the slope of the function,

$$v(\theta_B) = \theta_B - b + aK - aK\Phi\left[\frac{\tau_\theta}{\sqrt{\tau_\varepsilon}}\left(\theta_B - y + \frac{\sqrt{\tau_\theta + \tau_\varepsilon}}{\tau_\theta}\Phi^{-1}\left[\frac{u_{r_B}(\phi_A) - u_{\rho_B}(\phi_A)}{u_{R_B}(\phi_A) - u_{\rho_B}(\phi_A)}\right]\right)\right]$$

$$\frac{dv(\theta_B)}{d\theta_B} = 1 - aK\frac{\tau_\theta}{\sqrt{\tau_\varepsilon}}\phi(\cdot)$$

If the slope is always positive, then the solution is unique, since $\phi(\cdot) \leq \frac{1}{\sqrt{2\pi}}$. Then, a condition sufficient to guarantee a unique solution is

$$\frac{\tau_\theta}{\sqrt{\tau_\varepsilon}} \leq \frac{\sqrt{2\pi}}{aK}$$

This is the same condition that Bebchuck and Golstein (2011) found in their paper.

Now, let us shift the analysis to the behavior of the conditional payoff function of the banks. After receiving a signal x and when all players use a switching strategy at x^*, their conditional expected payoff is given by

$$U(x, \theta^*) = (1 - \Phi(\sqrt{\tau_\theta + \tau_\sigma}(\theta^* - x)))(1 + R) + \Phi(\sqrt{\tau_\theta + \tau_\sigma}(\theta^* - x))\rho$$

Where θ^* is defined as the equilibrium threshold.
According to Morris and Shin (2004), we can find a unique strategy that survives the iterative deletion of dominating strategies, if the following three properties are satisfied:

- Continuity: U is continuous
- Monotonicity: U is strictly increasing in the first argument (x) and strictly decreasing in the second argument (θ^*)
- Full Range: For any $\theta^* \in \mathbb{R} \cup \{-\infty, \infty\}$, $u(x, \theta^*) \rightarrow \rho$ as $x \rightarrow -\infty$, and $u(x, \theta^*) \rightarrow 1 + R$ as $x \rightarrow \infty$

It is easy to see that the first statement holds, because neither of the functions presents a discontinuity. The second argument holds, as a higher is the private signal (x) means a more probability of a good return $(1+R)$; also, a higher threshold (θ^*) means a less probable favorable outcome, therefore a smaller conditional expected utility. The third condition is also easy to follow from the second condition. Then the unique solution is given by $U(\theta^*) = 1+r$, with $U(\theta^*) = u(\theta^*, \theta^*)$.

With these two results, we can say that the equilibrium for this game is well defined and is unique, if $\frac{\tau_\theta}{\sqrt{\tau_\varepsilon}} \leq \frac{\sqrt{2\pi}}{aK}$.

5 Principal Results

- If banks are risk neutral (i.e. $u(x) = x$) and $\rho_i = 0$, the interconnection between banks has no effect on the thresholds and each region defines them exactly as in the original paper by Bebchuk and Goldstein (2011)

  Our results thus ilustrate the that role of cross regional deposits is significant only in conjunction with risk averseness. These two ideas are the main innovations included in this work. When banks are risk neutral, the extra income they receive from cross regional deposits will play no role in their investment decisions, as the extra utility they obtain from it, is equal at all levels of consumption. When we include risk averseness, this is not the case and depending of their consumption level, banks will change its investment decisions.

  Let us look at the expressions u_{R_i}, u_{r_i} and u_{ρ_i}:

$$u_{r_i}(\phi_{-i}) = u(1 + r_i + d(\phi_{-i} - r_i)) = 1 + r_i + d(\phi_{-i} - r_i)$$

$$u_{R_i}(\phi_{-i}) = u(1 + R_i + d(\phi_{-i} - R_i)) = 1 + R_i + d(\phi_{-i} - R_i)$$

$$u_{\rho_i}(\phi_{-i}) = u(\rho_i + d(1 + \phi_{-i} - \rho_i)) = d(1 + \phi_{-i})$$

  Then, we have

$$u_{R_i}(\phi_{-i}) - u_{r_i}(\phi_{-i}) = 1 + R_i + d(\phi_{-i} - R_i) - (1 + r_i + d(\phi_{-i} - r_i))$$

$$\Rightarrow u_{R_i}(\phi_{-i}) - u_{r_i}(\phi_{-i}) = (1 + R_i) - (1 - r_i) - d(1 + R_i) + d(1 + r_i)$$

$$u_{R_i}(\phi_{-i}) - u_{\rho_i}(\phi_{-i}) = 1 + R_i + d(\phi_{-i} - R_i) - d(1 + \phi_{-i})$$

$$\Rightarrow u_{R_i}(\phi_{-i}) - u_{\rho_i}(\phi_{-i}) = 1 + R_i - d(1 + R_i)$$

Now, let us define $\hat{R_i} = R_i - d(1+R_i)$ as the clean return of lending to firms (the return minus the fraction paid to the other region) and $\hat{r_i} = r_i - d(1+r_i)$ as the clean return of investing in the risk-free asset. Then,

$$\Rightarrow u_{R_i}(\phi_{-i}) - u_{r_i}(\phi_{-i}) = (1+\hat{R_i}) - (1-\hat{r_i})$$
$$\Rightarrow u_{R_i}(\phi_{-i}) - u_{\rho_i}(\phi_{-i}) = 1+\hat{R_i}$$

And replacing this, into the formula for θ_B^*, we get the same expression of Bebchuck and Goldstein (2011)

$$\Rightarrow \theta_B^* = b - aK - aK\Phi\left[\frac{\tau_\theta}{\sqrt{\tau_\varepsilon}}\left(\theta_B^*(\phi_A) - y - \frac{\sqrt{\tau_\theta+\tau_\varepsilon}}{\tau_\theta}\Phi^{-1}\left[1 - \frac{1+\hat{r_B}}{1+\hat{R_B}}\right]\right)\right]$$
$$\Rightarrow \theta_B^* = b - aK - aK\Phi\left[\frac{\tau_\theta}{\sqrt{\tau_\varepsilon}}\left(\theta_B^*(\phi_A) - y + \frac{\sqrt{\tau_\theta+\tau_\varepsilon}}{\tau_\theta}\Phi^{-1}\left[\frac{1+\hat{r_B}}{1+\hat{R_B}}\right]\right)\right]$$

- Let us write $\theta_B^*(\phi_A)$ when the precision of private information goes to infinity (i.e. $\sigma_\varepsilon \to 0$ which implies $\tau_\varepsilon \to \infty$)

  The main motivation to study the problem when the precision of private information goes to infinity, is that both the private and public threshold converge to the same point $(\theta_B^*(\phi_A))$.

It is easy to see that when $\tau_\epsilon \to \infty$, the threshold converges to:

$$\theta_B^*(\phi_A) = b - aK - aK\left[\frac{u_{R_B}(\phi_A) - u_{r_B}(\phi_A)}{u_{R_B}(\phi_A) - u_{\rho_B}(\phi_A)}\right] \tag{7}$$

Or

$$\theta_B^*(\phi_A) = b - aK + aK\left[\frac{u_{r_B}(\phi_A) - u_{\rho_B}(\phi_A)}{u_{R_B}(\phi_A) - u_{\rho_B}(\phi_A)}\right] \tag{8}$$

Another way of obtaining this result, is that when precision goes to infinity, banks are under the belief that the proportion of banks that will lend is uniform. Then, with probability $(1 - \frac{b-\theta^*}{aK})$ lending will be successful and using this we can get:

$$(1+r) = \left(1 - \frac{b - \theta_B*(\phi_A)}{aK}\right)u_{R_B}(\phi_A) + \left(\frac{b-\theta_B*(\phi_A)}{aK}\right)u_{\rho_B}(\phi_A)$$

From which we can also obtain (7).

- We define contagion, as a bad realization of ϕ, in the region that plays first, which increases the probability of a credit freeze in the region that plays second (i.e. $\theta^*_B(\phi_A)$ is increasing in ϕ_A).We now determine the conditions under which contagion from region A to region B occurs (when the precision of the private information goes to infinity)

  We will first analyze the sign of the derivative of $\theta^*_B(\phi_A)$ to ϕ_A. To ease notation we will drop all the B subscripts from R_i,r_i and ρ_i. Also, we will use the second expression for θ^*_B:

$$\theta^*_B(\phi_A) = b - aK + aK\left[\frac{u_{r_B}(\phi_A) - u_{\rho_B}(\phi_A)}{u_{R_B}(\phi_A) - u_{\rho_B}(\phi_A)}\right]$$

So,

$$\begin{aligned}
\frac{d\theta^*_B(\phi_A)}{d\phi_A} &= aK\frac{d}{d\phi_A}\left[\frac{u_r(\phi_A) - u_\rho(\phi_A)}{u_R(\phi_A) - u_\rho(\phi_A)}\right] \\
&= aK\left[\frac{d}{d\phi_A}\Big(\frac{u_r(\phi_A)}{u_R(\phi_A) - u_\rho(\phi_A)}\Big) - \frac{d}{d\phi_A}\Big(\frac{u_\rho(\phi_A)}{u_R(\phi_A) - u_\rho(\phi_A)}\Big)\right] \\
&= aK\left[\Big(\frac{u'_r(\phi_A)(u_R(\phi_A) - u_\rho(\phi_A)) - u_r(\phi_A)u'_R(\phi_A) + u_r(\phi_A)u'_\rho(\phi_A)}{(u_R(\phi_A) - u_\rho(\phi_A))^2}\Big)\right. \\
&\left. - \Big(\frac{u'_\rho(\phi_A)(u_R(\phi_A) - u_\rho(\phi_A)) - u_\rho(\phi_A)u'_R(\phi_A) + u_\rho(\phi_A)u'_\rho(\phi_A)}{(u_R(\phi_A) - u_\rho(\phi_A))^2}\Big)\right] \\
&= aK\left[\Big(\frac{u'_r(\phi_A)u_R(\phi_A) - u'_r(\phi_A)u_\rho(\phi_A) - u_r(\phi_A)u'_R(\phi_A) + u_r(\phi_A)u'_\rho(\phi_A)}{(u_R(\phi_A) - u_\rho(\phi_A))^2}\Big)\right. \\
&\left. - \Big(\frac{u'_\rho(\phi_A)u_R(\phi_A) - u'_\rho(\phi_A)u_\rho(\phi_A) - u_\rho(\phi_A)u'_R(\phi_A) + u_\rho(\phi_A)u'_\rho(\phi_A)}{(u_R(\phi_A) - u_\rho(\phi_A))^2}\Big)\right] \\
&= aK\left[\Big(\frac{u'_r(\phi_A)u_R(\phi_A) - u'_r(\phi_A)u_\rho(\phi_A) - u_r(\phi_A)u'_R(\phi_A) + u_r(\phi_A)u'_\rho(\phi_A)}{(u_R(\phi_A) - u_\rho(\phi_A))^2}\Big)\right. \\
&\left. + \Big(\frac{-u'_\rho(\phi_A)u_R(\phi_A) + u_\rho(\phi_A)u'_R(\phi_A)}{(u_R(\phi_A) - u_\rho(\phi_A))^2}\Big)\right] \\
&= aK\left[\frac{u_R(\phi_A)(u'_r(\phi_A) - u'_\rho(\phi_A)) + u_r(\phi_A)(u'_\rho(\phi_A) - u'_R(\phi_A))}{(u_R(\phi_A) - u_\rho(\phi_A))^2}\right. \\
&\left. + \frac{u_\rho(\phi_A)(u'_R(\phi_A) - u'_r(\phi_A))}{(u_R(\phi_A) - u_\rho(\phi_A))^2}\right]
\end{aligned}$$

We then must show that the numerator is negative. To do this we will add and subtract

$u_r'(\phi_A)$ into the $u_r(\phi_A)(u_\rho'(\phi_A) - u_R'(\phi_A))$. Then,

$$
\begin{aligned}
&= u_R(\phi_A)(u_r'(\phi_A) - u_\rho'(\phi_A)) + u_r(\phi_A)(u_\rho'(\phi_A) - u_r'(\phi_A) + u_r'(\phi_A) - u_R'(\phi_A)) \\
&+ u_\rho(\phi_A)(u_R'(\phi_A) - u_r'(\phi_A)) \\
&= u_R(\phi_A)(u_r'(\phi_A) - u_\rho'(\phi_A)) + u_r(\phi_A)(u_\rho'(\phi_A) - u_r'(\phi_A)) + u_r(\phi_A)(u_r'(\phi_A) - u_R'(\phi_A)) \\
&+ u_\rho(\phi_A)(u_R'(\phi_A) - u_r'(\phi_A)) \\
&= (u_r(\phi_A) - u_\rho(\phi_A))(u_r'(\phi_A) - u_R'(\phi_A)) - (u_R(\phi_A) - u_r(\phi_A))(u_\rho'(\phi_A) - u_r'(\phi_A))
\end{aligned}
$$

For any given ϕ_A each of the terms is positive, because u is increasing in consumption (i.e. $u_\rho(\phi_A) \leq u_r(\phi_A) \leq u_R(\phi_A)$) and the marginal utility of consumption is decreasing (i.e. $u_\rho'(\phi_A) \geq u_r'(\phi_A) \geq u_R'(\phi_A)$). Considering this, and if both of the following conditions are satisfied, then there will be contagion:

– $u_r(\phi_A) - u_\rho(\phi_A) \leq u_R(\phi_A) - u_r(\phi_A)$: Under this condition we need that the premium gain over the risk free, when projects are successful, is greater than the loss of utility of choosing them over the risk free when they fail

– $u_r'(\phi_A) \leq \frac{u_R'(\phi_A) + u_\rho'(\phi_A)}{2}$: This condition means that the average marginal utility of funding firms projects has to be greater than the marginal utility of investing in the risk free asset

We have two different forces that influence the sign of the impact of changes in ϕ_A:

– One could be called the income effect, which corresponds to a positive effect and the closer the coefficient $\frac{u_r(\phi) - u_\rho(\phi)}{u_R(\phi) - u_\rho(\phi)}$ is to 1 the less incentives banks have to invest in the risky asset

– The other one can be called the risk aversion effect, this can be associated with the variation of risk aversion at different levels of consumption. To have this effect, when ϕ is low, banks should be more risk averse than when there is a good realization of ϕ

Finally, we write the derivative

$$\frac{d\theta_B^*(\phi_A)}{d\phi_A} = aK\Bigg[\frac{(u_r(\phi_A) - u_\rho(\phi_A))(u_r'(\phi_A) - u_R'(\phi_A))}{(u_R(\phi_A) - u_\rho(\phi_A))^2} - \frac{(u_R(\phi_A) - u_r(\phi_A))(u_\rho'(\phi_A) - u_r'(\phi_A))}{(u_R(\phi_A) - u_\rho(\phi_A))^2}\Bigg] \tag{9}$$

- At the limit, the threshold is increasing in r_B.

  The monetary authority can decrease the risk free rate and make a credit freeze less likely (by reducing θ_B^*). This kind of policy has been a part of traditional monetary policy tools used by Central Banks to stimulate credit markets. Note that despite the rate cuts introduced by the authority, there will still be a range of macroeconomic fundamentals where inefficient credit freezes will arise (i.e. we will have $\theta_B^* > b - aK$).

$$\theta_B^*(\phi_A) = b - aK + aK\left[\frac{u_{r_B}(\phi_A) - u_{\rho_B}(\phi_A)}{u_{R_B}(\phi_A) - u_{\rho_B}(\phi_A)}\right]$$
$$\frac{d\theta_B^*(\phi_A)}{dr_B} = aK\frac{d}{dr_B}\left[\frac{u_{r_B}(\phi_A) - u_{\rho_B}(\phi_A)}{u_{R_B}(\phi_A) - u_{\rho_B}(\phi_B)}\right]$$
$$\frac{d\theta_B^*(\phi_A)}{dr_B} = aK\left[\frac{u_{r_B}'(\phi_A)}{u_{R_B}(\phi_A) - u_{\rho_B}(\phi_A)}\right] \geq 0$$

- At the limit, the threshold is decreasing in K.

  The monetary authorities can inject capital into the banks, or directly to firms, in order to make a credit freeze less likely (by reducing θ_B^*). As before, despite the injections of capital there will still exist a range of realizations of θ_B that allow for an inefficient credit freeze.

$$\theta_B^*(\phi_A) = b - aK + aK\left[\frac{u_{r_B}(\phi_A) - u_{\rho_B}(\phi_A)}{u_{R_B}(\phi_A) - u_{\rho_B}(\phi_A)}\right]$$
$$\frac{d\theta_B^*(\phi_A)}{dK} = -a + a\left[\frac{u_{r_B}(\phi_A) - u_{\rho_B}(\phi_A)}{u_{R_B}(\phi_A) - u_{\rho_B}(\phi_A)}\right]$$
$$\frac{d\theta_B^*(\phi_A)}{dr_i} = -a\left(1 - \left[\frac{u_{r_B}(\phi_A) - u_{\rho_B}(\phi_A)}{u_{R_B}(\phi_A) - u_{\rho_B}(\phi_A)}\right]\right) \leq -a \leq 0$$

  As in this problem, $u_{r_B}(\phi_A) \leq u_{R_B}(\phi_A)$ for all values of ϕ_A.

- A direct consequence of the last results, and while all stated conditions follow (i.e.: Banks are risk averse and there is contagion between regions), is that the central bank can decrease the risk free rate or increase the capital, to mitigate the impact of bad realizations of ϕ in the other region.

Risk free rate

So, let us look at the change in the risk free rate the authority needs to introduce to compensate the impact of ϕ_A, changing it to ϕ'_A. Let us define r_B as the actual risk free rate and r'_B as the new risk free rate

$$\theta^*_B(\phi_A) = b - aK + aK\left[\frac{u_{r_B}(\phi_A) - u_{\rho_B}(\phi_A)}{u_{R_B}(\phi_A) - u_{\rho_B}(\phi_A)}\right]$$
$$\theta'^*_B(\phi'_A) = b - aK + aK\left[\frac{u_{r'_B}(\phi'_A) - u_{\rho_B}(\phi'_A)}{u_{R_B}(\phi'_A) - u_{\rho_B}(\phi'_A)}\right]$$

Doing $\theta^*_B(\phi_A) = \theta'^*_B(\phi'_A)$, we obtain that

$$\frac{u_{r'_B}(\phi'_A) - u_{\rho_B}(\phi'_A)}{u_{R_B}(\phi'_A) - u_{\rho_B}(\phi'_A)} = \frac{u_{r_B}(\phi_A) - u_{\rho_B}(\phi_A)}{u_{R_B}(\phi_A) - u_{\rho_B}(\phi_A)}$$
$$\Rightarrow u_{r'_B}(\phi'_A) - u_{\rho_B}(\phi'_A) = \frac{u_{R_B}(\phi'_A) - u_{\rho_B}(\phi'_A)}{u_{R_B}(\phi_A) - u_{\rho_B}(\phi_A)}(u_{r_B}(\phi_A) - u_{\rho_B}(\phi_A))$$

Then we can write,

$$\Rightarrow u_{r'_B}(\phi'_A) = u_{\rho_B}(\phi'_A) + (u_{r_B}(\phi_A) - u_{\rho_B}(\phi_A))\frac{u_{R_B}(\phi'_A) - u_{\rho_B}(\phi'_A)}{u_{R_B}(\phi_A) - u_{\rho_B}(\phi_A)} \tag{10}$$

In order to neutralize the effect of a change in the deposits rate, the central bank must move the risk free rate maintaining the ratio between the utilities of the different outcomes. This relationship is not linear and will depend on the shape of the utility function.

Injection of Capital

Similarly, we calculate the injection of capital the authority needs to introduce to compensate the impact of ϕ_A, changing it to ϕ'_A. Let us define α as proportion of extra capital being injected to the banks (i.e. the final capital will be $K(1+\alpha)$)

$$\theta^*_B(\phi_A) = b - aK + aK\left[\frac{u_{r_B}(\phi_A) - u_{\rho_B}(\phi_A)}{u_{R_B}(\phi_A) - u_{\rho_B}(\phi_A)}\right]$$
$$\theta'^*_B(\phi'_A) = b - aK(1+\alpha) + aK(1+\alpha)\left[\frac{u_{r_B}(\phi'_A) - u_{\rho_B}(\phi'_A)}{u_{R_B}(\phi'_A) - u_{\rho_B}(\phi'_A)}\right]$$

Doing $\theta^*_B(\phi_A) = \theta'^*_B(\phi'_A)$, we obtain that

$$(1+\alpha)\Big[1 - \frac{u_{r_B}(\phi'_A) - u_{\rho_B}(\phi'_A)}{u_{R_B}(\phi'_A) - u_{\rho_B}(\phi'_A)}\Big] = 1 - \frac{u_{r_B}(\phi_A) - u_{\rho_B}(\phi_A)}{u_{R_B}(\phi_A) - u_{\rho_B}(\phi_A)}$$
$$\Rightarrow (1+\alpha)\Big[\frac{u_{R_B}(\phi'_A) - u_{r_B}(\phi'_A)}{u_{R_B}(\phi'_A) - u_{\rho_B}(\phi'_A)}\Big] = \frac{u_{R_B}(\phi_A) - u_{r_B}(\phi_A)}{u_{R_B}(\phi_A) - u_{\rho_B}(\phi_A)}$$

Then we can write,

$$\Rightarrow \alpha = \frac{u_{R_B}(\phi_A) - u_{r_B}(\phi_A)}{u_{R_B}(\phi'_A) - u_{r_B}(\phi'_A)} \frac{u_{R_B}(\phi'_A) - u_{\rho_B}(\phi'_A)}{u_{R_B}(\phi_A) - u_{\rho_B}(\phi_A)} - 1 \tag{11}$$

The proportion of capital injected by the monetary authority must be proportional to the difference between the utility under the new conditions (ϕ_A) and the old conditions (ϕ_A). Again, the relationship will depend on the shape of the utility function.

- One policy implemented by the monetary authorities in the financial crisis was to inject capital into the banks by buying illiquid assets. In this model, we can find conditions under which an injection of capital of c in exchange for the deposits, could generate a decrease in the threshold.

  We have that if the return on deposits is ϕ_A, the threshold is given by

$$\theta^*_B(\phi_A) = b - aK + aK\left[\frac{u_{r_B}(\phi_A) - u_{\rho_B}(\phi_A)}{u_{R_B}(\phi_A) - u_{\rho_B}(\phi_A)}\right]$$

  Now suppose there is an injection c of capital in exchange for the deposits (i.e. $\theta^*_B(0)$)

$$\theta^{c*}_B(0) = b - a(K+c) + a(K+c)\left[\frac{u_{r_B}(0) - u_{\rho_B}(0)}{u_{R_B}(0) - u_{\rho_B}(0)}\right]$$

  We need to have $\theta^{c*}_B(0) \leq \theta^*_B(\phi_A)$, so

$$K(1+\frac{c}{K})\bigg(-1 + \left[\frac{u_{r_B}(0) - u_{\rho_B}(0)}{u_{R_B}(0) - u_{\rho_B}(0)}\right]\bigg) \leq K\bigg(-1 + \left[\frac{u_{r_B}(\phi_A) - u_{\rho_B}(\phi_A)}{u_{R_B}(\phi_A) - u_{\rho_B}(\phi_A)}\right]\bigg)$$
$$\Rightarrow (1+\frac{c}{K})\bigg(\left[\frac{u_{R_B}(0) - u_{r_B}(0)}{u_{R_B}(0) - u_{\rho_B}(0)}\right]\bigg) \geq \bigg(\left[\frac{u_{R_B}(\phi_A) - u_{r_B}(\phi_A)}{u_{R_B}(\phi_A) - u_{\rho_B}(\phi_A)}\right]\bigg)$$

$$\Rightarrow c \geq K\bigg(\left[\frac{u_{R_B}(0) - u_{r_B}(0)}{u_{R_B}(\phi_A) - u_{\rho_B}(\phi_A)}\right]\left[\frac{u_{R_B}(\phi_A) - u_{r_B}(\phi_A)}{u_{R_B}(0) - u_{\rho_B}(0)}\right] - 1\bigg) \tag{12}$$

Looking at this condition, we find that not every program destined to buy illiquid assets may have a positive effect in the credit market. If the value at which the assets are bought does not compensate the decrease in future consumption, then this kind of policy could increase the probability of a credit market freeze.

6 Concluding Remarks

A study conducted by Bijlsma, Dubovik and Straathof (2013) estimated that OECD countries' industrial production growth was 5.5% lower during 2008 and 21% lower during 2009 because of the credit crunch that followed the global financial crisis. The magnitude of this results gives relevance to continue studying credit freezes. In this thesis, we developed a model to study the impact of risk averseness and banking networks on them. Specifically, we focus on inefficient credit market freezes, as these are situations where it would be socially optimal for banks to extend loans to operating firms, but due to the lack of coordination and self-fulfilling expectations, financial institutions will end up avoiding to lend. In this setting, we can identify the circumstances under which a credit freeze will arise and find its relationship to the amount and return of cross regional deposits. Once we have identified this, we can study its impact and how the monetary authority could react against these shocks.

The fact that in this model banks are risk averse, gives room for them to hold cross-regional deposits, as with them they can diversify their sources of income and protect themselves against regional shocks. But having these deposits also impacts the probability of a credit freeze, and we can identify two channels where they operate. The first one is an income effect, which operates on the absolute utility level. The closer the returns of the successful risky projects and the risk free rate are, the more probable is the occurrence of a credit freeze. So, for example, if the return of deposits is low, then the difference of return has a more significant role, but if the return of the deposits is very high, then the differences loses relevance. The second channel we identify is the risk aversion channel, which has a bigger relevance when risk aversion decreases as consumption increases.

After identifying these channels, we can find the conditions under which there could be contagion between regions. We define contagion as the fact that bad realizations of returns in one region (probably due to a credit freeze), could be passed onto the next region augmenting the probability of a credit freeze there. To have contagion in our model, we need to have a risk aversion effect of deposits that exceeds the income effect of them.

The analysis then shifts to the way that the monetary authorities can react to this inter-regional dependence. Our results are in line with those presented by Bebchuck and Goldstein in their original model and economic intuition. This means that if the monetary authority decreases the risk free rate, they will decrease the probability of a credit freeze. Also, injection of capital into the financial system will help to decrease the probability of a credit market freeze. Using these two facts, we can study the relationship between changes in the deposit rate received by banks and the two policies previously discussed. Another analysis we conducted, is the impact of injecting capital to the banks by buying their illiquid assets. The main result from the analysis was that the value at which these assets are bought, has an important role in the result of this policy.

One of the main limitations of this work, is that banks cannot optimally determine their investment portfolio, as they had to invest their capital in one of the two alternatives presented. Also, the fact that the amount deposited by each bank is exogenously determined, leaves room for future extensions, as we could determine the optimal level of interrelation between regions, in order to minimize the possibility of contagion. Another possible extension of this model could be a modification of the deposit contracts. By allowing early withdrawals, the regions could increase the amount of capital available to lend in case the fundamentals are below the threshold.

Other improvements to this model, could be the possibility of analyzing the reactions of the first region, specifically by including another noisy signal that could give the banks some information regarding the fundamentals of the other region. Also, the fact that most of the conclusions are studied when private information is very precise, generates an opportunity to enrich this work by allowing for more noisy signals. In this same context, the inclusion of new distributions to the variables and signals, could also provide robustness to the analysis presented.

Finally, the model developed in this work not only lets us develop an analytical framework to study how cross regional deposits play a role, but also to study how the monetary authorities should react to them. This work is far from conclusive and many more interesting results could be obtained by introducing innovations to the framework presented. Globalization, banking networks and credit freezes are situations that will continue to occur in the economy, so new studies in this subjects will continue to be helpful.

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