Performance Enhancement in OFDM Systems with ICI Utilizing the Improved Double Jump Linear Combination Pulse

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***Abstract*—It is well-know that carrier frequency offset in the subcarriers of orthogonal frequency division multiplexing (OFDM) leads to intercarrier interference (ICI); therefore, system degradation occurs. Nevertheless, this issue may effectively be mitigated via the pulse shaping filter method. In this manuscript, we present the improved double jump linear combination (IDJLC) function. With the goal of minimizing the bit error rate (BER), we then optimize it by employing its extra design parameters. Finally, our filter is compared with some of the best and most recent filters found in the literature in terms of ICI, signal to interference ratio, signal to noise ratio and normalized frequency offset requirements at a BER threshold of 10^{-3}, frequency profile, and average elapsed time. In general, the performance of the system is improved by using the IDJLC pulse.**

***Index Terms*—bit error rate, carrier frequency offset, improved double jump linear combination filter, intercarrier interference, orthogonal frequency division multiplexing.**

I. Introduction

Nowadays, orthogonal frequency division multiplexing (OFDM) has been considered as the modulation technique by several standards, for instance, digital subscriber line (DSL), digital video broadcasting (DVB), power line communication (PLC), long-term evolution (LTE), wireless fidelity (WiFi), and worldwide interoperability for microwave (WiMAX) [1], [2]. Furthermore, OFDM has been proposed for future communication systems, including the next generation passive optical networks (NG-PON) [3], evolution of WiFi (WiGig) [4], and 5^{th} generation cellular networks (5G) [5].

As any technology, OFDM exhibits both advantages and disadvantages [6]. On the one hand, OFDM is highlighted by its high spectral efficiency, robustness against intersymbol interference as well as fading, easy implementation, low sensitivity to time synchronization errors, among others. On the other hand, OFDM is very sensitive to phase noise and frequency offset, and has a large peak-to-average power ratio. In this manuscript, we will focus on frequency offset sensitivity.

The frequency offset can come from frequency differences between transmitter and receiver local oscillators, Doppler spread, and distortion in the wireless channel [2], [7]. The impact of carrier frequency offset on OFDM subcarriers has been firstly studied by Pollet et al. [8], showing that it mainly leads to intercarrier interference (ICI) and, consequently, system degradation. In this regard, numerous techniques, such as frequency domain equalization [9], frequency error correction [10], ICI self-cancellation scheme [11], pilot insertion [12], and receiver side windowing [13], have been developed and proved. In this paper, we will focus on the pulse shaping filter method.

From classical Nyquist-I pulses, which only depend on the roll-off factor [14]–[16], to modern pulse shaping functions, which are characterized by additional degrees of freedom for a given roll-off factor [17]–[21], the OFDM tolerance to carrier frequency offset has been increased. Among the first, the double jump filters are the best in terms of ICI, signal to interference ratio (SIR), and bit error rate (BER) [15], [16]. Recently, the improved double jump 1 (IDJ1) pulse has been introduced [21], displaying that its system performance is superior to the rest of pulses. However, the design and implementation of better functions are still imperative in order to overcome the increase of higher error-free data rates in next-generation networks [3]–[5].

In this manuscript, we present the improved double jump linear combination (IDJLC) filter, which is later optimized for the purpose of the BER minimization. To conclude, we contrasted the IDJLC pulse with the most relevant Nyquist-I pulses in terms of the ICI, SIR, BER, and time complexity. The proposed filter outperforms the other functions in all of the considered performance metrics except the latter. These discoveries are explained by analyzing the frequency response of each filter.

The remainder of the paper is organized as follows: in Section II, we present the received OFDM signal and performance metrics. Section III reveals the IDJLC pulse and its optimal design parameters. Section IV shows the comparison between the proposed filter and the most popular pulse shaping functions in terms of various performance metrics. We report conclusions in Section V.

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TABLE I
FREQUENCY RESPONSE OF THE BEST AND RECENT NYQUIST-I PULSES.

Function	*Spectra*
Improved parametric metric linear combination (IPLC) [17]	$exp[-\epsilon(\pi fT)^2]\{sinc(fT)[\mu sinc(\alpha fT)+(1+\mu)sinc^2(\frac{\alpha fT}{2})]\}^\gamma$
Sinc parametric exponential (SPE) [18]	$sinc^2(bfT)[sinc(fT)\frac{2\beta fT sin(\pi\alpha fT)+2cos(\pi\alpha fT)-1}{(\beta fT)^2+1}]^\gamma$ with $\beta=\frac{\pi\alpha}{ln2}$
Sinc parametric linear (SPL) [18]	$sinc^2(bfT)[sinc(fT)sinc(\alpha fT)]^p$
Sinc parametric linear combination (SPLC) [19]	$sinc^\gamma(\frac{bfT}{\pi})sinc(fT)[\mu sinc(\alpha fT)+(1+\mu)sinc^2(\frac{\alpha fT}{2})]$
Sinc exponential (SE) [20]	$exp[-\gamma(fT)^2]sinc(\beta fT)\frac{1-2cos(\pi\alpha fT)}{(3\alpha fT)^2-1}$
IDJ1 [21]	$exp[-\beta(fT)^2]sinc(fT)[(1-\alpha)cos(\pi\alpha fT)+\alpha sinc(\alpha fT)]$
IDJLC	$sinc(fT)[(1-\alpha\beta)cos(\pi\alpha fT)+\alpha\beta sinc(\alpha fT)]exp[-\gamma(fT)^2]$

II. RECEIVED OFDM SIGNAL AND PERFORMANCE METRICS

After frequency down-conversion, the complex-valued input to the OFDM demodulator can be written as [14], [15]

$$y(t)=\sum_{m=0}^{N-1}c_m p(t)exp\{j[2\pi(m/T+\Delta f)t]\}+n(t), \quad (1)$$

with N, c_m, $p(t)$, T, Δf, and $n(t)$ representing the number of subcarriers, symbol constellation, Nyquist-I pulse that limits each OFDM symbol, OFDM symbol period, frequency offset, and additive white Gaussian noise (AWGN), respectively. The frequency offset might be due to dispersion and Doppler spread in the wireless channel, or oscillator frequency detuning from the received carrier frequency. Meanwhile, the AWGN is added to have a realistic approach [2], [6], [7]. Notice that we do not take into account the invariant phase shift since it may be suppressed via pilot-assisted phase-noise correction [22], which is always incorporated in the majority of OFDM modems for wireless channel equalization [2], [6], [7].

Various performance metrics are employed along the paper. First, for an unmodulated OFDM symbol, the mean of ICI power acquires the form of [14], [15]

$$ICI\approx\sum_{m=0,m\neq N/2}^{N-1}\left|P\left(\frac{m-N/2}{T}+\Delta f\right)\right|^2, \quad (2)$$

where $P(f)$ is the Fourier transform of $p(t)$, i.e., $P(f)=\mathcal{F}\{p(t)\}$. In this context, the average signal power to average ICI power ratio, known as SIR, is also utilized. It is given by [14], [15]

$$SIR=\frac{|P(\Delta f)|^2}{ICI}. \quad (3)$$

Finally, when the binary phase shift keying (BPSK) modulates each subcarrier, the BER is expressed as [23]

$$BER=1-(1-BER_{sym})^N, \quad (4a)$$

$$BER_{sym}=\frac{1}{2}\{Q[P(\Delta f)(1+SIR^{-1/2})(2SNR)^{1/2}] +Q[P(\Delta f)(1-SIR^{-1/2})(2SNR)^{1/2}]\}, \quad (4b)$$

where BER_{sym} is the BER per OFDM symbol and $Q[.]$ denotes the Gaussian co-error function. As mentioned, the common phase noise is excluded. Unlike ICI and SIR, the BER can correspond to experimental observations because AWGN is considered. In regards to the forward error correction (FEC) limit, we adopt a BER threshold of 10^{-3} as our previous researches [21], [24], [25]. It is worth to mention that (2), (3), and (4) demonstrate how the system performance may be improved by using different pulse shaping filters.

III. IMPROVED DOUBLE JUMP LINEAR COMBINATION FUNCTION

For OFDM to perform adequately, the orthogonality between its subcarriers must be kept during symbol transmission all the way to the detector, namely, the Nyquist-I criterion must be fulfilled [26]. At the frequency domain, all Nyquist-I pulses are characterized by [17], [23]

$$P(f)=\begin{cases}1, & f=0\\ 0, & f=\pm 1/T,\pm 2/T,...\end{cases}. \quad (5)$$

Eq. (5) implies that the pulse shaping function spectrum possesses both unit-amplitude at the frequency subcarrier and spectral nulls at the separation between subcarriers.

In this manuscript, we introduce the IDJLC filter to achieve a performance improvement in OFDM-based systems corrupted by carrier frequency offset. Following the methodology proposed in [17], [19], our pulse results from an exponential function times a linear combination of double jump functions and, thus, has two additional degrees of freedom, β and γ, for a certain roll-off factor, α. The double jump filters were chosen since they outperform the rest of the general pulse shaping filters in terms of the BER [15], [16]. Moreover, the additional design parameters are defined for any real number, whereas α may take values between 0 and 1. The frequency response of the IDJLC function, along with some of the best and recent functions found in the literature, is exposed in Table I. We present the rest of the results by setting α to 0.22, following the 3rd Generation Partnership Project (3GPP) LTE [27], [28], which of course adopts OFDM as modulation scheme [29], [30]. Fig. 1 depicts the frequency profile of our filter with (a) β as parameter while γ is fixed to 1, and (b) γ as parameter while β is fixed to -32. We selected these fixed values by their proximity to the optimal values as will be seen later. Fig. 1(a) reveals that for positive values of β, the main-lobe increases, whereas the magnitude of the side-lobes downward increases as β increases. When β is negative, nevertheless, as β decreases, the central-lobe decreases, but lateral-lobes

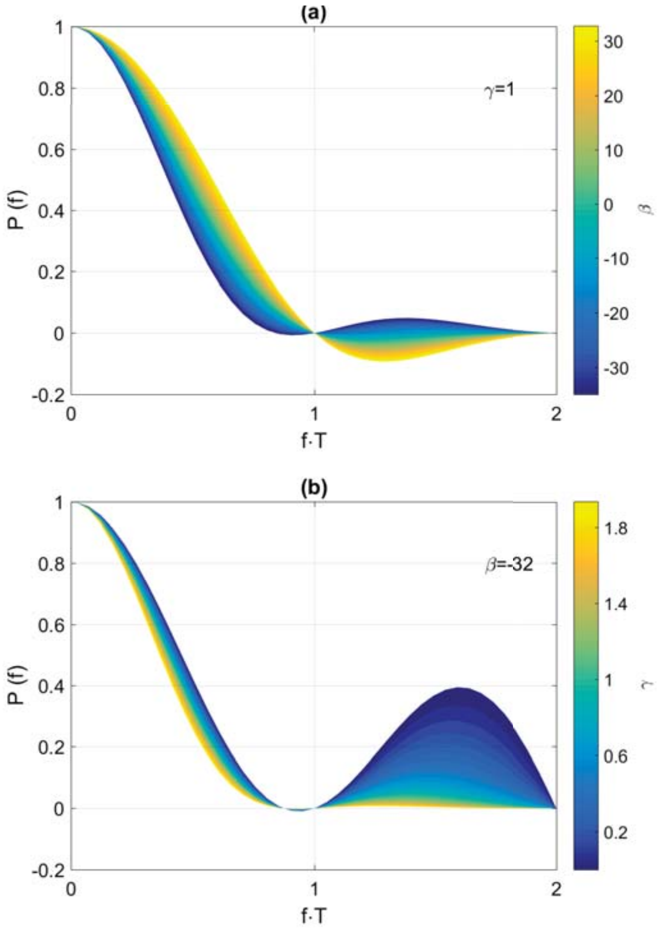

Fig. 1. Frequency response of the IDJLC filter with (a) β as parameter while γ is equal to 1, and (b) γ as parameter while β is equal to -32.

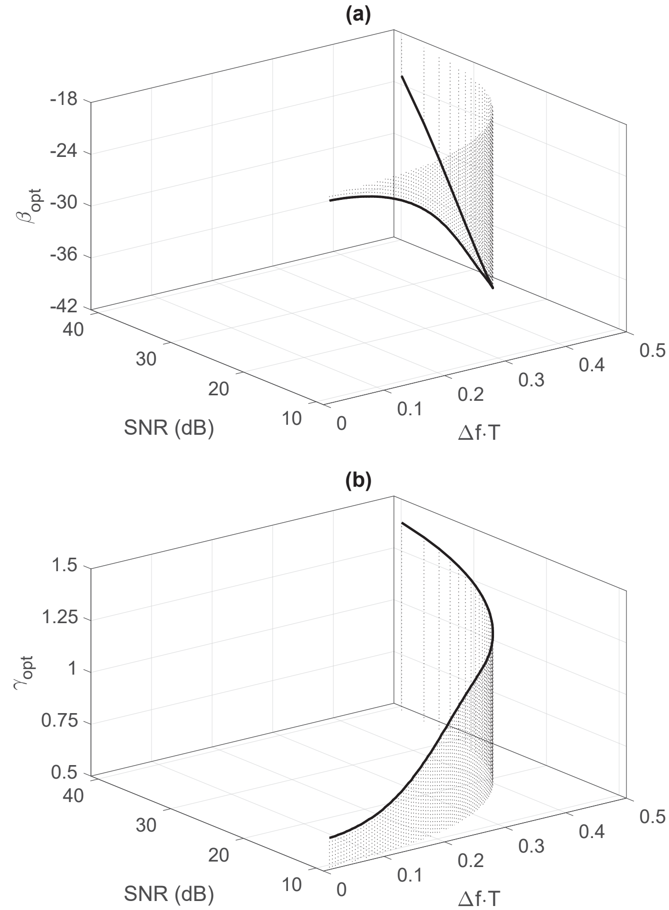

Fig. 2. At the FEC limit, the optimum (a) β and (b) γ as a function of the SNR and frequency offset times OFDM symbol period.

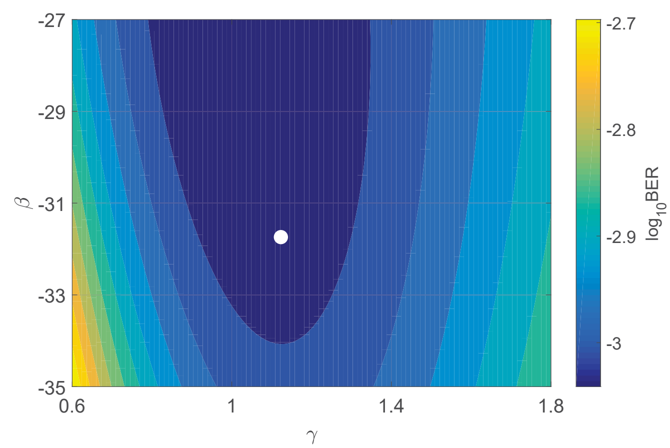

Fig. 3. For $\Delta fT = 0.25$ and SNR=12 dB, the BER in terms of β and γ. The white point represents the system performance with the optimal values of the extra design parameters.

increase upward. Furthermore, Fig. 1(b) shows that the main-lobe as well as the side-lobes increase as γ decreases. Negative γs are excluded because these completely degrade the system performance. In this context, a Nyquist-I pulse with not only a broad central-lobe but also smaller positive lateral-lobes is main design goal [14], [15].

A. Optimization

As stated in (4) and our pulse spectra, the BER can be minimized through the optimal extra design parameters, knowing the number of subcarriers, SNR, frequency offset, OFDM symbol period, and roll-off factor. In the following, an OFDM signal with 64 subcarriers is considered for comparison purposes [17]–[21], and as mentioned, α is equal to 0.22. At the FEC limit, namely, in the worst scenario, by using the $fminsearch$ MATLAB function [31], the optimum β and γ as a function the SNR and normalized frequency offset are therefore plotted in Figs. 2(a) and 2(b), respectively. This algorithm employs the derivative-free method of Lagarias et al. [32] to find the minimum of an unconstrained multi-variable function, such as the BER. Evidently, the optimal degrees of freedom depend on the SNR and frequency offset times the OFDM symbol period. Owing to the unpredictable by an OFDM modem of the amplitude and phase noises, we choose the mean of these values, which are $\beta_{opt} = -31.7507$ and $\gamma_{opt} = 1.1242$, as the best choices throughout the paper. Notice that when $\Delta f \cdot T$ is greater than or equal to 0.5, there is intercarrier interference and, consequently, the FEC

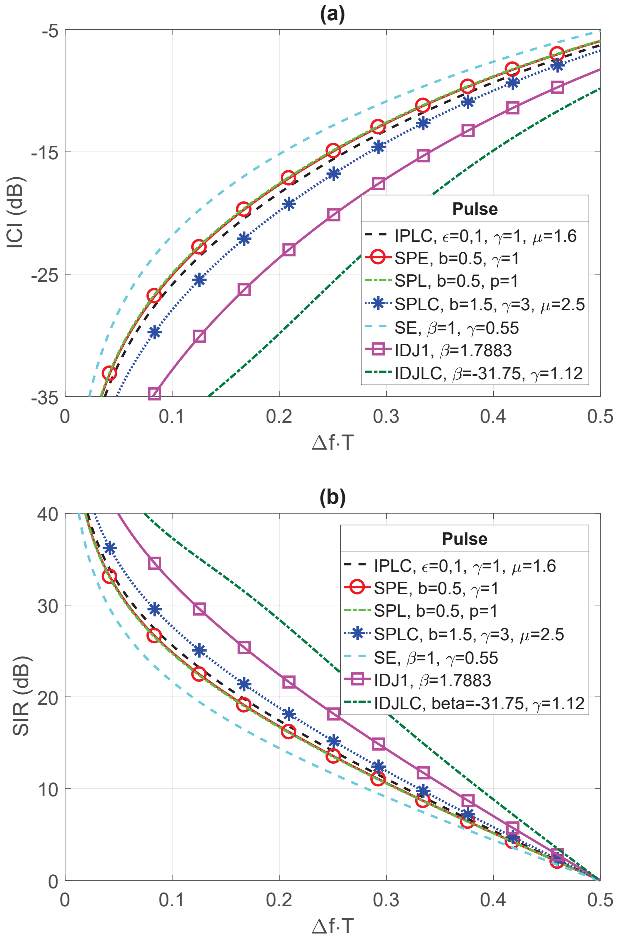

Fig. 4. (a) ICI and (b) SIR in terms of the normalized frequency offset for various Nyquist-I pulses.

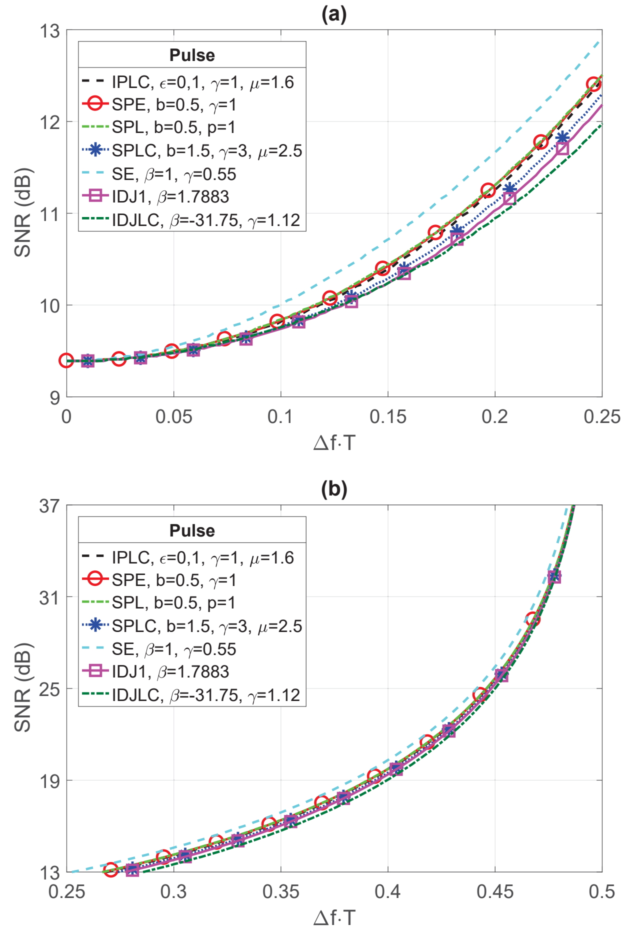

Fig. 5. At the FEC limit, the SNR and normalized frequency offset requirements with the pulse shaping filter as parameter for (a) low and (b) high ICIs.

limit is unattainable [33], [34]. In order to verify the previous result, Fig. 3 displays the BER in terms of β and γ if a given application imposes $\Delta f \cdot T = 0.25$ and SNR=12 dB. It is clear that the BER is minimized thanks to the optimum values of the design parameters, see the white point in Fig. 3. This trend is the same for other amplitude and phase noises.

IV. System performance

In the current section, we carry out the comparison of the studied Nyquist-I pulses, see Table I, where the IDJLC filter is included, in terms of ICI, SIR, BER, frequency response, and time complexity. The design parameters of the rest of pulse shaping functions are the ones that enhance the system performance [17]–[21], which were obtained via numerous numerical simulations in [17]–[20] and a similar methodology in [21].

First of all, ICI and SIR against normalized frequency offset for the different pulse shaping functions are shown in Figs. 4(a) and 4(b), respectively. As the frequency offset increases, ICI increases while SIR decreases. By utilizing our pulse, both performance metrics are improved, whereas the SE function exhibits the worst results. In the absence of frequency offset, ICI tends to $-\infty$ dB [14], [15], [23] and, hence, SIR= ∞ dB, see (3). In addition, as mentioned, these results do not make sense when $\Delta f \cdot T \geq 0.5$ because the BER reaches its maximum value.

Afterwards, at the threshold of BER= 10^{-3}, Fig. 5 depicts the SNR and frequency times OFDM symbol period requirements with the Nyquist-I pulse as parameter for (a) low and (b) high ICIs. Bellow a SNR of 9.5 dB, the system performance is limited by AWGN. In this context, as the SNR increases, the frequency offset slowly increases for low ICIs, see Fig. 5(a), but rapidly increases for high ICIs, see Fig. 5(b). Beyond $\Delta f \cdot T = 0.5$, the FEC limit may obviously not be accomplished. Between these boundaries, once again, the IDJLC filter outperforms the other filters, which presents more relevance for medium ICIs. Of course, without carrier frequency offset, the BER remains invariant to the pulse shaping function technique.

We finally attribute the superiority of the proposed pulse among the rest of the considered Nyquist-I pulses via its frequency profile, revealed in Fig. 6. Our function is characterized by being the only one that possesses negligible positive sidelobes, which can be interpreted as constructive interferences.

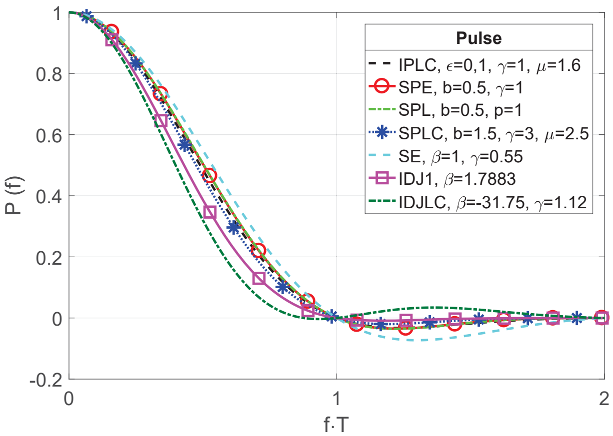

Fig. 6. Nyquist-I pulses spectra.

The other filters also expose negligible lateral-lobes, but these are negative and, therefore, result in destructive interferences. Regarding the main lobe of the all pulses are no-narrow, allowing an easy synchronization between local oscillators in the transmitter and receiver, namely, a successful transmission.

A. Complexity evaluation

In order to reduce the OFDM modem complexity, a pulse shaping filter characterized by an easy design and implementation is extremely desired [35]. We measure the pulse-time complexity based on the methodology firstly developed by Kamal et al. [19]. It observes the elapsed time for any function, measuring the execution time of its frequency response given the frequency and OFDM symbol period via the tic and toc MATLAB commands [36]. The tic function records an internal time of the computer system and, then, the elapsed time is displayed with the toc function. At the zero frequency, the average elapsed times for the studied filters are estimated over 100 runs, which are illustrated in Table II. Our computer system specifications are listed in Table III and, evidently, condition these results. Based on the fact of the spectral-expression complexity, see Table I, the SE and SPLC pulses take the least and longest times, respectively. In regards to our pulse, it consumes the third minimum time, but as demonstrated, it outperforms the other functions in terms of ICI, SIR, and BER.

TABLE II
AVERAGE ELAPSED TIME OF THE PULSE SHAPING FILTER SPECTRA.

Function	Time (μs)
IPLC	8.9159
SPE	6.6925
SPL	8.8944
SPLC	11.629
SE	3.6389
IDJ1	6.2959
IDJLC	6.4374

TABLE III
COMPUTER SYSTEM SPECIFICATIONS.

Title	Technology
Processor	Intel(R) Core(TM) i5
Speed	2.6 GHz
RAM	4 GB

V. CONCLUSION

We initially introduced the IDJLC pulse. It comes from an exponential function times a linear combination of double jump functions and, hence, is characterized by two extra degrees of freedom, called as β and γ, for a certain roll-off factor, α. To minimize the BER, the optimal design parameters of our filter are determined according to $\alpha = 0.22$ and 64 subcarriers. These correspond to $\beta_{opt} = -31.7507$ and $\gamma_{opt} = 1.1242$, which are also verified for particular amplitude and phase noises. To conclude, we contrasted the proposed pulse with the most recent Nyquist-I pulses via various performance metrics. On the one hand, the IDJLC function outperforms the rest of functions in terms of ICI, SIR, and SNR and frequency offset times OFDM symbol period requirements at the FEC limit. On the other hand, the OFDM modem design and implementation may be simplified by utilizing the SE filter, but the IDJLC filter is also suitable for OFDM systems due to its minimum time complexity. Nevertheless, this complexity evaluation could be confirmed through the popular big O notation, which does not depends on the characteristics of the system. All of the results are explained by the study of the pulse spectra. In particular, our function has not only a broad central-lobe but also negligible positive lateral-lobes, namely, the most closest frequency profile.

In addition, future works can be direct towards its possible implementation. In this sense, both a dispersive channel which originates frequency-selectivity and a matched filter for optimal reception must be consider. Also, the analysis must be done at the discrete-time domain in order to evidence the effects of the filter digitalization, and the design and implementation complexity must take into account a filter-bank for optimal reception.

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