Induced Generalized Ordered Weighted Logarithmic Aggregation Operators

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***Abstract*—We present the induced generalized ordered weighted logarithmic aggregation (IGOWLA) operator. It is an extension of the generalized ordered weighted logarithmic aggregation (GOWLA) operator. The IGOWLA operator uses order-induced variables that modify the reordering mechanism of the arguments to be aggregated. The main advantage of the induced process is the consideration of the complex attitude of the decision makers. We study some properties of the IGOWLA operator, such as idempotency, commutativity, boundedness and monotonicity. Finally we present an illustrative example of a group decision-making procedure using a multi-person analysis and the IGOWLA operator in the area of innovation management.**

Keywords— OWA operator; logarithmic aggregation operators; induced aggregation operators; group decision-making

I. INTRODUCTION

The wide range of problems that aggregation operators consider has attracted much attention of the literature, especially in the areas of economy, statistics and engineering [1]–[3]. Nowadays, the literature presents an extensive amount of aggregation operator [4]. One of the most well known methods of aggregation is the OWA operator. The parameterized families proposed in the OWA operator include the minimum, the maximum and the average. Since its appearance, the OWA operator has been applied extensively in diverse applications such as expert systems, group decision making, neural networks, data base systems, fuzzy systems, among others [5], [6].

A common extension of the OWA operator developed by Yager and Filev [7], it is called the induced ordered weighted aggregation (IOWA) operator. This extension allows a broader treatment of complex information applying an alternative reordering process, i.e. instead of a descending ordering of the arguments; a set of order-induced variables dictates the order of the aggregation. The characteristics of the IOWA operator have attracted much attention, motivating a varied range of applications [8], [9]. For example, Chen and Chen [10] studied the use of fuzzy numbers, Wei and Zhao [11] considered intuitionistic fuzzy information, Xu [12] and Xian et al. [13] developed induced aggregation operators with linguistic information, and Merigó and Casanovas [14] with distance measures.

In 2010 Zhou and Chen [15] proposed a generalization of the ordered weighted geometric averaging (OWGA) operator based on an optimal model. The new operator was named generalized ordered weighted logarithmic aggregation (GOWLA) operator. It introduces a set of parameterized families including the step generalized ordered weighted logarithmic averaging (Step-GOWLA) operator, the window generalized ordered weighted logarithmic averaging (Window-GOWLA) operator, the S-GOWLA, among others. The objective of this paper is the presentation of the induced generalized ordered weighted logarithmic aggregation (IGOWLA) operator. It is an extension of the optimal deviation model developed by Zhou and Chen [15], with the addition of order-induced variables that modify the reordering mechanism of the arguments, it is designed to consider a broader representation of the complex attitude of the decision makers.

Additionally, we propose an illustrative example of a multi-person decision making analysis in the field of innovation management. The application is designed to evaluate a strategic decision making process, where a series of experts need to evaluate the performance of new concepts to develop with a highly complex attitudinal character of the management. The results show a difference in the aggregation ranking when applying ordered-induced variables instead of using traditional operators. The operator could be useful for other decision-making applications in business, such as human resource management, strategic decision-making and marketing.

The paper is organized as follows. In Section II we present basic concepts of the OWA, IOWA and GOWLA operators, in which our propositions are founded. Section III presents the IGOWLA operator, its main concepts, properties and families. Section IV presents an illustrative application of a decision-making procedure utilizing the new operator. Finally, Section V summarizes the concluding remarks of the paper.

II. PRELIMINARIES

A. The OWA operator

The ordered weighted averaging operator, introduced by Yager [16] proposes a family of aggregation operators that have been used in a plethora of applications, see [5]. The OWA operator can be defined as follows:

Definition 1. An OWA operator is a mapping OWA: $R^n \rightarrow R$, that has an associated n weighting vector $w_j = (w_n)^T$, where $w_j \in [0,1]$, and the sum of the weights is 1. According to:

$$OWA(a_1, a_2, \ldots, a_n) = \sum_{j=1}^{n} w_j b_j \qquad (1)$$

where b_j is the jth largest of the arguments a_i.

It has been demonstrated that the OWA operator is commutative, idempotent, bounded and monotonic [16].

B. The induced OWA operator

The induced ordered weighted averaging operator introduced by [7] presents an extension of the OWA operator. This extension allows a reordering process defined by order-induced variables u_i, rather than the traditional ordering constructed from the values of the a_i arguments.

Definition 2. An IOWA operator of dimension n is a mapping $IOWA: R^n \rightarrow R$ associated to a weighting vector W of dimension n such that the sum of the weights is equal to 1, $w_j \in [0,1]$, and a set of order-inducing variables u_i, following the next formula:

$$IOWA(u_1, a_1, \ldots, u_n, a_n) = \sum_{j=1}^{n} w_j b_j \qquad (2)$$

where $(b_1, b_2, \ldots, b_n)$ is $(a_1, a_2, \ldots, a_n)$ reordered in decreasing values of the u_i. Note that the u_i are the order-inducing variables, and the a_1 are the argument variables.

C. GOWLA operator

The generalized ordered weighted logarithmic aggregation (GOWLA) operator developed by Zhou and Chen [15] introduces a parameterized family of aggregation operators including the step-GOWLA operator, the window-GOWLA operator, the S-GOWLA operator and the GOWHLA operator. The generalized ordered weighted logarithmic averaging (GOWLA) operator can be defined as follows:

Definition 3. A GOWLA operator of dimension n is a mapping $GOWLA: \Omega^n \rightarrow \Omega$ demarcated by an associated weighting vector W of dimension n, satisfying that $w_j \in [0,1]$ for all j, the sum of the weghts is 1 and a parameter λ that moves between $(-\infty,\infty) - \{0\}$according to the next formula:

$$GOWLA(a_1, a_2, \ldots, a_n) = exp\left\{\left(\sum_{j=1}^{n} w_j \left(lnb_j\right)^{\lambda}\right)^{\frac{1}{\lambda}}\right\} \qquad (3)$$

where b_j is the jth largest of the arguments $a_1, a_2, \ldots, a_n$. Observe that $lna_j \geq 0$, in that case, $exp(lna_j) \geq exp(0)$, so $a_j \geq 1$ following the notation in [15], $\Omega\ \{x \mid x \geq 1, x \in R\}$. Being x all the arguments a to be included in the aggregation process.

III. The Induced GOWLA Operator

The induced GOWLA operator (IGOWLA) is an extension of the GOWLA operator; the main distinction is the previous reordering step, i.e. the IGOWLA operator is not defined by the values of the arguments a_i, but by order-induced variables u_i, this means the position of the arguments a_i will be determined by the values of the u_i [17]. This extension enables an even more generalized ordering process, where decision-making could consider wider and complex conditions. The IGOWLA operator is defined as follows:

Definition 4: An IGOWLA operator of dimension n is a mapping $IGOWLA: \Omega^n \rightarrow \Omega$ defined by an associated weighted vector W of dimension n satisfying that the sum of the weights is 1 and $w_j \in [0,1]$, a set of order-inducing variables u_i, according to the formula:

$$IGOWLA(u_1, a_1, u_2, a_2, \ldots, u_n, a_n) = exp\left\{\left(\sum_{j=1}^{n} w_j \left(lnb_j\right)^{\lambda}\right)^{\frac{1}{\lambda}}\right\} \qquad (4)$$

where λ is a parameter such that $(-\infty,\infty) - \{0\}$, and $(b_1, \ldots, b_n)$ is simply $(a_1, \ldots, a_n)$ reordered in decreasing values of the u_i, the u_i are the order-inducing variables, and the a_i are the argument variables. Note that in this paper we follow the original argument where $\Omega\ \{x \mid x \geq 1, x \in R\}$.

Example 1. Assume the following collection of arguments set by their respective order-inducing variables (u_1,a_1): (4, 30), (2, 80), (7, 10), (5, 60). Let us assume that W (0.1, 0.3, 0.2, 0.4) and λ=2, the aggregation will result as:

$IGOWLA$ (⟨4,30⟩, ⟨2,80⟩, ⟨7,10⟩, ⟨5,60⟩) =

$exp\ \{((0.1 \times (ln10))^2 + ((0.3 \times (ln60))^2 + ((0.2 \times (ln30))^2 + ((0.4 \times (ln80))^2)^{(1/2)}\} = 51.6158$

It is observable, that the order-inducing variables u_1, affect the order of the argument variables a_1 in decreasing order.

The IGOWLA operator is a generalization of a mean operator. Therefore it is commutative, idempotent, bounded and monotonic. These properties can be proven as follows:

Theorem 1. Commutativity: Let the function f be the IGOWLA operator. Then

$$f(u_1, a_1, \ldots, u_n, a_n) = f(u_1, e_1, \ldots, u_n, e_n) \qquad (5)$$

where $(\langle u_1,a_1\rangle, \ldots, \langle u_n,a_n\rangle)$ is any given permutation of the arguments $(\langle u_1,e_1\rangle, \ldots, \langle u_n,e_n\rangle)$.

Proof. Let

$$f(u_1, a_1, \ldots, u_n, a_n) = exp\left\{\left(\sum_{j=1}^{n} w_j \left(lnb_j\right)^{\lambda}\right)^{\frac{1}{\lambda}}\right\} \qquad (6)$$

and

$$f\left(u_1,e_1,\ldots,u_n,e_n\right)=exp\left\{\left(\sum_{j=1}^{n}w_j\left(lnd_j\right)^{\lambda}\right)^{\frac{1}{\lambda}}\right\} \tag{7}$$

Since ($\langle u_1,a_1\rangle$, …, $\langle u_n,a_n\rangle$) is a permutation of($\langle u_1,e_1\rangle$, …, $\langle u_n,e_n\rangle$), then b_j, for all j, where b_j and d_j are the pairs (u_i,a_i) and (u_i,e_i) with the jth largest u_i. Therefore,

$$f\left(u_1,a_1,\ldots,u_n,a_n\right)=f\left(u_1,e_1,\ldots,u_n,e_n\right) \tag{8}$$

The theorem is proved.

Theorem 2. Idempotency: Let f be the IGOWLA operator and if $a_i=a$, for all i, then, $f(u_1, a_1,\ldots, u_n, a_n) = a$.

Proof. Since $a_i = a$ and the sum of w_j is 1, we get

$$\begin{aligned}f\left(u_1,a_1,\ldots,u_n,a_n\right)=exp\left\{\left(\sum_{j=1}^{n}w_j\left(lnb_j\right)^{\lambda}\right)^{\frac{1}{\lambda}}\right\}=\\ exp\left\{\left(\sum_{j=1}^{n}w_j\left(lna\right)^{\lambda}\right)^{\frac{1}{\lambda}}\right\}=\\ exp\left\{lna\left(\sum_{j=1}^{n}w_j\right)^{\frac{1}{\lambda}}\right\}=a\end{aligned} \tag{9}$$

The theorem is proved.

Theorem 3. Boundedness: Let f be the IGOWLA operator. Therefore

$$min\{a_i\}\le f\left(u_1,a_1,\ldots,u_n,a_n\right)\le max\{a_i\} \tag{10}$$

Proof. $max\ \{a_i\}= a_{max}$ and $min\ \{a_i\} =a_{min}$. Then

$$\begin{aligned}f\left(u_1,a_1,\ldots,u_n,a_n\right)=exp\left\{\left(\sum_{j=1}^{n}w_j\left(lnb_j\right)^{\lambda}\right)^{\frac{1}{\lambda}}\right\}\le\\ exp\left\{\left(\sum_{j=1}^{n}w_j\left(lna_{max}\right)^{\lambda}\right)^{\frac{1}{\lambda}}\right\}\le\\ exp\left\{\left(lna_{max}\right)\left(\sum_{j=1}^{n}w_j\right)^{\frac{1}{\lambda}}\right\}\le a_{max}\end{aligned} \tag{11}$$

and

$$\begin{aligned}f\left(u_1,a_1,\ldots,u_n,a_n\right)=exp\left\{\left(\sum_{j=1}^{n}w_j\left(lnb_j\right)^{\lambda}\right)^{\frac{1}{\lambda}}\right\}\ge\\ exp\left\{\left(\sum_{j=1}^{n}w_j\left(lna_{min}\right)^{\lambda}\right)^{\frac{1}{\lambda}}\right\}\ge\\ exp\left\{\left(lna_{min}\right)\left(\sum_{j=1}^{n}w_j\right)^{\frac{1}{\lambda}}\right\}\ge a_{min}\end{aligned} \tag{12}$$

then

$$a_{min}\le f\left(u_1,a_1,\ldots,u_n,a_n\right)\le a_{max} \tag{13}$$

Theorem is proved.

Theorem 4. Monotonicity: Let f be the IGOWLA operator. If $a_i \ge c_i$ for all i, then

$$f\left(u_1,a_1,\ldots,u_n,a_n\right)\ge f\left(u_1,c_1,\ldots,u_n,c_n\right) \tag{14}$$

Proof. Let

$$f\left(u_1,a_1,\ldots,u_n,a_n\right)=exp\left\{\left(\sum_{j=1}^{n}w_j\left(lnb_j\right)^{\lambda}\right)^{\frac{1}{\lambda}}\right\} \tag{15}$$

and

$$f\left(u_1,c_1,\ldots,u_n,c_n\right)=exp\left\{\left(\sum_{j=1}^{n}w_j\left(lnd_j\right)^{\lambda}\right)^{\frac{1}{\lambda}}\right\} \tag{16}$$

Consider f (u_1,a_1), …, (u_n,a_n), we then take the natural logarithm of $f(u_1,a_1)$, …, (u_n,a_n) two times, then we get:

$$ln\left(lnf\left(u_1,a_1,\ldots,u_n,a_n\right)\right)=\frac{ln\left(\sum_{j=1}^{n}w_j\left(lnb_j\right)^{\lambda}\right)}{\lambda} \tag{17}$$

following, we take the partial derivative with respect to b_j

$$\frac{\partial ln\left(lnf\right)}{\partial b_j}=\frac{1}{\lambda}\bullet\frac{1}{\sum_{j=1}^{n}w_j\left(lnb_j\right)^{\lambda}}\bullet w_j\bullet\lambda\left(lnb_j\right)^{\lambda-1}\bullet\frac{1}{b_j}\ge 0 \tag{18}$$

Since $\frac{\partial ln(lnf)}{\partial b_j} \geq 0$, then $ln(lnf)$ is monotonic with respect to b_j, that is, f is monotonic with respect to b_j. Since $a_i \geq c_i$ for all i, so $b_j \geq d_j$ for all j. The theorem is proved.

IV. Group Decision Making with the IGOWLA operator

A. Decision making process

The literature on decision making exhibits a plethora of techniques and methods that could be suitable to apply with the IGOWLA operator (see e.g., [18]–[21]). In this paper we focus our attention in a decision making application, specifically in the selection of strategies using a multi-person analysis in the area of innovation management.

An interesting issue to study when dealing with innovation management in a given a company is the selection of the best product to develop from a portfolio of new products [22]. The effectiveness in which an organization manages its new products portfolio is often a key determinant of competitive advantage [23]. Specifically, portfolio management deals with the allocation of scarce resources of the enterprise (money, time, people, machinery, etc.) on potential projects under uncertain conditions. Furthermore, new products must align correctly to business objectives and balance in both risk and timespan.

The process to follow in the selection of strategies with the IGOWLA operator with multi-person decision making can be summarized as follows:

Step 1. Let $A=\{A_1, A_2,\ldots, A_h\}$ be a set of limited options or alternatives, $S=\{S_1, S_2,\ldots, S_i\}$ a set of finite states of nature or characteristics, conforming the payoff matrix, $(a_{hi})_{m\times n}$. Let $E=\{E_1, E_2,\ldots, E_q\}$ be a finite set of decision makers. Assume that the selected decision makers have a different level of importance such that $X = (x_1, x_2,\ldots, x_p)$ is the vector that defines the weight of each decision maker in the problem, where the sum of all x_k from k = 1 until p is equal to 1 and $x_k \in [0,1]$. Then each decision maker must provide a pay-off matrix $(a_{hi})^{k}_{m\times n}$.

Step 2. Calculate the order-inducing variables $(u_{hi})_{m\times n}$ to be used in the payoff matrix for each alternative h and state of the nature i. Find the weighting vector $W = (w_1, w_2,\ldots, w_n)$ satisfying the IGOWLA operator definitions and define the parameter λ to be applied in the aggregation. Note that the weighting vectors could be obtained using any classic method treating probabilistic information.

Step 3. Use the weighted average (WA) to aggregate the information of the decision makers E, by using the decision-makers weighting vector X. The result will then be the collective payoff matrix $(a_{hi})^{k}_{m\times n}$, therefore $a_{hi} = \sum_{k=1}^{p} x\left(a_{hi}\right)^{k}$.

Step 4. Solve for IGOWLA operator as described in (4). Consider that λ value is usually set in 1; however, any other value could be used depending on the problem analyzed and the properties of the operator.

Step 5. Establish a ranking of the alternatives; analyze the results to the specific problem and generate a decision making approach.

V. Group Decision Making with the IGOWLA operator

A. Illustrative example

This section presents an illustrative example regarding a strategic decision making procedure in product portfolio management using a multi-person analysis and the IGOWLA operator. Note that other business-decision making applications could be developed in the area of innovation management, such as knowledge management, project management, organization and structure among others [24].

Step 1. Assume that a company engaged in the production and marketing of fast moving consumer goods must select a new product to develop from their portfolio of five potential enhanced beverage concepts. Then we have five alternatives h:

- A_1 Sport: vitamin C plus electrolytes
- A_2 Energy: vitamin C plus caffeine
- A_3 Recover: vitamin B5, B6 and B12
- A_4 Diet sport: vitamin C plus electrolytes, no sugar
- A_5 AntiOx: manganese plus vitamin B3

In order to select the concept to be developed, the company chooses experts in different areas of the business to give their opinion. Based on the literature on innovation management, the company sets 6 key factors to be analyzed in the selection process, i.e. six states of nature i:

- S_1 Expected benefits
- S_2 Alignment to business
- S_3 Less development cost
- S_4 Technical viability
- S_5 Minimum risk
- S_6 Less time to market

The experts are divided in three subgroups (Tables I –VI). The first group comprises two experts in the area of engineering. The second has two experts in the area of marketing and sales. And the remaining two financial experts belong to the third group. In total there is a set of six experts q. From the scale of 1 to 100 each expert must give their opinion about the expected performance of each product h based on the key factors i selected by the direction. In order to correctly develop the aggregation process, we must generate first a multi-aggregation process in which the opinions of the groups can be concentrated. This process allows us to visualize the information of each group separately. Then we need to aggregate all the groups in a sole collective group payoff matrix. Finally use the IGOWLA operator to generate the final results of the aggregation process and aid the direction board in the selection of the most suitable alternative.

Step 2. Due to the complexity of the information analyzed, the administration generates a set of order-inducing variables: *U* (7, 5, 4, 2, 10, 9). The experts consider a weighting vector *IGOWLA* (0.1, 0.1, 0.1, 0.2, 0.1, 0.4).

Step 3. The *p* weighting vectors *X* that represent the importance of each expert *q* in the analysis are the following: First group of experts X_1 = (0.4, 0.6), the second group of experts X_2 = (0.7, 0.3), and the third group X_3 = (0.5, 0.5). For the collective matrix we have X_4 = (0.3, 0.4, 0.3). With this information we can obtain inter-medium results by first aggregating the opinions of the three groups of experts, the results are shown in Tables VII, VIII and IX Following, we use the weighted average to aggregate the three subgroups into a collective payoff matrix $(a_{hi})^{k}_{m\times n}$. The results are shown in Table X.

Step 4. Using the IGOWLA, we aggregate the collective information and obtain final results. Tables XI and XII show the results of the aggregations.

Step 5. In order to generate a decision we must establish a ranking of the performance of each product. The ordering of alternatives is presented in Table XIII. Note that the symbol "}" represents "preferred to". Also note that for each of the selected aggregation operator, a different ranking can be assembled, leading to distinct decision making process.

TABLE I. PAYOFF MATRIX – EXPERT 1.

	S1	*S2*	*S3*	*S4*	*S5*	*S6*
A_1	80	80	100	100	100	100
A2	20	40	40	60	20	60
A3	80	100	80	60	80	100
A4	80	100	80	100	80	100
A5	40	40	60	40	60	40

TABLE II. PAYOFF MATRIX – EXPERT 2.

	S1	*S2*	*S3*	*S4*	*S5*	*S6*
A1	100	80	100	80	100	100
A2	20	60	60	20	20	20
A3	80	40	40	100	60	100
A4	80	80	100	80	100	100
A5	40	60	60	60	40	40

TABLE III. PAYOFF MATRIX – EXPERT 3.

	S1	*S2*	*S3*	*S4*	*S5*	*S6*
A1	100	100	80	80	100	80
A2	20	40	60	20	60	40
A3	100	100	100	80	40	40
A4	100	80	80	100	80	100
A5	40	40	40	60	40	60

TABLE IV. PAYOFF MATRIX – EXPERT 4.

	S1	*S2*	*S3*	*S4*	*S5*	*S6*
A1	80	80	100	100	80	80
A2	60	20	40	60	40	60
A3	60	60	80	40	100	60
A4	100	100	80	80	100	80
A5	60	60	40	60	40	60

TABLE V. PAYOFF MATRIX – EXPERT 5.

	S1	*S2*	*S3*	*S4*	*S5*	*S6*
A1	80	100	80	80	100	100
A2	20	20	60	20	40	60
A3	60	40	60	100	100	100
A4	80	80	80	100	80	80
A5	60	60	40	40	40	40

TABLE VI. PAYOFF MATRIX – EXPERT 6.

	S1	*S2*	*S3*	*S4*	*S5*	*S6*
A1	100	80	80	80	100	100
A2	60	60	40	40	20	60
A3	60	80	40	80	100	60
A4	80	80	100	80	80	100
A5	60	60	40	40	40	60

TABLE VII. PAYOFF MATRIX – GROUP 1 (EXPERT 1 AND 2).

	S1	*S2*	*S3*	*S4*	*S5*	*S6*
A1	92.00	80.00	100.00	88.00	100.00	100.00
A2	20.00	52.00	52.00	36.00	20.00	36.00
A3	80.00	64.00	56.00	84.00	68.00	100.00
A4	80.00	88.00	92.00	88.00	92.00	100.00
A5	40.00	52.00	60.00	52.00	48.00	40.00

TABLE VIII. PAYOFF MATRIX – GROUP 2 (EXPERT 3 AND 4).

	S1	*S2*	*S3*	*S4*	*S5*	*S6*
A1	94.00	94.00	86.00	86.00	94.00	80.00
A2	32.00	34.00	54.00	32.00	54.00	46.00
A3	88.00	88.00	94.00	68.00	58.00	46.00
A4	100.00	86.00	80.00	94.00	86.00	94.00
A5	46.00	46.00	40.00	60.00	40.00	60.00

TABLE IX. PAYOFF MATRIX – GROUP 3 (EXPERT 5 AND 6).

	S1	*S2*	*S3*	*S4*	*S5*	*S6*
A1	90.00	90.00	80.00	80.00	100.00	100.00
A2	40.00	40.00	50.00	30.00	30.00	60.00
A3	60.00	60.00	50.00	90.00	100.00	80.00
A4	80.00	80.00	90.00	90.00	80.00	90.00
A5	60.00	60.00	40.00	40.00	40.00	50.00

TABLE X. COLLECTIVE PAYOFF MATRIX.

	S1	*S2*	*S3*	*S4*	*S5*	*S6*
A1	92.20	88.60	88.40	84.80	97.60	92.00
A2	30.80	41.20	52.20	32.60	36.60	47.20
A3	77.20	72.40	69.40	79.40	73.60	72.40
A4	88.00	84.80	86.60	91.00	86.00	94.60
A5	48.40	52.00	46.00	51.60	42.40	51.00

TABLE XI. AGGREGATED RESULTS 1.

	AM	*MIN*	*MAX*	*OWA*	*IOWA*	*IGOWLA* $\lambda = -1$	*IGOWLA* $\lambda = 1$
A1	90.60	84.80	97.60	88.66	88.66	88.53	88.57
A2	40.10	30.80	52.20	36.96	37.96	37.08	37.38
A3	74.07	69.40	79.40	72.50	75.5	75.37	75.41
A4	88.50	84.80	94.60	87.20	88.88	88.80	88.82
A5	48.57	42.40	52.00	46.70	49.82	49.66	49.72

TABLE XII. AGGREGATED RESULTS 2.

	IGOWLA $\lambda = 2$	*IGOWLA* $\lambda = 3$	*GOWLA* $\lambda = -1$	*GOWLA* $\lambda = 1$	*GOWLA* $\lambda = 2$	*GOWLA* $\lambda = 3$
A1	88.59	88.61	88.53	88.57	88.59	88.61
A2	37.53	37.69	35.99	36.32	36.49	36.66
A3	75.43	75.45	72.39	72.43	72.44	72.46
A4	88.84	88.85	87.12	87.15	87.16	87.17
A5	49.75	49.77	46.45	46.54	46.58	46.62

TABLE XIII. RANKING OF THE PERFORMANCE OF THE CONCEPTS TO BE DEVELOPED.

AM	$A_1 \succ A_4 \succ A_3 \succ A_5 \succ A_2$	IGOWLA $\lambda = 2$	$A_4 \succ A_1 \succ A_3 \succ A_5 \succ A_2$
MIN	$A_1 \succ A_4 \succ A_3 \succ A_5 \succ A_2$	IGOWLA $\lambda = 3$	$A_4 \succ A_1 \succ A_3 \succ A_5 \succ A_2$
MAX	$A_1 \succ A_4 \succ A_3 \succ A_2 \succ A_5$	GOWLA $\lambda = -1$	$A_1 \succ A_4 \succ A_3 \succ A_5 \succ A_2$
OWA	$A_1 \succ A_4 \succ A_3 \succ A_5 \succ A_2$	GOWLA $\lambda = 1$	$A_1 \succ A_4 \succ A_3 \succ A_5 \succ A_2$
IOWA	$A_4 \succ A_1 \succ A_3 \succ A_5 \succ A_2$	GOWLA $\lambda = 2$	$A_1 \succ A_4 \succ A_3 \succ A_5 \succ A_2$
IGOWLA $\lambda = -1$	$A_4 \succ A_1 \succ A_3 \succ A_5 \succ A_2$	GOWLA $\lambda = 3$	$A_1 \succ A_4 \succ A_3 \succ A_5 \succ A_2$
IGOWLA $\lambda = 1$	$A_4 \succ A_1 \succ A_3 \succ A_5 \succ A_2$		

Overall results show that the ordering process varies depending on the operator used to analyze the arguments. For our specific problem, based on the opinion of six experts and the attitude of the direction, the best concepts to develop are product A_1 (Sport) and A_4 (Diet sport). It is also observable that the induced operators display different rankings than the traditional ones, therefore indicating that the order-induced variables affect the ranking of the arguments.

VI. CONCLUSIONS

In this paper, we have introduced a new aggregation operator named the IGOWLA operator. It is an extension of the optimal deviation model GOWLA operator; therefore it shares the same principal characteristics. The main improvement of the IGOWLA operator is the inclusion of order-induced variables in the reordering process of the arguments, therefore offering a wider representation of a complex attitude of the decision makers. Furthermore, we have studied different properties of the IGOWLA operator, such as the commutativity, idempotency, boundedness and monotonicity.

The IGOWLA operator is an approach to assist group decision making. The operator can be applied in a wide range of scientific areas, such as statistics, economics and engineering. In the present paper, we present an illustrative example of the usage of the IGOWLA operator with a multi-person analysis to assess a strategic decision making process in the area of innovation management, specifically in new product portfolio management. The example has proven the IGOWLA operator to be useful when the procedure involves the opinion of diverse experts with diverse backgrounds, including a complex attitudinal character of the direction board.

Further research needs to be conducted. Firstly, we need to deepen in the mathematical approach of the logarithmic properties. Secondly, the development of further extensions to assess uncertain information, i.e. fuzzy numbers, linguistic variables and interval numbers, distance measures such as the Hamming or Euclidean distance, heavy aggregations among other complex formulations. And finally, considering new decision making problems, in diverse fields of knowledge.

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