INTERNATIONAL JOURNAL OF CLIMATOLOGY
Int. J. Climatol. **37**: 849–860 (2017)
Published online 28 April 2016 in Wiley Online Library
(wileyonlinelibrary.com) DOI: 10.1002/joc.4744

RMetS
Royal Meteorological Society

The temporal fractality of precipitation in mainland Spain and the Balearic Islands and its relation to other precipitation variability indices

Oliver Meseguer-Ruiz,[a,c,*] Jorge Olcina Cantos,[b] Pablo Sarricolea[c,d] and Javier Martín-Vide[c]
[a] *Department of Historical and Geographical Sciences, University of Tarapacá, Arica, Chile*
[b] *Interuniversitary Institute of Geography, University of Alicante, Spain*
[c] *Group of Climatology, University of Barcelona, Spain*
[d] *Department of Geography, University of Chile, Santiago, Chile*

ABSTRACT: Recent years have seen a rise in research into the behaviour of precipitation variability on account of the application of new statistical techniques with a longstanding tradition in other fields. Fractal is a word used to refer to regular objects or processes that cannot be defined by the classical Euclidian mathematics. The fractal dimension of the temporal distribution of precipitation (*D*) is an indicator of the property of self-similarity in rainfall distribution at different time intervals. While its spatial meaning has previously been developed extensively and is well defined, the interpretation of the concept of fractality applied to the temporal distribution is abstract. The overarching goal of this article is to give climatic significance to this indicator. To this end, data logged at 10-min intervals from 44 weather stations in mainland Spain and the Balearic Islands for the period from 1997 to 2010 has been employed. The *D* values obtained ranged between 1.4499 for the observatory in Ibiza and 1.6039 for the observatory in Jaca. The fractal dimension presents a significant and good negative correlation (−0.55) with the concentration index (CI), and a significant and good positive correlation (0.67) with entropy. The correlation of *D* with other traditionally used indices, such as the coefficient of variation or the consecutive disparity index is very limited, as this indicator is more focused on the distribution of precipitation intervals than on the total accumulated rainfall over a given period. In an endeavour to develop multivariate models that explain the behaviour of *D*, only two-variable models can be obtained, which account for most of the variability and that involve the CI or entropy. Self-similarity is therefore associated with the regular recurrence of precipitation intervals, which is more evident in those observatories with higher *D* values.

KEY WORDS fractal dimension; temporal distribution; precipitation; self-similarity; Spain

Received 28 July 2015; Revised 14 March 2016; Accepted 20 March 2016

1. Introduction

The variability of the climate system in general, and atmospheric variables in particular, is notable on any time scale that is considered. This applies to all climate variables, regardless of whether or not there are patterns in their chronological behaviour. On the Iberian Peninsula, on account of its specific position between an ocean and an almost inland sea, and in the border area between areas dominated by subtropical anticyclones, in the south, and westerly winds and polar front storms, to the north (Martin-Vide and Olcina Cantos, 2001), the variable whose records show greatest dispersion is precipitation, both in the amounts accumulated and in the temporal distribution thereof. This renders it an interesting subject, as confirmed by a large number of studies that have been undertaken (Rodríguez-Puebla *et al.*, 1998; Sáenz *et al.*, 2001; Goodess and Jones, 2002; Martin-Vide and Lopez-Bustins, 2006; Gonzalez-Hidalgo *et al.*, 2009; de Luis *et al.*, 2010; Rodríguez-Puebla and Nieto, 2010; Casanueva *et al.*, 2014).

The Mediterranean Sea, a crucial agent in atmospheric dynamics affecting the Iberian Peninsula, plays a primordial role, introducing a large number of distinctive features in the field of study. Amongst said features, it should be noted that it occupies a large area measuring approximately 2.5 million square kilometers between Europe and Africa; and it is only connected to a limited extent to the Atlantic Ocean by way of the Strait of Gibraltar. In turn, it is subdivided into two sub-basins, the Eastern Mediterranean and the Western Mediterranean, connected via the Strait of Sicily. On account of its relatively small size and its geographic location that is almost landlocked, the Mediterranean is highly sensitive and responds rapidly to atmospheric forcing and/or anthropogenic influences (Pionello, 2012). Moreover, it presents its own unique atmospheric behaviour, given the protection and insulation provided by the relief surrounding its basin (Pionello, 2012). Population growth, climate change and overexploitation are placing tremendous pressure on

* Correspondence to: O. Meseguer-Ruiz, Departamento de Ciencias Históricas y Geográficas, Facultad de Educación y Humanidades, Universidad de Tarapacá 18 de Septiembre #2222, Arica, Chile. E-mail: oliver.meseguer@gmail.com

the Mediterranean environment and on its ecosystems and resources. Furthermore, it is a region where oceanic processes also take place, but on a much smaller scale that those that occur elsewhere, such as the formation of deep waters, which help maintain the thermohaline circulation cell in dimensions on a par with the sub-basins, as is the case in the planetary belt at ocean level (Pionello, 2012). This reality bears a strong influence on lending the Mediterranean region distinctive climatic features.

If the accumulated rainfall from year to year is taken into account, there is very high variability, which is a characteristic of the Mediterranean climate that affects an important area of the studied region. Using Gibraltar's observatory, at the southern tip of the Iberian Peninsula, as a reference point, which has the oldest precipitation records on the Iberian Peninsula, which date back to the late 18th century, annual records in the region of 2000 mm in the mid-19th century can be found, compared with just over 350 mm in the early 1980s. The huge interannual variability of the total accumulated precipitation over the series is evident, in which the wettest year accounted for more than five times the precipitation of the driest year (Wheeler and Martín-Vide, 1992, Martín-Vide, 2008). Many other regions on the Iberian Peninsula show a similar variability to that mentioned herein, particularly in the south and southeast of the peninsula, whereas in the northernmost points, variability does not present such marked values. However, this dispersion is not only reflected in one spot over the years, but is reflected spatially, and considering the seasonal distribution of precipitation, a wide variety of seasonal precipitation patterns in Spain can be obtained (Martin-Vide and Estrada Mateu, 1998; Rodríguez-Puebla *et al.*, 2001; De Luis *et al.*, 2010). During the last years, wet periods have become longer over most of Europe, characterized by more abundant precipitation (Zolina *et al.*, 2010; Zolina, 2014). Similar realities have been identified also in the United States, where an increase in precipitation coming from intense rain events have been demonstrated (Groisman and Knight, 2008).

1.1. Fractals as a basis of study

In palaeoclimatology, studying cores taken from ice caps plays a key role in determining what the planet's climate was like in the past (Pelletier, 1997; Valdez-Cepeda *et al.*, 2003). This experiment conducted in Antarctica (the EPICA project), based on an ice core 3190-m thick, has allowed climate connections to be established over the last 740,000 years, and, on the basis of the fractal analysis, for information to be supplied regarding the evolution of glacial cycles (King, 2005). Previous studies have associated the information obtained from ice cores in Antarctica with historic climatic data obtained from marine deposits at the bottom of the sea (Raidl, 1996; Sahay and Sreenivasan, 1996). Fractal analysis of the data obtained provides evidence regarding the connections between climate data behaviour in several places, such as Hungary (Bodri, 1994) or on the Kamchatka Peninsula based on volcanic eruptions of an explosive origin in the last 10,000 years (Gusev *et al.*, 2003), or, on a smaller time scale, on the basis of records of sediments in floodplains in the Po Valley (Italy) (Mazzarella and Rapetti, 2004).

Fractal analysis methodology has also been applied in recent decades to climate-related studies, and some of their variables (temperature, precipitation and atmospheric pressure, among others) have fractal behaviour, related to both space and time, to the point that they determine the persistence through time of these variables and their respective interdependence (Rehman, 2009; Tuček *et al.*, 2011; Nunes *et al.*, 2013). On the basis of the datasets of three major climate variables (temperature, precipitation and atmospheric pressure) and the variability thereof month by month and between seasons, regional climate models are not capable of developing a good climate forecast at local level, as they only work with averaged amounts on the basis of time series. In the same vein, other forecasting models that incorporate fractals are more reliable as they take more climate dynamics into consideration, improving the existing models on a regional scale (Rangarajan and Sant, 1997, 2004).

In recent years, new contributions have emerged that offer different methods with which to analysis of the temporal behaviour of the data generated by climate models and real climatic series obtained from the records of weather station networks. These approaches combine the analysis of fractal data, the monitoring of real data flows and model-generated data to detect deviations in the intrinsic correlation between the series of observed data and data forecast by the model. Therefore, forecasts developed by regional climate models and the corresponding data measurements observed in a network of sensors reveal that this approach allows differences in behaviour to be determined between the data observed and the data derived using models. This shows that there is still room for improvement in climate change models, regardless of whether they can be improved due to other significant facts, such as more accurate parameter setting of certain processes, or a better consideration of the cloud cover and soil moisture, and that the concepts based on fractal theory can contribute in that respect (Nunes *et al.*, 2011).

In short, many of the uses given to fractals in climatology studies have focused on forecast methodologies as regards meteorological and climate models, and the validation thereof, as well as the fractal nature of the spatial precipitation fields. However, there are not many in which these principles are applied to the purely dynamic behaviour of the climate system, but they are by no means non-existent, as discussed below. By the very definition of a fractal object, it is easy to be inclined to think that the application of Mandelbrot's principles has focused on the spatial distribution of precipitation, following patterns that would fit fractal objects. The question has even been directly raised if it is actually possible to apply a fractal approach to it (Sivakumar, 2001). The discovery of these new realities has allowed advances to be made in precipitation models, which have significantly enhanced the existing ones (Chou, 2003), simulating rain fields in line with the property of multifractality, which confirms the scale invariance of this phenomenon. The fractal properties of spatial distribution and accumulated precipitation amounts have therefore been demonstrated. To delve deeper into this knowledge, it must be ascertained whether the temporal distribution of precipitation follows these same principles.

1.2. The temporal fractality of precipitation

The concepts of fractal theory have a more intuitive application of precipitation in terms of their spatial rather than temporal distribution. When reference was made to spatial and fractal distribution, it may be held, almost automatically, that a precipitation field can have a fractal form. If the details are examined, it can be verified that one part represents the whole, maintaining self-similarity or scale invariance; nevertheless, in the case of the temporal fractality of precipitation, the concept is more difficult to understand. Firstly, the scaling application is to verify if rain has been accumulated at different time intervals of a given duration. It must be ascertained whether this behaviour is repeated at time intervals of greater or lesser duration. Rainfall shows high variability in a wide range of temporal and spatial scales. The substantial fluctuations and high temporal variability of precipitation renders its statistical and mathematical processing more complex than in the case of other variables.

A plethora of models has been developed in hydrology based on the fractal properties of the temporal and spatial distribution of precipitation (Zhou, 2004; Khan and Siddiqui, 2012). These models of hydrological processes in basins are considerably more useful when they can be extrapolated across spatial and temporal scales. This problem of scale transfer, in other words, the description and forecast of the characteristics and processes at a different scale to that in which observations and measurements are performed, has become the focus of much research today in hydrology and other fields (Strahler, 1977). Indeed, these types of dynamics have been identified in studies in mainland Spain on the basis of long series (90 years) of annual accumulated precipitation, and their analysis reveals that the distribution of this variable is in line with fractal distribution (Oñate Rubalcaba, 1997). The values obtained, with an average fractal dimension of 1.32 for the whole territory, are in the same order of magnitude as the fractal dimensions obtained from other macrometeorological and palaeoclimatic records.

Obtaining the fractal dimension of annual precipitation implies the existence or non-existence of a pattern in this variable, establishing the degree of persistence and the significance and sign of the observed trends by means of the Mann–Kendall non-parametric test, thereby identifying changes in the temporal behaviour of precipitation. Such is the case studied in the east of the province of La Pampa (Argentina), where a local analysis was conducted that could define the patterns of the forecasts made by the IPCC-AR4 in greater detail (Pérez *et al.*, 2009). A similar study was undertaken in Venezuela (Amaro *et al.*, 2004) on the basis of data from ten weather stations with annual precipitation values in line with a fractal distribution. Thanks to these findings, climate changes at different time scales in Venezuela can be predicted, as demonstrated in the aforementioned study.

In other regions of the world, such as the Shandong Peninsula in China, where the problem of accessing increasingly scarce water resources is on the rise and is one of the areas of greatest and most rapid development in the Asian giant, knowledge of precipitation patterns is a crucial issue for future development, and the studies that allow this integrate fractality elements in their analysis (Rehman and Siddiqi, 2009; Gao and Hou, 2012). In the same vein as multifractal models, applications have also been undertaken in studies on the Iberian Peninsula, such as the case of Córdoba, in southern Spain (Dunkerley, 2008; García-Marín *et al.*, 2008). However, it has been demonstrated that extreme precipitation is in line with even more complex models than multifractal ones, as they are affected by limited periods, such as very short durations or very long return periods (Veneziano and Furcolo, 2002; Veneziano *et al.*, 2006; Langousis *et al.*, 2009). The temporal resolution applied plays a decisive role in this type of study, because the analysis with hourly data, on the one hand, and with daily data, on the other, already causes changes in the values of the fractal dimensions, which also hinges on the most characteristic precipitation of each place (Olsson *et al.*, 1992; García Marín, 2007; López Lambraño, 2012). Furthermore, the analysis of fractality allows better analysis methods as regards precipitation frequencies to be ascertained, coinciding with previously mentioned studies (Gao and Hou, 2012), even allowing different types of precipitation patterns in a specific region to be defined based solely on this methodology (Dunkerley, 2010; Reiser and Kutiel, 2010; Kutiel and Trigo, 2014).

In most studies on scaling properties in the precipitation process, multifractal behaviour has been researched without taking the different rain-generation mechanisms involved into account. Nevertheless, it is common knowledge that rain processes are linked to certain scales determined by climatological features, as well as by regional and local weather characteristics. One of the implications drawn from these connections is the possibility that the multifractal parameters of rainfall may depend on the overriding precipitation-generating mechanism; the synoptic origins of precipitation bear an influence on the fractal dimension values obtained. Fractal analysis techniques have been applied to rainfall data recorded in the Barcelona metropolitan area in the period 1994–2001, as well as to a selection of rainfall episodes recorded in the same city in the period 1927–1992. This influence is also revealed in the analysis of the effects of seasonality in the multifractal behaviour of rainfall in Barcelona (Rodríguez *et al.*, 2013) and fractal behaviour in Catalonia (Meseguer-Ruiz and Martín-Vide, 2014).

In other areas of the Mediterranean region, studies have been undertaken in which the value of the fractal dimension has been determined (Ghanmi *et al.*, 2013). In this case, the fractal dimension for different time series at different resolutions (at 5-min intervals and daily) with various durations (2.5 years for the first, 137 years for the second) was calculated. Three self-similar structures were

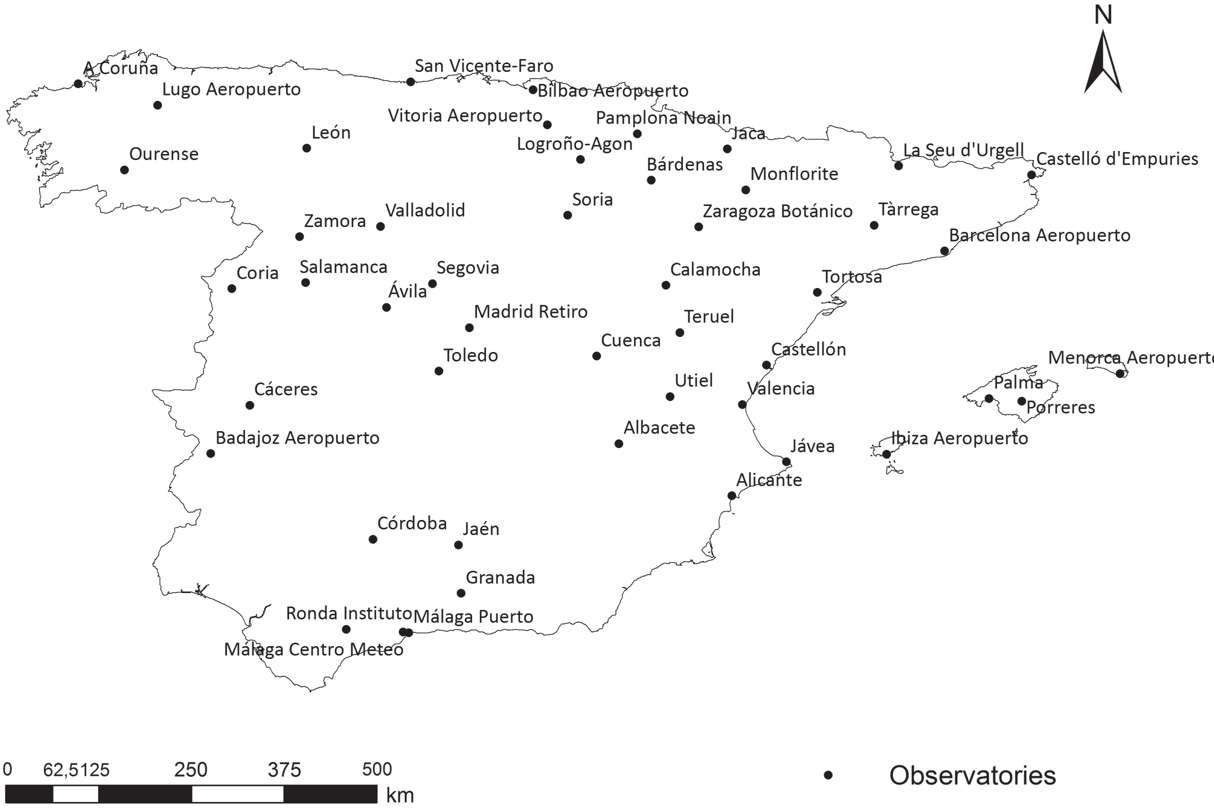

Figure 1. Location of the observatories used.

identified: the microscale (from 5 min to 2 days), with a fractal dimension of 1.44, the mesoscale (from 2 days to 1 week) and the synoptic scale (from 1 week to 8 months), with a fractal dimension of 1.9. Kalauzi *et al.* (2009) propose a comparative study of the fractal dimension be conducted, not only of precipitation but also of other climatic variables, in a Mediterranean environment, Veneto (Italy), and a completely different area, the province of Pastaza, in the Ecuadorian Amazon. In this case, the pace at which the principle of self-similarity is reproduced in each series was determined, that means that a period has been identified in which the behaviour of precipitation and other climatic variables are reproduced in time. It was much slower in the province of Pastaza (4.4 years), modulated by El Niño-Southern Oscillation (ENSO), than in the Mediterranean region of Veneto (10.3 years), where the influence of the solar activity cycle can be felt, although it must be confirmed. Another area where similar work has been carried out is the region of Tamil Nadu, at the southeastern tip of the Indian subcontinent (Selvi and Selvaraj, 2011). In this study, the fractal dimension was determined using data logged between 1902 and 2008 (unspecified time resolution) with the Hurst method, obtaining a D (fractal dimension) value of 1.7895.

2. Data and methodology

As in the case of fractal objects (Mandelbrot, 1976), scale-invariant processes and systems do not possess a scale that characterizes them. Bearing this in mind, a fractal process is one in which the basic process itself takes place on different scales, in other words, on a scale in which one part reproduces the whole. Therefore, fractal geometry and the fractal dimension (such as an incomplete dimension) are known to be a valuable tool that allows the form of the objects to be described. It has gained widespread use and acceptance in many fields of natural sciences including geography, ecology and the new technologies applied to geographic information (Goodchild, 1980; Goodchild and Mark, 1987; Peitgen *et al.*, 1992; Hastings and Sugihara, 1994; Tuček *et al.*, 2011).

The databases of 48 observatories in the network of automatic stations belonging to the Spanish Meteorological Agency (AEMet) were used; 10-min-resolution precipitation data were obtained from said databases. A total of 75 observatories were initially available, but the series with a missing value exceeding 15% were omitted. The series employed in this study comprise records verified by AEMet. In the end, the subject area was covered satisfactorily, as presented in Figure 1.

Moreover, a common time period was selected for observatories in which the quality of the series was guaranteed, settling on the period of analysis 1997–2010.

The fractal dimension was calculated according to the box-counting method, in the following manner. On the basis of 10-min resolution rainfall data records, the period of 10 min was considered to be the baseline interval unit in order to perform the analysis. The periods outlined below were established, which contain 1, 2, 3, 6, 12, 18, 24, 36, 48, 72, 144 and 288 unit intervals, i.e. periods of 10, 20 and

Table 1. Values obtained following box counting for Ávila.

Length of the interval (l) (h)	ln (l)	Number of intervals with precipitation (N)	ln (N)
0.166	−1.792	17478	9.769
0.333	−1.099	11443	9.345
0.5	−0.693	9052	9.110
1	0	6146	8.724
2	0.693	4328	8.373
3	1.099	3590	8.186
4	1.386	3123	8.046
6	1.792	2595	7.861
8	2.079	2263	7.724
12	2.485	1892	7.545
24	3.178	1370	7.223
48	3.871	993	6.901

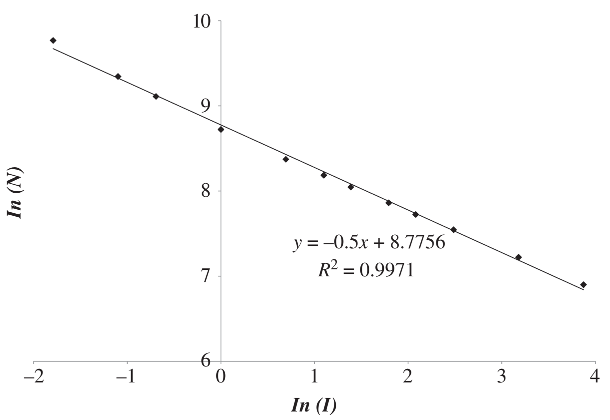

Figure 2. Regression line that provides the D value for Ávila.

30 min, 1, 2, 3, 4, 6, 8, 12, 24 and 48 h respectively, and the number of them that recorded a precipitation amount was calculated. The D value of the temporal distribution of precipitation was defined on the basis of the slope of the regression line, resulting from representing the pairs of values obtained from natural logarithms of 1, the extent or length of the interval, and N, the number of intervals with precipitation. In fact, the logarithms of these pairs of values for each observatory are aligned with notable approximation. D is determined by $1 + \alpha$, where α is the absolute value of the slope of the regression line. The observatory in Ávila corresponds to what is shown in Table 1 and in Figure 2.

Many indices that quantify the variability of a series of numerical data do not consider the order of the series values. However, the chronological order of the values constitutes an essential feature of the temporal behaviour of the element taken into consideration. By way of example, any rainfall series whose chronological sequence is considered and the same values sorted in an ascending or descending order have the same measurement and the same deviation type, and therefore the same coefficient of variation, but their climatic meaning is significantly different. For the ordered series, it would be a question of lower temporal irregularity, although its variability, determined by the classical statistical parameters (deviation type, variance, coefficient of variation, etc.), is the same as the first case. In conclusion, indices should be available to assess the temporal irregularity, bearing in mind the chronological order of the values of the climatic series. These indices quantify the jumps between the consecutive values, which in the case of rainfall can be expressed as the difference between the consecutive totals, in absolute value so that the differences equivalent to the opposite sign are not cancelled out. The ratios between the consecutive totals can also be considered, in other words, a multiplicative scheme, which in the case of precipitation is preferable to an additive scheme (Pérez-Cueva *et al.*, 2001). A multiplicative scheme of this type is the one defined by the Consecutive Disparity Index (S) (Martin-Vide *et al.*, 2001), which is calculated on the basis of the following formula:

$$S = \frac{1}{n-1} \sum_{i=1}^{n-1} \left| \ln \frac{P_{i+1}}{P_i} \right| \tag{1}$$

The concentration index (CI) is defined as an approximation of the Gini Index, a numerical representation of the inequalities shown by the Lorenz curve, used to express the degree of concentration of a specific magnitude in a portion of a given population. In this case, the CI is used to quantify the importance of wet days compared with the total accumulated rainfall in a time series (Martín-Vide, 2004).

The coefficient of variation (CV) is used to refer to the ratio between the magnitude of the mean and the variability of the variable in question. Its formula expresses the standard deviation as a percentage of the arithmetic mean, showing a better understanding of the degree of variability than standard deviation. To avoid mistakes in interpretation, this coefficient requires all values to be positive. As is widely known, the higher the value of the coefficient of variation, the greater the heterogeneity of the variable's values. Conversely, the lower the coefficient of variation, the closer to the mean of the series values. The coefficient of variation is generally used in annual-resolution precipitation studies, because as it depends on the mean, it cannot be zero nor, as far as possible, close to zero, as its meaning would be distorted. It is for this reason that, in subtropical climates, it is not generally used at anything less than annual temporal resolutions, because the averages of the summer months can come close to 0 mm (Pérez-Cueva *et al.*, 2001).

The CI is calculated using daily data for the period 1951–1990 from many weather stations across the study area, not all the same to the currently used to calculate D. Changing from one kind of station to another (from traditional to automatic) may cause inhomogeneities in the series, so the authors preferred to maintain the reference data from other periods, which are long enough and has been already published and accepted. The same happened with the annual records for CV and S for the period 1940–1994.

The concept of entropy (H) was introduced by Shanon (1948) to refer to the degree of disorder implicit in a series, or to ascertain the noise level in said series, apart from the variability itself. The entropy of an isolated system never decreases, because isolated systems gravitate towards thermodynamic equilibrium, a state with maximum entropy. However, those systems that are not isolated can note a decrease in their entropy. As entropy is a function of a particular state, the change in a system's entropy is the same for any process that evolves from an initial state to a final given state. Entropy thus defined constitutes a parameter characteristic of the variable's distribution. Entropy corresponding to a variable with unimodal Gaussian distribution will have a lower entropy than that of a bimodal distribution. It is therefore an indicator of the amplitude of the non-periodic components of a signal. Many studies show that the distribution of precipitation in recent years has become more irregular due to climate change and intensive human activity (Liu *et al.*, 2013). The estimate of precipitation distribution holds extraordinary importance in understanding the water cycle and is crucial for managing water resources. Better knowledge of the irregular behaviour of precipitation can be gained through examining the entropy of different precipitation series and their evolution (Liu *et al.*, 2013). Hao and Singh (2013) use an entropy-based analysis to create a model of the distribution of the maximum rainfall accumulated over the year. The entropy analysis can be addressed from a multi-scale perspective to investigate the changes in the complexity of the rainfall-runoff processes due to human activity, and to facilitate the selection of rainfall-runoff models that take self-similarity into account (Chou, 2012), which is closely related to the fractal processes within the temporal distribution of precipitation. In the same vein, it has been shown that the internal complexity of the series increases as the temporal series studied increases (Chou, 2011, 2014). Nevertheless, for temporal precipitation series (and runoff) on different scales, findings characterized by low complexity and high predictability were obtained, which provides a reference point to determine the appropriate time scale for the analysis and the prediction of precipitation and runoff values. The same method was used in the basin of the Yellow River (China) to determine the patterns of the temporal variability of precipitation (Liu *et al.*, 2008) over the period 1960–2006. The findings show that entropy holds a good, positive correlation with longitude, increasing from west to east, with the highest in the stations nearest the sea, and where the amounts are higher. Similar findings were obtained in another study carried out in a nearby area, in Xinjiang (Zhao *et al.*, 2011). However, in other regions of the world, such as the Middle East, entropy is indeed associated with latitude, and not longitude, as regards precipitation distribution (Mathbout *et al.*, 2014, 2015). In a closer region to the field of study, and also of a Mediterranean nature, Montesarchio *et al.* (2011) use entropy to define thresholds based on which episodes of high hourly rainfall intensity can occur. Therefore, higher values of H link observatories where high precipitation vales in every interval are recorded and, at the same time, longer dry spells. In Barcelona's observatory, high-entropy values in certain periods is linked to the rise in observations far from the mean (Rodriguez *et al.*, 1999). In the northeast of Catalonia, in the northeast of the Iberian Peninsula, a study was carried out that concludes that entropy of a precipitation series in this region is associated with a higher amount of values far from the mean of each series (Lana *et al.*, 2009).

Table 2. D values for all the observatories used.

Observatory	D	Observatory	D
A Coruña	1.5629	Málaga Centro Meteo	1.5595
Albacete	1.4941	Málaga Puerto	1.5376
Alicante	1.4710	Menorca Aeropuerto	1.4680
Ávila	1.5000	Monflorite	1.5223
Badajoz Aeropuerto	1.5183	Ourense	1.5704
Barcelona Aeropuerto	1.5071	Palma	1.4988
Bárdenas	1.4933	Pamplona Noain	1.5487
Bilbao Aeropuerto	1.5827	Porreres	1.4966
Cáceres	1.5464	Madrid-Retiro	1.5432
Calamocha	1.4805	Ronda Instituto	1.5832
Castelló d'Empuries	1.5161	Salamanca	1.5075
Castellón	1.5075	San Vicente-Faro	1.5839
Córdoba Aeropuerto	1.5605	Segovia	1.5105
Coria	1.5644	Soria	1.5190
Cuenca	1.5468	Tàrrega	1.4732
Granada	1.5414	Teruel	1.4856
Ibiza Aeropuerto	1.4499	Toledo	1.5047
Jaca	1.5848	Tortosa	1.5167
Jaén	1.5573	Utiel	1.5058
Jávea	1.5101	Valencia	1.5258
La Seu d'Urgell	1.5036	Valladolid	1.5261
León	1.5578	Vitoria Aeropuerto	1.5559
Logroño-Agon	1.4961	Zamora	1.5020
Lugo Aeropuerto	1.6039	Zaragoza Botánico	1.5154

The Persistence Index P_{11} is defined as the probability of a rainfall episode occurring (in this case, every 10 min) after another rain episode (Martín-Vide and Gomez, 1999). The persistence index (P_{11}) refers to the likelihood of a rainfall interval followed by another rainfall interval occurring.

3. Findings

D values were obtained for all the observatories (Table 2), and have been correlated directly (using Pearson's r) with the values of CI, CV, S, H and P_{11}.

The values of CI, CV and S were taken from the corresponding reference papers, whereas the values of H and P_{11} were expressly calculated for each observatory in this study (Table 3).

The CI is an index calculated from a series of daily-resolution data and that measures the degree of concentration of accumulated precipitation on certain days; in particular, it assesses the weight of the rainiest days compared with the total number of days with rain. The value of Pearson's r between D and CI is −0.55, with a p-value of 0.012. There is therefore a significant and good negative correlation between both variables. The relationship between these two indices can be seen below (Figure 3). The equation of the resulting regression line is:

$$y = -0.3657x + 1.7492 \tag{2}$$

Table 3. Values of CI, CV, S, H and P_{11} for the different observatories.

Observatory	CI	CV	S	H	P11
A Coruña	0.56	0.172	0.18	10.02280	0.650
Albacete	0.59	0.275	0.35	8.63761	0.671
Alicante	0.68	0.324	0.38	8.33006	0.661
Ávila	0.60	0.270	–	9.28513	0.690
Badajoz Aeropuerto	–	0.260	0.25	9.12476	0.668
Barcelona Aeropuerto	0.65	0.254	0.29	8.56308	0.745
Bárdenas	–	–	–	9.00131	0.632
Bilbao Aeropuerto	–	–	–	9.97164	0.697
Cáceres	0.57	0.262	0.29	9.26560	0.714
Calamocha	–	–	–	8.59422	0.708
Castelló d'Empuries	–	–	–	8.45508	0.708
Castellón	–	–	–	8.76485	0.713
Córdoba	0.58	0.389	0.34	9.20925	0.722
Coria	–	–	–	9.26907	0.677
Cuenca	0.56	0.281	0.30	9.10585	0.685
Granada	0.56	0.240	0.25	9.12952	0.672
Ibiza	–	–	–	–	0.705
Jaca	–	–	–	–	0.704
Jaén	–	0.342	0.32	–	0.690
Jávea	–	–	–	–	0.691
La Seu d'Urgell	–	–	–	9.07186	0.642
León	0.57	0.230	0.25	9.04730	0.663
Logroño-Agon	0.59	0.205	0.22	9.17594	0.718
Lugo Aeropuerto	–	–	–	10.27887	0.703
Málaga Centro Meteo	–	–	–	8.96813	0.713
Málaga Puerto	–	0.376	0.36	8.86448	0.783
Menorca Aeropuerto	–	–	–	8.96585	0.708
Monflorite	–	–	–	9.08762	0.769
Ourense	0.55	0.250	–	9.75776	0.664
Palma	–	0.258	0.31	8.82552	0.682
Pamplona Noain	0.58	0.190	–	9.70724	0.675
Porreres	–	–	–	8.96199	0.676
Madrid-Retiro	0.60	0.266	0.31	9.08248	0.675
Ronda Instituto	–	0.250	–	9.26839	0.696
Salamanca	0.57	–	0.23	9.30993	0.655
San Vicente-Faro	–	–	–	9.74783	0.677
Segovia	–	–	–	9.43729	0.682
Soria	0.56	0.206	0.21	9.45052	0.663
Tàrrega	–	–	–	8.78301	0.684
Teruel	–	–	–	8.96255	0.731
Toledo	–	0.232	0.26	8.74777	0.702
Tortosa	0.69	0.319	0.41	8.72664	0.694
Utiel	–	–	–	9.05445	0.642
Valencia	0.70	0.373	0.42	8.69059	0.687
Valladolid	0.58	0.256	0.26	9.36129	0.715
Vitoria Aeropuerto	–	–	–	9.91099	0.665
Zamora	–	0.314	0.30	9.17706	0.703
Zaragoza Botánico	0.62	0.263	0.30	9.05581	0.680

The confidence limits of this regression line are -0.3657 ± 0.1317 and 1.7492 ± 0.0790. Thus, the fractal dimension can be expressed in terms of the CI in the following manner:

$$D = -0.3657 \times \mathrm{CI} + 1.7492 \qquad (3)$$

S allows the order of the series values to be considered, which is not taken into account for other indices, such as CV. The chronological order of the values constitutes a key feature of the temporal behaviour of precipitation. This index is calculated on the basis of annual values. D and S have been linearly correlated, yielding a value of -0.21 for Pearson's r, with a p-value of 0.336; the correlation between both variables is therefore not statistically significant.

CV is used to refer to the relationship between the size of the mean and the variability of the variable in question. The data on the basis of which it is calculated are annual. There is no linear correlation between these two indices, given that the value of Pearson's r -0.1, is not significant, as the p-value is 0.626.

H is an index employed to gauge the degree of disorder implicit in a series, or to gauge the level of noise in said series, beyond its actual variability.

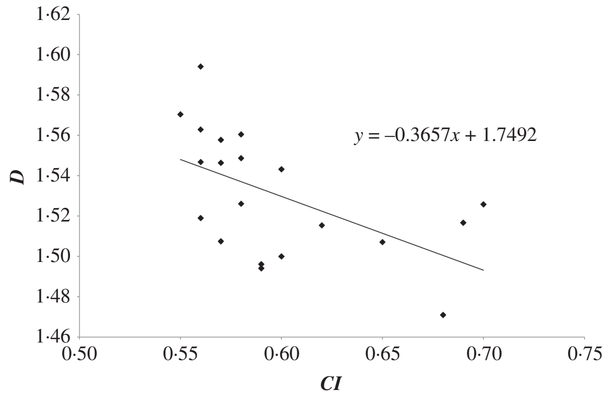

Figure 3. Linear relationship between CI and D for the 20 observatories studied.

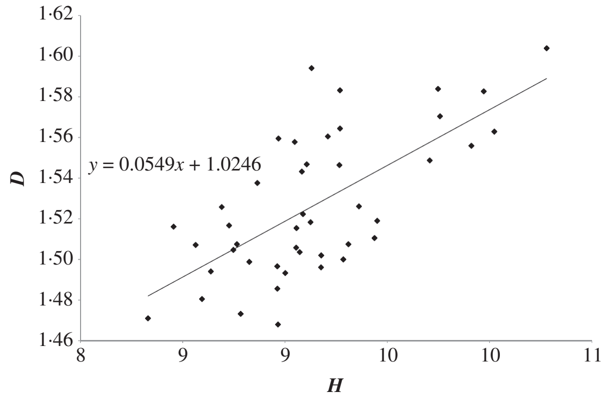

Figure 4. Linear relationship between H and D for the 44 observatories studied.

As mentioned previously, the internal complexity of the series increases as the temporal series studied increases (Chou, 2011, 2014). Nevertheless, for temporal precipitation series (and runoff) on different scales, findings characterized by low complexity and high predictability were obtained, which provides a reference point to determine the appropriate time scale for the analysis and the prediction of precipitation and runoff values.

D is closely related to entropy behaviour, with a strong mutual correlation between them (the value of Pearson's r is 0.67). The H and D values hold a good direct relationship (Figure 4).

The regression line between both variables is as follows:

$$y = 0.0549x + 1.0246 \tag{4}$$

The confidence limits of the regression line are 0.05749 ± 0.0094 and 1.0246 ± 0.0857. D can therefore be expressed as a variable that is dependent on H:

$$D = 0.0549 \times H + 1.0246 \tag{5}$$

Of all the indices with which the fractal dimension was correlated, H is the one with which the strongest linear relationship is held, coinciding with the index whose series present higher temporal resolution.

The P_{11} is used to calculate the probability of a rainfall interval followed by another rainfall interval occurring. There is no linear correlation between D and P_{11} because the value of Pearson's r is −0.02, which is not significant, with a p-value of 0.890.

Table 4. Relationship between the fractal dimension and other indices.

Index	r	95% Significant	Data resolution	Equation
CI	−0.55	Yes	Daily	$y = -0.3657x + 1.7492$
S	−0.21	No	Annual	N/A
CV	−0.10	No	Annual	N/A
H	+0.67	Yes	10-min	$y = 0.0549x + 1.0246$
P_{11}	−0.02	No	10-min	N/A

Table 5. Mutual correlations between the different indices (italicized values represent significant Pearson's r values, with a p-value lower than 0.05).

P_{11}	0.24	*0.44*	−0.21	0.28
S	*0.79*	*0.89*	*−0.78*	
H	*−0.74*	−0.60		
CV	0.58			

In short, after correlating the fractal dimension with five indices that explain the temporal behaviour of precipitation, a statistically significant correlation is obtained with some of them and not with others. These findings are summarized in Table 4.

As some indices are mutually correlated (Table 5), they will not be considered independent, and will therefore not all be included in the same model of multiple correlation.

Based on these findings, models of more than two variables that explain the D value as a dependent variable due to the existence of mutual correlations cannot be obtained; the information added by them would therefore be redundant. Hence, those models whose two variables are not mutually correlated, yielding various regression planes, were selected. Four models were selected that constitute four regression planes in which D is a function of two of the indices that are independent of one another (Table 6).

The models that present greater correlation with the variability of D are model 1 and model 2, with r values of 0.620 and 0.630, respectively. Model 2's mean squared error is also the lowest, i.e. 0.0256. It is therefore assumed that the best regression plane that explains the variability of D is model 2. Models 1, 2 and 3 present a confidence level exceeding 95%, as they yield levels of probability of 0.020, 0.017 and 0.048, respectively. The statistical significance of model 4 lies below the confidence level of 90%, and is therefore omitted. The dispersion of the different models obtained is shown in Figure 5, where it is quite evident that there is no model which is clearly representative of the fractal dimension including two independent variables.

4. Discussion and conclusions

The fractal dimension D and the CI (Martín-Vide, 2004) show a good negative correlation (−0.55), and significant

Table 6. Different regression planes that explain the fractal dimension.

Model	Indep. variables	Regression plane	Correlation	Mean squared error	Degree of probability
1	CI and CV	$D = 1.787 - 0.485\text{CI} + 0.134\text{CV}$	0.620	0.0259	0.020
2	CV and H	$D = 0.992 + 0.149\text{CV} + 0.055H$	0.630	0.0256	0.017
3	CI and P_{11}	$D = 1.752 - 0.365\text{CI} - 0.004P_{11}$	0.548	0.0272	0.048
4	S and P_{11}	$D = 1.524 - 0.104\text{S} + 0.048P_{11}$	0.216	0.0294	0.618

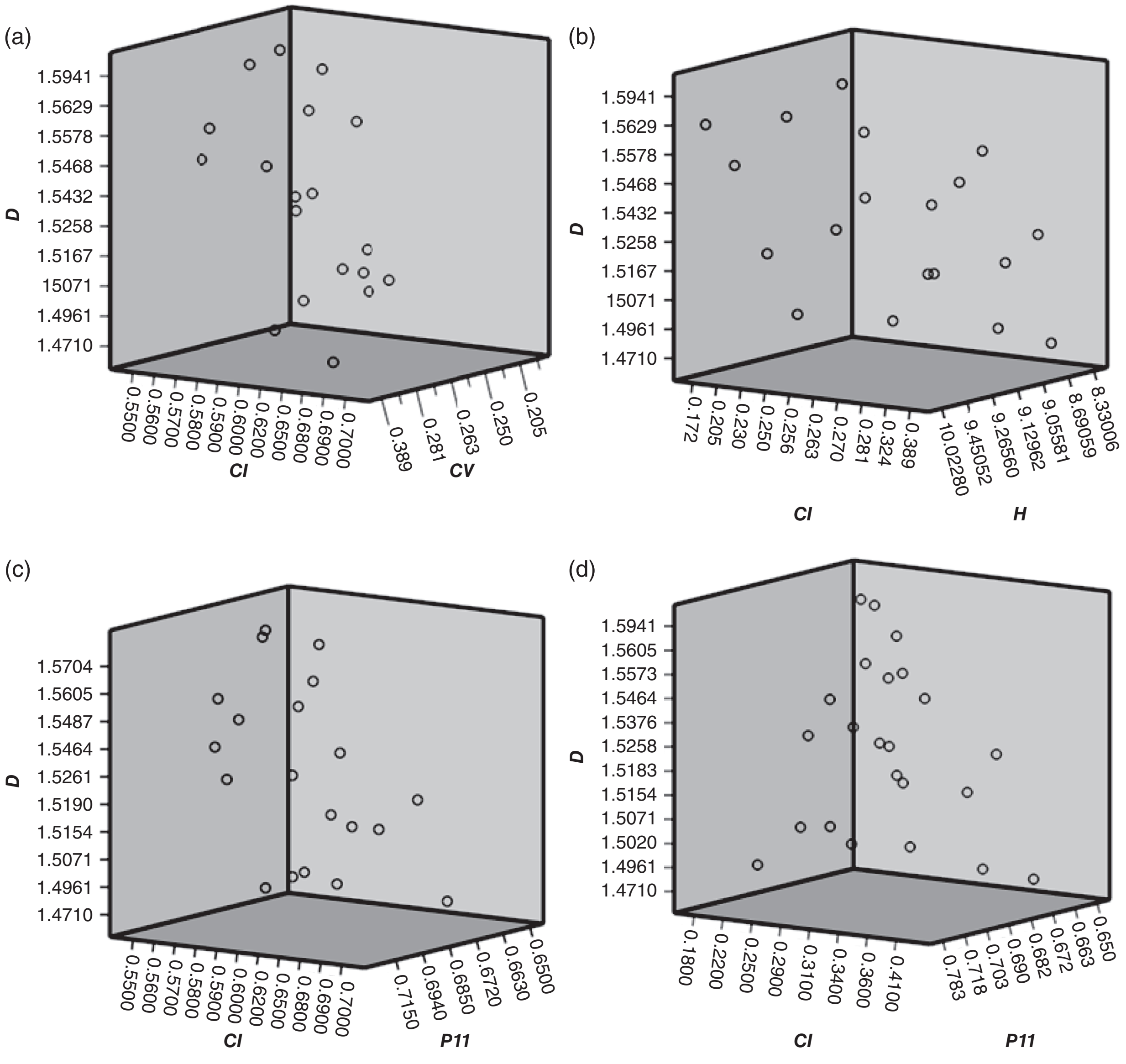

Figure 5. Representation of the dispersion of the points of agreement in models 1 (a), 2 (b), 3 (c) and 4 (d).

at 99%. The CI represents the weight of the rainiest days compared with the total number of days with rain in a series. Therefore, it can be stated that D bears a negative correlation with CI, which means that the higher the value of CI, the lower the value of D; in other words, a high daily concentration of rainfall produces a low D value, low temporal self-similarity. This is clearly seen in the fact that the highest CI values on the Iberian Peninsula occur in the eastern region, where more than 200 mm of rainfall can be accumulated in less than 24 h; it is also where the D values are lowest. Hence, a good distribution of the rainfall amount amongst the rainy days, in other words, a low CI value, raises the D value. These results are consistent insofar as rain of a convective nature is shown to be dominant in the eastern areas of the field of study compared with rain of frontal origin, which is more frequent in the north and east (Martín-Vide, 2004; Casanueva *et al.*, 2014). However, the difference in the resolution of the data used (daily and at 10-min intervals) implies there is still a part of the fractal dimension of precipitation that is not explained using CI, which is also noted in the fact that the value of Pearson's r is not equal to

-1. Some of the existing difference corresponds to the fact that the fractal dimension does not take the accumulated amounts into account, but considers whether or not the phenomenon occurs.

The annual consecutive disparity index, *S*, and *D* bear a low negative (-0.21) and not significant correlation, so it can be said that the relationship between the two indices is non-existent. *S* considers the order of the values applied here to annual rainfall series. *D* was calculated on the basis of 10-min interval values, so, from the outset, it is difficult to expect a satisfactory correlation between them given the difference in time scales of the analysis and, above all, the conceptual difference. In one case, the frequency and temporal distribution of the phenomenon is analyzed and, in the other, the variation of their consecutive amounts is examined (Martin-Vide *et al.*, 2001).

The annual coefficient of variation, CV, and the fractal dimension, *D*, do not have a statistical relationship, as the value of Pearson's *r* between the two is -0.1. The coefficient of variation was calculated on the basis of accumulated rainfall values on an annual basis. It could also be calculated monthly, though the mean values close to 0 mm in some observatories in the south of Spain in July and in other summer months would have stripped it of statistical significance. It can hence be affirmed that the annual CV and D are not mutually correlated. The first refers to the annual amounts and the second to the temporal distribution of the 10-min and 10-min-plus intervals with precipitation, regardless of the accumulated amount (Pérez-Cueva *et al.*, 2001).

Entropy, *H*, and the fractal dimension, *D*, have a good positive correlation (the value of Pearson's *r* is 0.67) and significant at 99%; therefore, the higher the entropy, the higher the fractal dimension. This would imply that the fractal dimension would reflect some of the existing disorder (or noise) in a series, regardless of the actual variability. *D* is calculated by counting those fine intervals in which precipitation is recorded, regardless of the amount accumulated, and it is somehow indicative of the 'recurrence' of precipitation over time; it would coincide with entropy in this aspect. Higher *H* values occur in observatories in the north of the peninsula, where precipitation is higher and more continuous. A higher *H* value means that the disorder of a series is higher, and this makes sense since, as it concerns data taken at 10-min intervals, the number of intervals with precipitation is much closer to the number of intervals without precipitation, making this disorder higher. In observatories in the Mediterranean region, where precipitation is more scarce and more concentrated over time, the value of *H* is lower, in other words, the disorder of the series is lower, as much more intervals without rain that those with rain are recorded, and therefore the degree of disorder (or noise) that the intervals with rain can introduce will be limited. The highest *H* values are logged in those observatories where *D* is, in turn, higher, situated in regions of the field of study where higher amounts of precipitation are accumulated over the year (north of the Iberian Peninsula). This result partly overlaps with the one presented by Liu *et al.* (2008) and by Zhao *et al.* (2011), because the observatories located further to the east, on the Mediterranean coast, present lower values. The effect of the sea on entropy is therefore not well defined in this case. The results are also somewhat consistent with the findings obtained by Mathbout *et al.* (2014, 2015) in the eastern Mediterranean, as the correlation obtained between entropy and latitude is high, something that is not replicated for the Iberian Peninsula. The findings agree with those presented in the paper by Rodriguez *et al.* (1999), as the highest values are obtained in those observatories where precipitation shows a highly irregular nature, these results being the same as those obtained for a larger field of study (Lana *et al.*, 2009).

D and the P_{11} of the rainy intervals are not related (Pearson's *r* of 0.13), therefore it can be inferred that the probability of a rainfall interval occurring after another rainfall interval does not influence the value of the fractal dimension because, in calculating this indicator, the persistence of precipitation in brief time intervals does not bear an influence.

Choosing a multivariate model of two variables rather than one with more variables corresponds to the fact that many of the indices that have been used in an endeavour to determine which precipitation characteristics explain the fractal dimension are connected to one another and therefore must not be included in the model at the same time, as the information they would contribute would be redundant. Four two-variable models were found, all of which were possible. The model with CI and CV as independent variables and the model with H and CV as independent variables show good correlations, of 0.62 and 0.63, with levels of probability of 0.020 and 0.017, respectively, with *D*, as a dependent variable, thereby defining the best regression planes. While the second presents a better correlation, so it can be assumed that the fractal dimension would be most effectively explained on the basis of entropy and the coefficient of variation, the significance provided by the CI should not be overlooked. Although the explanation provided by the coefficient of variation is low in terms of direct correlation with the fractal dimension, in both cases it complements the information regarding entropy and the CI.

The fractal dimension, *D*, therefore appears as an indication of the regular recurrence of precipitation, i.e. that the rain episodes are repeated regularly over time on different scales. This affirmation is consistent with the significant correlations obtained, positive with entropy and negative with the CI. It is the so-called self-similarity that would be reflected in higher *D* values.

Acknowledgements

The study was funded by PRECABAL project (CGL2011-29263-C02-01) from the Ministry of Science of the Spanish Government, FPU scholarship programme from the Ministry of Education of the Spanish Government, Group of Climatology (2014SGR300) from the University of Barcelona, Agencia Estatal de Meteorologia (AEMet).

References

Amaro IR, Demey JR, Macchiavelli R. 2004. Aplicación del análisis R/S de Hurst para estudiar las propiedades fractales de la precipitación en Venezuela. *Interciencia* **29–11**: 617–620.

Bodri L. 1994. Fractal analysis of climatic data: mean annual temperature records in Hungary. *Theor. Appl. Climatol.* **49**: 53–57.

Casanueva A, Rodríguez-Puebla C, Frías MD, González-Reviriego N. 2014. Variability of extreme precipitation over Europe and its relationships with teleconnection patterns. *Hydrol. Earth Syst. Sci.* **18**: 709–725.

Chou Y. 2003. *Short-term Rainfall Prediction using a Multifractal Model*. Massachusetts Institute of Technology, Master's Degree Thesis: Cambridge.

Chou CM. 2011. Wavelet-based multi-scale entropy analysis of complex rainfall time series. *Entropy* **13**: 241–253.

Chou CM. 2012. Applying multiscale entropy to the complexity analysis of rainfall-runoff relationships. *Entropy* **14**: 945–957.

Chou CM. 2014. Complexity analysis of rainfall and runoff time series based on sample entropy in different temporal scales. *Stoch. Env. Res. Risk A.* **28**: 1401–1408.

De Luis M, Brunetti M, Gonzalez-Hidalgo JC, Longares LA, Martin-Vide J. 2010. Changes in seasonal precipitation in the Iberian Peninsula during 1946–2005. *Glob. Planet. Chang.* **74**: 27–33.

Dunkerley D. 2008. Rain event properties in nature and in rainfall simulation experiments: a comparative review with recommendations for increasingly systematic study and reporting. *Hydrol. Process.* **22**: 4415–4435.

Dunkerley DL. 2010. How do the rain rates of sub-events intervals such as the maximum 5- and 15-min rates (I_5 or I_{30}) relate to the properties of the enclosing rainfall event? *Hydrol. Process.* **24**: 2425–2439.

Gao M, Hou X. 2012. Trends and multifractals analyses of precipitation data from Shandong Peninsula, China. *Am. J. Environ. Sci.* **8**(3): 271–279.

García Marín AP. 2007. *Análisis Multifractal de Series de Datos Pluviométricos en Andalucía*. Universidad de Córdoba: Córdoba, Spain.

García-Marín AP, Jiménez-Hornero FJ, Ayuso-Muñoz JL. 2008. Universal multifractal description of an hourly rainfall time series from a location in southern Spain. *Atmosfera* **21**(4): 347–355.

Ghanmi H, Bargaoui Z, Mallet C. 2013. Investigation of the fractal dimension of rainfall occurrence in a semi-arid Mediterranean climate. *Hydrol. Sci. J.* **58**(3): 483–497.

Gonzalez-Hidalgo JC, Lopez-Bustins JA, Štepánek P, Martin-Vide J, de Luis M. 2009. Monthly precipitation trends on the Mediterranean fringe of the Iberian Peninsula during the second half of the twentieth century (1951–2000). *Int. J. Climatol.* **29**: 1415–1429.

Goodchild MF. 1980. Fractals and the accuracy of geographical measures. *Math. Geol.* **12**(2): 85–98.

Goodchild MF, Mark DM. 1987. The fractal nature of geographic phenomena. *Ann. Assoc. Am. Geogr.* **77**(2): 265–278.

Goodess CM, Jones PD. 2002. Links between circulation and changes in the characteristics of Iberian rainfall. *Int. J. Climatol.* **22**: 1593–1615.

Groisman PY, Knight RW. 2008. Prolonged dry episodes over the conterminous United States: new tendencies emerging during the last 40 years. *J. Clim.* **21**: 1850–1862.

Gusev AA, Ponomoreva VV, Braitseva OA, Melekestsev IV, Sulerzhitsky LD. 2003. Great explosive eruptions on Kamchatka during the last 10000 years: Self-similar irregularity of the output of volcanic products. *J. Geophys. Res.-Solid Earth* **108**: Art. No. 2126.

Hao Z, Singh VP. 2013. Entropy-based method for extreme rainfall analysis in Texas. *J. Geophys. Res.* **118**: 263–273, doi: 10.1029/2011JD017394.

Hastings HM, Sugihara G. 1994. *Fractals: A User's Guide for the Natural Sciences*. Oxford University Press: Oxford, UK.

Kalauzi A, Cukic M, Millá H, Bonafoni S, Biondi R. 2009. Comparison of fractal dimension oscillations and trends of rainfall data from Pastaza Province, Ecuador and Veneto, Italy. *Atmos. Res.* **93**: 673–679.

Khan MS, Siddiqui TA. 2012. Estimation of fractal dimension of a noisy time series. *Int. J. Comput. Appl.* **45**(10): 1–6.

King MR. 2005. Fractal analysis of eight glacial cycles from an Antarctic ice core. *Chaos Soliton. Fract.* **25**: 5–10.

Kutiel H, Trigo RM. 2014. The rainfall regime in Lisbon in the last 150 years. *Theor. Appl. Climatol.* **118**: 387–403, doi: 10.1007/s00704-013-1066-y.

Lana X, Burgueño A, Martínez MD, Serra C. 2009. A review of statistical analyses on monthly and daily rainfall in Catalonia. *Tethys* **6**: 15–29.

Langousis A, Veneziano D, Furcolo P, Lepore C. 2009. Multifractal rainfall extremes: theoretical analysis and practical estimation. *Chaos Soliton. Fract.* **39**: 1182–1194.

Liu Q, Yang Z, Baoshan C. 2008. Spatial and temporal variability of annual precipitation during 1016–2006 in Yellow River Basin, China. *J. Hydrol.* **361**: 330–338.

Liu B, Chen X, Lian Y, Wu L. 2013. Entropy-based assessment and zoning of rainfall distribution. *J. Hydrol.* **490**: 32–40.

López Lambraño AA. 2012. *Análisis Multifractal y Modelación de la Precipitación*. Universidad Autónoma de Querétaro: Querétaro, Mexico.

Mandelbrot BB. 1976. *The Fractal Geometry of Nature*. W.H. Freeman and Company: New York, NY.

Martín-Vide J. 2004. Spatial distribution of a daily precipitation concentration index in Peninsular Spain. *Int. J. Climatol.* **24**: 959–971.

Martín-Vide J. 2008. La nueva realidad del calentamiento global. Un decálogo del cambio climático. *Scripta Nova, Revista Electrónica de Geografía y Ciencias Sociales* **12**(270): 23.

Martín-Vide J, Estrada Mateu J. 1998. Una nueva propuesta metodológica de regímenes pluviométricos estacionales para la Península Ibérica. *Nimbus* **1–2**: 85–92.

Martín-Vide J, Gomez L. 1999. Regionalization of peninsular Spain based on the length of dry spells. *Int. J. Climatol.* **19**: 537–555.

Martin-Vide J, Lopez-Bustins JA. 2006. The Western Mediterranean Oscillation and rainfall in the Iberian Peninsula. *Int. J. Climatol.* **26**: 1455–1475.

Martín-Vide J, Olcina Cantos J. 2001. *Climas y tiempos de España*. Alianza Editorial: Madrid, Spain.

Martin-Vide J, Prohom M, Bohigas M, Peña JC, Esteban P, Montserrat D. 2001. Índices de irregularidad temporal y dimensión fractal de la precipitación anual en España. In *El Tiempo y el Clima*, Pérez-Cueva AJ, López Baeza E, Tamayo Carmona J (eds). Publicaciones de la Asociación Española de Climatología: Valencia, Spain.

Mathbout S, López-Bustins JA, Luterbacher J, Martín-Vide J. 2014. Precipitation trends and variability within the Eastern Mediterranean region for the 1961–2012. Presented at the *EMS Annual Meeting Abstracts*, vol. 11.

Mathbout S, López-Bustins JA, Martín-Vide J. 2015. Study of precipitation variability based on entropy and seasonality over the Eastern Mediterranean. Presented at the *Geophysical Research Abstracts, EGU General Assembly*, vol. 17.

Mazzarella A, Rapetti F. 2004. Scale-invariance laws in the recurrence interval of extreme floods: an application to the upper Po river valley (northern Italy). *J. Hydrol.* **288**: 264–271.

Meseguer-Ruiz O, Martín-Vide J. 2014. Análisis de la fractalidad temporal de la precipitación en Cataluña, España (2010). *Invest. Geogr.* **47**: 41–52.

Montesarchio V, Ridolfi E, Russo F, Napolitano F. 2011. Rainfall threshold definition using an entropy decision approach and radar data. *Nat. Hazards Earth Syst. Sci.* **11**: 2061–2074, doi: 10.5194/nhess-11-2061-2011.

Nunes SA, Romani LAS, Avila AMH, Coltri PP, Traina C, Cordeiro RLF, De Sousa EPM, Traina AJM. 2011. Fractal-based analysis to identify trend changes in multiple climate time series. *J. Inform. Data Manage.* **2**: 51–57.

Nunes SA, Romani LAS, Avila AMH, Coltri PP, Traina C, Cordeiro RLF, De Sousa EPM, Traina AJM. 2013. Analysis of large scale climate data: how well climate change models and data from real sensor networks agree? In *Proceedings of the IW3C2 WWW 2013 Conference*, Schwabe D, Almeida V, Glaser H, Baeza-Yates R, Moon S (eds) IW3C2: Rio de Janeiro, Brasil.

Olsson J, Niemczynowicz J, Berndtsson R, Larson M. 1992. An analysis of the rainfall time structure by box counting – some practical implications. *J. Hydrol.* **137**: 261–277.

Oñate Rubalcaba JJ. 1997. Fractal analysis of climatic data: annual precipitation records in Spain. *Theor. Appl. Climatol.* **56**: 83–87.

Peitgen HO, Jürgens H, Saupe D. 1992. *Chaos and Fractals: New Frontiers of Science*. Springer: New York, NY.

Pelletier JD. 1997. Analysis and modelling of the natural variability of climate. *J. Clim.* **10**: 1331–1342.

Pérez SP, Sierra EM, Massobrio MJ, Momo FR. 2009. Análisis fractal de la precipitación anual en el este de la Provincia de la Pampa, Argentina. *Rev. Climatol.* **9**: 25–31.

Pérez-Cueva AJ, López Baeza E, Tamayo CJ. 2001. *El Tiempo del Clima*. Publicaciones de la Asociación Española de Climatología: Valencia, Spain.

Pionello P. 2012. *The Climate of the Mediterranean Region. From the Past to the Future*. Elsevier Insights: London.

Raidl A. 1996. Estimating the fractal dimension, K-2-entropy, and the predictability of the atmosphere. *Czechoslov. J. Phys.* **46**: 296–328.

Rangarajan G, Sant DA. 1997. A climate predictability index and its applications. *Geophys. Res. Lett.* **24**: 1239–1242.

Rangarajan G, Sant DA. 2004. Fractal dimensional analysis of Indian climatic dynamics. *Chaos Soliton. Fract.* **19**: 285–291.

Rehman S. 2009. Study of Saudi Arabian climatic conditions using Hurst exponent and climatic predictability index. *Chaos Soliton. Fract.* **39**: 499–509.

Rehman S, Siddiqi AH. 2009. Wavelet based Hurst exponent and fractal dimensional analysis of Saudi climatic dynamics. *Chaos Soliton. Fract.* **40**: 1081–1090.

Reiser H, Kutiel H. 2010. Rainfall uncertainty in the Mediterranean: intraseasonal rainfall distribution. *Theor. Appl. Climatol.* **100**: 105–121.

Rodríguez R, Llasat MC, Wheeler D. 1999. Analysis of the Barcelona precipitation series 1850–1991. *Int. J. Climatol.* **19**: 787–801.

Rodríguez R, Casas MC, Redaño A. 2013. Multifractal analysis of the rainfall time distribution on the metropolitan area of Barcelona (Spain). *Meteorog. Atmos. Phys.* **121**: 181–187.

Rodríguez-Puebla C, Nieto S. 2010. Trends of precipitation over the Iberian Peninsula and the North Atlantic Oscillation under climate change conditions. *Int. J. Climatol.* **30**: 1807–1815.

Rodríguez-Puebla C, Encinas AH, Nieto S, Garmendia J. 1998. Spatial and temporal patterns of annual precipitation variability over the Iberian Peninsula. *Int. J. Climatol.* **18**: 299–316.

Rodríguez-Puebla C, Encinas AH, Sáenz J. 2001. Winter precipitation over the Iberian Peninsula and its relationship to circulation indices. *Hydrol. Earth Syst. Sci.* **5**(2): 233–244.

Sáenz J, Zubillaga J, Rodríguez-Puebla C. 2001. Interannual variability of winter precipitation in northern Iberian Peninsula. *Int. J. Climatol.* **21**: 1503–1513.

Sahay JD, Sreenivasan KR. 1996. The search for a low-dimensional characterization of a local climate system. *Philos. Trans. R. Soc.* **354**: 1715–1750.

Selvi T, Selvaraj S. 2011. Fractal dimension analysis of Northeast monsoon of Tamil Nadu. *Univers J Environ Res Technol* **1**(2): 219–221.

Shanon CE. 1948. A mathematical theory of communication. *Bell Syst. Tech. J.* **27**: 623–656.

Sivakumar B. 2001. Is a chaotic multi-fractal approach for rainfall possible? *Hydrol. Process.* **15**: 943–955.

Strahler AN. 1977. *Geografía Física*. Ediciones Omega: Barcelona, España.

Tuček P, Marek L, Paszto V, Janoška Z, Dančák M. 2011. Fractal perspectives of GIScience based on the leaf shape analysis. In *GeoComputation Conference Proceedings*, University College London, London, 169–176.

Valdez-Cepeda RD, Hernandez-Ramirez D, Mendoza B, Valdes-Galicia J, Maravilla D. 2003. Fractality of monthly extreme minimum temperature. *Fractals* **11**: 137–144.

Veneziano D, Furcolo P. 2002. Multifractality of rainfall and scaling of intensity-duration-frequency curves. *Water Resour. Res.* **38**(12): 1306, doi: 10.1029/2001WR000372.

Veneziano D, Langousis A, Furcolo P. 2006. Multifractality and rainfall extremes: a review. *Water Resour. Res.* **42**: W06D15, doi: 10.1029/2005WR004716.

Wheeler D, Martín-Vide J. 1992. Rainfall characteristics of Mainland Europe's most southerly stations. *Int. J. Climatol.* **12**: 96–76.

Zhao C, Ding Y, Ye B, Yao S, Zhao Q, Wang Z, Wang Y. 2011. An analyses of long-term precipitation variability based on entropy over Xinjiang, northwestern China. *Hydrol. Earth Syst. Sci. Discuss.* **8**: 2975–2999.

Zhou X. 2004. *Fractal and Multifractal Analysis of Runoff Time Series and Stream Networks in Agricultural Watersheds*. Virginia Polytechnic Institute and State University: Blacksburg, Virginia.

Zolina O. 2014. Multidecadal trends in the duration of wet spells and associated intensity of precipitation as revealed by a very dense observational German network. *Environ. Res. Lett.* **9**: 025003, doi: 10.1088/1748-9326/9/2/025003.

Zolina O, Simmer C, Gulev SK, Kollet S. 2010. Changing structure of European precipitation: longer wet periods leading to more abundant rainfalls. *Geophys. Res. Lett.* **37**: L06704, doi: 10.1029/2010GL042468.

