Computing embedding numbers and branches of orders via extensions of the Bruhat-Tits tree

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Let k be a local field and let be the two-by-two matrix algebra over k. In our previous work, we developed a theory that allows the computation of the set of maximal orders in containing a given suborder. This set is given as a subgraph of the Bruhat-Tits (BT)-tree that is called the branch of the order. Branches have been used to study the global selectivity problem and also to compute local embedding numbers. They can usually be described in terms of two invariants. To compute these invariants explicitly, the strategy in our past work has been visualizing branches through the explicit representation of the BT-tree in terms of balls in k. This is easier for orders spanning a split commutative subalgebra, i.e. an algebra isomorphic to k × k. In fact, we have successfully used this idea in the past to compute embedding numbers for the split algebra. In the present work, we develop a theory of branches over field extensions that can be used to extend our previous computations to orders spanning a field. We use the same idea to compute branches for orders generated by arbitrary pairs of non-nilpotent pure quaternions, generalizing previous results due to the first author and Saavedra. We assume throughout that char(k)?2.