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Bonferroni induced heavy operators in ERM decision-making: A case on large companies in Colombia

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ABSTRACT

Averaging aggregation operators analyse a set of data providing a summary of the results. This study focuses on the Bonferroni mean and the induced and heavy aggregation operators. The aim of the work is to present new aggregation operators that combine these concepts forming the Bonferroni induced heavy ordered weighted average and several particular formulations. This approach represents Bonferroni means with order inducing variables and with weighting vectors that can be higher than one. The paper also develops some extensions by using distance measures forming the Bonferroni induced heavy ordered weighted average distance and several particular cases. The study ends with an application in a large companies risk management problem in Colombia. The main advantage of this approach is that it provides a more general framework for analysing the data in scenarios where the numerical values may have some complexities that should be assessed with complex attitudinal characters.

1. Introduction

The enterprise risk management (ERM) has been around for some years but the formal definition has been developed by the Committee of Sponsoring Organizations (COSO) in 2004 [1]. In this approach, the company has to identify, assess and manage risk as a corporate perspective and not as a department-by-department perspective [2]. Some of the advantages that the ERM has is that reduce capital costs, the volatility of earnings and stock prices and helps to create synergy in different risk management activities [2–4].

A way to improve the results of the ERM is adding the knowledge and expectations that the decision makers have about the future scenarios. One of the techniques that have been used to aggregate information in decision making process [5] is the Ordered Weighted Average (OWA) operator developed by Yager [6]. It has been used in a great deal of applications [7–11] and developed a lot of extensions [12–16].

The originality of this paper is that introduce in the same formulation a complex formulation that takes four different aggregation operators into one. The operators that the paper focuses are the Bonferroni OWA (BON-OWA) [17], induced OWA (IOWA) [18], heavy OWA (HOWA) [19] and OWA distance (OWAD) [20] operators. The BON-OWA operator [17] can compare the arguments in multiple ways and capture the interrelationship. This operator has multiple extensions such as uncertain linguistic weights [21], interval values [22], distance operators [23,24] and some applications in multiple criteria decision making [25], multiple attribute decision making [26] and group decision making [27] has been made.

The IOWA operator [18] has as main characteristic that it is able to reorder the arguments using order-inducing variable depending on values associated to arguments. This operator has been studied using fuzzy measures, fuzzy numbers and many extensions have been made [28]. On other hand, the HOWA operator [19] has as main characteristic that the weighting vector is not bounded to one. Doing this is

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possible to generate new scenarios to under or overestimate the results according to the expectation and knowledge of the future scenarios of the decision makers. Some applications and extensions have been developed [29]. Finally, The OWAD operator [30] is an operator that can be used when two sets of information want to be compared and an ordered weighting vector is used. Among the applications there are in information systems, transparency and access to information laws and decision-making process [13,31,32].

The aim of the paper is to present new operators that use the main characteristics of the four previously specified techniques. In this sense the Bonferroni induced heavy ordered weighted average distance (BON-IHOWAD) operator is introduced and some of the families that are obtained when not all the characteristics of the main operator need to be used. This occurs in cases that the problem is easy to solve or that not many characteristics need to be taken into account to the final decision. In these situations we may get the Bonferroni heavy ordered weighted average distance (BON−HOWAD) operator, Bonferroni induced ordered weighted average distance (BON-IOWAD) operator and some others. Also, among the main advantages the BON-IHOWAD operator has is that it can include in one formulation the expectations and knowledge of the decision maker in the result. Thus, is possible to generate new scenarios that cannot be seen if only the historical data is taken into account or using the aggregation process proposed of each operator separately.

Even though, traditional methods for dealing with ERM provide a rational philosophy, the heterogeneity of its organizational dynamics are difficult to determine used them. Thus, this new method allows dealing with heterogeneity and uncertainty by information asymmetry in ERM [32]. Hence, the BON-IHOWAD operator and families' operators have been applied in an ERM problem for large Colombians companies in order to generate new scenarios that compare the main goals in ERM with expected benefits through the interrelationship between main risks. Firstly, this application allows us to observe how the managers responsible of risk management prioritize goals for each sector to get expected benefits. Secondly, this will help the decision maker in understanding their actual situation and improve the strategy that they are applying with the objective of obtain more profits.

The remainder of the paper is organized as follows. Section 2 presents the basic concepts of the OWA operator, distance operators and Bonferroni operators. Section 3 introduced the new propositions of operators based in Bonferroni, IOWA, HOWA and distances operators and in Section 4 a practical case of ERM in large Colombian companies is presented. Finally, Section 5 summarizes the main conclusions of the paper.

2. Preliminaries

This section presented some basic concepts that have been used throughout the paper, including the aggregation operators, Bonferroni operators and distance operators.

2.1. Aggregation operators

One of the methods that have been used for aggregate information is the OWA operator [6]. The main characteristic of this operator is the reordering step with the possibility to obtain the maximum operator and the minimum operator. Since its presentation many extension have been developed and applied in many problems [9,33,34]. The definition is as follows.

Definition 1. An OWA operator of dimension n is a mapping $OWA : R^n \rightarrow R$ with an associated weight vector W of dimension n such that $\sum_{j=1}^{n} w_j = 1$ and $w_j \in [0, 1]$, according tothe following formula:

$$OWA(a_1, a_2, \ldots, a_n) = \sum_{j=1}^{n} w_j b_j, \tag{1}$$

where b_j is the *jth* largest element of the collection a_i.

Also, it is noteworthy that decisions within the OWA operators can be given under four criteria:

1. Optimistic criterion. It is based on the assumption that the most favorable state was presented, so that one should select the most favorable result of each alternative and the results obtained selecting the most favorable of all. So that this criterion is based on a maxim that is formulated

$$Decision = Max\{E_i\} = Max[Max\{a_j\}]$$

2. Pessimistic or Wald criterion. It argues that the decision-maker must select the alternative that provides a higher level of security, so that our decision should be the most favorable outcome of the most unfavorable for each alternative. This method is known as Max Min and its formula is

$$Decision = Max\left\{E_i\right\} = Max[Min\left\{a_j\right\}]$$

3. Hurwicz criterion. It is to ponder with an optimistic coefficient and another pessimist to the best and worst case respectively, then add the values and choose that alternative that proposes a greater result. The formula for this criterion is

$$Decision = Max\left\{E_j\right\} = Max[\alpha Max\left\{a_j\right\} + (1-\alpha) Min\left\{a_j\right\}]$$

where $\alpha + (1 - \alpha) = 1$.

4. Laplace criterion. It is based on the principle of insufficient reason, so that the same degree of probability is associated with the different scenarios, as long as there are no indications of the opposite. The formula is

$$Decision = Max\left\{E_j\right\} = Max[\left({}^1\!/\!_n\right) \sum_{j=1}^{n} a_j]$$

One of the extension of the OWA operator that put attention in the way of the reordering step is made, is the Induced OWA (IOWA) operator [18]. In this operator, the weighting vector affects the arguments based in the order-induced vector, such that:

Definition 2. An IOWA operator of dimension n is an application $IOWA : R^n \times R^n \to R$ that has a weighting vector associated, W of dimension n where the sum of the weights is 1 and $w_j \in [0, 1]$, where an induced set of ordering variables are included (u_i) such that the formula is:

$$IOWA(\langle u_1, a_1\rangle, \langle u_2, a_2\rangle, \ldots, \langle u_n, a_n\rangle) = \sum_{j=1}^{n} w_j b_j, \quad (2)$$

where b_j is the a_i value of the OWA pair $< u_i, a_i >$ having the jth largest u_i. u_i is the order inducing variable and a_i is the argument variable.

Another extension that put attention in the weighting vector is the Heavy OWA (HOWA) operator [19]. This operator consider that the total value of the weighting vector can be lower or higher than 1 and the definition is:

Definition 3. A heavy aggregation operator is an extension of the OWA operator for which the sum of weights is bounded by n. Thus, an HOWA operator is a map $R^n \to R$ that is associated with a weight vector w, with $w_j \in [0, 1]$ and $1 \le \sum_{j=1}^{n} w_j \le n$, such that:

$$HOWA(a_1, a_2, \ldots, a_n) = \sum_{j=1}^{n} w_j b_j, \quad (3)$$

where b_j is the jth largest element of the collection $a_{1,} a_2, \ldots, a_n$ and the sum of the weights w_j is bounded to n or can be unbounded if the weighting vector W, $-\infty \le \sum_{j=1}^{n} w_j \le \infty$. Additionally, according to Yager [19] and Merigó and Casanovas [35] the OWA operator is found when the sum of the weights is one. The total operator is found when the sum of the weights is n. The minimum is found when $w_n = 1$, $w_j = 0$ for all $j \ne n$ and the sum of the weights is one, which is called Descending HOWA (DHOWA) operator. For obtaining the maximum and the average criteria, we could find it from different aggregations as the weighting vector can be higher than 1. We should note that these results could also be obtained for the Ascending HOWA (AHOWA) operator.

It is also possible to add the characteristics of the IOWA operator and the HOWA operator in one unified formulation, this operator is known as the Induced Heavy OWA (IHOWA) operator. This operator uses the reordering step of the IOWA operator (based in order-induced variables) and the values of the weights are not bounded to 1, the definition is as follows [35].

Definition 4. An IHOWA operator of dimension n is a mapping $IHOWA : R^n x R^n \to R$ that has an associated weighting vector W of dimension n with $w_j \in [0, 1]$ and $1 \le \sum_{j=1}^{n} w_j \le n$, such that:

$$IHOWA(\langle u_1, a_1\rangle, \ldots, \langle u_n, a_n\rangle = \sum_{j=1}^{n} w_j b_j, \quad (4)$$

where b_j is the a_i value of the IHOWA pair $\langle u_i, a_i \rangle$ having the jth largest u_i, u_i is the order-inducing variable, a_i is the argument variable and the sum of the weights w_j is bounded to n or can be unbounded if the weighting vector W, $-\infty \le \sum_{j=1}^{n} w_j \le \infty$.

2.2. Distance OWA operators

The Hamming distance is useful when there are two sets that want to be compared [36]. This technique has been applied to in fuzzy set theory [37,38]. The general expression is:

Definition 5. A normalized Hamming distance of dimension n is a mapping $NHD : [0, 1]^n x[0, 1]^n \to [0, 1]$, such that:

$$NHD(A, B) = \left(\frac{1}{N} \sum_{i=1}^{n} \left| a_i - b_i \right| \right), \quad (5)$$

where a_i and b_i are the ith arguments of sets A and B, respectively.

The Hamming distance can be combined with the OWAD operator, in order to generate the Hamming OWAD operator. With this operator is possible to generate the maximum and the minimum operator. The definition is as follows [38,39]:

Definition 6. An OWAD operator of dimension n is a mapping $OWAD : [0, 1]^n x[0, 1]^n \to [0, 1]$ that has an associated weighting vector w, with $\sum_{j=1}^{n} w_j = 1$ and $w_j \in [0, 1]$, such that:

$$OWAD(\langle \mu_1, \mu_1^{(k)}\rangle, .., \langle \mu_n, \mu_n^{(k)}\rangle) = \sum_{j=1}^{n} w_j D_j, \quad (6)$$

where D_j is the *jth* largest of the differences $|x_i - y_i|$, and $|x_i - y_i|$ is the argument variable represented in the form of individual distances.

Another extension can be done if the weighting vector is not bounded to 1. This operator is called heavy ordered weighted-average distance (HOWAD) operator and is defined as [40]:

Definition 7. A HOWAD operator of dimension n is a mapping $HOWAD: R^n x R^n \rightarrow R$ that has an associated weighting vector W with $w_i \in [0, 1]$ and $1 \leq \sum_{j=1}^{n} w_j \leq n$, such that

$$HOWAD(\langle x_1, y_1\rangle, \langle x_2, y_2\rangle, \ldots, \langle x_m, y_m\rangle) = \sum_{j=1}^{n} w_j D_j, \tag{7}$$

where D_j is the *jth* largest of the differences $|x_i - y_i|$, $|x_i - y_i|$ is the argument variable represented in the form of individual distances, and the weighting vector can take values that satisfy $-\infty \leq \sum_{j=1}^{n} w_j \leq \infty$ to under- or overestimate the results according to the information available.

Also note that if using order-inducing variables does the reordering step the OWAD operator becomes the Induced OWAD (IOWAD) operator. The definition is as follows [34]:

Definition 8. An IOWAD operator of dimension n is a mapping $IOWAD: R^n x R^n x R^n \rightarrow R$ that has an associated weighting vector W such that $w_j \in [0, 1]$ and $W = \sum_{j=1}^{n} w_j = 1$, according to the following formula

$$IOWAD(\langle u_1, x_1, y_1\rangle, \ldots, \langle u_n, x_n, y_n\rangle) = \sum_{j=1}^{n} w_j b_j \tag{8}$$

where b_j is the $|x_1 - y_1|$ value of the IOWAD triplet $\langle u_i, x_i, y_i\rangle$ having the *jth* largest u_i, u_i is the order-inducing variable and $|x_1 - y_1|$ is the argument variable represented in the form of individual distances.

Finally, if we unify the reordering step of the IOWAD and the weighting vector of the HOWAD, we obtain the induced heavy OWAD (IHOWAD) operator. The definition is as follows [41]:

Definition 9. An IHOWAD operator of dimension n is a mapping $IHOWAD: R^n x R^n x R^n \rightarrow R$ that has an associeated weighting vector W with $w_i \in [0, 1]$ and $1 \leq \sum_{j=1}^{n} w_j \leq n$, such that:

$$IHOWAD(\langle u_1, x_1, y_1\rangle, \ldots, \langle u_n, x_n, y_n\rangle) = \sum_{j=1}^{n} w_j D_j \tag{9}$$

where D_j is the $|x_1 - y_1|$ value of the IHOWAD triplet $\langle u_i, x_i, y_i\rangle$ having the *jth* largest u_i, u_i is the order-inducing variable and $|x_1 - y_1|$ is the argument variable represented in the form of individual distances.

2.3. BON-OWA operators

The Bonferroni mean was proposed by Bonferroni [42] and many extensions have been developed like using linguistic variable [21], fuzzy sets [15], interval values [22] and so on. One of the extensions is with the OWA operator, which was presented by Yager [17] as the Bonferroni OWA operator as follows.

Definition 10. The Bonferroni OWA is a mean type aggregation operator. It can be defined by using the following expression.

$$BON - OWA(a_1, \ldots, a_n) = \left(\frac{1}{n}\sum_i a_i^r OWA_W\left(V^i\right)\right)^{\frac{1}{r+q}} \tag{10}$$

where $\left(V^i\right)$ is the vector of all a_j except a_i. Let W be an OWA weighing vector of dimension $n-1$ with components $w_i \in [0, 1]$ when $\sum_i w_i = 1$. Then, we can define this aggregation as $OWA_W\left(V^i\right) = \left(\sum_{j=1}^{n-1} w_i a_{\pi_k(j)}\right)$, where $a_{\pi_k(j)}$ is the largest element in the n – 1 tuple V^i = $(a_1, \ldots, a_{i\text{-}1}, a_{i+1}, \ldots, a_n)$.

Note that if $w_i = \frac{1}{n-1}$ for all i, $OWA_W\left(V^i\right) = \left(\frac{1}{n-1}\sum_{j=1, j\neq i}^{n} a_j^q\right)$, and Eq. (10) becomes the classical Bonferroni mean.

Definition 11. A BON-OWAD distance for two sets $X = \{x_1, x_2, \ldots, x_n\}$ and $Y = \{y_1, y_2, \ldots, y_n\}$ is given by:

$$BON-OWAD(\langle x_1,y_1\rangle,\ldots,\langle x_n,y_n\rangle)=\left(\frac{1}{n}\sum_i D_i^r OWAD_{w_i}\left(V^i\right)\right)^{\frac{1}{r+q}} \quad (11)$$

where $\left(V^i\right)$ is the vector of all $\left|x_j-y_j\right|$ except $\left|x_i-y_i\right|$. Let W be an OWA weighing vector of dimension $n-1$ with components $w_i \in [0,1]$ when $\sum_i w_i=1$. Then, we can define this aggregation as $OWAD_W\left(V^i\right)=\left(\sum_{j=1}^{n-1} w_i D_{\pi_k(j)}\right)$, where $D_{\pi_k(j)}$ is the largest element in the n – 1 tuple $V^i=\left(\langle x_1,y_1\rangle,\ldots,\left\langle x_{i-1},y_{i-1}\right\rangle,\left\langle x_{i+1},y_{i+1}\right\rangle,\ldots,\langle x_n,y_n\rangle\right)$.

Note that if $w_i=\frac{1}{n-1}$ for all i, $OWAD_{w_i}\left(V^i\right)=\left(\frac{1}{n-1}\sum_{j=1,\ j\neq i}^{n} D_j^q\right)$, and Eq. (11) becomes Bonferroni mean distance [20].

3. New proposition of BON-OWA operators with IOWA, HOWA and Hamming distance

One extension that can be done for the BON-OWA operator is if a reordering step according to order-inducing variables and a weighting vector not bounded to 1 are applied. This operator is called the Bonferroni Induced Heavy OWA (BON-IHOWA) operator and can be defined as follows.

Definition 12. The Bonferroni IHOWA (BON-IHOWA) is a mean type aggregation operator that has an associated weighting vector W with $w_i \in [0,1]$ and $1 \le \sum_{j=1}^{n} w_j \le n$, such that

$$BON-IHOWA(\langle u_1,a_1\rangle,\ldots,\langle u_n,a_n\rangle)=\left(\frac{1}{n}\sum_i a_i^r IHOWA_W\left(V^i\right)\right)^{\frac{1}{r+q}} \quad (12)$$

where $\left(V^i\right)$ is the vector of all a_j except a_i. Let W be an OWA weighing vector of dimension $n-1$ with components $w_i \in [0,1]$ when $1 \le \sum_{j=1}^{n} w_j \le n$, where the weights are associated according to the largest value of u_i and u_i is the order-inducing variable. Likewise, the sum of the weights w_j is bounded to n or can be unbounded if the weighting vector $W=-\infty \le \sum_{j=1}^{n} w_j \le \infty$. Then, we can define this aggregation as $IHOWA_W\left(V^i\right)=\left(\sum_{j=1}^{n-1} w_i a_{\pi_k(j)}\right)$, where $a_{\pi_k(j)}$ is the largest element in the n – 1 tuple V^i = $V^i=\left(\langle u_1,a_1\rangle,\ldots,\left\langle u_{i-1},a_{i-1}\right\rangle,\left\langle u_{i+1},a_{i+1}\right\rangle,\ldots,\langle u_n,a_n\rangle\right)$.

Note that if $w_i=\frac{1}{n-1}$ for all i, $IHOWA_{w_i}\left(V^i\right)=\left(\frac{1}{n-1}\sum_{j=1,\ j\neq i}^{n} a_j^q\right)$, and Eq. (12) becomes Bonferroni induced heavy mean.

Example 1. We have assumed that IHOWA pair's $\left\langle \mu_i, a_i\right\rangle$ is given by $A=0.2,\ 0.5,\ 0.9,\ 0.8$ and $U=3,\ 6,\ 1,\ 5$. W^* is the weighting vector of the a_i associated with α_i whose components v_{ij}. Here we shall let W^*, instead of being 1, it is 1.9 and it has the following values: $W^*=0.4,\ 0.3,\ 0.7,\ 0.5$. The ordered OWA pair's is $\left\langle 6,\ 0.5\right\rangle,\ \left\langle 5,\ 0.8\right\rangle,\ \left\langle 3,\ 0.2\right\rangle,\ \left\langle 1,\ 0.9\right\rangle$, that is the ordered list a_i is $0.5,\ 0.8,\ 0.2,\ 0.9$. We take $r=q=1$. In addition: $V^1=(0.8+0.2+0.9)$, $V^2=(0.5+0.2+0.9)$, $V^3=(0.5+0.8+0.9)$ and $V^4=(0.5+0.8+0.2)$. Using this get:

$$IHOWA_{v_1}\left(V^1\right)=0.3\times(0.8+0.2+0.9)=0.57$$

$$IHOWA_{v_3}\left(V^3\right)=0.5\times(0.5+0.8+0.9)=1.10$$

$$BON-IHOWA=\left(\frac{1}{4}\times((0.3\times 0.57)+((0.4\times 0.64)+((0.5\times 1.10)++((0.7\times 1.05))\right)^{0.5}=0.6542$$

Note that it is possible to distinguish two particular cases. The first one, if the weighting vector is equal to 1, then the BON-IHOWA becomes the BON-IOWA, so.

Definition 13. The Bonferroni IOWA (BON-IOWA) is a mean type aggregation operator that can be defined as follows.

$$BON-IOWA(\langle u_1,a_1\rangle,\ldots,\langle u_n,a_n\rangle)=\left(\frac{1}{n}\sum_i a_i^r IOWA_W\left(V^i\right)\right)^{\frac{1}{r+q}} \quad (13)$$

where $\left(V^i\right)$ is the vector of all a_j except a_i. Let W be an OWA weighing vector of dimension $n-1$ with components $w_i \in [0, 1]$ when $\sum_i w_i = 1$, where the weights are associated according to the largest value of u_i and u_i is the order-inducing variable. Then, we can define this aggregation as $IHOWA_W\left(V^i\right) = \left(\sum_{j=1}^{n-1} w_i a_{\pi_k(j)}\right)$, where $a_{\pi_k(j)}$ is the largest element in the n – 1 tuple $V^i = V^i = \left(\langle u_1, a_1\rangle, \ldots, \langle u_{i-1}, a_{i-1}\rangle, \langle u_{i+1}, a_{i+1}\rangle, \ldots, \langle u_n, a_n\rangle\right)$.

Note that if $w_i = \frac{1}{n-1}$ for all i, $IOWA_{w_i}\left(V^i\right) = \left(\frac{1}{n-1}\sum_{\substack{j=1\\ j\neq i}}^{n} a_j^q\right)$, and Eq. (13) becomes Bonferroni induced mean.

Example 2. We have assumed that IOWA pair's $\langle \mu_i, a_i\rangle$ is given by $\langle 3,\ 0.2\rangle$, $\langle 6,\ 0.5\rangle$, $\langle 1,\ 0.9\rangle$, $\langle 5,\ 0.8\rangle$. w_i is the weighting vector of the $\langle \mu_i, a_i\rangle$ associated with α_i whose components v_{ij}. Here we shall let $\alpha_1 = 0.4,\ \alpha_2 = 0.3,\ \alpha_3 = 0.7\ and\ \alpha_4 = 0.5$. The ordered OWA pair's is $\langle 6,\ 0.5\rangle$, $\langle 5,\ 0.8\rangle$, $\langle 3,\ 0.2\rangle$, $\langle 1,\ 0.9\rangle$, that is the ordered list a_i is 0.5, 0.8, 0.2, 0.9. We take $r = q = 1$. In addition: $V^1 = (0.8 + 0.2 + 0.9)$, $V^2 = (0.5 + 0.2 + 0.9)$, $V^3 = (0.5 + 0.8 + 0.9)$ and $V^4 = (0.5 + 0.8 + 0.2)$. Using this get:

$$IOWA_{v_1}\left(V^1\right) = 0.4 \times (0.8 + 0.2 + 0.9) = 0.76$$

$$IOWA_{v_3}\left(V^3\right) = 0.7 \times (0.5 + 0.8 + 0.9) = 1.54$$

$$BON - IOWA = \left(\frac{1}{4} \times ((0.5 \times 0.76) + ((0.8 \times 0.48) + ((0.2 \times 1.54) + + ((0.9 \times 0.75))\right)^{0.5} = 0.6608$$

Also if in the reordering step $u_i = 1/n$, then the BON-HOWA operator is obtained, being the formulation.

Definition 14. The Bonferroni HOWA (BON–HOWA) is a mean type aggregation operator that has an associated weighting vector W with $w_i \in [0, 1]$ and $1 \leq \sum_{j=1}^{n} w_j \leq n$, such that

$$BON - HOWA(a_1, \ldots, a_n) = \left(\frac{1}{n}\sum_i a_i^r HOWA_W\left(V^i\right)\right)^{\frac{1}{r+q}} \tag{14}$$

where $\left(V^i\right)$ is the vector of all a_j except a_i. Let W be an OWA weighing vector of dimension $n-1$ with components $w_i \in [0, 1]$ when $1 \leq \sum_{j=1}^{n} w_j \leq n$. Thus, the sum of the weights w_j is bounded to n or can be unbounded if the weighting vector $W = -\infty \leq \sum_{j=1}^{n} w_j \leq \infty$. Then, we can define this aggregation as $HOWA_W\left(V^i\right) = \left(\sum_{j=1}^{n-1} w_i a_{\pi_k(j)}\right)$, where $a_{\pi_k(j)}$ is the largest element in the n – 1 tuple $V^i = V^i = (a_1, \ldots, a_{i-1}, a_{i+1}, \ldots, a_n)$.

Note that if $w_i = \frac{1}{n-1}$ for all i, $HOWA_{w_i}\left(V^i\right) = \left(\frac{1}{n-1}\sum_{\substack{j=1\\ j\neq i}}^{n} a_j^q\right)$, and Eq. (14) becomes Bonferroni heavy mean.

Another extension is when the Bonferroni means are combined with heavy weighted average (HWA) operators [35], this operator is called the Bonferroni HWA (BON-HWA) operator. The formulation is:

Definition 15. The Bonferroni HWA (BON-HWA) is a mean type aggregation operator that has an associated weighting vector W with $w_i \in [0, 1]$ and $1 \leq \sum_{j=1}^{n} w_j \leq n$, such that

$$BON - HWA(a_1, \ldots, a_n) = \left(\frac{1}{n}\sum_i a_i^r HWA_W\left(V^i\right)\right)^{\frac{1}{r+q}} \tag{15}$$

where $\left(V^i\right)$ is the vector of all a_j except a_i. Let W be weighted average vector of dimension $n-1$ with components $w_i \in [0, 1]$ when $1 \leq \sum_{j=1}^{n} w_j \leq n$. Thus, the sum of the weights w_j is bounded to n or can be unbounded if the weighting vector $W = -\infty \leq \sum_{j=1}^{n} w_j \leq \infty$.

Then, we can define this aggregation as $HWA_W\left(V^i\right) = \left(\sum_{j=1}^{n-1} w_i a_{\pi_k(j)}\right)$, where $a_{\pi_k(j)}$ is the largest element in the n – 1 tuple V^i = $V^i = (a_1, \ldots, a_{i-1}, a_{i+1}, \ldots, a_n)$.

Note that if $w_i = \frac{1}{n-1}$ for all i, $HWA_{w_i}\left(V^i\right) = \left(\frac{1}{n-1}\sum_{j=1, j \neq i}^{n} a_j^q\right)$, and Eq. (15) becomes Bonferroni heavy mean.

Example 3. We have assumed that BON−HOWA is given by 0.2, 0.5, 0.9, 0.8. W^* is the weighting vector of the a_i associated with α_i whose components v_{ij}. Here we shall let W^*, instead of being 1, it is 1.9 and it has the following values: $W^* = 0.4,\ 0.3,\ 0.7,\ 0.5$. We take $r = q = 1$. In addition: $V^1 = (0.5 + 0.9 + 0.8)$, $V^2 = (0.2 + 0.9 + 0.8)$, $V^3 = (0.2 + 0.5 + 0.8)$ and $V^4 = (0.2 + 0.5 + 0.9)$. Using this get:

$$HOWA_{v_1}\left(V^1\right) = 0.4 \times (0.5 + 0.9 + 0.8) = 0.88$$

$$HOWA_{v_3}\left(V^3\right) = 0.7 \times (0.2 + 0.5 + 0.8) = 1.05$$

$$BON - HOWA = \left(\frac{1}{4} \times ((0.2 \times 0.88) + ((0.5 \times 0.57) + ((0.9 \times 1.05) + +((0.8 \times 0.80))\right)^{0.5} = 0.7151$$

In the paper is presented a new extension of the BON-OWAD operator [23]. In this new operator the characteristics of the IOWA operator and the HOWA operator are included in one formulation, calling it the Bonferroni Induced Heavy Ordered Weighted Average (BON-IHOWA) operator. The main advantage of this new operator is that can include in one single formulation more information of the decision maker and solve more complex problems. Another important aspect is that if the problem is not that complex it is possible to use some of the particular cases that are included in the operator. The definition of the BON-IHOWAD is as follows.

Definition 16. A BON-IHOWAD distance for two sets $X = \{x_1, x_2, \ldots, x_n\}$ and $Y = \{y_1, y_2, \ldots, y_n\}$ that has an associated weighting vector W with $w_i \in [0, 1]$ and $1 \leq \sum_{j=1}^{n} w_j \leq n$, such that

$$BON - IHOWAD\left(\langle u_1, x_1, y_1\rangle, \ldots, \langle u_n x_n, y_n\rangle\right) = \left(\frac{1}{n}\sum_i D_i^r IHOWAD_{w_i}\left(V^i\right)\right)^{\frac{1}{r+q}} \tag{16}$$

where $IHOWAD_{w_i}\left(V^i\right) = \left(\frac{1}{n-1}\sum_{j=1, j \neq i}^{n} D_j^q\right)$ with $\left(V^i\right)$ being the vector of all $\left|x_j - y_j\right|$ except $\left|x_i - y_i\right|$ and w_i being an $n-1$ vector W_i associated with α_i whose components w_{ij} are the IHOWAD weights. Let W be an IHOWAD weighing vector of dimension $n-1$ with components $w_i \in [0, 1]$ and $1 \leq \sum_{j=1}^{n} w_j \leq n$ where the weights are associated according to the largest value of u_i and u_i is the order-inducing variable.

Let W be an OWA weighing vector of dimension $n-1$ with components $w_i \in [0, 1]$ when $1 \leq \sum_{j=1}^{n} w_j \leq n$, where the weights are associated according to the largest value of u_i and u_i is the order-inducing variable. Likewise, the sum of the weights w_j is bounded to n or can be unbounded if the weighting vector $W = -\infty \leq \sum_{j=1}^{n} w_j \leq \infty$. Then, we can define this aggregation as $IHOWAD_W\left(V^i\right) = \left(\sum_{j=1}^{n-1} w_i D_{\pi_k(j)}\right)$, where $a_{\pi_k(j)}$ is the largest element in the n – 1 tuple V^i = $V^i = \left(\langle u_1, x_1, y_1\rangle, \ldots, \left\langle u_{i-1}, x_{i-1}, y_{i-1}\right\rangle, \left\langle u_{i+1}, x_{i+1}, y_{i+1}\right\rangle, \ldots, \langle u_n, x_n, y_n\rangle\right)$.

Note that if $w_i = \frac{1}{n-1}$ for all i, $IHOWAD_{w_i}\left(V^i\right) = \left(\frac{1}{n-1}\sum_{j=1, j \neq i}^{n} a_j^q\right)$, and Eq. (16) becomes Bonferroni induced heavy mean distance.

Example 4. We have assumed that BON-IHOWAD pair's $\left\langle \mu_i, x_i, y_i\right\rangle$ is given by $X = (0.2,\ 0.5,\ 0.9,\ 0.8)$, $Y = (0.5,\ 0.6,\ 0.1,\ 0.9)$ and induced variable are $U = (3,\ 6,\ 1,\ 5)$. W^* is the weighting vector of the $\left\langle \mu_i, x_i, y_i\right\rangle$ associated with α_i whose components v_{ij}. Here we shall let W^*, instead of being 1, it is 1.9 and it has the following values: $W^* = 0.4,\ 0.3,\ 0.7,\ 0.5$. The ordered OWA pair's is $\left\langle 6, \left|0.5 - 0.6\right|\right\rangle$, $\left\langle 5, \left|0.8 - 0.9\right|\right\rangle$, $\left\langle 3, \left|0.2 - 0.5\right|\right\rangle$, $\left\langle 1, \left|0.9 - 0.1\right|\right\rangle$ that is the ordered list x_i, y_i is $\left|0.5 - 0.6\right|$, $\left|0.8 - 0.9\right|$, $\left|0.2 - 0.5\right|$, $\left|0.9 - 0.1\right|$. We take $r = q = 1$. In addition: $V^1 = (\left|0.8 - 0.9\right| + \left|0.2 - 0.5\right| + \left|0.9 - 0.1\right|)$, $V^2 = (\left|0.5 - 0.6\right| + \left|0.2 - 0.5\right| + \left|0.9 - 0.1\right|)$, $V^3 = (\left|0.5 - 0.6\right| + \left|0.8 - 0.9\right| + \left|0.9 - 0.1\right|)$ and $V^4 = (\left|0.5 - 0.6\right| + \left|0.8 - 0.9\right| + \left|0.2 - 0.5\right|)$. Using this get:

$$IHOWAD_{v_1}\left(V^1\right) = 0.3 \times (\left|0.8 - 0.9\right| + \left|0.2 - 0.5\right| + \left|0.9 - 0.1\right|) = 0.36$$

$$IHOWAD_{v_2}\left(V^2\right) = 0.4\times(\left|0.5-0.6\right| + \left|0.2-0.5\right| + \left|0.9-0.1\right|) = 0.48$$

$$IHOWAD_{v_3}\left(V^3\right) = 0.5\times(\left|0.5-0.6\right| + \left|0.8-0.9\right| + \left|0.9-0.1\right|) = 0.50$$

$$IHOWAD_{v_4}\left(V^4\right) = 0.7\times(\left|0.5-0.6\right| + \left|0.8-0.9\right| + \left|0.2-0.5\right|) = 0.35$$

$$BON-IHOWAD = \left(\frac{1}{4}\times\left(\left(0.36\times\left|0.5-0.6\right|\right) + \left(\left(0.48\times\left|0.8-0.9\right|\right) + \left(\left(0.50\times\left|0.2-0.5\right|\right) + + \left(\left(0.35\times\left|0.9-0.1\right|\right)\right)\right)\right)\right)^{0.5} = 0.5634$$

It is possible generate two particular cases. The first one when the weights are equal to 1, then the BON-IOWAD operator is obtained and is defined as:

Definition 17. A BON-IOWAD distance for two sets $X = \{x_1, x_2, \ldots, x_n\}$ and $Y = \{y_1, y_2, \ldots, y_n\}$ such that

$$BON-IOWAD(\langle u_1, x_1, y_1\rangle, \ldots, \langle u_n x_n, y_n\rangle) = \left(\frac{1}{n}\sum_i D_i^r IOWAD_{w_i}\left(V^i\right)\right)^{\frac{1}{r+q}} \tag{17}$$

where $IOWAD_{w_i}\left(V^i\right) = \left(\frac{1}{n-1}\sum_{j=1, j\neq i}^{n} D_j^q\right)$ with $\left(V^i\right)$ being the vector of all $\left|x_j - y_j\right|$ except $\left|x_i - y_i\right|$ and w_i being an $n-1$ vector W_i associated with α_i whose components w_{ij} are the IOWAD weights, where the weights are associated according to the largest value of u_i and u_i is the order-inducing variable

Let W be an OWA weighing vector of dimension $n-1$ with components $w_i \in [0, 1]$ when $\sum_i w_i = 1$, where the weights are associated according to the largest value of u_i and u_i is the order-inducing variable. Then, we can define this aggregation as $IOWAD_W\left(V^i\right) = \left(\sum_{j=1}^{n-1} w_i D_{\pi_k(j)}\right)$, where $D_{\pi_k(j)}$ is the largest element in the n – 1 tuple V^i = $V^i = \left(\langle u_1, x_1, y_1\rangle, \ldots, \left\langle u_{i-1}, x_{i-1}, y_{i-1}\right\rangle, \left\langle u_{i+1}, x_{i+1}, y_{i+1}\right\rangle, \ldots, \langle u_n, x_n, y_n\rangle\right)$

Note that if $w_i = \frac{1}{n-1}$ for all i, $IOWAD_{w_i}\left(V^i\right) = \left(\frac{1}{n-1}\sum_{j=1, j\neq i}^{n} D_j^q\right)$, and Eq. (17) becomes Bonferroni induced mean distance.

Example 5. We have assumed that BON-IOWAD pair's $\left\langle \mu_i, x_i, y_i\right\rangle$ is given by $X = (0.2,\ 0.5,\ 0.9,\ 0.8)$, $Y = (0.5,\ 0.6,\ 0.1,\ 0.9)$ and induced variable are $U = (3,\ 6,\ 1,\ 5)$. w_i is the weighting vector of the $\left\langle \mu_i, x_i, y_i\right\rangle$ associated with α_i whose components v_{ij}. Here we shall let $\alpha_1 = 0.4$, $\alpha_2 = 0.3$, $\alpha_3 = 0.7\, and\, \alpha_4 = 0.5$. The ordered OWA pair's is $\left\langle 6,\ \left|0.5-0.6\right|\right\rangle$, $\left\langle 5,\ \left|0.8-0.9\right|\right\rangle$, $\left\langle 3,\ \left|0.2-0.5\right|\right\rangle$, $\left\langle 1,\ \left|0.9-0.1\right|\right\rangle$ that is the ordered list x_i, y_i is $\left|0.5-0.6\right|$, $\left|0.8-0.9\right|$, $\left|0.2-0.5\right|$, $\left|0.9-0.1\right|$. We take $r = q = 1$. In addition: $V^1 = (\left|0.8-0.9\right| + \left|0.2-0.5\right| + \left|0.9-0.1\right|)$, $V^2 = (\left|0.5-0.6\right| + \left|0.2-0.5\right| + \left|0.9-0.1\right|)$, $V^3 = (\left|0.5-0.6\right| + \left|0.8-0.9\right| + \left|0.9-0.1\right|)$ and $V^4 = (\left|0.5-0.6\right| + \left|0.8-0.9\right| + \left|0.2-0.5\right|)$. Using this get:

$$IOWAD_{v_1}\left(V^1\right) = 0.4\times(\left|0.8-0.9\right| + \left|0.2-0.5\right| + \left|0.9-0.1\right|) = 0.48$$

$$IOWAD_{v_2}\left(V^2\right) = 0.3\times(\left|0.5-0.6\right| + \left|0.2-0.5\right| + \left|0.9-0.1\right|) = 0.36$$

$$IOWAD_{v_3}\left(V^3\right) = 0.7\times(\left|0.5-0.6\right| + \left|0.8-0.9\right| + \left|0.9-0.1\right|) = 0.70$$

$$IOWAD_{v_4}\left(V^4\right) = 0.5\times(\left|0.5-0.6\right| + \left|0.8-0.9\right| + \left|0.2-0.5\right|) = 0.25$$

$$BON-IOWAD = \left(\frac{1}{4}\times\left(\left(\left|0.5-0.6\right|\times 0.48\right) + \left(\left(\left|0.8-0.9\right|\times 0.36\right) + \left(\left(\left|0.2-0.5\right|\times 0.70\right) + + \left(\left(\left|0.9-0.1\right|\times 0.25\right)\right)\right)\right)\right)^{0.5} = 0.5590$$

Also if the order-inducing variable is according to $u_i = 1/n$, then the BON−HOWAD operator remains. The formulation is:

Definition 18. A BON−HOWAD distance for two sets $X = \{x_1, x_2, \ldots, x_n\}$ and $Y = \{y_1, y_2, \ldots, y_n\}$ that has an associated weighting vector W with $w_i \in [0, 1]$ and $1 \leq \sum_{j=1}^{n} w_j \leq n$, such that

$$BON-HOWAD(\langle x_1, y_1\rangle, \ldots, \langle x_n, y_n\rangle) = \left(\frac{1}{n}\sum_i D_i^r HOWAD_{w_i}\left(V^i\right)\right)^{\frac{1}{r+q}} \tag{18}$$

where $HOWAD_{w_i}\left(V^i\right)=\left(\frac{1}{n-1}\sum_{\substack{j=1\\ j\neq i}}^{n} D_j^q\right)$ with $\left(V^i\right)$ being the vector of all $\left|x_j-y_j\right|$ except $\left|x_i-y_i\right|$ and w_i being an $n-1$ vector W_i associated with α_i whose components w_{ij} are the HOWAD weights. Let W be an HOWA weighing vector of dimension $n-1$ with components $w_i \in [0,1]$ and $1\leq\sum_{j=1}^{n}w_j\leq n$.

Let W be an OWA weighing vector of dimension $n-1$ with components $w_i\in[0,1]$ when $1\leq\sum_{j=1}^{n}w_j\leq n$. Thus, the sum of the weights w_j is bounded to n or can be unbounded if the weighting vector $W=-\infty\leq\sum_{j=1}^{n}w_j\leq\infty$. Then, we can define this aggregation as $HOWAD_W\left(V^i\right)=\left(\sum_{j=1}^{n-1}w_iD_{\pi_k(j)}\right)$, where $D_{\pi_k(j)}$ is the largest element in the n – 1 tuple $V^i = V^i=\left(\langle x_1,y_1\rangle,\ldots,\left\langle x_{i-1},y_{i-1}\right\rangle,\left\langle x_{i+1},y_{i+1}\right\rangle,\ldots,\langle x_n,y_n\rangle\right)$.

Note that if $w_i=\frac{1}{n-1}$ for all i, $HOWAD_{w_i}\left(V^i\right)=\left(\frac{1}{n-1}\sum_{\substack{j=1\\ j\neq i}}^{n} D_j^q\right)$, and Eq. (18) becomes Bonferroni heavy mean distance.

Definition 19. A BON-HWAD distance for two sets $X=\{x_1,x_2,\ldots,x_n\}$ and $Y=\{y_1,y_2,\ldots,y_n\}$ that has an associated weighting vector W with $w_i\in[0,1]$ and $1\leq\sum_{j=1}^{n}w_j\leq n$, such that

$$BON-HWAD(\langle x_1,y_1\rangle,\ldots,\langle x_n,y_n\rangle)=\left(\frac{1}{n}\sum_{i}D_i^rHWAD_{w_i}\left(V^i\right)\right)^{\frac{1}{r+q}} \tag{19}$$

where $HWAD_w\left(V^i\right)=\left(\frac{1}{n-1}\sum_{\substack{j=1\\ j\neq i}}^{n} D_j^q\right)$ with $\left(V^i\right)$ being the vector of all $\left|x_j-y_j\right|$ except $\left|x_i-y_i\right|$ and w_i being an $n-1$ vector W_i associated with α_i whose components w_{ij} are the HWAD weights. Let W be an HWAD weighing vector of dimension $n-1$ with components $w_i\in[0,1]$ and $1\leq\sum_{j=1}^{n}w_j\leq n$.

Let W be weighted average vector of dimension $n-1$ with components $w_i\in[0,1]$ when $1\leq\sum_{j=1}^{n}w_j\leq n$. Thus, the sum of the weights w_j is bounded to n or can be unbounded if the weighting vector $W=-\infty\leq\sum_{j=1}^{n}w_j\leq\infty$. Then, we can define this aggregation as $HWAD_W\left(V^i\right)=\left(\sum_{j=1}^{n-1}w_iD_{\pi_k(j)}\right)$, where $D_{\pi_k(j)}$ is the largest element in the n – 1 tuple $V^i = V^i=\left(\langle x_1,y_1\rangle,\ldots,\left\langle x_{i-1},y_{i-1}\right\rangle,\left\langle x_{i+1},y_{i+1}\right\rangle,\ldots,\langle x_n,y_n\rangle\right)$.

Note that if $w_i=\frac{1}{n-1}$ for all i, $HWAD_{w_i}\left(V^i\right)=\left(\frac{1}{n-1}\sum_{\substack{j=1\\ j\neq i}}^{n} D_j^q\right)$, and Eq. (19) becomes Bonferroni heavy mean distance.

Also if the weights are equal to 1 and the order inducing variable is $u_i=1/n$, the BON-OWAD operator remains. The definition is presented in [23].

Example 6. We have assumed that BON–HOWAD is given by $X=(0.2,\ 0.5,\ 0.9,\ 0.8)$, $Y=(0.5,\ 0.6,\ 0.1,\ 0.9)$. W^* is the weighting vector of the a_i associated with α_i whose components v_{ij}. Here we shall let W^*, instead of being 1, it is 1.9 and it has the following values: $W^*=0.4,\ 0.3,\ 0.7,\ 0.5$. We take $r=q=1$. In addition: $V^1=(\left|0.5-0.6\right|+\left|0.9-0.1\right|+\left|0.8-0.9\right|)$, $V^2=(\left|0.2-0.5\right|+\left|0.9-0.1\right|+\left|0.8-0.9\right|)$, $V^3=(\left|0.2-0.5\right|+\left|0.5-0.6\right|+\left|0.8-0.9\right|)$ and $V^4=(\left|0.2-0.5\right|+\left|0.5-0.6\right|+\left|0.1-0.9\right|)$. Using this get:

$$HOWAD_{v_1}\left(V^1\right)=0.4\times(\left|0.5-0.6\right|+\left|0.9-0.1\right|+\left|0.8-0.9\right|)=0.40$$

$$HOWAD_{v_2}\left(V^2\right)=0.3\times(\left|0.2-0.5\right|+\left|0.9-0.1\right|+\left|0.8-0.9\right|)=0.36$$

$$HOWAD_{v_3}\left(V^3\right)=0.7\times(\left|0.2-0.5\right|+\left|0.5-0.6\right|+\left|0.8-0.9\right|)=0.35$$

Table 1
Main goal set in ERM.

	Goals
G_1	Guarantee continuity of operation
G_2	Comply with internal and external rules
G_3	Guarantee availability and quality of information
G_4	Keep the good will
G_5	Prevent economic losses
G_6	Protect people

$$HOWAD_{v_4}\left(V^4\right) = 0.5 \times (\left|0.2-0.5\right| + \left|0.5-0.6\right| + \left|0.1-0.9\right|) = 0.60$$

$$BON-HOWAD = \left(\frac{1}{4} \times \left(\left(\left|0.2-0.5\right| \times 0.40\right) + \left(\left(\left|0.5-0.6\right| \times 0.36\right) + \left(\left(\left|0.9-0.1\right| \times 0.35\right) + \left(\left(\left|0.8-0.9\right| \times 0.80\right)\right)\right)^{0.5} = 0.6244$$

4. Case study

We have proposed a new set of methods combining OWA operators, distance measure, Bonferroni means to provide a new tools for decision-making process. To developed case study, we used the enterprise risk management (ERM) in large companies in Colombia survey. We have studied four main sectors: Trading and consumer, Service, Manufacturing and Extractive. We have focused our analysis on main goals in ERM, main types of risk and main benefits in ERM. Using these new methods, we have analysed the preference, comparison, order and relation information that has a decision-maker according to their expectations about ERM. The main idea is to establish interrelationship and continuous comparison between goals, types of risks and expected benefits in ERM in large companies in Colombia. Thus, it is desired to determine which of the goals is prioritized by the managers through the interrelationship and continuous comparison using the different types of risks and the expected benefits as weighted vectors.

4.1. Enterprise risk management

The Committee of Sponsoring Organizations of the Treadway Commission [42] proposes a comprehensive framework that allows identifying, analysing and evaluating a group of risks from the interrelation of eight components: internal environment, goal setting, event identification, risk assessment, risk response, control activities, information and communication and monitoring. Large companies use this framework by their simplicity and easy understanding. However, the management of the information requires a great amount of time and responsible monitoring for its execution. This framework suggests how these components can be applied within company to pursuit organizational goals across all levels of the organization [43].

In Colombia, the technical standard that adapts the COSO guidelines is NTC 31,000 [44], which defines the guidelines, valuation methods and evaluation tools for corporate risk management. Likewise, NTC−OHSAS 18,001 [45] have been formulated to identify, assess and evaluate the occupational safety and health risks and the NTC-ISO 14,001 standard [46] that establishes environmental management requirements. However, the application of these frameworks and norms depends on the flexibility and integration of the information by the companies, which can be evidenced in a management of the risk at a traditional level or level of progressive management or level of strategic risk management. Therefore, making a sensible and responsible risk management requires the total commitment of the company, rather than having a regulation or established frameworks. Recently, a research has shown a descriptive study about risk management processes in privates enterprise in Bogota, in which is highlighted the main implications in health and labour safe [47]. This study stands out the importance of enterprise risk management.

In spite of all these frames and tools, it is difficult to know within dynamic environment, (1) which kinds of risks are preferred over the others, (2) how each risk can be interrelated between them and (3) the expectation of fulfilment between goals raised and expected benefits. Thus, the ERM is an uncertain bet that is affected by the internal and external changes of the company.

4.2. Mathematical application

Now, we present the mathematical application of the algorithm proposed above. The main advantage of on using BON-OWA operators is that they can interrelate, order and overestimate information according to attitude of decision-maker. The application is focused on knowing the degree of importance of goals on ERM and its interrelationship with expected benefits and types of risk in large industries in Colombia.

Step 1. Using the data set of the enterprise risk management (ERM) in large companies in Colombia survey, we have taken the main goal set in ERM, which are common for four large economic sectors trading and consumer sector, service sector, manufacturing sector and extractive sector (see Table 1). They are identified as G_1, G_2, G_3, G_4, G_5 and G_6. It is to remain that each goal is considered a property.

Step 2. Using the data set, we have determined what goal is the most relevant in each sector according to type of risk. Here, it is evident that some risks are more important than others depending on the sector (see Tables 2–5). With this data, we are able to make a continuous comparison and establish interrelationship between each goals taking into account different types of risk for each sector as characteristics in the same formulation.

Step 3. To make technical continuous comparison and interrelationship between each goal for each sector, we have used BON-OWAD, BON−HOWAD, BON-IOWAD and BON-HIOWAD. The main idea is make a continuous comparison between goals using types of risks. In this sense, we have taken as weighted vector w expected benefits in ERM Finance Benefit (FB), Organizational Improvement (OI) and Decision-making Benefits (DMB) (see Tables 6 and 8). It is noteworthy that Heavy OWA is characterized by the degree of expectation, i.e.,

Table 2
Trading and consumer sector.

	Strategic	Operational	Financial	Technological	Socials	Environmental	Labour	Legal	Physics	Insurable
G_1	0,176	0,321	0,286	0,071	0,036	0,107	0,071	0,143	0,036	0,000
G_2	0,059	0,071	0,071	0,000	0,000	0,000	0,036	0,000	0,000	0,000
G_3	0,000	0,071	0,071	0,000	0,000	0,036	0,000	0,036	0,000	0,000
G_4	0,059	0,071	0,036	0,036	0,036	0,036	0,071	0,000	0,000	0,000
G_5	0,353	0,107	0,214	0,000	0,000	0,071	0,107	0,036	0,000	0,000
G_6	0,000	0,000	0,036	0,000	0,000	0,071	0,107	0,071	0,036	0,000

Table 3
Service sector.

	Strategic	Operational	Financial	Technological	Socials	Environmental	Labour	Legal	Physics	Insurable
G_1	0,100	0,229	0,157	0,100	0,014	0,129	0,143	0,157	0,014	0,029
G_2	0,057	0,114	0,071	0,071	0,000	0,071	0,043	0,071	0,014	0,000
G_3	0,043	0,043	0,029	0,029	0,000	0,014	0,057	0,029	0,000	0,014
G_4	0,014	0,029	0,014	0,029	0,000	0,014	0,014	0,000	0,014	0,000
G_5	0,029	0,114	0,114	0,029	0,029	0,043	0,029	0,029	0,014	0,000
G_6	0,043	0,129	0,057	0,000	0,014	0,100	0,143	0,086	0,029	0,000

These weighted vectors represent the expectations of managers in ERM.

Table 4
Manufacturing sector.

	Strategic	Operational	Financial	Technological	Socials	Environmental	Labour	Legal	Physics	Insurable
G_1	0,118	0,235	0,176	0,000	0,000	0,059	0,059	0,176	0,000	0,059
G_2	0,000	0,176	0,118	0,000	0,000	0,059	0,118	0,059	0,000	0,000
G_3	0,059	0,118	0,118	0,059	0,118	0,059	0,000	0,000	0,000	0,000
G_4	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
G_5	0,118	0,118	0,059	0,000	0,000	0,000	0,000	0,059	0,000	0,000
G_6	0,118	0,118	0,059	0,000	0,000	0,059	0,235	0,059	0,000	0,059

Table 5
Extractive sector.

	Strategic	Operational	Financial	Technological	Socials	Environmental	Labour	Legal	Physics	Insurable
G_1	0,143	0,357	0,143	0,000	0,000	0,214	0,071	0,143	0,000	0,000
G_2	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
G_3	0,000	0,071	0,000	0,000	0,000	0,071	0,071	0,000	0,000	0,000
G_4	0,071	0,000	0,000	0,000	0,000	0,000	0,071	0,071	0,000	0,000
G_5	0,000	0,000	0,071	0,000	0,000	0,071	0,071	0,000	0,000	0,000
G_6	0,071	0,286	0,214	0,000	0,071	0,357	0,214	0,071	0,000	0,000

Table 6
Weighted vector for BON-OWAD.

	W_1	W_2	W_3	W_4	W_5	W_6	W_7	W_8	W_9	W_{10}
FB	0,011	0,019	0,022	0,056	0,107	0,119	0,122	0,122	0,185	0,237
OI	0,010	0,022	0,022	0,051	0,114	0,121	0,127	0,130	0,175	0,229
DMB	0,010	0,020	0,023	0,052	0,111	0,121	0,128	0,131	0,174	0,230

Descending HOWA (DHOWA) operator is found when $w_n = 1$, $w_j = 0$ for all $j \neq n$ and the sum of the weights is one and Ascending HOWA (AHOWA) operator is found from different aggregations as the weighting vector can be higher than 1 [35]. Likewise, Yager [19] proposed that HOWA weighted vectors can be obtained using weighting vectors allocations, which is called "*the push up*" allocation and is denote W_{pu}. This method is emphasized the larger values in the aggregate and thus we try to allocate as much of the available magnitude to the elements at the top of the weighting vector W [19]. Due to Heavy OWA is characterized by the degree of expectation its weighted vectors have some adaptations using W_{pu} allocation (see Tables 7 and 9). *Step 4*. In order to unify the continuous comparison and interrelationship between each goal, we have used the maximum similarity sub-relations [48]. This method allows us to obtain the similarity relation [S ˜] from technical continuous comparison explained in step 3. Thus, we found a dissimilarity fuzzy relation [D ˜] through its complement, where [D ˜] = |1−[S ˜]. From [D ˜] we obtain a symmetric and reflexive matrix through the determined α level, where α = n [23]. Finally,

Table 7
Weighted vector for BON−HOWAD.

	W_1	W_2	W_3	W_4	W_5	W_6	W_7	W_8	W_9	W_{10}	Ave
FB	0,256	0,496	0,388	0,116	0,047	0,256	0,248	0,225	0,039	0,023	2,093
OI	0,295	0,558	0,426	0,124	0,054	0,310	0,318	0,279	0,054	0,023	2,442
DMB	0,287	0,543	0,411	0,124	0,047	0,302	0,310	0,264	0,054	0,023	2,364

Table 8
Weighted vector for BON-IOWAD.

	W_1	W_2	W_3	W_4	W_5	W_6	W_7	W_8	W_9	W_{10}
FB	0,122	0,237	0,185	0,056	0,022	0,122	0,119	0,107	0,019	0,011
OI	0,121	0,229	0,175	0,051	0,022	0,127	0,130	0,114	0,022	0,010
DMB	0,121	0,230	0,174	0,052	0,020	0,128	0,131	0,111	0,023	0,010

Table 9
Weighted vector for BON-HIOWAD.

	W_1	W_2	W_3	W_4	W_5	W_6	W_7	W_8	W_9	W_{10}	Ave
FB	0,256	0,496	0,388	0,116	0,047	0,256	0,248	0,225	0,039	0,023	2,093
OI	0,295	0,558	0,426	0,124	0,054	0,310	0,318	0,279	0,054	0,023	2,442
DMB	0,287	0,543	0,411	0,124	0,047	0,302	0,310	0,264	0,054	0,023	2,364
Order	6	1	2	7	8	4	3	5	9	10	

Table 10
α levels to determine the maximum similarity sub-relations.

	Trading and consumer sector						Service sector					
	FB		OB		DMB		FB		OB		DMB	
α	≥	≤	≥	≤	≥	≤	≥	≤	≥	≤	≥	≤
BON-OWAD	0,996	0,977	0,996	0,977	0,996	0,977	0,995	0,977	0,995	0,977	0,995	0,977
BON-HOWAD	0,994	0,967	0,993	0,963	0,993	0,964	0,993	0,979	0,993	0,977	0,993	0,977
BON-IOWAD	0,995	0,965	0,995	0,965	0,995	0,965	0,995	0,977	0,995	0,977	0,995	0,977
BON-IHOWAD	0,993	0,949	0,992	0,945	0,992	0,946	0,993	0,972	0,992	0,970	0,992	0,971
	Manufacturing sector						Extractive sector					
	FB		OB		DMB		FB		OB		DMB	
α	≥	≤	≥	≤	≥	≤	≥	≤	≥	≤	≥	≤
BON-OWAD	0,994	0,977	0,994	0,977	0,994	0,977	1,000	0,977	1,000	0,977	1,000	0,977
BON-HOWAD	0,992	0,976	0,991	0,974	0,991	0,975	0,999	0,983	0,999	0,979	0,999	0,980
BON-IOWAD	0,993	0,977	0,992	0,977	0,992	0,977	1,000	0,977	1,000	0,977	1,000	0,977
BON-IHOWAD	0,989	0,968	0,999	0,965	0,999	0,965	0,999	0,965	0,999	0,959	0,999	0,960

The results obtained are analysed and it is possible to observe how is the company being oriented and how the goals in ERM contributes to benefits according each sector.

4.3. Results

In this section we can observe the main results obtained of the application. We have used the new methods proposed, BON−HOWAD, BON-IOWAD AND BON-IHOWAD. These methods allow making continuous aggregations, multiple comparisons and distance measures between each argument. Furthermore, these operators have specific characteristics by the combination of different methods. The main advantage of BON−HOWAD over the others BON-OWA operators is that *Heavy part*, which allows the weighting vector to range between the OWA operator and the total operator, i.e., the restrictions on the associated weighting vector are diminished. The main advantage of BON-IOWAD is that *Induce part*, which allows taking OWA pairs in which one component induces a ordering over the second components, i.e., the reordering step is induced by another mechanism which depends upon the values of their associated order-inducing variables. The main advantage of BON IHOWAD is that IHOWA part combines the characteristics of HOWA and IOWA that allow using at the same time an *inducing order* and a *sum of the weights* that can range from 1 to n.

Based on methods proposed above, we have used information of the enterprise risk management in large companies in Colombia survey in order to compare and show the sort of goals in ERM using the different types of risks and the expected benefits as weighted vectors in four main large sectors. We have shown the maximum similarity sub-relations matrices to understand the presentation of results after mathematical operation (see [Appendix 1]). We have obtained the maximum similarity sub-relations, which can be expressed in max (≥) and min (≤) terms (Table 10). Thus, using the maximum similarity sub-relations matrices is determined the average sub-relation to obtained the main results from each method for each sector, which can observe in Tables 11–14.

Firstly, when it is used BON-OWAD, it is noteworthy that the differences between expected benefits within each sector are not significant. However, the importance of goals has an important variation. In this case, we have only taken into account continuous aggregations, multiple comparisons and distance measures between each argument.

Secondly, when it is used BON−HOWAD, it is noteworthy that the differences between expected benefits within each sector have changed. Likewise, the importance of goals has an important variation. In this case, we have also taken into account the relaxing of weighted vector.

Thirdly, when it is used BON-IOWAD, it is noteworthy that the differences between expected benefits within each sector are not significant. However, the importance of goals has an important variation in compare with BON−HOWAD. In this case, we have also taken into account the induced component. On first thought, the results obtained seem to be similar although there are significant difference depending on method used and the sector studied. Thus, it is to highlight those theses methods allows carrying out continuous aggregations and multiple comparisons of the information. Besides, when using the Hamming distance the positive results are located at the lower limit.

Table 11
BON-OWAD.

	Trading and consumer sector			Service sector			Manufacturing sector			Extractive sector		
	FB	OB	DMB	FB	OB	DMB	FB	OB	DMB	FB	OB	DMB
G_1	0,983	0,983	0,983	0,988	0,988	0,988	0,989	0,988	0,989	0,997	0,996	0,996
G_2	0,986	0,986	0,986	0,990	0,990	0,990	0,990	0,990	0,990	0,996	0,996	0,996
G_3	0,990	0,990	0,990	0,991	0,991	0,991	0,989	0,989	0,989	0,997	0,996	0,996
G_4	0,989	0,989	0,989	0,991	0,990	0,990	0,989	0,989	0,989	0,997	0,996	0,996
G_5	0,988	0,988	0,988	0,990	0,990	0,990	0,990	0,990	0,990	0,993	0,993	0,993
G_6	0,985	0,985	0,985	0,984	0,984	0,984	0,982	0,982	0,982	0,986	0,986	0,986

Table 12
BON−HOWAD.

	Trading and consumer sector			Service sector			Manufacturing sector			Extractive sector		
	FB	OB	DMB	FB	OB	DMB	FB	OB	DMB	FB	OB	DMB
G_1	0,975	0,973	0,973	0,983	0,981	0,981	0,984	0,982	0,982	0,993	0,991	0,991
G_2	0,980	0,978	0,979	0,985	0,984	0,984	0,986	0,985	0,985	0,991	0,989	0,990
G_3	0,986	0,985	0,985	0,987	0,986	0,986	0,984	0,982	0,983	0,993	0,991	0,991
G_4	0,984	0,983	0,984	0,986	0,985	0,985	0,984	0,983	0,983	0,993	0,991	0,991
G_5	0,982	0,981	0,981	0,985	0,984	0,984	0,986	0,985	0,985	0,986	0,983	0,983
G_6	0,985	0,984	0,984	0,984	0,983	0,983	0,981	0,980	0,980	0,987	0,984	0,985

Table 13
BON-IOWAD.

	Trading and consumer sector			Service sector			Manufacturing sector			Extractive sector		
	FB	OB	DMB	FB	OB	DMB	FB	OB	DMB	FB	OB	DMB
G_1	0,976	0,976	0,976	0,984	0,984	0,984	0,983	0,983	0,983	0,992	0,992	0,992
G_2	0,980	0,980	0,980	0,987	0,987	0,987	0,986	0,986	0,986	0,991	0,991	0,991
G_3	0,985	0,986	0,986	0,989	0,989	0,989	0,985	0,985	0,985	0,993	0,993	0,993
G_4	0,980	0,980	0,980	0,988	0,988	0,988	0,983	0,983	0,983	0,993	0,993	0,993
G_5	0,976	0,977	0,977	0,986	0,986	0,986	0,988	0,988	0,988	0,986	0,986	0,986
G_6	0,977	0,977	0,977	0,983	0,983	0,983	0,983	0,983	0,983	0,984	0,984	0,984

Table 14
BON-IHOWAD.

	Trading and consumer sector			Service sector			Manufacturing sector			Extractive sector		
	FB	OB	DMB	FB	OB	DMB	FB	OB	DMB	FB	OB	DMB
G_1	0,965	0,962	0,963	0,977	0,976	0,976	0,975	0,982	0,982	0,982	0,979	0,980
G_2	0,971	0,969	0,969	0,981	0,979	0,979	0,980	0,982	0,982	0,982	0,979	0,980
G_3	0,979	0,977	0,978	0,984	0,983	0,983	0,978	0,985	0,985	0,985	0,982	0,983
G_4	0,971	0,969	0,969	0,982	0,981	0,981	0,975	0,985	0,985	0,985	0,982	0,983
G_5	0,966	0,963	0,964	0,979	0,978	0,978	0,983	0,971	0,971	0,971	0,966	0,967
G_6	0,968	0,966	0,967	0,981	0,980	0,980	0,981	0,973	0,973	0,973	0,969	0,969

Fourthly, when it is used BON-IHOWAD, it is noteworthy that the differences between expected benefits within each sector are significant and the importance of goals has a critical variation. Hence, these methods allow comparing the feasibility and usefulness according to expectations of decision-makers, when 1) the weighted vector has a range from 1 to n and 2) when the weighted vector is induced to its reorder.

We have depicted the results in order to comprehend the information behaviour. It is noteworthy that the results are ordered in ascending because the lowest data is the most important goal to achieve the expected benefit. When using BON-OWAD operator the importance and interrelationship of the expected benefits are similar for each sector, i.e. all benefits have the equal relevance and all goals works in the same way. Likewise, the most important goals for each sector are G_2 for one sector and G_3 for three sectors and the least important goal is G_6 (see Fig. 1). When using BON−HOWAD operator the importance and interrelationship of the expected benefits change for each sector. It is noteworthy that goal G_3 is the most important for tow sectors and goal G_5 is the most important for the other ones.

Here, none benefit has more relevance than the others into each goal. However, it is marked an important variation between each of goal (see Fig. 2). When using BON-IOWAD operator the importance and interrelationship of the expected benefits are similar for each sector, such as BON-OWAD operator. However, there are some variations in the importance of goals where G_1 is the most important goal for three sectors and G_6 for one sector and the least important goals are G_4 and G_3 for three sector and G_5 for one sector (see Fig. 3). When using BON-IHOWAD operator the importance and interrelationship of the expected benefits change for each sector, such as BON−HOWAD. It is noteworthy that the importance of blue bar (finance benefit) becomes more pronounced among the others. Here, the relevance of one of the benefits is remarkable. As the same previous one, the most important and least goals are not the same of the others methods (see Fig. 4).One of the main contributions is to understand how can be related goals, benefits through risk management and the expectations and attitude of managers. Risk management enables resources to be effectively allocated and used, improved operational efficiency and effectiveness, enriched organizational learning, and enhanced organizational flexibility [44]. These aspects are directly

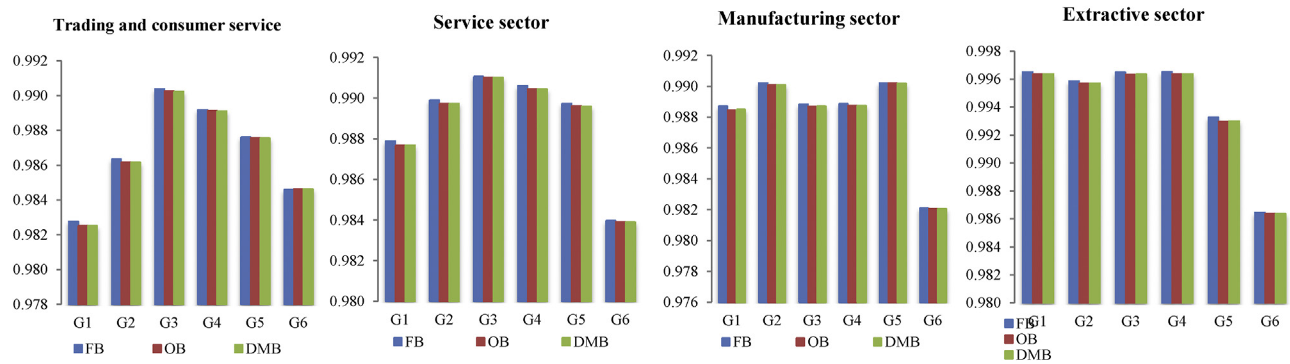

Fig. 1. BON-OWAD.

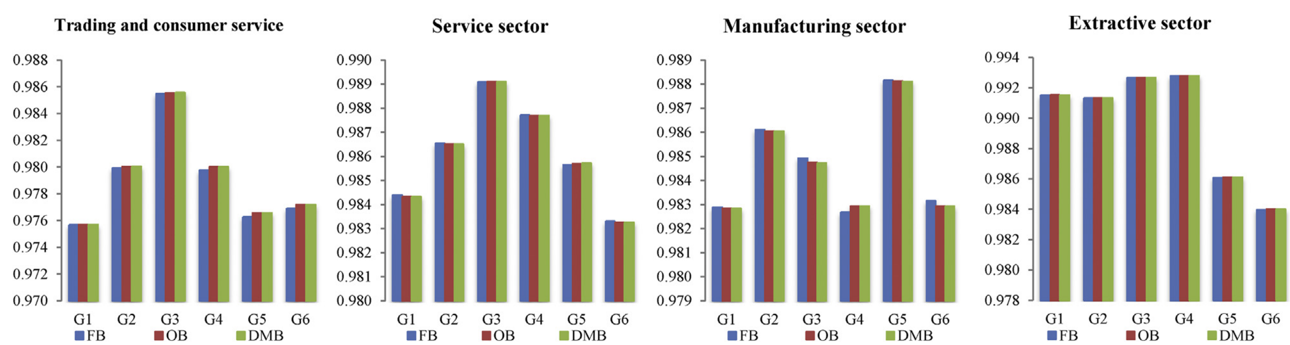

Fig. 2. BON–HOWAD.

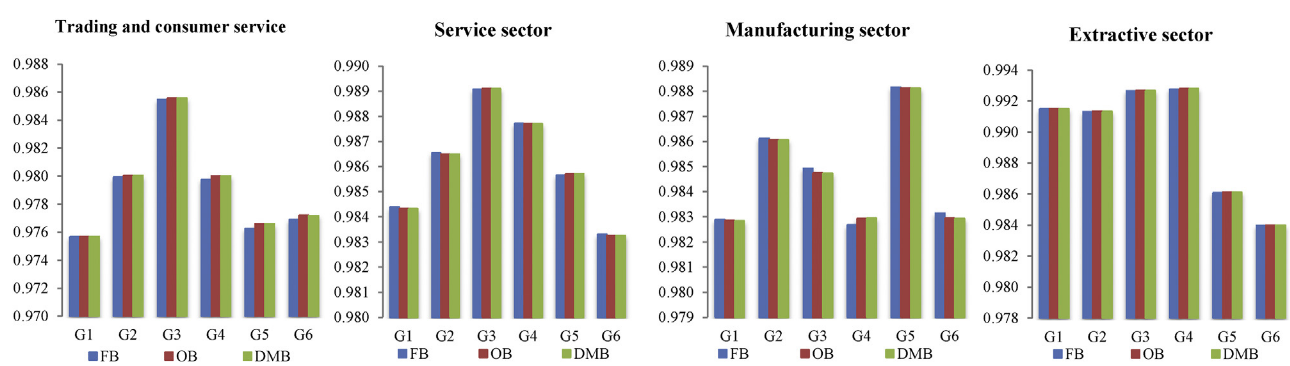

Fig. 3. BON-IOWAD.

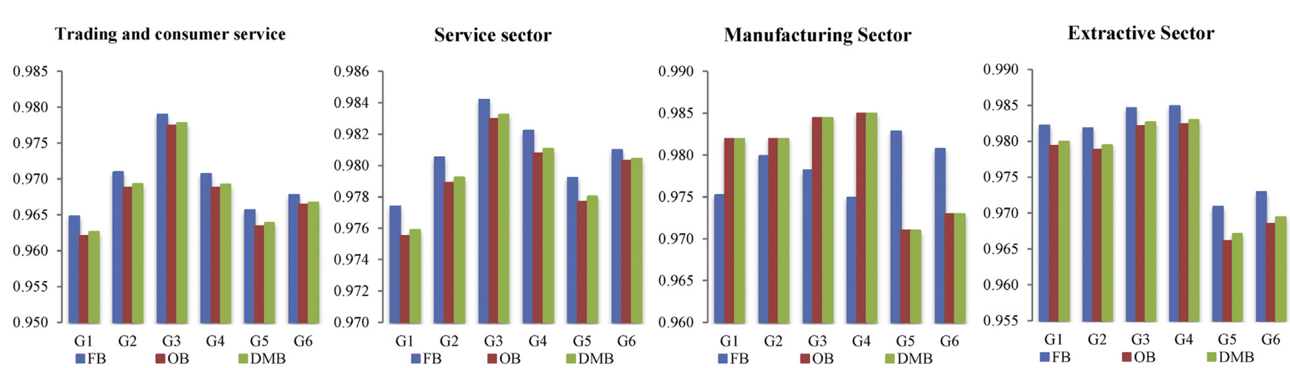

Fig. 4. BON-IHOWAD.

related to strategy and organizational aims, which are integrated into a system that identifies, qualifies, evaluates and designs measures of treatment, implementation, monitoring and evaluation [42].

In this sense, the applications enable to make continuous aggregation and multiple comparison of available information taking into account their distance, range and ordered induce. Likewise, it is important to note that with the use of different values in the weight vector and reordering the assigned weight to the argument based in the induced vector, it is possible to generate new scenarios that

Table 15
Trading and consumer service order.

	BON-OWAD			BON-HOWAD			BON-IOWAD			BON-IHOWAD		
	FB	OB	DMB	FB	OB	DMB	FB	OB	DMB	FB	OB	DMB
1	G_3	G_3	G_3	G_3	G_3	G_3	G_3	G_3	G_3	G_3	G_3	G_3
2	G_4	G_4	G_4	G_6	G_6	G_6	G_2	G_2	G_2	G_4	G_4	G_4
3	G_5	G_5	G_5	G_4	G_4	G_4	G_4	G_4	G_4	G_2	G_2	G_2
4	G_2	G_2	G_2	G_5	G_5	G_5	G_6	G_6	G_6	G_6	G_6	G_6
5	G_6	G_6	G_6	G_2	G_2	G_2	G_5	G_5	G_5	G_5	G_5	G_5
6	G_1	G_1	G_1	G_1	G_1	G_1	G_1	G_1	G_1	G_1	G_1	G_1

Table 16
Service sector order.

	BON-OWAD			BON-HOWAD			BON-IOWAD			BON-IHOWAD		
	FB	OB	DMB	FB	OB	DMB	FB	OB	DMB	FB	OB	DMB
1	G_3	G_3	G_3	G_3	G_3	G_3	G_3	G_3	G_3	G_3	G_3	G_3
2	G_4	G_4	G_4	G_4	G_4	G_4	G_4	G_4	G_4	G_4	G_4	G_4
3	G_2	G_2	G_2	G_2	G_2	G_2	G_2	G_2	G_2	G_2	G_2	G_2
4	G_5	G_5	G_5	G_5	G_5	G_5	G_5	G_5	G_5	G_5	G_5	G_5
5	G_1	G_1	G_1	G_6	G_6	G_6	G_6	G_6	G_6	G_6	G_6	G_6
6	G_6	G_6	G_6	G_1	G_1	G_1	G_1	G_1	G_1	G_1	G_1	G_1

Table 17
Manufacturing sector order.

	BON-OWAD			BON-HOWAD			BON-IOWAD			BON-IHOWAD		
	FB	OB	DMB	FB	OB	DMB	FB	OB	DMB	FB	OB	DMB
1	G_5	G_5	G_5	G_5	G_5	G_5	G_5	G_5	G_5	G_5	G_3	G_3
2	G_2	G_2	G_2	G_2	G_2	G_2	G_2	G_2	G_2	G_6	G_4	G_4
3	G_1	G_3	G_1	G_1	G_4	G_3	G_3	G_3	G_3	G_2	G_1	G_1
4	G_3	G_4	G_3	G_3	G_1	G_4	G_1	G_1	G_1	G_3	G_2	G_2
5	G_4	G_1	G_4	G_4	G_3	G_1	G_4	G_4	G_4	G_1	G_6	G_6
6	G_6	G_6	G_6	G_6	G_6	G_6	G_6	G_6	G_6	G_4	G_5	G_5

using the traditional statistical formulations cannot be achieved. With the changes in these two vectors is possible to add the knowledge, aptitude and expectations of the decision maker into the final results obtained by the formulations. These is important to take into account considering that the decision-makers not always have a rational behaviour and sometimes overreact or underreact when they receive certain information [49]. Thus, in this application the distance reflects the differences between information of each risk according to each goal; the range and ordered induce reflect the attitude of manager on what expect from risk management; and continues aggregation and multiple comparison reflect the interrelationship of each of the variables. Hence, it is possible to observe which goals are prioritized to achieve the expected benefits. Now, it is explained which goals are prioritized in each economic sector selected in Colombia for each method applied.

In Table 15 is shown the order of trading and consumer service. It is observed that the prioritized goal for each method applied is G_3 (Guarantee availability and quality of information). It is noteworthy that in the ordering in positions 2, 3, 4 and 5 has great variation for each method, in which is found goals G_2, G_4, G_5 and G_6. Finally, the goal less prioritized is G_1 (Guarantee continuity of operation). Thus, in this sector, it is shown that in order to achieve the expected benefits the most important is to Guarantee availability and quality of information. This feature reflects that it focuses on affects the feedback, learning and accumulation of knowledge by companies. Also, a non-favourable aspect is not to prioritize the G_3 goal, since it negatively day-to-day management.

In Table 16 is shown the order of service sector. Initially, it is observed that all methods prioritize same goals. Results obtained are homogeneous for each method used. This ordering is given by the attitude of managers that show the range and induce of their perceptions. On the one hand, methods show the prioritized goal is G_3 (Guarantee availability and quality of information) followed by G_4 (Keep the good will) and the ordering in positions 3, 4 and 5 has great variation for each method, in which is found goals G_2 and G_5. Finally, the goals less prioritized are G_1 (Guarantee continuity of operation) and G_6 (Protect people). Thus, in this sector, it is shown that in order to achieve the expected benefits the most important is to Guarantee availability and quality of information and Keep the good will. These features suggest that they focus on affects the feedback, learning and accumulation of knowledge by companies and organizational reputation. Also, a non-favourable aspect is not to prioritize the G_1 and G_6 goals, since it negatively affects the guarantee operational performance and protection of their employees.

In Table 17 is shown the order of manufacturing sector. In this sector, BON-OWAD, BON−HOWAD and BON-IOWAD show the prioritized goal for each method applied is G_5 (Prevent economic losses) followed by G_2 (Comply with internal and external rules) and the ordering in positions 3, 4 and 5 has great variation for each method, in which is found goals G_1, G_3 and G_4. Finally, the goal less prioritized is G_6 (Protect people). On the other hand, BON-IHOWAD there is a great variation in the order of each goal. The prioritize goal are G_5 (Prevent economic losses) and G_3 (Guarantee availability and quality of information) followed by G_4 (Keep the good will) and G_6 (Protect people). Likewise, the ordering in positions 3, 4 and 5 has great variation for each method, in which is found goals G_1, G_2, G_3 and G_6. Finally, the goals less prioritized are G_4 (Keep the good will) and G_5 (Prevent economic losses). Hence, in this sector, it is shown that in order to achieve the expected benefits the most important is also to the solvency by companies and the feedback, learning and accumulation of knowledge. Also, a non-favourable aspect is not to prioritize the G_5 goal, since it negatively affects protection of their employees.

Table 18
Extractive sector order.

	BON-OWAD			BON-HOWAD			BON-IOWAD			BON-IHOWAD		
	FB	OB	DMB	FB	OB	DMB	FB	OB	DMB	FB	OB	DMB
1	G_1	G_1	G_1	G_1	G_1	G_1	G_3	G_3	G_3	G_3	G_3	G_3
2	G_3	G_2	G_2	G_3	G_3	G_3	G_4	G_4	G_4	G_4	G_4	G_4
3	G_4	G_3	G_3	G_4	G_4	G_4	G_1	G_1	G_1	G_1	G_1	G_1
4	G_2	G_4	G_4	G_2	G_2	G_2	G_2	G_2	G_2	G_2	G_2	G_2
5	G_5	G_5	G_5	G_6	G_6	G_6	G_5	G_5	G_5	G_6	G_6	G_6
6	G_6	G_6	G_6	G_5	G_5	G_5	G_6	G_6	G_6	G_5	G_5	G_5

Table 19
Prioritize goals.

Sector	The most prioritized		The least prioritized	
Trading and consumer	Guarantee availability and quality of information		Guarantee continuity of operation	
Service	Guarantee availability and quality of information		Guarantee continuity of operation	Guarantee continuity of operation
Manufacturing	Prevent economic losses	Guarantee availability and quality of information	Prevent economic losses	Keep the good will
Extractive	Guarantee availability and quality of information		Prevent economic losses	

In Table 18 is shown the order of extractive sector. In this sector, BON-OWAD and BON−HOWAD show the prioritized goal for each method applied is G_1 (Guarantee continuity of operation) followed by G_3 (Guarantee availability and quality of information) and the ordering in positions 3, 4 and 5 has great variation for each method, in which is found goals G_4, G_2 and G_5. Finally, the goal less prioritized is G_6 (Protect people). BON-IOWAD and BON-IHOWAD show the prioritized goal for each method applied is G_3 (Guarantee availability and quality of information) followed by G_4 (Keep the good will) and the ordering in positions 3, 4 and 5 to BON-IOWAD has great variation, in which is found goals G_1, G_2 and G_5. Finally, the goal less prioritized is G_6 (Protect people). On the other hand, the ordering in positions 3, 4 and 5 to BON-IHOWAD has also great variation, in which is found goals G_1, G_2 and G_6. Finally, the goal less prioritized is G_5 (Prevent economic losses). Thus, in this sector, the information related is more consistent and evidence a great coincidence between goals and benefits for each method.

Finally, it is highlighted that the variation of results depends on attitudinal character of manager and not by the sort of information. Likewise, it is observed that BON-IHOWAD is the most robust method, since it combines all characteristics the each method aggregated. In this sense, it is important to highlight that these methods allow comparing and interrelating continuously with specific characteristics. As result, we can identify the most and the least prioritized goal (see Table 19).

Methods are shown that the most important goals in ERM are Guarantee availability and quality of information and Prevent economic losses and the least important goals are Guarantee continuity of operation, keep the good will and prevent economic losses. This suggests that there are risks more o less relevant at the moment to make a decision in ERM orientation. Also, it is observed how large companies in Colombia prioritize goals to manage risk in search of good operational and economic results. Finally, it is obtained an integral view between risks, goals and benefits, which allows determining aspects among planning, implementation and expected results.

As we can see, by the use of different aggregation operators the results that are obtained and the ranking of the most to least prioritized goal may change. This is important to consider because it is a clear example of how the results of the same problem can differ considerably depending if the subjective information that the decision maker has about the future expectations and knowledge of the market. In this sense, it is possible to affirm that when the problem that is analysed is complex the information provided by the decision maker makes a difference when it is included or not in the final results, and also, if the case is simpler using this information or not can sometimes give the same results, so using one of the families of the BON-IHOWAD that doesn't include all the aggregation processes will give the same results in a simpler formulation.

5. Conclusions

We have studied several mathematical methods that allow proposing a new set of tools in aggregation theory. We have focused on Bonferroni means, OWA operators and Hamming distance to develop operators that are able to aggregate, interrelate, compare and rank information multiple and continuously according to the preferences and attitude of the decision maker. The operators proposed are called BON−HOWAD, BON-IOWAD AND BON-IHOWAD. Each operator has a characteristic by the combination of different methods. For BON−HOWAD the main advantage is associated with the *heavy part*, which allows diminishing the restrictions on the associated weighting vector and there is a range between the OWA operator and the total operator. For BON-IOWAD the main advantage is associated with i*nduce part*, which allows doing the reordering step through an induced mechanism that is dependent upon the values of their associated order-inducing variables. For BON-IHOWAD the main advantage is given by IHOWA part, which combines the characteristics of HOWA and IOWA in the same formulation.

We have proposed a case study for mathematical application. Case study is focused on enterprise risk management (ERM) in large companies in Colombia survey. This application analyse the preference, comparison, order and relation information that has a decision-maker according to their expectations about ERM. We have taken into account main goals in ERM, main types of risk and main benefits in ERM for four main sectors: Trading and consumer, Service, Manufacturing and Extractive. Finally, we have used the new methods to establish a multiple comparison and interrelationship between goals, risk and benefits in ERM in large companies in Colombia. Results are shown in three parts. Firstly, it is highlighted the main results for each method used and their implications. Secondly, results are depicted in order to comprehend the information behaviour. Figures have shown the variation and the prioritization between goals to get expected benefits for each sector. Thirdly, we have shown the order for each sector. Here, the application allows reflecting the differences between information of each risk according to each goal, the attitude of manager on what expect from risk management and the interrelationship of each of the variables. Thus, it is shown that the most important goals in ERM are Guarantee availability and quality of information and Prevent economic losses and the least important goals are Guarantee continuity of operation, keep the good will and prevent economic losses.

In ERM the methods more used are focused on quantitative and qualitative. These methods seek risk measure, such as: "*probability* (the likelihood that a threat will occur), *impact* (the loss that will occur if the threat is realized) and *risk exposure* (the magnitude of a risk based on current values of probability and impact)" [42]. Likewise, these methods provide a rational philosophy but the heterogeneity of its organizational dynamics is difficult to determine used them [50]. According to Arena, Arnaboldi and Azzone [50] this heterogeneity is explained at the highest level by differing risk rationalities, information asymmetry and their potential to challenge the conceptualization of uncertainty. Thus, this new method allows dealing with heterogeneity and uncertainty by information asymmetry, since new method is being able to aggregate, multiple compare and order information continuously in a real application in order to identify the relation and prioritization between goals, risk and benefits for large firms in Colombia. This allows aggregate different sort of information in the decision-making process and strategy planning process for ERM. Finally, it was possible to identify how the final results can change when the complete operator is used (taking into account all the different aggregation processes) or only some of the families of the same.

In future research, we expect to expand these methods into other business areas [51], such as, personal selection, investment and opportunities of business. Also, the development of new aggregation operators using probabilities [52], moving averages [53], support vector machines [54], prioritized operators [55,56], fuzzy numbers, linguistic variables and new applications in group decision making are expected.

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Appendix 1.

The maximum similarity sub-relations matrices with BON-OWAD

See Tables A1–A16.

Table A1
Trading and consumer sector.

	Finance benefits					Organizational improvement					Decision-making benefits				
	G_1	G_2	G_3	G_4	G_5	G_1	G_2	G_3	G_4	G_5	G_1	G_2	G_3	G_4	G_5
G_1	1	0,977	0,989	0,987	0,986	1	0,977	0,988	0,987	0,986	1	0,977	0,988	0,987	0,986
G_2	0,977	1	0,996	0,994	0,988	0,977	1	0,996	0,994	0,988	0,977	1	0,996	0,994	0,988
G_3	0,989	0,996	1	0,993	0,987	0,988	0,996	1	0,993	0,987	0,988	0,996	1	0,993	0,987
G_4	0,987	0,994	0,993	1	0,987	0,987	0,994	0,993	1	0,987	0,987	0,994	0,993	1	0,987
G_5	0,986	0,988	0,987	0,987	1	0,986	0,988	0,987	0,987	1	0,986	0,988	0,987	0,987	1
G_6	0,987	0,991	0,992	0,992	0,977	0,987	0,991	0,992	0,992	0,977	0,987	0,991	0,992	0,992	0,977

Table A2
Service sector.

	Finance benefits					Organizational improvement					Decision-making benefits				
	G_1	G_2	G_3	G_4	G_5	G_1	G_2	G_3	G_4	G_5	G_1	G_2	G_3	G_4	G_5
G_1	1	0,985	0,987	0,988	0,990	1	0,985	0,987	0,988	0,990	1	0,985	0,987	0,988	0,990
G_2	0,985	1	0,992	0,993	0,994	0,985	1	0,992	0,992	0,994	0,985	1	0,992	0,992	0,994
G_3	0,987	0,992	1	0,995	0,994	0,987	0,992	1	0,995	0,994	0,987	0,992	1	0,995	0,994
G_4	0,988	0,993	0,995	1	0,994	0,988	0,992	0,995	1	0,993	0,988	0,992	0,995	1	0,994
G_5	0,990	0,994	0,994	0,994	1	0,990	0,994	0,994	0,993	1	0,990	0,994	0,994	0,994	1
G_6	0,987	0,991	0,989	0,988	0,977	0,987	0,991	0,989	0,987	0,977	0,987	0,991	0,989	0,987	0,977

Table A3
Manufacturing sector.

	Finance benefits					Organizational improvement					Decision-making benefits				
	G_1	G_2	G_3	G_4	G_5	G_1	G_2	G_3	G_4	G_5	G_1	G_2	G_3	G_4	G_5
G_1	1	0,988	0,985	0,985	0,993	1	0,988	0,984	0,984	0,993	1	0,988	0,984	0,985	0,993
G_2	0,988	1	0,988	0,988	0,993	0,988	1	0,988	0,988	0,993	0,988	1	0,988	0,988	0,993
G_3	0,985	0,988	1	0,989	0,993	0,984	0,988	1	0,989	0,993	0,984	0,988	1	0,989	0,993
G_4	0,985	0,988	0,989	1	0,994	0,984	0,988	0,989	1	0,994	0,985	0,988	0,989	1	0,994
G_5	0,993	0,993	0,993	0,994	1	0,993	0,993	0,993	0,994	1	0,993	0,993	0,993	0,994	1
G_6	0,985	0,987	0,984	0,984	0,977	0,985	0,987	0,984	0,983	0,977	0,985	0,987	0,984	0,983	0,977

Table A4
Extractive sector.

	Finance benefits					Organizational improvement					Decision-making benefits				
	G_1	G_2	G_3	G_4	G_5	G_1	G_2	G_3	G_4	G_5	G_1	G_2	G_3	G_4	G_5
G_1	1	0,994	0,999	0,999	0,999	1	0,994	0,999	0,999	0,999	1	0,994	0,999	0,999	0,999
G_2	0,994	1	1,000	1,000	1,000	0,994	1	1,000	1,000	1,000	0,994	1	1,000	1,000	1,000
G_3	0,999	1,000	1	1,000	1,000	0,999	1,000	1	1,000	1,000	0,999	1,000	1	1,000	1,000
G_4	0,999	1,000	1,000	1	1,000	0,999	1,000	1,000	1	1,000	0,999	1,000	1,000	1	1,000
G_5	0,999	1,000	1,000	1,000	1	0,999	1,000	1,000	1,000	1	0,999	1,000	1,000	1,000	1
G_6	0,996	0,992	0,993	0,993	0,977	0,996	0,992	0,993	0,993	0,977	0,996	0,992	0,993	0,993	0,977

The maximum similarity sub-relations matrix with BON-HOWAD

Table A5
The maximum similarity sub-relations matrix with BON-HOWAD. Trading and consumer sector.

	Finance benefits					Organizational improvement					Decision-making benefits				
	G_1	G_2	G_3	G_4	G_5	G_1	G_2	G_3	G_4	G_5	G_1	G_2	G_3	G_4	G_5
G_1	1	0,967	0,984	0,981	0,980	1	0,963	0,982	0,979	0,978	1	0,964	0,982	0,980	0,979
G_2	0,967	1	0,994	0,992	0,982	0,963	1	0,993	0,991	0,981	0,964	1	0,993	0,991	0,981
G_3	0,984	0,994	1	0,990	0,982	0,982	0,993	1	0,989	0,980	0,982	0,993	1	0,990	0,981
G_4	0,981	0,992	0,990	1	0,982	0,979	0,991	0,989	1	0,980	0,980	0,991	0,990	1	0,980
G_5	0,980	0,982	0,982	0,982	1	0,978	0,981	0,980	0,980	1	0,979	0,981	0,981	0,980	1
G_6	0,980	0,987	0,989	0,988	0,984	0,979	0,987	0,988	0,987	0,983	0,980	0,987	0,988	0,987	0,983

Table A6
Service sector.

	Finance benefits					Organizational improvement					Decision-making benefits				
	G_1	G_2	G_3	G_4	G_5	G_1	G_2	G_3	G_4	G_5	G_1	G_2	G_3	G_4	G_5
G_1	1	0,979	0,981	0,983	0,986	1	0,977	0,979	0,982	0,985	1	0,977	0,979	0,982	0,985
G_2	0,979	1	0,988	0,989	0,992	0,977	1	0,987	0,988	0,991	0,977	1	0,987	0,989	0,991
G_3	0,981	0,988	1	0,993	0,991	0,979	0,987	1	0,993	0,990	0,979	0,987	1	0,993	0,990
G_4	0,983	0,989	0,993	1	0,991	0,982	0,988	0,993	1	0,990	0,982	0,989	0,993	1	0,990
G_5	0,986	0,992	0,991	0,991	1	0,985	0,991	0,990	0,990	1	0,985	0,991	0,990	0,990	1
G_6	0,981	0,987	0,984	0,982	0,015	0,980	0,986	0,982	0,980	0,977	0,980	0,986	0,983	0,981	0,016

Table A7
Manufacturing sector.

	Finance benefits					Organizational improvement					Decision-making benefits				
	G_1	G_2	G_3	G_4	G_5	G_1	G_2	G_3	G_4	G_5	G_1	G_2	G_3	G_4	G_5
G_1	1	0,982	0,978	0,978	0,990	1	0,981	0,976	0,975	0,989	1	0,981	0,976	0,976	0,989
G_2	0,982	1	0,983	0,983	0,990	0,981	1	0,981	0,981	0,989	0,981	1	0,981	0,982	0,989
G_3	0,978	0,983	1	0,984	0,990	0,976	0,981	1	0,982	0,989	0,976	0,981	1	0,983	0,990
G_4	0,978	0,983	0,984	1	0,992	0,975	0,981	0,982	1	0,991	0,976	0,982	0,983	1	0,991
G_5	0,990	0,990	0,990	0,992	1	0,989	0,989	0,989	0,991	1	0,989	0,989	0,990	0,991	1
G_6	0,979	0,982	0,977	0,976	0,986	0,977	0,980	0,976	0,974	0,985	0,978	0,980	0,976	0,975	0,015

Table A8
Extractive sector.

	Finance benefits					Organizational improvement					Decision-making benefits				
	G_1	G_2	G_3	G_4	G_5	G_1	G_2	G_3	G_4	G_5	G_1	G_2	G_3	G_4	G_5
G_1	1	0,988	0,997	0,997	0,997	1	0,986	0,996	0,996	0,996	1	0,986	0,996	0,997	0,996
G_2	0,988	1	0,999	0,999	0,999	0,986	1	0,999	0,999	0,999	0,986	1	0,999	0,999	0,999
G_3	0,997	0,999	1	0,999	0,999	0,996	0,999	1	0,999	0,999	0,996	0,999	1	0,999	0,999
G_4	0,997	0,999	0,999	1	0,999	0,996	0,999	0,999	1	0,999	0,997	0,999	0,999	1	0,999
G_5	0,997	0,999	0,999	0,999	1	0,996	0,999	0,999	0,999	1	0,996	0,999	0,999	0,999	1
G_6	0,992	0,983	0,986	0,986	0,986	0,990	0,979	0,983	0,983	0,977	0,980	0,987	0,988	0,987	0,983

Table A9
The maximum similarity sub-relations matrix with BON-IOWAD. Trading and consumer sector.

	Finance benefits					Organizational improvement					Decision-making benefits				
	G_1	G_2	G_3	G_4	G_5	G_1	G_2	G_3	G_4	G_5	G_1	G_2	G_3	G_4	G_5
G_1	1	0,965	0,987	0,985	0,975	1	0,965	0,986	0,985	0,976	1	0,965	0,986	0,985	0,976
G_2	0,965	1	0,995	0,993	0,979	0,965	1	0,995	0,993	0,979	0,965	1	0,995	0,993	0,979
G_3	0,987	0,995	1	0,992	0,979	0,986	0,995	1	0,992	0,979	0,986	0,995	1	0,992	0,979
G_4	0,985	0,993	0,992	1	0,978	0,985	0,993	0,992	1	0,979	0,985	0,993	0,992	1	0,979
G_5	0,975	0,979	0,979	0,978	1	0,976	0,979	0,979	0,979	1	0,976	0,979	0,979	0,979	1
G_6	0,971	0,980	0,982	0,981	0,977	0,972	0,981	0,983	0,981	0,977	0,972	0,981	0,983	0,981	0,977

Table A10
Service sector.

	Finance benefits					Organizational improvement					Decision-making benefits				
	G1	G2	G3	G4	G5	G1	G2	G3	G4	G5	G1	G2	G3	G4	G5
G1	1	0,981	0,983	0,988	0,983	1	0,981	0,983	0,988	0,983	1	0,981	0,983	0,988	0,983
G2	0,981	1	0,989	0,992	0,990	0,981	1	0,990	0,992	0,990	0,981	1	0,990	0,992	0,990
G3	0,983	0,989	1	0,995	0,990	0,983	0,990	1	0,995	0,990	0,983	0,990	1	0,995	0,990
G4	0,988	0,992	0,995	1	0,990	0,988	0,992	0,995	1	0,990	0,988	0,992	0,995	1	0,990
G5	0,983	0,990	0,990	0,990	1	0,983	0,990	0,990	0,990	1	0,983	0,990	0,990	0,990	1
G6	0,986	0,990	0,987	0,986	0,977	0,986	0,990	0,987	0,986	0,977	0,986	0,990	0,987	0,986	0,977

Table A11
Manufacturing sector.

	Finance benefits					Organizational improvement					Decision-making benefits				
	G1	G2	G3	G4	G5	G1	G2	G3	G4	G5	G1	G2	G3	G4	G5
G1	1	0,985	0,983	0,978	0,984	1	0,985	0,983	0,978	0,984	1	0,985	0,983	0,978	0,984
G2	0,985	1	0,986	0,983	0,986	0,985	1	0,986	0,983	0,986	0,985	1	0,986	0,983	0,986
G3	0,983	0,986	1	0,983	0,985	0,983	0,986	1	0,983	0,985	0,983	0,986	1	0,983	0,985
G4	0,978	0,983	0,983	1	0,988	0,978	0,983	0,983	1	0,988	0,978	0,983	0,983	1	0,988
G5	0,984	0,986	0,985	0,988	1	0,984	0,986	0,985	0,988	1	0,984	0,986	0,985	0,988	1
G6	0,988	0,989	0,987	0,987	0,977	0,988	0,989	0,987	0,986	0,977	0,988	0,989	0,987	0,986	0,977

Table A12
Extractive sector.

	Finance benefits					Organizational improvement					Decision-making benefits				
	G1	G2	G3	G4	G5	G1	G2	G3	G4	G5	G1	G2	G3	G4	G5
G1	1	0,985	0,998	0,997	0,997	1	0,985	0,998	0,997	0,997	1	0,985	0,998	0,997	0,997
G2	0,985	1	1,000	0,999	0,999	0,985	1	1,000	0,999	0,999	0,985	1	1,000	0,999	0,999
G3	0,998	1,000	1	0,999	1,000	0,998	1,000	1	0,999	1,000	0,998	1,000	1	0,999	1,000
G4	0,997	0,999	0,999	1	0,999	0,997	0,999	0,999	1	0,999	0,997	0,999	0,999	1	0,999
G5	0,997	0,999	1,000	0,999	1	0,997	0,999	1,000	0,999	1	0,997	0,999	1,000	0,999	1
G6	0,991	0,983	0,986	0,986	0,977	0,991	0,983	0,986	0,986	0,977	0,991	0,983	0,986	0,986	0,977

Table A13
The maximum similarity sub-relations matrix with BON-IHOWAD. Trading and consumer sector.

	Finance benefits					Organizational improvement					Decision-making benefits				
	G1	G2	G3	G4	G5	G1	G2	G3	G4	G5	G1	G2	G3	G4	G5
G1	1	0,949	0,981	0,978	0,964	1	0,945	0,979	0,976	0,962	1	0,946	0,979	0,976	0,962
G2	0,949	1	0,993	0,990	0,970	0,945	1	0,992	0,989	0,968	0,946	1	0,992	0,990	0,968
G3	0,981	0,993	1	0,988	0,970	0,979	0,992	1	0,987	0,968	0,979	0,992	1	0,988	0,968
G4	0,978	0,990	0,988	1	0,969	0,976	0,989	0,987	1	0,967	0,976	0,990	0,988	1	0,967
G5	0,964	0,970	0,970	0,969	1	0,962	0,968	0,968	0,967	1	0,962	0,968	0,968	0,967	1
G6	0,959	0,972	0,975	0,973	0,977	0,956	0,970	0,973	0,971	0,977	0,957	0,970	0,973	0,971	0,977

Table A14
Service sector.

	Finance benefits					Organizational improvement					Decision-making benefits				
	G1	G2	G3	G4	G5	G1	G2	G3	G4	G5	G1	G2	G3	G4	G5
G1	1	0,972	0,975	0,983	0,975	1	0,970	0,974	0,981	0,974	1	0,971	0,974	0,981	0,974
G2	0,972	1	0,985	0,989	0,985	0,970	1	0,984	0,988	0,984	0,971	1	0,984	0,988	0,985
G3	0,975	0,985	1	0,993	0,985	0,974	0,984	1	0,992	0,984	0,974	0,984	1	0,992	0,984
G4	0,983	0,989	0,993	1	0,985	0,981	0,988	0,992	1	0,984	0,981	0,988	0,992	1	0,984
G5	0,975	0,985	0,985	0,985	1	0,974	0,984	0,984	0,984	1	0,974	0,985	0,984	0,984	1
G6	0,980	0,985	0,982	0,980	0,977	0,978	0,984	0,980	0,978	0,977	0,978	0,984	0,980	0,978	0,977

Table A15
Manufacturing sector.

	Finance benefits					Organizational improvement					Decision-making benefits				
	G1	G2	G3	G4	G5	G1	G2	G3	G4	G5	G1	G2	G3	G4	G5
G1	1	0,978	0,975	0,968	0,977	1	0,976	0,973	0,966	0,975	1	0,977	0,974	0,966	0,975
G2	0,978	1	0,980	0,975	0,979	0,976	1	0,979	0,974	0,978	0,977	1	0,979	0,974	0,978
G3	0,975	0,980	1	0,975	0,978	0,973	0,979	1	0,973	0,977	0,974	0,979	1	0,974	0,977
G4	0,968	0,975	0,975	1	0,982	0,966	0,974	0,973	1	0,981	0,966	0,974	0,974	1	0,981
G5	0,977	0,979	0,978	0,982	1	0,975	0,978	0,977	0,981	1	0,975	0,978	0,977	0,981	1
G6	0,983	0,985	0,981	0,980	0,977	0,981	0,983	0,979	0,978	0,977	0,981	0,983	0,979	0,978	0,977

Table A16
Extractive sector.

	Finance benefits					Organizational improvement					Decision-making benefits				
	G_1	G_2	G_3	G_4	G_5	G_1	G_2	G_3	G_4	G_5	G_1	G_2	G_3	G_4	G_5
G_1	1	0,969	0,995	0,994	0,994	1	0,965	0,994	0,993	0,993	1	0,966	0,994	0,994	0,993
G_2	0,969	1	0,999	0,998	0,999	0,965	1	0,999	0,998	0,998	0,966	1	0,999	0,998	0,998
G_3	0,995	0,999	1	0,998	0,999	0,994	0,999	1	0,998	0,999	0,994	0,999	1	0,998	0,999
G_4	0,994	0,998	0,998	1	0,998	0,993	0,998	0,998	1	0,998	0,994	0,998	0,998	1	0,998
G_5	0,994	0,999	0,999	0,998	1	0,993	0,998	0,999	0,998	1	0,993	0,998	0,999	0,998	1
G_6	0,981	0,965	0,970	0,972	0,977	0,978	0,959	0,965	0,967	0,977	0,979	0,960	0,966	0,968	0,977

References

[1] A. Hiles, Enterprise risk management, in: Defin. Handb. Bus. Contin. Manag., John Wiley & Sons, Inc, Hoboken, NJ, USA, 2011, pp. 1–21.
[2] R.E. Hoyt, A.P. Liebenberg, The value of enterprise risk management, J. Risk Insur. 78 (2011) 795–822.
[3] J. Lam, The CRO is her to stay, Risk Manag. 48 (2001) 16.
[4] M.S. Beasley, R. Clune, D.R. Hermanson, Enterprise risk management: an empirical analysis of factors associated with the extent of implementation, J. Account. Public Policy. 24 (2005) 521–531.
[5] F. Blanco-Mesa, J.M. Merigó, A.M. Gil-Lafuente, Fuzzy decision making: a bibliometric-based review, J. Intell. Fuzzy Syst. 32 (2017) 2033–2050.
[6] R.R. Yager, On ordered weighted averaging aggregation operators in multicriteria decision making, IEEE Trans. Syst. Man Cybern. 18 (1988) 183–190.
[7] S. Zeng, J.M. Merigó, W. Su, The uncertain probabilistic OWA distance operator and its application in group decision making, Appl. Math. Model. 37 (2013) 6266–6275.
[8] J.M. Merigó, A.M. Gil-Lafuente, A method for decision making based on generalized aggregation operators, Int. J. Intell. Syst. 28 (2013) 453–473.
[9] P. Liu, Y. Wang, Interval neutrosophic prioritized OWA operator and its application to multiple attribute decision making, J. Syst. Sci. Complex. 29 (2016) 681–697.
[10] Z.S. Xu, Q.L. Da, The uncertain OWA operator, Int. J. Intell. Syst. 17 (2002) 569–575.
[11] Y. Xu, H. Wang, The induced generalized aggregation operators for intuitionistic fuzzy sets and their application in group decision making, Appl. Soft Comput. 12 (2012) 1168–1179.
[12] X.-P. Jiang, G.-W. Wei, Some Bonferroni mean operators with 2-tuple linguistic information and their application to multiple attribute decision making, J. Intell. Fuzzy Syst. 27 (2014) 2153–2162.
[13] F. Blanco-Mesa, A.M. Gil-Lafuente, J.M. Merigó, New aggregation operators for decision-making under uncertainty: an applications in selection of entrepreneurial opportunities, Technol. Econ. Dev. Econ. 24 (2018) 335–357.
[14] E. León-Castro, E. Áviles-Ochoa, A.M. Gil-Lafuente, Exchange rate usd/mxn forecast through econometric models, time series and howma operators, Econ. Comput. Econ. Cybern. Stud. Res. 50 (2016) 135–150.
[15] M. Xia, Z. Xu, B. Zhu, Generalized intuitionistic fuzzy Bonferroni means, Int. J. Intell. Syst. 27 (2012) 23–47.
[16] Y. He, H. Chen, Z. He, L. Zhou, Multi-attribute decision making based on neutral averaging operators for intuitionistic fuzzy information, Appl. Soft Comput. 27 (2015) 64–76.
[17] R.R. Yager, On generalized Bonferroni mean operators for multi-criteria aggregation, Int. J. Approx. Reason. 50 (2009) 1279–1286.
[18] R.R. Yager, D.P. Filev, Induced ordered weighted averaging operators, IEEE Trans. Syst. Man Cybern. B Cybern. 29 (1999) 141–150.
[19] R.R. Yager, O.W.A. Heavy, Fuzzy Operators, Optim, Decis. Mak. 1 (2002) 379–397.
[20] J.M. Merigó, D. Palacios-Marqués, P. Soto-Acosta, Distance measures, weighted averages, OWA operators and Bonferroni means, Appl. Soft Comput. 50 (2017) 356–366.
[21] P. Liu, L. Rong, Y. Chu, Y. Li, Intuitionistic linguistic weighted Bonferroni mean operator and its application to multiple attribute decision making, Scientific World Journal. 2014 (2014) 1–13.
[22] Z. Xu, Q. Chen, A multi-criteria decision making procedure based on interval-valued intuitionistic fuzzy bonferroni means, J. Syst. Sci. Syst. Eng. 20 (2011) 217–228.
[23] F. Blanco-Mesa, J.M. Merigó, J. Kacprzyk, Bonferroni means with distance measures and the adequacy coefficient in entrepreneurial group theory, Knowledge-Based Syst. 111 (2016) 217–227.
[24] J.M. Merigó, The probabilistic weighted averaging distance and its application in group decision making, Kybernetes 42 (2013) 686–697.
[25] X. Gou, Z. Xu, H. Liao, Multiple criteria decision making based on Bonferroni means with hesitant fuzzy linguistic information, Soft Comput. 21 (2017) 6515–6529.
[26] Z. Liu, P. Liu, Intuitionistic uncertain linguistic partitioned Bonferroni means and their application to multiple attribute decision-making, Int. J. Syst. Sci. 48 (2017) 1092–1105.
[27] D. Liang, Z. Xu, A.P. Darko, Projection model for fusing the information of pythagorean fuzzy multicriteria group decision making based on geometric Bonferroni mean, Int. J. Intell. Syst. 32 (2017) 966–987.
[28] E. León-Castro, E. Avilés-Ochoa, J.M. Merigó, Induced heavy moving averages, Int. J. Intell. Syst. 33 (2018) 1823–1839.
[29] E. León-Castro, E. Avilés-Ochoa, J.M. Merigó, A.M. Gil-Lafuente, Heavy moving averages and their application in econometric forecasting, Cybern. Syst. 49 (2018) 26–43.
[30] J.M. Merigó, A.M. Gil-Lafuente, Decision making techniques with similarity measures and OWA operators, SORT – Stat. Oper. Res. Trans. 36 (2012) 81–102.
[31] J.M. Merigó, M. Casanovas, Decision-making with distance measures and induced aggregation operators, Comput. Ind. Eng. 60 (2011) 66–76.
[32] L.A. Pérez-Arellano, E. León-Castro, E. Avilés-Ochoa, J.M. Merigó, Prioritized induced probabilistic operator and its application in group decision making, Int. J. Mach. Learn. Cybern. (2017), http://dx.doi.org/10.1007/s13042-017-0724-2.
[33] R.R. Yager, J. Kacprzyk, The Ordered Weighted Averaging Operators: Theory and Applications, 1st ed., Kluwer Academic Publishers, New York, 1997.
[34] J.M. Merigó, M. Casanovas, Decision making with distance measures and linguistic aggregation operators, Int. J. Fuzzy Syst. 12 (2010) 190–198.
[35] J.M. Merigó, M. Casanovas, Induced and uncertain heavy OWA operators, Comput. Ind. Eng. 60 (2011) 106–116.
[36] R.W. Hamming, Error detecting and error correcting codes, Bell Syst. Tech. J. 29 (1950) 147–160.
[37] A. Kaufmann, Introduction to the Theory of Fuzzy Subsets, v1 ed., Academic Press, New York, 1975.
[38] Z. Xu, J. Chen, Ordered weighted distance measure, J. Syst. Sci. Syst. Eng. 17 (2008) 432–445.
[39] J.M. Merigó, A.M. Gil-Lafuente, New decision-making techniques and their application in the selection of financial products, Inf. Sci. 180 (2010) 2085–2094.
[40] J.M. Merigó, M. Casanovas, S. Zeng, Distance measures with heavy aggregation operators, Appl. Math. Model. 38 (2014) 3142–3153.
[41] J.M. Merigo, M. Casanovas, Induced and heavy aggregation operators with distance measures, J. Syst. Eng. Electron. 21 (2010) 431–439.
[42] C. Bonferroni, Sulle medie multiple di potenze, Boll. Dell Unione Mat. Ital. 5 (3-4) (1950) 267–270.
[43] E. O'Donnell, Enterprise risk management: a systems-thinking framework for the event identification phase, Int. J. Account. Inf. Syst. 6 (2005) 177–195.
[44] ICONTEC, Risk management: Principles and guidelines, Colombian Institute of Technical Standards and Certification, Colombia, Colombia, 2009 www.icontec.org.
[45] ICONTEC, Guide to the Identification of Hazards and the Assessment of Risks Is Occupational Health and Safety, Colombian Institute of Technical Standards and Certification, Colombia, 2012 www.icontec.org.
[46] ICONTEC, Environmental Management Systems: Requirements With Guidance for Use, Colombian Institute of Technical Standards and Certification, Colombia, 2004 https://www.iso.org/obp/ui#iso:std:iso:14001:ed-2:v1:es.
[47] J. Cifuentes-Valenzuela, F. Blanco-Mesa, L.G. Gutiérrez-Bernal, E. Oviedo, M.R. Godoy-Rodríguez, J.M. Mora-Álvarez, Assessment of occupational safety and health development in large private companies in Bogota, Colombia, in: R.C. Mejía-Quijano, M.A. Nuñez-Patiño, I. Martins (Eds.), Bus. Risk Manag. Colomb. Mex. Argentina, Editorial EAFIT, Medellín, 2017, p. 217.
[48] J. Gil-Aluja, Elements for a Theory of Decision in Uncertainty, Kluwer Academic Publishers, Dordrecht, 1999.
[49] W.F. De Bondt, R.H. Thaler, Further evidence on investor overreaction and stock market seasonality, J. Finance 42 (1987) 557–581.
[50] M. Arena, M. Arnaboldi, G. Azzone, The organizational dynamics of enterprise risk management, Accounting, Organ. Soc. 35 (2010) 659–675.
[51] S. Laengle, G. Loyola, J.M. Merigó, Mean-variance portfolio selection with the ordered weighted average, IEEE Trans. Fuzzy Syst. 25 (2017) 350–362.

[52] M. Casanovas, A. Torres-Martínez, J.M. Merigó, Decision making in reinsurance with induced OWA operators and Minkowski distances, Cybern. Syst. 47 (2016) 460–477.
[53] J.M. Merigó, R.R. Yager, Generalized moving averages, distance measures and OWA operators, Int. J. Uncert. Fuzz. Knowl.-Based Syst. 21 (2013) 533–559.
[54] S. Maldonado, J.M. Merigó, J. Miranda, Redefining support vector machines with the ordered weighted average, Knowl.-Based Syst. 148 (2018) 41–46.
[55] E. Avilés-Ochoa, E. León-Castro, L.A. Pérez-Arellano, J.M. Merigó, Government transparency measurement through prioritized distance operators, J. Intell. Fuzzy Syst. 34 (2018) 2783–2794.
[56] J.M. Merigó, Fuzzy decision making using immediate probabilities, Comput, Comput. Ind. Eng. 58 (4) (2010) 651–657.