Optim Lett (2017) 11:855–874
DOI 10.1007/s11590-016-1061-1

ORIGINAL PAPER

Alternative equilibria in two-period ultimatum bargaining with envy

Gino Loyola[1]

Received: 4 January 2016 / Accepted: 28 June 2016 / Published online: 7 July 2016

Abstract A two-period ultimatum bargaining game is developed in which parties experience an envy-type externality coming from the surplus captured by their counterparts. Our assumptions on envy levels and outside opportunities allow us to characterize a richer set of bargaining outcomes than that identified by the prior literature, which includes a novel agreement equilibrium which we label Type I agreement. As this novel agreement solution is delivered by a negotiation resembling a one-shot ultimatum game, only characteristics of the second-moving player shape the sources of bargaining power. This property contrasts with that of Type II agreement—an agreement solution previously reported by related literature—in which characteristics of both players influence negotiating strengths. Numerical simulations are performed to illustrate the interplay between envy, impatience rates and outside opportunities as well as the degree of inequity generated by each agreement type.

Keywords Ultimatum game · Envy · Negative externality · Negotiation breakdown

1 Introduction

Over the last three decades, one of the more vibrant research lines in experimental economics has been concerned with ultimatum-game experiments (see the extensive survey by Güth and Kocher [7]). In particular, most recent works have focused on studying how feelings and emotional states, like envy, anger, and others, can help explain the puzzling results coming from this kind of experiments [6,10,19–21,23].

✉ Gino Loyola
gloyola@fen.uchile.cl

[1] Department of Management Control, University of Chile, Diagonal Paraguay 257, of. 1903, Santiago, Chile

In this vein, this paper proposes a two-stage ultimatum bargaining model in which players suffer a negative externality arising from envy. In particular, we assume that each negotiator experiences envy because of the surplus stake captured by his/her rival. Using this model we identify (i) the conditions that make an agreement or a negotiation breakdown more likely, (ii) the properties of a potential agreement, and (iii) the sources of bargaining power.

More specifically, we show that the combination of envy and positive outside opportunities modifies the classical two-stage ultimatum game in such a way that we are able to characterize a richer set of bargaining outcomes than that identified by the previous literature. In this modified game, three different outcomes become possible, depending on the size of what we will call the *modified* surplus at stake. The first of these possible outcomes, a permanent negotiation breakdown, will occur if this surplus is too small to cover the *modified* discounted outside opportunities of both parties. The second possible outcome, denoted a Type I agreement without delay, will result if the surplus is relatively moderate (higher than modified discounted outside opportunities but lower than *modified* undiscounted ones). The third and final possibility, dubbed a Type II agreement solution without delay, will be observed if the surplus is relatively high (exceeding the modified undiscounted outside opportunities).

The main contribution of our work is concerned with Type I agreement—a negotiating solution not reported previously—, which in our model emerges because moderate values of the modified surplus at stake produce a dual outcome: a breakdown in the second-period but an agreement in the first-period. As a consequence, the first-period negotiation resembles a one-shot ultimatum game in which the individual characteristics—envy, outside opportunity, impatience—of *only* the second-moving player influence the sources of bargaining power.

In the specific case of envy, we establish that such emotion plays a key role in shaping the outcomes of the bargaining game. If envy levels are high, disagreement is more likely; indeed, if global envy, defined as the product of the players' individual sensitivities to this type of externality, is high enough, negotiations will break down altogether. However, conditional on reaching an agreement, envy is paradoxically a source of bargaining power since it constrains negotiator strategies in a way reminiscent of analyses in the existing literature on commitment devices. In the context of ultimatum-type games, this envy-based commitment tactic can be exploited by the negotiator playing the role of responder, who can credibly threaten to reject an offer regarded as being too generous to his/her counterpart.

Our model describes in detail a complex interaction of various sources of bargaining strength, which includes, besides the individual characteristics of the parties, the role these parties perform as a proposer and/or a responder in the relevant game originating each agreement type. To gain insights into under which conditions which of these negotiating advantages finally dominate, we conduct several numerical simulations. In particular, these exercises illustrate the opposite roles played in bargaining outcomes by envy and impatience, as envy weakens the positive 'cost-of-haggling' effect exerted by impatience rates on the likelihood of reaching an agreement in sequential negotiations. The simulations also suggest that the region of Type II agreements is larger than that of Type I solutions and show that our novel Type I agreement is in general more generous to the first-moving negotiator.

This paper is related to the game-theory literature exploring the role of other-regarding elements in the preferences of negotiators involved in bargaining processes of an ultimatum-type, such as envy [12], fairness [11,18,24], inequity aversion [5], reciprocity [4], and trust [1]. Among this literature, the model that most resembles our analysis is that proposed by Kirchsteiger in [12], which develops a two-period ultimatum game in which players also suffer a negative envy-driven utility effect from the surplus obtained by their counterpart. However, due to differences in the setting design, the Kirchsteiger's model is unable to analyze the variety of bargaining outcomes examined here. One such difference is that that model does not incorporate the role played by outside opportunities, simply normalizing them to zero, and as a result, it is only able to characterize a single agreement, in contrast to our two-fold agreement solution. A second difference is that, whereas our goal is to propose a model that yields insights into real-world dynamic bargaining situations with envy, Kirchsteiger's main objective is to explain certain 'anomalies' frequently observed in ultimatum bargaining *experiments*. This leads Kirchsteiger to adopt conversion factors (between real and experimental money) instead of discount factors. As a consequence, although his single agreement is close to our Type II equilibrium, both agreement solutions become equivalent only under some particular symmetric environments.

The present article also has connections with the literature studying the possibility of a breakdown in a distributive bargaining game with externalities. These works, however, propose models that differ from ours in some critical dimensions. Whereas Laengle and Loyola [14] develop a *static* setting based on the classic Nash demand game, Laengle and Loyola [13] posit a *one-period* ultimatum game. Furthermore, neither of these studies examine the roles played by outside opportunities or rates of impatience. As a consequence of these modelling differences, our framework provides a richer analysis of the conditions making more likely an agreement or disagreement, which also allows us to examine the interplay of different sources of bargaining power when an envy-type externality is involved.

More indirectly, this work also relates to research on bargaining with externalities but in an environment rather different to ours, in which one seller negotiates with two potential buyers that suffer externalities between both of them. Under this framework, Chowdhury [2] assumes that buyers with symmetric externalities enjoy a first-mover advantage in the selling process, one of the outcomes being a disagreement if externality levels are sufficiently high. However, the symmetry assumption implies that, unlike our setup, his model is unable to identify sources of differentiated bargaining powers between the buyers. By contrast, [8,9] assume that the seller is the first-mover of the game and that asymmetric externalities between buyers are possible. Although their models deliver a delay of the agreement if externality levels are large enough, they cannot characterize a definitive negotiation breakdown or, alternatively, a permanent disagreement.

The structure of the remainder of this article is as follows. Section 2 proposes a two-stage ultimatum bargaining game with envy. Section 3 fully characterizes the set of equilibria of the game for all parameter values and the bargaining outcomes derived from them. Section 4 discusses the properties and intuition of the equilibria and the outcomes, reinterpreting the agreement/disagreement conditions in terms of classical conditions stated by the existing literature and identifying the interplay of different sources of bargaining strengths. Section 5 presents a series of numerical simulations to illustrate our main findings. Section 6 sets out the main conclusions of our analysis. Finally, all proofs are collected in the Appendix.

2 The model

Two players, A (he) and B (she), are to distribute a surplus of size π. The bargaining mechanism to be used for the distribution is a (potentially) two-period procedure with the following schedule.

- At $t = 1$, player A makes an offer $(x_1, \pi - x_1)$, where x_1 is the fraction of the surplus he gets and $\pi - x_1$ is the fraction his counterpart gets. Player B can accept or reject the offer. If she accepts it, each party ends with his/her respective fraction of the surplus and the negotiation is over; if she rejects it, the negotiation goes into a second period.
- At $t = 2$, player B makes a counteroffer $(x_2, \pi - x_2)$, where $\pi - x_2$ is the fraction she gets and x_2 is the fraction her counterpart gets. Player A can accept or reject the counteroffer. If he accepts it, each party gets his/her respective fraction; if he rejects it, each party receives his/her outside opportunity $U_i \geq 0$ for $i = A, B$.

We assume that player i has a discount factor $\delta_i \in (0, 1)$, which implicitly and inversely reflects the impatience degree of player i.

In case of agreement $(x_t, \pi - x_t)$ in period t, the payoff functions of the two parties are described by:

$$u_t^A(x_t; \theta_A) = x_t - \theta_A(\pi - x_t),$$

and

$$u_t^B(x_t; \theta_B) = (\pi - x_t) - \theta_B x_t,$$

for $t = 1, 2$, where $\theta_i \geq 0$ is player i's sensitivity to an envy-type externality generated by the surplus fraction going to his/her counterpart.

3 The equilibrium

In this and the following sections we will look for insights into three sets of questions:

- In equilibrium, what are the conditions under which players do or do not reach an agreement?
- If an agreement is struck, what is each player's equilibrium share? And what are each party's sources of bargaining power?
- In what period is the agreement arrived at? Is there a delay?

To address these questions, we must first characterize the equilibrium set of our bargaining game. To this end, let us begin by considering the two following conditions:

$$U_A(1+\theta_B) + U_B(1+\theta_A) \leq \pi(1-\theta_A\theta_B) \tag{1}$$
$$\delta_A U_A(1+\theta_B) + \delta_B U_B(1+\theta_A) \leq \pi(1-\theta_A\theta_B). \tag{2}$$

We also define the following upper bound:

$$\overline{x_1} \equiv \frac{\pi - \delta_B U_B}{1+\theta_B}.$$

The next two propositions fully characterize the subgame perfect equilibria (SPE) of this game for all parameter values.

Proposition 1 *Assume that (1) is not satisfied. Then the SPE of the dynamic bargaining game with envy is described by the following pair of strategies:*

- *At $t = 1$, player A offers*

$$x_1^* = \begin{cases} \overline{x_1} & if\ (2)\ is\ verified \\ \overline{x_1} + \varepsilon_A & otherwise \end{cases}$$

  for some $\varepsilon_A > 0$.
  At $t = 2$, he accepts player B's counteroffer if

$$x_2 \geq \frac{U_A + \theta_A \pi}{1+\theta_A},$$

  otherwise he rejects it.
- *At $t = 1$, player B accepts player A 's offer if*

$$x_1 \leq \overline{x_1},$$

  otherwise she rejects it.
  At $t = 2$, she counteroffers

$$x_2^* = \frac{U_A + \theta_A \pi}{1+\theta_A} - \varepsilon_B$$

  for some $\varepsilon_B > 0$.

From this proposition we can directly derive the outcome of this bargaining game when condition (1) is not satisfied as follows.

Corollary 1 *If condition (1) is not satisfied, two bargaining outcomes are possible:* (i) (*Type I agreement*). *If condition (2) holds, an agreement is reached at $t = 1$ in which the shares obtained by A and B are $\overline{x_1}$ and $\pi - \overline{x_1}$, respectively.* (ii) (*Negotiation breakdown*). *If condition (2) does not hold, a disagreement occurs in both periods.*

Note that if the global envy of the game is high enough, no agreement will be reached. This observation is formalized in the following statement.

Corollary 2 *Let Θ be the level of global envy of the game, defined as*

$$\Theta \equiv \theta_A \theta_B.$$

Then, if $\Theta > 1$, a negotiation breakdown occurs.

We now characterize the equilibrium of our dynamic bargaining model when condition (1) is met. We begin by defining the following upper bound:

$$\overline{\overline{x_1}} \equiv \frac{\pi - \delta_B \left[\pi - \frac{(1+\theta_B)(U_A+\theta_A\pi)}{1+\theta_A}\right]}{1+\theta_B}.$$

Proposition 2 *Assume that condition (1) is satisfied. Then the SPE of the dynamic bargaining game with envy is described by the following pair of strategies:*

- *At $t = 1$, player A offers*

  $$x_1^* = \overline{\overline{x_1}}.$$

  At $t = 2$, he accepts player B's counteroffer if

  $$x_2 \geq \frac{U_A + \theta_A \pi}{1+\theta_A},$$

  and rejects it otherwise.
- *At $t = 1$, player B accepts player A's offer if*

  $$x_1 \leq \overline{\overline{x_1}}$$

  and rejects it otherwise.
  At $t = 2$, she counteroffers

  $$x_2^* = \frac{U_A + \theta_A \pi}{1+\theta_A}. \tag{3}$$

We now derive the outcome of the game from the last proposition when condition (1) is satisfied. This is formally stated as follows.

Corollary 3 (Type II agreement) *If condition (1) holds, an agreement is reached at $t = 1$. The shares obtained in the agreement by A and B are $\overline{\overline{x_1}}$ and $\pi - \overline{\overline{x_1}}$, respectively.*

4 Interpretation and properties of the equilibrium

In this section we discuss the intuition behind the bargaining outcomes generated by the equilibrium of our dynamic game, especially through the comparison between the two agreement types previously characterized. We also contrast our findings with the closest previous literature, stressing that our Type I agreement constitutes the main contribution of the present work. Finally, we provide an interpretation of the conditions for an agreement or disagreement that is consistent with existing interpretations in the literature on bargaining *without* envy.

To perform all this analysis, we previously define the following additional notation. Let $\widetilde{U}_i$ be *player i's modified outside opportunity*, defined as

$$\widetilde{U}_i \equiv U_i(1+\theta_j), \tag{4}$$

for all $i, j = A, B, i \neq j$, and $\widetilde{\pi}$ be the *modified surplus*, defined as

$$\widetilde{\pi} \equiv (1-\theta_A\theta_B)\pi. \tag{5}$$

Also let $\widetilde{U}_{\delta_i}$ be the *discounted player i's modified outside opportunity*, defined as

$$\widetilde{U}_{\delta_i} \equiv \delta_i \widetilde{U}_i, \tag{6}$$

for $i = A, B$.

We first focus on explaining and comparing the two agreement types, and after that, we analyze the negotiation breakdown outcome.

4.1 Agreement outcomes

Using the notation above introduced, we characterize the two possible agreement solutions as follows.

Type I agreement This type of agreement takes place whenever

$$\widetilde{U}_{\delta_A} + \widetilde{U}_{\delta_B} \leq \widetilde{\pi} < \widetilde{U}_A + \widetilde{U}_B, \tag{7}$$

i.e., when the modified surplus at stake takes a moderate value (i.e., equal to or greater than the sum of the discounted modified outside opportunities, but smaller than the simple sum of modified outside opportunities). Bargaining powers are then *compatible*, which ensures that an equilibrium exists and both players reach what we have defined as a negotiated Type I agreement solution. Furthermore, since this agreement is struck in period $t = 1$, we say that there is *no delay*.

It is worthy to remark that this type of agreement is the most interesting finding of this paper since it has not been reported previously. In particular, notice that the two-period version of the ultimatum game with envy proposed by Kirchsteiger in [12] is unable to deliver this type of agreement. This is because Kirchsteiger normalizes outside opportunities to zero, which, in the context of our setup, implies that condition (7) can never be met due to $\widetilde{U} = \widetilde{U}_\delta = 0$ for both players. In contrast, our potential positive outside opportunities allow the possibility of $\widetilde{U} > \widetilde{U}_\delta$ for at least one of the negotiators, which in turn makes possible that $\widetilde{\pi}$ may take a moderate value, that is, a value in the range characterized by condition (7).

The main feature of Type I agreement is that *only* the characteristics of player B shape the sources of bargaining strength and weakness.[1] In particular, the equilibrium partition is such that

$$\frac{\partial \overline{x_1}}{\partial \theta_B} < 0, \frac{\partial \overline{x_1}}{\partial U_B} < 0 \quad \text{and} \quad \frac{\partial \overline{x_1}}{\partial \delta_B} < 0, \tag{8}$$

which means that player A's agreement share will be higher as long as either: (i) player B's envy level decreases, (ii) player B's outside opportunity decreases, or (iii) player B becomes more impatient. For player B's agreement share these effects are the opposite.

Type II agreement This type of agreement arises whenever

$$\widetilde{U}_A + \widetilde{U}_B \leq \widetilde{\pi}, \tag{9}$$

i.e., when the modified surplus at stake is sufficiently high (i.e., equal to or greater than the sum of the modified outside opportunities). Then the bargaining powers are *compatible* as with Type I equilibrium but the negotiated solution the players arrive at is a Type II agreement. Still, since this agreement is closed at period $t = 1$, there is *no delay*.

It is important to remark that, contrary to Type I solution, Type II agreement is, under some symmetric environments, just the generalization of an agreement type already characterized by the related previous literature. As for this point, notice that although the model proposed by Kirchsteiger is very close to ours, two main differences must be highlighted.

First, as commented before, Kirchsteiger sets outside options to zero, which prevents him from getting our Type I equilibrium. Notice, however, that his assumptions of $\widetilde{U} = 0$ and $\widetilde{\pi} > 0$ (because he assumes θ to be strictly smaller than 1 for both players) imply in fact that our condition (9) is always satisfied under his setup.[2] Second, whereas the Kirchsteiger's model discounts money (or chips), our framework discounts utility. Although this difference seems rather subtle, it affects the equilibrium partition found in each model and avoids that, even assuming zero outside options, such partitions be the same under both settings. Despite those differences, and as a result of the close relationship between the two models, we can nevertheless recover the same equilibrium as Kirchsteiger does for some parameter values, in particular when outside opportunities are zero and discount factors are equal. In such symmetric environment, player B's equilibrium share becomes

$$\pi - \overline{\overline{x_1}} \equiv \frac{\pi \ (\theta_B(1 + \theta_A) + \delta(1 - \theta_A\theta_B))}{(1 + \theta_A)(1 + \theta_B)},$$

[1] This analysis excludes bargaining advantages coming from the parties' order of moves in the game.

[2] This is thus an alternative explanation of why the Kirchsteiger's model is able to arrive at only one of the two agreement types we here characterize.

which can be checked is identical to the expression (4.8) in [12] under the parameter values above indicated.[3]

It is important to note that, contrary to a Type I agreement, the sources of bargaining strength and weakness in a Type II solution stem from the characteristics of *both* players. In particular, the equilibrium sharing rule for this agreement type is such that

$$\frac{\partial \overline{\overline{x_1}}}{\partial \theta_A} > 0, \quad \frac{\partial \overline{\overline{x_1}}}{\partial U_A} > 0, \quad \frac{\partial \overline{\overline{x_1}}}{\partial \theta_B} < 0, \quad \frac{\partial \overline{\overline{x_1}}}{\partial \delta_B} < 0, \tag{10}$$

which implies that player A's agreement share is higher if it increases either (i) his envy level, or (ii) his outside opportunity, but is lower if player B either (i) has a higher envy level or (ii) becomes less impatient.

4.2 Intuition and comparison of the two agreement types

The main observation when comparing both agreement types is that whereas in a Type I agreement the surplus distribution depends only on player B's characteristics, in a Type II agreement the sharing rule is also determined by those of player A. This is due to the fact that the alternative scenario to an agreement in period 1 is different for the two agreement types.

More specifically, in a Type I agreement the alternative scenario to an agreement in period 1 is a *disagreement* in period 2 (see proof of Proposition 1). Thus, when applying backward induction, the subgame played in the first period becomes strategically equivalent to a *one-shot* ultimatum game with envy in which outside opportunities are gotten at $t = 2$, and thus, they are given in present value terms by the pair $(\delta_A U_A, \delta_B U_B)$. As a consequence, when computing the equilibrium of the negotiation played at period $t = 1$, from a strategic viewpoint it is relevant the role each negotiator plays *only* in that period: player A as a *proposer* and player B as a *responder*. This produces the following sources of bargaining power. On the one side, A enjoys a *last-proposer advantage* that allows him in principle to extract all the remaining surplus at stake after giving the outside opportunity $\delta_B U_B$ to his counterpart. On the other side, B has three sources of bargaining power that can counterbalance that strategic advantage of A. A first source of strength comes from the outside opportunity of player B in her role as the *last responder* of the relevant game, which ensures her, in *future* value terms a minimum payoff of U_B. A second source of bargaining power is the discount factor, which reflects the fact that as long as B is less impatient, she will be more willing to rebuff low offers and wait for her outside opportunity at $t = 2$, making in turn A to be more generous. Lastly, player B exhibits a source of bargaining strength coming from her envy level θ_B, which, as we explain in more detail in Sect.

[3] Note, however, that under the same parameter values, our Type I agreement is unable to recover the Kirchsteiger's equilibrium since its equilibrium partition is such that

$$\pi - \overline{x_1} = \frac{\theta_B \pi}{1 + \theta_B}.$$

4.3, constitutes a sort of *commitment device* that makes the envious player a tougher negotiator. In Type I agreement, nevertheless, given the equivalence of the game with a one-shot ultimatum bargaining, only the responder can use the externality generated by envy as a sort of *envy-based last-responder advantage*. As a result, the agreement benefits (damages) the responder (proposer) as long as θ_B increases.

The result of the interaction among all these sources of bargaining power is an equilibrium sharing rule that only depends on player B's characteristics in the way above described by expression (8).

In a Type II agreement, on the other hand, the alternative scenario to an agreement in period 1 is an *agreement* in period 2 (see proof of Proposition 2). Thus, the subgame played in this period is a *two-stage* ultimatum bargaining under an offer-counteroffer scheme with envy and positive outside opportunities. As a consequence, when characterizing the equilibrium of the first-period negotiation, from a strategic perspective it is now relevant the role each negotiator plays in the *two* stages, that is, like a proposer (A at $t = 1$ and B at $t = 2$) as well as like a responder (B at $t = 1$ and A at $t = 2$). This fact produces the following sources of bargaining power. On the one side, B has three sources of negotiating strengths. First, she enjoys now the last-proposer advantage of the game, which allows her, as a starting point, to extract in the second period all the remaining surplus at stake after giving the outside opportunity to her counterpart. Second, the patience degree of B is also a source of negotiating advantage (reflected through discount factor δ_B), as her position like responder in the first period gives a sufficiently patient negotiator B the possibility of rejecting low offers and counteroffering in the second period. Third, B can take advantage of her envy level θ_B as a commitment tactic also in her role as a responder of the first-period negotiation, which operates as a credible threat of rejecting low A's offers and counteroffering in the subsequent stage. On the other side, player A has two sources of bargaining power. A first negotiating strength is given by his outside opportunity U_A in his role as a last responder in the whole game. In addition, player A can also exploit the possibility of using his envy parameter θ_A as a commitment device in his role as a responder of the second-period negotiation.

The interaction of all these bargaining power sources makes that in Type II equilibrium, unlike Type I, characteristics of *both* parties shape the agreement partitions, in accordance with that described by expression (10). In the concrete case of envy, this phenomenon implies that two opposite negotiating advantages coming from this negative externality are present: an *envy-based last-responder advantage* (player A) and an *envy-based first-responder advantage* (player B). As a result, whereas player i has greater bargaining power due to his/her own envy parameter θ_i, his/her position is weakened as his/her rival's envy parameter θ_j increases.

We end this subsection with an alternative illustration of the different strategic nature underlying the two agreement types. To do that, we restrict the environment to the assumptions adopted by the previous literature on classic ultimatum-type games so that envy is absent, i.e., $\theta_A = \theta_B = 0$, and outside opportunities are normalized to zero, i.e., $U_A = U_B = 0$. Under this environment, the Type II agreement shares are such that

$$\overline{\overline{x_1}} = \pi(1 - \delta_B),$$

which coincides with the agreement sharing rule yielded by the classic *two-stage* ultimatum game. Notice, nevertheless, that this equilibrium cannot be recovered from Type I agreement, whose equilibrium shares under the same parameter values are such that

$$\overline{x_1} = \pi,$$

which represents in fact the agreement sharing rule delivered by the classic *one-shot* ultimatum game.

4.3 Negotiation breakdown

A disagreement will occur in our bargaining game whenever

$$\widetilde{\pi} < \widetilde{U}_{\delta_A} + \widetilde{U}_{\delta_B}. \tag{11}$$

Thus, if the modified surplus at stake is sufficiently low (i.e., smaller than the sum of the present value of modified outside opportunities), bargaining powers become *incompatible* and negotiation breaks down. And since in our model the negotiation breaks down in both periods, we say that there is a *permanent* disagreement.

It is worthy to note that, like Type I agreement, this negotiation breakdown is neither possible in the setup proposed by Kirchsteiger in [12] due to his assumptions on outside opportunities and envy parameters. Specifically, these assumptions imply that $\widetilde{U}_\delta = 0$ for the two players and that $\widetilde{\pi} > 0$, which makes impossible disagreement condition (11) to be satisfied.

Notice that when envy levels increase, the disagreement region becomes larger since it can easily be established that

$$\frac{\partial \widetilde{\pi}}{\partial \theta_i} < 0 \quad \text{and} \quad \frac{\partial \widetilde{U}_{\delta_j}}{\partial \theta_i} > 0 \quad \text{for all } \; i, j = A, B, i \neq j.$$

Also, note that condition (11) is a modified version of the (dis)agreement condition found in the existing literature. In fact, it can be interpreted as the converse of a sufficient condition ensuring the existence of the so-called *contract zone* [3], but in a bargaining problem *modified* by the participation of envious players.[4] As pointed out when analyzing properties of the agreement types, we argue that this modification generates insights similar to those identified by earlier theoretical models of *commitment tactics* in bargaining situations [3,15]. In particular, we claim that envy can also be thought of as an element that constrains the negotiator's capacity to make concessions to his/her counterpart. In the context of ultimatum-types games, this commitment device is exercised by the player who performs the role of responder, as envy can induce him/her to credibly threaten to reject offers giving too much to the proposer.

[4] In the terminology of [17], the (strictly) converse of inequality (11) would be a sufficient condition for an *essential* bargaining problem, i.e., the existence of a nonempty set of payoff pairs from potential agreements such that both parties are better off than they would be if a disagreement takes place.

Thus, although at first sight envy can be seen as a weakness, it in fact constitutes a source of bargaining power.[5]

The similarity in the roles played by envy and commitment tactics is easily appreciated if we consider that both of them modify outside opportunities. Thus, the modification of outside opportunities from U_i to $\widetilde{U_i}$ in our framework is comparable to the case of commitment devices as outlined by Nobel laureate economist Thomas C. Schelling in his classic 1960 bargaining theory essay [22]. Referring to outside opportunities as reserve prices, Schelling argues that "...the process of discovery and revelation [of the reservation prices] becomes quickly merged with the process of creating and discovering commitments; the commitments permanently *change*, for all practical purposes, the 'true' reservation prices." ([22], pp. 27, emphasis added).

4.4 Inequity of agreements

A related question also of importance is whether or not one of the two agreement types is more generous to a given player. This concern is especially pertinent because our previous analysis revealed, for the two agreement types, a complex interaction among different bargaining power sources for each negotiator.

To address this question and determine the degree of inequity of a given agreement solution, we define Ψ_k as the *bias ratio of Type k agreement* to be given by

$$\Psi_I \equiv \frac{\overline{x_1}}{\pi - \overline{x_1}}, \tag{12}$$

and

$$\Psi_{II} \equiv \frac{\overline{\overline{x_1}}}{\pi - \overline{\overline{x_1}}}. \tag{13}$$

Hence, if $\Psi_k = 1$ then the Type k agreement involves a perfectly equitable sharing rule; if $\Psi_k > 1$ the agreement is biased in favor of player A; and if $\Psi_k < 1$, the bias favors player B.

For the sake of simplicity, we now add an assumption of symmetry for θ, δ, and U.[6] Under this assumption, the bias ratio of Type I agreement is

$$\Psi_I = \frac{\pi - \delta U}{\theta\pi + \delta U} \geqslant 1.$$

[5] This property is evident above when analyzing how agreement shares depend on the θ parameters in expressions (8) and (10).

[6] This assumption is adopted for simplicity. It should be clear that a symmetric environment of this class does not 'neutralize' any source of bargaining power between players in any of the two agreement types. This is because parties have different order of moves, and consequently, they exploit different negotiating advantages (see the discussion presented in Sect. 4.2). Notice, nevertheless, that assuming a common θ will allow us to explore through numerical simulations which envy-based advantage eventually dominates in Type II agreement, either that of the first-responder (player B), or that of the last-responder (player A).

This is true by (2), the condition ensuring a Type I agreement. By contrast, under the same symmetry assumption for a Type II agreement we get

$$\Psi_{II} = \frac{\pi(1-\delta(1-\theta)) + \delta U}{\pi(\theta+\delta(1-\theta)) - \delta U},$$

which according to the parameters of the model can be greater than, less than or equal to 1. Thus, whereas a Type I agreement never favors player B, such a bias is possible in a Type II agreement. This provides a first approximation of the degree of inequity of both types of agreements, suggesting that while the interaction of bargaining power sources in Type I agreement benefits unambiguously player A, this interaction produces a 'more fair' negotiation in Type II solution. This result will be examined further in the next section on simulations.

5 Numerical illustration

This section presents the results of a series of numerical simulations carried out to illustrate the influence of envy, impatience and outside opportunities on the game outcomes.

If we assume, for simplicity, complete symmetry between the parties such that $\theta_A = \theta_B = \theta$, $U_A = U_B = U$, and $\delta_A = \delta_A = \delta$, conditions (1) and (2) for agreement/disagreement become

$$\frac{U}{\pi} \leq \frac{1-\theta}{2} \equiv u(\theta) \tag{14}$$

and

$$\frac{U}{\pi} \leq \frac{1-\theta}{2\delta} \equiv u(\theta,\delta), \tag{15}$$

respectively.[7]

Based on these two conditions we can now characterize in Fig. 1 the agreement and disagreement regions yielded by our model. Two properties in particular are clearly apparent.

First, there is an inverse relationship between the envy level θ and the size of the two agreement regions. This is clearly illustrated if we consider two extreme cases. Thus, when $\theta = 0$, the regions are at their largest and a Type I agreement is feasible over the region $\frac{1}{2} < \frac{U}{\pi} \leq \frac{1}{2\delta}$ while a Type II agreement is possible if $\frac{U}{\pi} \leq \frac{1}{2}$. By contrast, when $\theta = 1$, both regions shrink to a mere point, and if in addition $U > 0$, the point disappears and the only possibility is a negotiation breakdown.[8] Second, as the rate of impatience increases (and thus δ decreases), the Type I agreement region becomes larger since $\frac{\partial u(\theta,\delta)}{\partial \delta} < 0$ for all θ.[9] This result is consistent with the idea that a negotiated solution is more likely in a dynamic bargaining framework when the

7 Since $\delta < 1$, it is easily confirmed that $u(\theta,\delta) > u(\theta)$.

8 This result is consistent with Corollary 2.

9 This property is only present in Type I agreement.

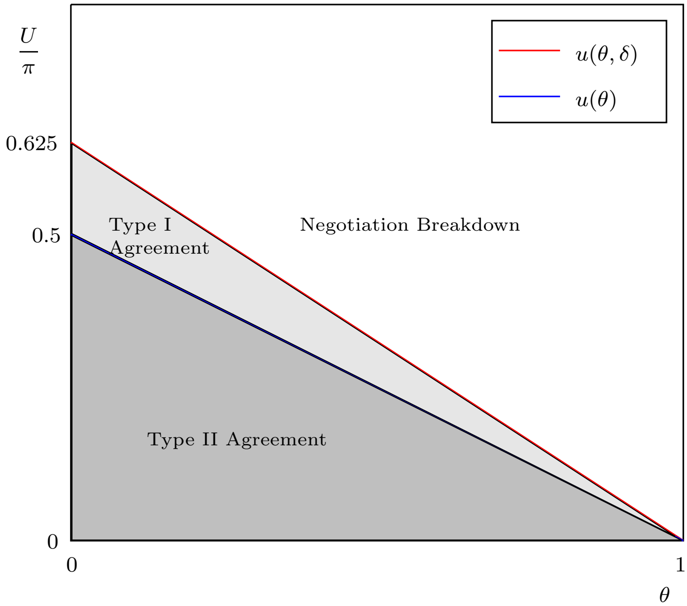

Fig. 1 Agreement and breakdown regions under symmetry, assuming $\delta = 0.8$. Type II agreement region consists of all pairs $(\theta, \frac{U}{\pi})$ on or below upper-bound curve $u(\theta)$ (*gray region*); Type I agreement region consists of all pairs above upper-bound $u(\theta)$ but on or below upper-bound curve $u(\theta, \delta)$ (*light gray region*); negotiation breakdown region consists of all pairs above upper bound $u(\theta, \delta)$ (*white region*)

cost of haggling increases, as impatience constitutes a source of *friction* that acts as an incentive for players to reach an agreement [16]. Note, however, that this positive impact of a lower δ on the agreement region is *decreasing* with the envy level θ, reflecting the fact that impatience and envy are *opposite* forces acting on the likelihood of an agreement/disagreement outcome.

A number of our simulations also examine the characteristics of the two agreement types' sharing rules. Two properties in particular are explored: (i) the comparative degree of inequity between the two types of agreements, and (ii) the role played by envy and outside opportunities. We start by computing Ψ_k, the bias ratio of a Type k agreement, as defined by Eqs. (12) and (13). As well as assuming symmetry of θ, δ and U, we suppose that $\pi = 1$ and $\delta = 0.8$. The main results of these simulations are set out in Table 1 and can be summarized as follows. First, as Fig. 1 also suggests, the region of Type II agreements largely exceeds that of Type I agreements. Second, the larger the value of θ, the higher the bias Ψ_{II}, suggesting that the net effect of an increase in the common envy level leads to an agreement biased in favor of the offerer, i.e., player A. This result means, therefore, that in Type II agreement the envy-based advantage is stronger for the last responder (player A) than for the first responder (player B). The commitment tactic because of envy, thus, seems to be more effective when involving a threat to break altogether the negotiations than a threat to make a counteroffer in a subsequent stage. Third, the larger the value of U, the higher the bias Ψ_{II}. As for this property, let us recall that in the context of Type II agreement,

Table 1 Bias ratio of agreements Ψ

	θ							
	0.1	0.3	0.4	0.5	0.6	0.7	0.8	1
U								
0	0.34[a]	0.51	0.59	0.67	0.74	0.81	0.88	*NB*
0.1	0.49	0.67	0.75	0.83	0.90	0.98	1.05	*NB*
0.2	0.67	0.86	0.94	1.03	1.11	*NB*	*NB*	*NB*
0.3	0.90	1.09	1.19	**1.03**	*NB*	*NB*	*NB*	*NB*
0.4	1.20	**1.09**	*NB*	*NB*	*NB*	*NB*	*NB*	*NB*
0.5	**1.20**[b]	*NB*	*NB*	*NB*	*NB*	*NB*	*NB*	*NB*
0.6	*NB*	*NB*	*NB*	*NB*	*NB*	*NB*	*NB*	*NB*
0.7	*NB*	*NB*	*NB*	*NB*	*NB*	*NB*	*NB*	*NB*

NB Negotiation breakdown
[a] Figures in regular typeface: Type II agreement
[b] Figures in bold typeface: Type I agreement

an increase of U means in fact an increase of player A's outside opportunity. Thus, this result just reflects the last responder's capacity to use his outside opportunity as a device to obtain a minimum stake of the surplus under negotiation.

Finally, as suggested by the analysis in Sect. 4.4, a Type I agreement seems to be more generous with player A than a Type II agreement, as constellations of parameters imply that whereas Ψ_I is always greater than 1, Ψ_{II} is in most of the cases smaller than 1. This finding is consistent with the fact that, although players' negotiating strengths depend on the type of agreement, in our novel Type I solution the interaction of the sources of such strengths benefits unambiguously player A. More specifically, this result reveals that the last-proposer advantage A enjoys in the kind of one-shot ultimatum game originating Type I agreement seems to be more powerful than the similar advantage B exhibits in the two-stage ultimatum game delivering Type II agreement.

6 Concluding remarks

We have proposed a two-period ultimatum bargaining model with envy in which each party suffers a negative payoff proportional to the surplus captured by his/her counterpart. As we consider more general assumptions regarding envy parameters and outside opportunities, our framework yields a richer set of bargaining outcomes than that delivered by existing close works. More specifically, our setting allows three possible outcomes: (i) a Type I agreement without delay, (ii) a Type II agreement without delay, and (iii) a permanent disagreement.

Among these outcomes, the main novelty is Type I agreement, a negotiated solution not reported previously, which contrasts with Type II agreement, a solution closer to the equilibrium of traditional versions of the two-stage ultimatum bargaining game. Whereas Type I agreement is the equilibrium reached in a negotiation strategically equivalent to a one-shot ultimatum game played in period 1, Type II agreement is the equilibrium in a first-period negotiation whose alternative scenario is the agreement of an ultimatum game in period 2. This difference between the two agreement types implies further differences in two dimensions: (i) which player's characteristics (envy level, outside opportunity and impatience degree) influence the distributive solutions, and (ii) which party is more highly favored.

Regarding the influence of the individual players' characteristics, whereas in a Type I agreement only the second-moving party's parameters play a role, in a Type II agreement parameters of both parties shape the final solution. More particularly, the envy level of the second-moving player is a source of her bargaining power in both agreement types (as with a commitment tactic) while the envy parameter of the first-moving player gives him a bargaining advantage only in the second type.

As for the degree of inequity of the two agreement types, analytical solutions and numerical simulations suggest that our novel Type I is more generous to the first-moving player than Type II. Although the inequity degree of the two agreement solutions is the consequence of a complex interaction of different bargaining power sources, the last result suggests, at least, two interesting facts. On the one hand, in an ultimatum game of the one-shot type, the proposer's strategic advantage in general dominates any negotiating advantage the responder can exploit from credibly rejecting, because of envy, a proposal too generous to her counterpart. On the other hand, although in a two-stage ultimatum game the last-proposer's advantage is weaker than that enjoyed by the proposer in the one-shot version of the game, it is enough to induce a priori a 'more fair' negotiation.

Finally, as regards the condition for a permanent negotiation breakdown, we restated it in terms of a *modified* relationship between outside opportunities and the surplus at stake, which is consistent with the classic condition found in the existing literature. Our results demonstrate that envy levels and impatience rates act as opposing forces on the probability of a disagreement. A breakdown is more likely when the two parties' envy levels are relatively high, which can be interpreted as a negotiation conducted under a polarized environment. On the other hand, an agreement is more likely when the impatience rates of both players are relatively high, which can be interpreted as implying larger haggling costs and therefore greater incentives for the parties to reach a negotiated solution.

Appendix

Proof of Proposition 1 By the backward induction principle, we proceed as follows: *Period* $t = 2$ At this stage player, A accepts player B's counteroffer if

$$x_2 - \theta_A(\pi - x_2) \geq U_A \Longleftrightarrow x_2 \geq \frac{U_A + \theta_A \pi}{1 + \theta_A},$$

and rejects it otherwise. In turn, player B chooses x_2 such that

$$\underset{x_2}{Max}\, \pi - (1 + \theta_B) x_2$$
$$s.t.$$

$$
\begin{aligned}
0 \leq x_2 &\leq \pi, \\
\pi - (1+\theta_B)x_2 &\geq U_B, \\
x_2 &\geq \frac{U_A + \theta_A \pi}{1+\theta_A}.
\end{aligned}
$$

This problem is equivalent to the following program:

$$
\underset{x_2}{Min}\,(1+\theta_B)x_2 \tag{16}
$$

$$
s.t.
$$

$$
\frac{U_A + \theta_A \pi}{1+\theta_A} \leq x_2 \leq \frac{\pi - U_B}{(1+\theta_B)}. \tag{17}
$$

The interval of x_2 defined by constraint (17) turns out to be empty because when (1) is not satisfied, it follows that

$$
\frac{U_A + \theta_A \pi}{1+\theta_A} > \frac{\pi - U_B}{(1+\theta_B)}.
$$

Hence, a negotiation breakdown would take place at $t = 2$. The optimal strategy for player B is therefore:

$$
x_2^* = \frac{U_A + \theta_A \pi}{1+\theta_A} - \varepsilon_B,
$$

for some $\varepsilon_B > 0$.
Period $t = 1$ At this stage, player B accepts player A's offer if

$$
\begin{aligned}
(\pi - x_1) - \theta_B x_1 &\geq \delta_B U_B \\
&\Longleftrightarrow \\
x_1 &\leq \frac{\pi - \delta_B U_B}{1+\theta_B} \equiv \overline{x_1},
\end{aligned}
$$

and rejects it otherwise. In turn, player A chooses x_1^* such that

$$
\underset{x_1}{Max} u_1^A = \begin{cases} (1+\theta_A)x_1 - \theta_A \pi & \text{if } x_1 \leq \overline{x_1} \\ \delta_A U_A & \text{otherwise} \end{cases}.
$$

If we make the substitution $x_1 = \overline{x_1}$, the previous problem implies that player A finally chooses $x_1^* = \overline{x_1}$ if

$$
\begin{aligned}
(1+\theta_A)\overline{x_1} - \theta_A \pi &\geq \delta_A U_A \\
&\Longleftrightarrow \\
\overline{x_1} &\geq \frac{\delta_A U_A + \theta_A \pi}{1+\theta_A}.
\end{aligned} \tag{18}
$$

Note that condition (18) holds if it is true that

$$\begin{aligned}\frac{\pi-\delta_B U_B}{1+\theta_B} &\geq \frac{\delta_A U_A+\theta_A\pi}{1+\theta_A}\\ &\Longleftrightarrow\\ \delta_A U_A(1+\theta_B)+\delta_B U_B(1+\theta_A) &\leq \pi(1-\theta_A\theta_B),\end{aligned}$$

which is in fact condition (2) identified in the main text. Thus, the optimal strategy of player A is described by

$$x_1^*=\begin{cases}\overline{x_1} & \text{if (2) is verified}\\ \overline{x_1}+\varepsilon_A & \text{otherwise}\end{cases}$$

for some $\varepsilon_A>0$. □

Proof of Corollary 1 (i) Since condition (1) is not satisfied and condition (2) is, it follows from Proposition 1 that the equilibrium path is such that at $t=1$, player A offers $x_1^*=\overline{x_1}$ and player B accepts it. (ii) Since conditions (1) and (2) are not satisfied, it follows from Proposition 1 that the equilibrium path is such that:

- At $t=1$, player A offers $x_1^*=\overline{x_1}+\varepsilon_A$ and player B rejects it.
- At $t=2$, player B counteroffers

$$x_2^*=\frac{U_A+\theta_A\pi}{1+\theta_A}-\varepsilon_B,$$

and player A rejects it. □

Proof of Corollary 2 If $\Theta>1$ then $\pi(1-\theta_A\theta_B)$ is negative. Thus, since $\delta_A>0$, $U_A\geq 0$ and $\theta_B\geq 0$ for all $i=A,B$, it is the case that

$$\pi(1-\theta_A\theta_B)<\delta_A U_A(1+\theta_B)+\delta_B U_B(1+\theta_A), \tag{19}$$

which is a sufficient condition for both conditions (1) and (2) not to hold. Applying Corollary 1, Part (ii), the statement is finally demonstrated. □

Proof of Proposition 2 By backward induction, we proceed as follows:
Period $t=2$

We follow a similar line of reasoning to that used in proof of Proposition 1. Thus, whereas the optimal decision rule for player A remains the same, player B solves the program described by Eqs. (16) and (17). Note, however, that now the interval for x_2 defined by constraint (17) is nonempty, which follows from assuming that condition (1) is satisfied.

Thus, the optimal strategy for player B is given by

$$x_2^*=\frac{U_A+\theta_A\pi}{1+\theta_A}.$$

Period $t = 1$

At this stage, player B accepts player A's offer if

$$\begin{aligned} (\pi - x_1) - \theta_B x_1 &\geq \delta_B \left[\pi - (1+\theta_B) x_2^*\right] \\ &\Longleftrightarrow \\ x_1 &\leq \frac{\pi - \delta_B \left[\pi - \frac{(1+\theta_B)(U_A+\theta_A \pi)}{1+\theta_A}\right]}{1+\theta_B} \equiv \overline{\overline{x_1}}, \end{aligned}$$

and rejects it otherwise. Player A chooses x_1^* such that

$$\underset{x_1}{Max} u_1^A = \begin{cases} (1+\theta_A)x_1 - \theta_A \pi & \text{if } x_1 \leq \overline{\overline{x_1}} \\ \delta_A \left[(1+\theta_A)x_2^* - \theta_A \pi\right] & \text{otherwise} \end{cases}.$$

If we make substitution $x_1 = \overline{\overline{x_1}}$, the above program implies that player A finally chooses $x_1^* = \overline{\overline{x_1}}$ as long as

$$(1+\theta_A)\overline{\overline{x_1}} - \theta_A \pi \geq \delta_A \left[(1+\theta_A)x_2^* - \theta_A \pi\right].$$

which after substituting x_2^* is equivalent to

$$\overline{\overline{x_1}} \geq \frac{\delta_A U_A + \theta_A \pi}{1+\theta_A}. \tag{20}$$

Note, however, that condition (20) holds if

$$\begin{aligned} \frac{\pi - \delta_B \left[\pi - \frac{(1+\theta_B)(U_A+\theta_A \pi)}{1+\theta_A}\right]}{1+\theta_B} &\geq \frac{\delta_A U_A + \theta_A \pi}{1+\theta_A} \\ &\Longleftrightarrow \\ \frac{U_A(1+\theta_B)(\delta_A - \delta_B)}{(1-\delta_B)} &\leq \pi(1-\theta_A \theta_B). \end{aligned}$$

Since condition (1) suffices for the above inequality to hold, checking that it is always satisfied is straightforward.[10] Hence, the optimal strategy of player A becomes

$$x_1^* = \overline{\overline{x_1}},$$

which completes the proof. □

Proof of Corollary 3 Since condition (1) holds, the equilibrium path according to Proposition 2 is such that at $t = 1$, player A offers $x_1^* = \overline{\overline{x_1}}$ and player B accepts it. □

[10] That is, since $\delta_i \in (0, 1)$ it is true that

$$U_A(1+\theta_B)\frac{\delta_A - \delta_B}{1-\delta_B} \leq U_A(1+\theta_B) + U_B(1+\theta_A) \leq \pi(1-\theta_A \theta_B),$$

where the last inequality is in fact condition (1).

References

1. Berg, J., Dickhaut, J., McCabe, K.: Trust, reciprocity, and social history. Games Econ. Behav. **10**(1), 122–142 (1995)
2. Chowdhury, P.R.: Externalities and bargaining disagreement. Econ. Lett. **61**, 61–65 (1998)
3. Crawford, V.: A theory of disagreement in bargaining. Econometrica **50**(3), 607–637 (1982)
4. Falk, A., Fischbacher, U.: A theory of reciprocity. Games Econ. Behav. **54**(2), 293–315 (2006)
5. Fehr, E., Schmidt, K.: A theory of fairness, competition, and cooperation. Q. J. Econ. **114**(3), 817–868 (1999)
6. Fischbacher, U., Hertwig, R., Bruhin, A.: How to model heterogeneity in costly punishment: insights from responders' response times. J. Behav. Decis. Making **26**(5), 462–476 (2013)
7. Güth, W., Kocher, M.: More than thirty years of ultimatum bargaining experiments: motives, variations, and a survey of the recent literature. J. Econ. Behav. Organ. **108**, 396–409 (2014)
8. Jehiel, P., Moldovanu, B.: Negative externalities may cause delay in negotiation. Econometrica **63**(6), 1321–1335 (1995)
9. Jehiel, P., Moldovanu, B.: Cyclical delay in bargaining with externalities. Rev. Econ. Stud. **62**(4), 619–637 (1995)
10. Kagel, J., Wolfe, K.: Tests of fairness models based on equity considerations in a three-person ultimatum game. Exp. Econ. **4**(3), 203–219 (2001)
11. Kahneman, D., Knetsch, J., Thaler, R.: Fairness and the assumptions of economics. J. Bus. **59**(4), 285–300 (1986)
12. Kirchsteiger, G.: The role of envy in ultimatum games. J. Econ. Behav. Organ. **25**(3), 373–389 (1994)
13. Laengle, S., Loyola, G.: The ultimatum game with externalities. Econ. Comput. Econ. Cybern. Stud. Res. **49**(4), 279–287 (2015)
14. Laengle, S., Loyola, G.: Bargaining and negative externalities. Optim. Lett. **6**, 421–430 (2012)
15. Muthoo, A.: A Bargaining model based on the commitment tactic. J. Econ. Theory **69**, 134–152 (1996)
16. Muthoo, A.: Bargaining Theory with Applications. Cambridge University Press, Cambridge (1999)
17. Myerson, R.: Game Theory: Analysis of Conflict. Harvard University Press, Cambridge (1991)
18. Nowak, M., Page, K., Sigmund, K.: Fairness versus reason in the ultimatum game. Science **289**(5485), 1773–1775 (2000)
19. Pfister, H.-R., Böhm, G.: Responder feelings in a three-player three-option ultimatum game: affective determinants of rejection behavior. Games **3**(1), 1–29 (2012)
20. Pillutla, M., Murnighan, J.: Unfairness, anger, and spite: emotional rejections of ultimatum offers. Organ. Behav. Human. Decis. Process. **68**(3), 208–224 (1996)
21. Sanfey, A., Rilling, J., Aronson, J., Nystrom, L., Cohen, J.: The neural basis of economic decision-making in the ultimatum game. Science **300**(5626), 1755–1758 (2003)
22. Schelling, T.C.: The Strategy of Conflict. Harvard University Press, Cambridge (1960)
23. van 't Wout, M., Kahn, R., Sanfey, A., Aleman, A.: Affective state and decision-making in the ultimatum game. Exp. Brain Res. **169**(4), 564–568 (2006)
24. Xie, W., Li, Y., Wang, Y., Li, K.: Responders' dissatisfaction may provoke fair offer. J. Econ. Interact. Coord. **7**(2), 197–207 (2012)