Article

Urban Analytics and City Science

EPB: Urban Analytics and City Science
2019, Vol. 46(3) 445–468

DOI: 10.1177/2399808317719071
journals.sagepub.com/home/epb
SAGE

# A quasi-equilibrium approach for market clearing in land use microsimulations

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## Abstract

A method for market clearing in land use models with a microsimulation approach for location choice of agents is proposed. The method, based on the Bid-auction theory and random utility models, assumes that agents individually adjust their perceived expected utility by observing market prices before entering auctions for a real estate good, hence modifying their overall willingness to pay for locations. The adjustment translates into a correction of each agent's bid level that follows the direction of supply-demand equilibrium, as they attempt to ensure their location. In each period, auctions for each available real estate good are simulated and prices are computed as the expected maximum bid of all agents in the market. The proposed method is tested for the city of Brussels, validated against real data and compared with results obtained when the bid adjustment is not included. Simulation results reproduce price trends that were observed in reality between the year 2001 and 2008, outperforming results obtained without a quasi-equilibrium bid adjustment approach. The proposed method is feasible to be implemented in large scale microsimulations and agent-based models because it does not require solving large fixed-point equilibrium problems.

## Keywords

Land use, agent based, market clearing, microsimulation, location choice

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## Introduction

Interventions on urban systems such as large real estate developments, modifications to the transport system and changes in urban policy are usually costly to implement and can benefit from models to forecast and evaluate their performance and effects in other elements of the system. Since Lowry (1964), land use models have been under constant development, following several different theoretical and methodological approaches to deal with this complex problem. A review of existing models and their methods can be found in Batty (2009), Iacono et al. (2008), and Wegener (2004, 2014).

Land use models dealing with urban real estate markets can be broadly classified according to three dimensions: the level of aggregation for agent treatment, the level of spatial aggregation and the degree of market representation (ranging from having no land-market to different mechanisms for market clearing). Some models treat agents in an aggregate way, defining homogeneous groups that can be characterized by average attributes, while other models treat their agents individually, these are usually known as microsimulation or as agent-based models (ABM, see Batty, 2007; Benenson and Torrens, 2004; Heppenstall et al., 2011). In a similar way, space can be divided in analysis zones of different sizes, from communes or traffic analysis zones to buildings, parcels or dwelling units, the latter being the most disaggregate level possible.

However, the most relevant difference across modeling approaches is the treatment of market clearing: how, and at which prices, agents are assigned to locations or units. Equilibrium models traditionally clear the market by finding a vector of prices that solves the equilibrium between supply (locations or units) and demand (agents) at an aggregate level. This approach, while consistent with microeconomic theory, usually requires a series of unrealistic assumptions like a cross-sectional treatment of time, instant and simultaneous relocation of all agents, and a perfect match between supply and demand. Moreover, solving the equilibrium problem will either require working with aggregate agents and space, or solving a very large fixed point problem with an excessively high computational cost. On the other hand, dynamic or disequilibrium models treat time as an explicit variable, usually representing it through a sequence of periods where the output of one period is the input of the next. Prices in this type of model are usually defined through hedonic methods or heuristics without solving a market equilibrium as usually defined in microeconomics. Microsimulation and ABM usually adopt this approach which, although more realistic, detailed and capable of representing the dynamics of city development and inertia of urban systems, deviates from classic urban economic theory. A more detailed analysis of this trade-off between theory, realism and functionality can be found in the works of Parker and Filatova (2008), Simmonds et al. (2013), and Wegener (2014).

With the increasing availability of data and computing resources, microsimulation models are becoming more relevant and attractive due to the possibility of representing individual agents and their complex interactions in a simple yet robust and flexible way. Moreover, agent-based microsimulation can easily account for the dynamics in the system, something that is hard to achieve in equilibrium models. However, there is a wide variety of approaches used to represent market dynamics, and no consensus in this regard has been reached yet. Exploring market clearing mechanisms that are grounded on microeconomic theory may help to better understand the underlying dynamics and simulation outcomes from an analytical point of view.

This paper proposes a method to model location choice and real estate prices simultaneously in a land use microsimulation context, defining a modeling framework that could also be applied to more general ABMs of urban phenomena (for an explanation of the differences between ABM and microsimulation models see Batty, 2009). The method is based on the bid-auction approach for location choice modeling (Ellickson, 1981; Martinez, 1992) and works under the assumption that real estate goods are traded in an auction market, where the best bid for a particular location determines both the located agent and the price or rent of the good. The proposed approach does not require solving for equilibrium, but estimates the maximum bid in each location and period by simulating the underlying auction process. Given exogenous supply levels, individual households adjust their willingness to pay as a reaction to the (observed) market conditions. This adjustment goes in the direction of an equilibrium (although it does not necessarily reach it), because it is obtained from solving a simplified approximation of microeconomic equilibrium conditions, and produces prices that are higher when goods are scarce and lower when goods are abundant. The approach allows the computation of market prices without bundling supply (real estate units) or demand (households) into aggregate types or categories, as required by equilibrium models. It is also not excessively expensive in terms of computational time, thanks to the estimation of the price as an expectation of the maximum bid over all agents instead of simulating individual transactions.

The proposed method is applied to a location choice model for the city of Brussels. Simulation outcomes are validated against real data, showing a better fit to observed market prices when the quasi-equilibrium bid adjustment is used.

The paper is organized as follows: We first review market clearing mechanisms used in equilibrium, ABM and dynamic disequilibrium models, to then propose our own quasi-equilibrium approach for this. We proceed to formulate a general framework for land use simulation which embeds the previously proposed market clearing mechanism; this framework is then used to simulate the city of Brussels. We conclude the paper by assessing the advantages and limitations of the proposed modeling approach and identifying possible further research.

## Market clearing

Real estate markets are particular and different from other markets because of the scarce nature of the traded goods and an almost-inelastic demand for them. Because of their spatial attributes, real estate goods have a quasi-unique nature: all locations are different because they cannot use the same space and their access to amenities and exposure to externalities will always be different. Besides the fact that only one consumer can use a location, the main implication of the quasi-unique nature of real estate goods is the fact that demand for them will be differentiated: the preference and willingness to pay of one consumer for a particular location will also be quasi-unique. Besides this, housing is a basic need and demand for it is essentially inelastic: a household cannot afford not to locate anywhere because it has to live somewhere. These particular characteristics generate a lot of competition between agents in the real estate market, especially if supply is relatively scarce.

The location choice problem can be described, in simple terms, as matching agents with locations in a coherent way. Two main modeling approaches have been proposed for this in the literature, the Bid-auction (maximum bid, see Ellickson, 1981) and the Choice approach (maximum utility, see McFadden, 1978). These two approaches have been proved to be equivalent under equilibrium conditions by Martinez (1992) but, otherwise, cannot account for the aforementioned conflicts (and resulting prices) that arise from competition for urban land. A market clearing mechanism, understood as the process of interaction between supply and demand that assigns goods to consumers (or vice-versa), is needed for this. The following subsections briefly review the main concepts behind market clearing for equilibrium and for dynamic disequilibrium models.

## Equilibrium models

From a classical urban economics perspective, market clearing is the process of finding the combination of location choices and equilibrium prices where every agent achieves maximum utility (Fujita, 1989) because, in a market context, conflicts are solved through prices (or bids) that are adjusted in order to discriminate between agents or goods (for a more detailed review on equilibrium conditions for urban location see Hurtubia, 2012). Under the Bid-auction paradigm, it is assumed that agents bid their willingness to pay for each location, with each seller selecting the highest bidder. If an agent is the best bidder for more than one location, it can discriminate between them by marginally (and homogeneously) reducing its bid until it remains the best bidder in only one. The bidder does this because it behaves as a utility (or consumer surplus) maximizer and it is indifferent between locations where it can achieve the maximum utility (Alonso, 1964), so it will try to pay the smallest possible amount for any of them. If an agent does not win any auction, it will increase its bid until it wins at least one auction. If the agent runs out of available budget and still cannot find a location it becomes ''homeless''. It is important to notice that an adjustment of the willingness to pay does not imply a change in the preferences of agents, but rather a re-assessment and adjustment of their expected utility level. The equilibrium condition in this case can be expressed as follows

$$\sum_i S_i P(h|i, B_{hi}(z_i)) = H_h \quad \forall h \tag{1}$$

where $S_i$ is the amount of supply (number of units) in each location $i$ and $H_h$ is the number of households of type $h$. $B_{hi}$ is the willingness to pay (or bid) of household $h$ for location $i$, which is a function of location attributes $(z_i)$. The term $P(h|i, B_{hi}(z_i))$ represents the probability of agent $h$ being the highest bidder for location $i$. The equilibrium condition of (1) implies finding a vector of bids $(B_{hi}, \forall h, i)$ ensuring all households are located in the existing supply while achieving maximum utility. This is analogous to finding the vector of dwelling prices that would solve the equilibrium under a Choice approach and it requires that total supply $(\sum_i S_i)$ must equal total demand $(\sum_h S_h)$.

Equilibrium models generally use equilibrium conditions that are similar to the one described by (1). This usually requires treating agents and locations in an aggregate way and to solve a fixed point problem. Examples of models doing this are RURBAN (Miyamoto and Kitazume, 1989) and MUSSA (Martínez, 1996; Martínez and Donoso, 2010) in the family of Bid-auction models, and TRANUS (De La Barra, 1980, 1989; De La Barra et al., 1984), MEPLAN (Echenique et al., 1990) and RELU-TRANS (Anas and Liu, 2007) from the family of models assuming a Choice location process.

## Dynamic disequilibrium

Dynamic disequilibrium models generally avoid solving equilibrium problems and introduce dynamics by modeling period-wise, accounting for time lags and feedback effects (mostly excess of supply or surplus and transport system performance) that make decisions in one period dependent on the outcome of previous periods. Examples of this type of models are DELTA (Hunt and Simmonds, 1993; Simmonds, 1999), PECAS (Hunt and Abraham, 2003), and IRPUD (Wegener, 2008). Some dynamic models do try to incorporate equilibrium solutions, for example, by solving a long-term inter-temporal equilibrium problem (Anas and Arnott, 1991) or by solving a series of cross-sectional equilibrium problems in each modeling period (Martínez and Hurtubia, 2006). These models, however, require several strong assumptions about market or user behavior and would need a very large amount of detailed time-series of data to be calibrated, rendering them very hard to implement for a real case study.

Microsimulation land use models generally adopt a dynamic disequilibrium approach. For example, UrbanSim (Waddell, 2002; Waddell et al., 2003) finds real estate prices in each simulation period through a hedonic price model estimated for the base year, but computed for each simulation period using updated explanatory variables. In cases of conflict like, for example, two agents choosing the same location, UrbanSim assigns it using a "*first-come, first-served*" approach (Waddell, 2010). The idea behind using such an approach is that neither sellers nor buyers have perfect information on the market and sellers will minimize risks by giving the location to the first buyer at the asking price. Newer developments in the UrbanSim framework introduce the effect of competition among agents into the price formation process and location assignment (Wang and Waddell, 2013) by adjusting prices in order to minimize the difference between supply and demand. A similar approach is used in ILUTE (Salvini and Miller, 2005) and was tested for the residential real estate market of Toronto in Farooq and Miller (2012).

ABMs also tend to adopt a dynamic disequilibrium approach for market clearing; a review of modeling approaches and applications can be found in the works of Huang et al. (2014), Matthews et al. (2007), and Torrens and Benenson (2005). Some ABMs incorporate complex interactions such as competition through real estate auctions and adjustment of the agent's expectations (Chen et al., 2011; Ettema, 2011; Filatova, 2015; Filatova et al., 2009; Parker and Filatova, 2008; Magliocca et al., 2011), explicitly modeling the market clearing process at an individual scale (for example, by simulating negotiations between real estate buyers and sellers) and reducing the gap between ABM and economic theory. In this context, large scale simulations and model result validations confirm that ignoring market elements such as competitive bidding might produce incorrect forecasts and policy recommendations (Sun et al., 2014). The approach proposed in this paper also attempts to account for these elements, by proposing a market clearing mechanism that works at an individual agent level, but is derived from market equilibrium conditions that are related to the one described by equation (1).

## A quasi-equilibrium approach to market clearing

A model for real estate market clearing is proposed. The model attempts to take into account the equilibrium forces that exist in market interactions, like demand or supply surplus (i.e. competition), the adjustment of expectations of agents and the corresponding adjustment of behavior. However, the model does not attempt to solve an equilibrium but, instead, proposes a disaggregate adjustment process with some outcomes, like the price, being the result of aggregate market interactions.

The proposed model has the following assumptions

- Interactions between agents take place in a discrete period framework. A period can be any amount of time large enough to account for a change in the levels of supply and demand.
- In each period, a group of agents enters the market looking for a location. Real estate supply for the same period is determined independently and does not necessarily satisfy demand.
- Real estate goods are transacted in auctions. All active agents (those looking for a location) are potential bidders for all locations.
- Agents do not have access to perfect information on the willingness to pay of other agents for each location, they can only infer them from the prices they observe in previous

periods. Before bidding for locations agents adjust their expectations according to this information.

- Auctions take place simultaneously, the best bidder gets the location. Prices are computed as the expected maximum bid.
- If an agent is the best bidder for more than one location, it chooses the one that provides maximum consumer surplus. Vacant locations and unlocated agents participate in new auctions until the market clears in a particular period. Remaining unlocated agents or unused units become part of the pool of active agents or vacant units for the next period.

Figure 1 describes the auction and market clearing sequence in the proposed model for a particular period $t$. All elements shown in gray boxes are exogenous to the process, including relocating agents, vacant units, and attributes of the location in the previous period. Agents adjust their willingness to pay after observing prices and location attributes in the previous period and bid for each location. The auctions determine who is the best bidder for each location and the clearing mechanism assigns agents to locations. If, due to conflicts, there are unlocated agents or empty units, a new set of auctions takes place. The process is repeated until all agents are located or all locations are occupied; this is a (relaxed) approximation to the classical equality condition between supply and demand, as described by equation (1). At the end of the sequence, any unlocated agents or empty units will become relocating agents

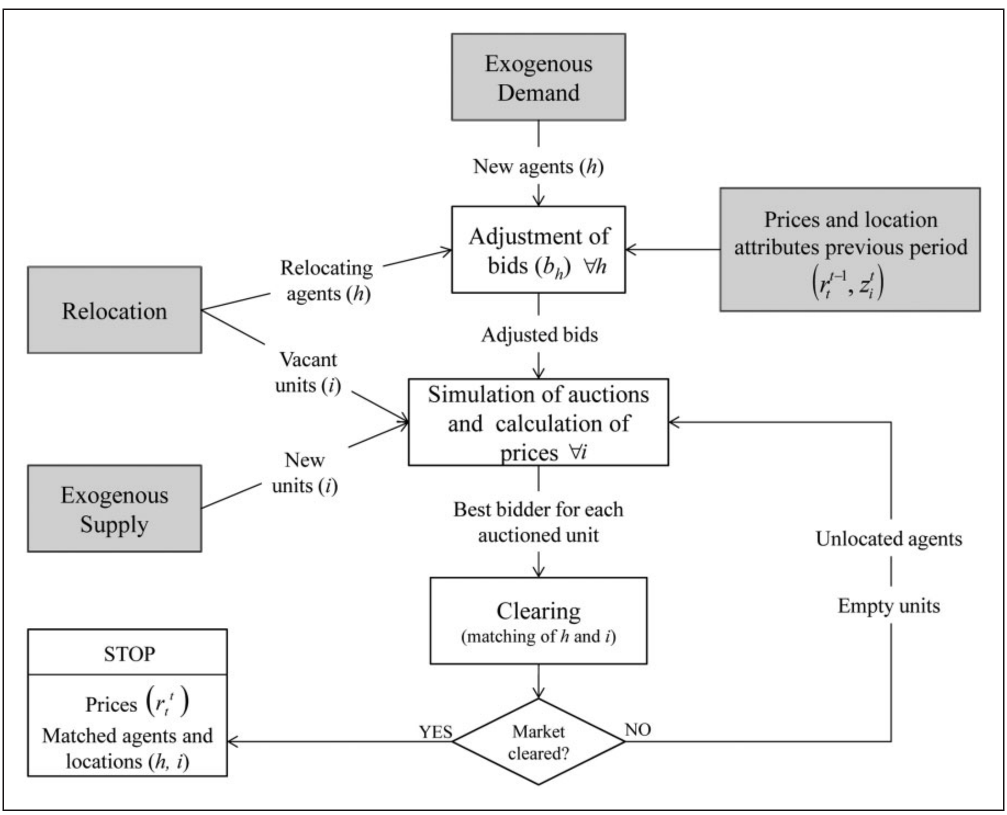


**Figure 1.** Algorithm for market clearing.

and vacant units respectively for the next simulation period. The overall modeling procedure and main components of the algorithm are described next.

### Location choice modeling under a bid auction approach

Ellickson (1981), following the work of Alonso (1964), proposed to model the household's bid (or willingness to pay) as a function of location attributes ($z_i$) and proposed to account for the unobserved heterogeneity in preferences across households by adding a random error term

$$\widetilde{B_{hi}} = B_h(z_i) + \varepsilon_h = B_{hi} + \varepsilon_h \tag{2}$$

The probability of a residential unit or location $i$ being occupied by $h$ is the probability of that particular household winning the auction against all other bidding households

$$P_{h/i} = Prob\big\{\ B_{hi} + \varepsilon_h > B_{h'i} + \varepsilon_{h'},\ \forall h' \neq h\big\}$$

If the error terms follow a Gumbel or Extreme Value distribution, the best bid probability can be expressed as

$$P(h|i) = \frac{\exp(\mu B_{hi})}{\sum_{g \in H} \exp\big(\mu B_{gi}\big)} \tag{3}$$

where $H$ is the set of bidding households. Under the auction market assumption, the price or rent ($r_i$) of a good is the value of the winning bid which, in a stochastic setting, can be expressed as an expectation

$$r_i = E\left(\max_h (B_{hi})\right) \tag{4}$$

The Extreme Value distribution assumption allows one to express the expected maximum bid of a location $i$ as (Ben-Akiva and Lerman, 1985)

$$r_i = \frac{1}{\mu} \ln\left(\sum_{g \in H} \exp\big(\mu B_{gi}\big)\right) + \frac{\gamma}{\mu} \tag{5}$$

where $\gamma$ is the Euler's constant. Notice that equation (5) allows writing the maximum bid probability of (3) as a function of the expected price, as follows

$$P(h|i) = \exp(\mu(B_{hi} - r_i)) \tag{6}$$

Simulation of auctions can be performed through Monte Carlo draws from the probability distribution described by equation (3) or (6).

### Adjustment of bids

Following Martinez (2000), we assume that the deterministic part of the bid function can be separated into two elements, therefore; for a particular period $t$

$$B^t_{hi} = b^t_h + b_{hi}\big(z^{t-1}_i, \beta\big) \tag{7}$$

where $b_h^t$ is the adjustment component that relates the bid with the utility level of the household and $b_{hi}$ is the hedonic part of the bid expressing the value a household $h$ gives to the attributes of a location $i$ through a set of parameters $\beta$. Because location has not taken place yet in $t$, only attributes from the previous period $(z_i^{t-1})$ are observable by the household. The functional form of (7) implies the assumption of a quasi-linear underpinning utility function, which allows for additive decomposition and simplifies the interpretation of each element (Martínez and Henríquez, 2007). We assume the preferences of households will remain constant in time; therefore the value of the hedonic part of a bid for a particular pair $(b_{hi})$ will remain constant unless there is a change in the attributes of the location $(z_i^t)$.

The adjustment of $b_h$ follows the logic of households changing their expectations given what they observe in the market and, therefore, increasing or decreasing their bids depending on their perceived chance of winning an auction. We define the perceived probability $(q)$ that agent $h$ has of winning the auction for location $i$ in period $t$ following the bid probability equation (6), but considering that only the prices of the previous period are observable

$$q^t(h|i) = \exp\left(B_{hi}^t - r_i^{t-1}\right) \tag{8}$$

For simplicity we assume the scale parameter $(\mu)$ to be equal to one. The expected probability (perceived by $h$) of winning any auction is the sum over all available supply $(S^t)$ of the perceived winning probabilities

$$q^t(h) = \sum_{i \in S^t} q^t(h|i) \tag{9}$$

Because demand for location is inelastic and agents need to locate somewhere, they try to make this probability to be equal to one. Therefore, by replacing (7) and (8) in (9), we get the following description of the bid adjustment that accounts for the expectations of the agent, given the market conditions described by $r^{t-1}$

$$\sum_{i \in S^t} q^t(h|i) = \sum_{i \in S^t} \exp\left(b_h^t + b_{hi}\left(z_i^{t-1}, \beta\right) - r_i^{t-1}\right) = 1 \tag{10}$$

Clearing $b_h^t$ from (10) we get

$$b_h^t = -\ln\left(\sum_{i \in S^t} \exp\left(b_{hi}\left(z_i^{t-1}, \beta\right) - r_i^{t-1}\right)\right) \tag{11}$$

The adjustment of $b_h^t$ is, to some extent, similar to adjusting or re-calibrating the alternative specific constants of a logit model, in order to capture unobserved factors that describe the market conditions of the forecast scenario (Train, 2009). An intuitive interpretation is that $b_h^t$ is the bid adjustment households perceive as necessary to attain a location without over or underbidding, given the available information.

It is important to notice that if supply $(S^t)$ is large, the sum of (11) will tend to be large as well. This ensures that, all things being equal, larger supply will always generate smaller values for $b_h$. This is consistent with the expected lower overall bids that should occur when supply is abundant. In the opposite case, if supply is scarce, $b_h$ will have bigger values. In general, supply surplus will generate low values of $b_h$ while a demand surplus scenario will trigger increases in the value of $b_h$.

Equation (10) is similar to equation (1) and to the equilibrium conditions found in models like MUSSA (Martínez, 1996). The bid adjustment of (11) follows the direction of a supply-demand equilibrium, as it is the result of each household (demand) attempting to ensure their location in some unit (supply). However, equation (11) does not represent an equilibrium problem because the attributes of the locations ($z_i^{t-1}$) and the observed prices will not change within period $t$; hence we call this a "quasi-equilibrium" solution. This means equation (11) is not a fixed point problem and, therefore, can be easily evaluated for each agent $h$ in each period $t$. It is important to notice that the solution of (11) will not ensure the location of an agent $h$ when the auctions take place.

The bid adjustment process described here is similar to the one proposed by Filatova et al. (2009) in the fact that both correct the willingness to pay of agents as a function of aggregate market conditions (supply or demand surplus). However, while agents in Filatova et al. (2009) participate in different sequential auctions where buyers and sellers correct their bids, asking prices proportionally to the level of demand surplus, the method proposed here derives the adjustment from quasi-equilibrium conditions, which are solved period-wise and are explicitly dependent of previous (observable) market prices. Comparison of both approaches is matter of future work.

## Simulation of auctions and calculation of prices

After the bid adjustments have been calculated for each agent, the auctions take place with the following equation describing the probability of agent $h$ winning the auction for location $i$

$$P^t(h|i) = \frac{\exp\left(b_h^t + b_{hi}\left(z_i^{t-1}, \beta\right)\right)}{\sum_{g\in H^t} \exp\left(b_g^t + b_{gi}\left(z_i^{t-1}, \beta\right)\right)} \tag{12}$$

A simulation is performed, generating an auction-outcome for each location $i$, where the highest bidder will be chosen following the cumulative probability distribution defined by (12). Prices are the expected maximum bid of each auction, considering the bids of all agents looking for a location ($H^t$) and the reference or asking price set by the seller. We model the asking price as the potential price the location will achieve if auctioned between all located households ($\overline{H^t}$). This is equivalent to computing the equilibrium price for each dwelling using the logsum expression of (5) but considering that new (re-locating) agents bid their adjusted willingness to pay

$$r_i^t = \ln\left(\sum_{g\in\overline{H^t}} \exp\left(B_{gi}\left(z_i^{t-1}\right)\right) + \sum_{h\in H^t} \exp\left(b_h^t + b_{hi}\left(z_i^{t-1}\right)\right)\right) \tag{13}$$

The inclusion of the asking price is equivalent to accounting for the potential bid of other actors in the market and it is consistent with the definition of equilibrium prices of (5). This generates a stable price dynamic, since the prices will not depend only on the bids of active agents but also on that of all potential bidders. The prices will react to scenarios of supply or demand surplus thanks to the inclusion of the bid adjustments ($b_h$).

The simultaneous simulation of auction outcomes and the use of expected prices are required for consistency with the economic framework described in the "Adjustment of bids" section. This also smoothens the simulation results and reduces path dependency, making prices less dependent on the iterative clearing process. Although the proposed approach does not attempt to model singular events such as housing bubbles, the convenience of ignoring potential path dependency should be reviewed in the future (see Ge, 2017).

## Clearing

Since all auctions are simulated simultaneously, it is possible to find agents that are best bidders for more than one location and agents that could not win any auction. The clearing process sorts agents and locations by solving conflicts and determines which agents and locations will go through a new sequence of auctions.

If an agent is the best bidder in more than one auction, it will choose the location that provides maximum consumer surplus ($CS$), defined as the difference between the willingness to pay for the location minus the (equilibrium) price defined in (13)

$$CS_{hi}^{t} = B_{hi}^{t} - r_{i}^{t} \tag{14}$$

If desired, a probabilistic approach can be applied here too, by computing the probability of choosing a particular location as

$$P(i|h) = \frac{\exp\left(B_{hi}^{t} - r_{i}^{t}\right)}{\sum_{j \in S(h)} \exp\left(B_{hj}^{t} - r_{j}^{t}\right)} \tag{15}$$

where $S(h)$ is the set of locations where agent $h$ was the highest bidder. The probability of (15) can be used to generate a simulated choice. Because all active agents bid simultaneously for all locations, the order in which winning bidders are drawn does not affect the outcome. After the selection is made, the agent and the location are taken out of the pool. After repeating this process for all locations, some of them will be empty because they were discarded by a winning agent and some agents will remain unlocated because they were not the highest bidders in any auction. This set of empty locations and unlocated agents enters a new auctioning sequence where the clearing process is repeated until all conflicts are solved (when either all agents are located or all locations occupied). Bids will be re-adjusted during the new auction sequences.

This mechanism has the advantage of simulating market clearing in a realistic way. The bid adjustment has the effect of avoiding under or over-bidding by households; therefore the best bidder for a particular location is also likely to be an agent that perceives a high utility in that particular location. This is consistent with economic theory and similar to the situation observed in equilibrium models, although the relation between the highest bid and the maximum utility is not absolute, due to the asymmetries of information and temporal lag in attribute perception.

The adjustment of the bids before entering the auctions can also be understood as a (simplified) way to model strategic behavior by households. Because of the simultaneous bid-adjustment for all households, each of them is less likely to overbid and end up winning auctions for unattractive locations or, in the opposite case, to underbid and end up losing systematically in all auctions. A more explicit model for strategic behavior could allow households to participate in different sequential auctions and re-adjust their bids in each iteration as a function of the observed winning prices. However, there is not enough empirical evidence describing how households adjust their individual expectations, and hence their bids, when losing an auction in reality. The proposed method aims at being an operational model for these processes without incurring excessive complexity or requiring very detailed assumptions about individual behavior.

## General framework for land use modeling

In order to test the proposed methodology, we insert the market clearing method described in the previous section within a broader framework for land use modeling, especially developed for this, called Random Utility Simulator of Household-Location and Urban Dynamics (RUSH-LoUD). RUSH-LoUD was programmed in Python (code is available upon request) and contains a series of modules accounting for different elements of urban dynamics: Demand, Supply, Transport, Relocation and Market Clearing. Figure 2 shows the different modules and how they interact in one simulation period. In this implementation, most sub-models are simplified and follow observed distributions instead of having a behavioral approach, with the exception of the residential market clearing (and location choice) model. However, the framework supports any behavioral assumption for each of the sub-models and could be improved in further implementations.

We consider an exogenous demand module that, for each simulation period, generates a set of new agents that enter the system, following control totals defined by official population statistics coming from Census projections. Relocating agents are drawn randomly in each period, following an exogenous (and fixed) relocation rate for the whole region. If there are unlocated agents from the previous period, they are added to the new relocating agent set. The new agents, together with those agents that are relocating, define the demand. The uniform sampling protocol for both models generates a set of new and relocating households with the same distribution of socioeconomic attributes of the observed population.

Supply is determined by a model that, in each period, generates enough supply to satisfy the total demand, with a spatial and building type distribution that follows that of the supply

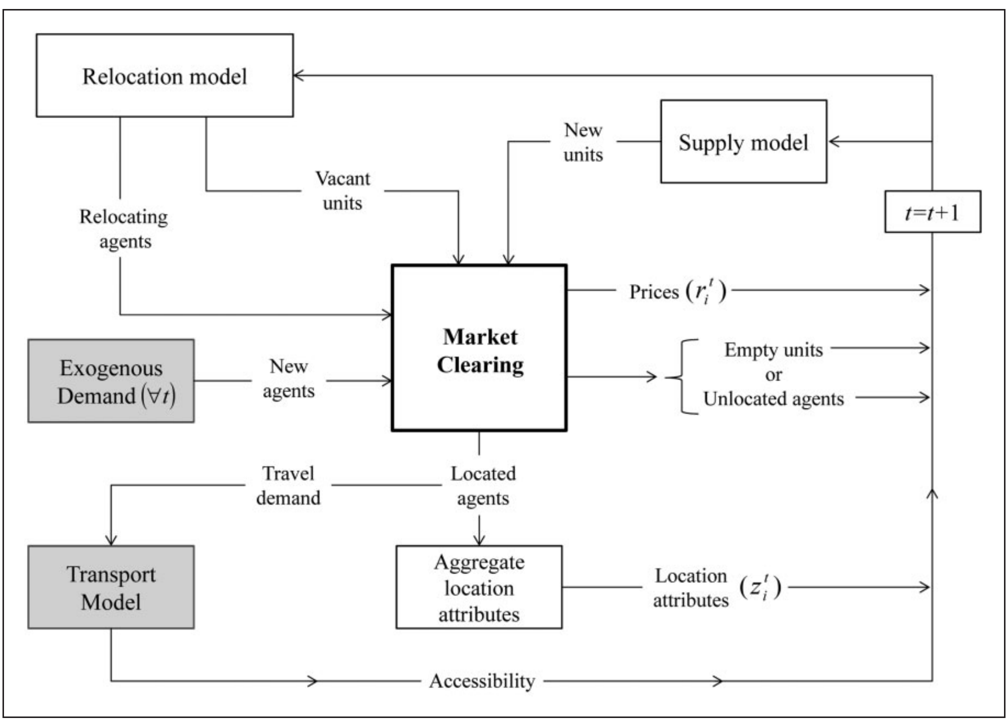


**Figure 2.** General modeling framework.

in the base year. This is a strong simplifying assumption but generates a supply distribution that is close enough to that observed in reality for the purposes of this experimental implementation. The supply model also keeps track of unoccupied supply and makes it available for the next period.

Demand and supply interact in the Market Clearing model, where the mechanism described in the previous section takes place and auctions are simulated. The result is a set of sorted agents and locations together with prices, updated attributes of the locations and a set of empty units or unlocated agents (depending on the market conditions). These elements are the main input of the simulation in the next period. Because in this particular implementation the supply model is set to satisfy total demand plus a structural (relatively small) vacancy, only empty units are possible.

The transport model is exogenous and provides accessibility measures that characterize the transport system in each location or zone. In this particular implementation, the results of a MATSim simulation (Nicolai and Nagel, 2015; Rieser et al., 2007) for the base year are used and kept constant for all periods. The accessibility is computed as the logsum of the travel (dis)utility from the zone of origin to every possible zone of destination. A more appropriate approach would be to re-compute accessibility measures after each simulation period, but this is very expensive in computational terms and it is left for future validations of the framework.

For simplicity (and due to the scarce data for job location), the current implementation simulates the dynamics for the residential real estate market only. Therefore, the non-residential attributes of the locations (number and distributions of jobs by type and zone) are computed for the base year and kept constant in the simulation periods.

If data allow, all the simplifying assumptions previously mentioned can be relaxed and replaced by behavioral models, although these should be carefully specified and estimated. For this implementation, the focus is placed on the behavioral models for household location and residential market clearing.

In summary, the current implementation of RUSH-LoUD will only model residential dynamics. Although reducing the complexity and scope of the simulation, this setting will allow us to test and evaluate the market clearing model as a proof of concept, avoiding confusion with the effects that could come from the non-residential market dynamics.

## Case study

The proposed model is implemented for the city of Brussels, where data have been collected in the context of the European research project SustainCity[1] (for an extensive description of this research project see Bierlaire et al., 2015). Table 1 summarizes the main data sources, most of which were obtained from the Belgian Statistical Office (StatBel). For a more detailed description of data collection and processing see Cabrita et al. (2015).

The study area considers an extended region around Brussels including 151 communes ($c$) subdivided in a total of 4,945 zones ($i$), covering an important part of the Dutch speaking region to the north (Flanders) and the French speaking region to the south (Wallonia). Dwelling alternatives ($v$) are classified in three types of houses (fully detached, semi-detached and attached) and one type of multi-unit building (apartments), generating a total of 19,780 possible locations (combinations of zones and building types). Dwellings are described by average attributes (price, surface) calculated by type of building and zone.

The area of study contains a total of 1,213,169 households. Figure 3 shows the distribution of households across the communes in the area of study for the base year. Central communes (the city of Brussels) concentrate the larger amount of located

**Table 1.** Data sources.

| Variables | Database | Source |
|---|---|---|
| Observed location of households and their characteristics | MOBEL | Hubert and Toint (2002) |
| Socioeconomic attributes of zones and communes | 2001 Population Census | StatBel (2001a) |
| Employment by activity type and commune | ONSS (2001) | ONSS (2001) |
| Average transaction prices of dwellings by commune | House Price Index 2001–2008 | StatBel (2001b, 2008) |

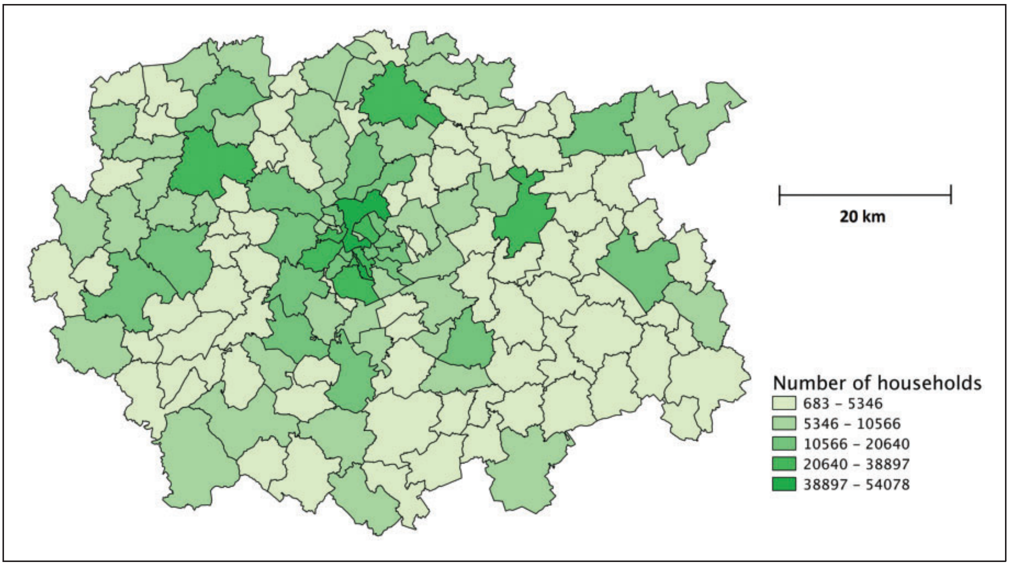


**Figure 3.** Number of households by commune, 2001.

households and are, at the same time, the most dense communes. Besides Brussels, the main urban agglomerations inside the area of study are Leuven (east of Brussels), Mechelen (north), and Aalst (west, northwest). Outer communes are less dense, with the less populated communes located southeast and southwest of Brussels city.

Given data availability, the modeling period-length is a year. For the base year, a synthetic population is generated (for details of the process see Farooq et al., 2013, 2015) where individual households are described in terms of their socioeconomic attributes and their location (building type and zone). For the following modeling periods, control totals coming from official estimations of population size are used to generate new households from a sample of the synthetic population. Households are characterized by their size, income level, number of children, number of workers, and the education level of its members. Table 2 describes the values for each attribute level.

The marginal distributions of attributes for the synthetic population are consistent with observed distributions coming from the census and other data sources. The synthetic population was also used to run the MATSim transport simulation for Brussels.

**Table 2.** Household attributes.

| Attribute | Levels |
|---|---|
| Income level of the household ($inc_h$) | 1 (0–1,859 Euros) |
| | 2 (745–1,859 Euros) |
| | 2 (1,860–3,099 Euros) |
| | 4 (3,100–4,958 Euros) |
| | 5 (>4,959 Euros) |
| Household size ($hh\_size_h$) | 1,2,3,4,5+ |
| Number of children ($children_h$) | 0,1,2+ |
| Number of workers ($workers_h$) | 0,1,2+ |
| Number of cars ($cars_h$) | 0,1,2,3+ |
| Number of people with university degree ($univ_h$) | 0,1,2+ |

**Table 3.** Bid function specification and estimation results.

| Parameter | location/Spatial attribute | × | household (hh) attribute | Estimate | *T* test |
|---|---|---|---|---|---|
| $ASC_2$ | – | | income 2 constant | −0.171 | −2.07 |
| $ASC_3$ | – | | income 3 constant | −0.461 | −4.1 |
| $ASC_4$ | – | | income 4 constant | 2.05 | 5.47 |
| $ASC_5$ | – | | income 5 constant | 2.19 | 5.68 |
| $\beta_{house}$ | dummy for houses (types 1, 2 or 3) | × | dummy $hh\_size_h > 2$ and $inc_h > 2$ | −0.128 | −2.7 |
| $\beta_{apartment}$ | dummy for apartment (type 4) | × | dummy $hh\_size_h > 2$ and $inc_h > 2$ | −0.702 | −3.88 |
| $\beta_{surface}$ | avg surface of dwelling *v* in zone *i* | × | logarithm of $hh\_size_h$ | 0.002 | 2.6 |
| $\beta_{high\text{-}inc}$ | % of hh's of income 4 and 5 in *c* | × | dummy for income $inc_h > 2$ | 3.97 | 3.21 |
| $\beta_{low\text{-}inc}$ | % of hh's of income 1 and 2 in *c* | × | dummy for income $inc_h > 3$ | −3.94 | −5.62 |
| $\beta_{education}$ | density of education jobs in *c* | × | dummy for $univ_h > 0$ | 0.356 | 2.8 |
| $\beta_{industry}$ | % of industry jobs in commune *c* | × | dummy for $inc_h > 3$ | −0.562 | −2.25 |
| $\beta_{service}$ | % of service jobs in zone *i* | × | dummy for $workers_h > 0$ | 0.046 | 2.31 |
| $\beta_{shopping}$ | density of retail jobs in zone *i* | × | dummy for income $inc_h > 2$ | 0.040 | 2.24 |
| $\beta_{pubtrans}$ | public transport $acces_i$ | × | dummy for $cars_h = 0$ | 0.257 | 2.72 |
| $\beta_{pubtrans2}$ | public transport $acces_i$ | × | dummy for $cars_h > 1$ | −0.249 | −2.46 |
| $\beta_{car\text{-}access}$ | car accessibility in zone *i* (MATSim) | × | dummy for $cars_h > 0$ | 0.007 | 1.9[a] |

[a]Parameter not significant at the 95% level.

## *Bid-auction model estimation*

Preference parameters (the vector $\beta$ in equation (7)) for the hedonic part of the bid function are estimated using a method that adjusts the choice probabilities (3) to the observed locations of households, while simultaneously reproducing observed prices as a function of the expected maximum bid (for details on the estimation method and the bid function specification, see Hurtubia and Bierlaire, 2014). A total of 1,346 observations from the MOBEL survey are used in the estimation. Table 3 shows the specification of the linear-in-parameters bid function and the results obtained through maximum likelihood estimation with the statistical software BIOGEME (Bierlaire, 2003; Bierlaire and Fetiarison, 2009).

All parameters have the expected signs. The scale parameter $\mu$ has been normalized to one. Socioeconomic agglomeration effects are explained by the positive value of $\beta_{\text{high-inc}}$ and the negative value of $\beta_{\text{low-inc}}$, meaning that middle and high income households prefer locations with a higher income distribution while high income households decrease their willingness to pay for a location where low income households are located in a zone. Presence of shopping, services, and education increase the willingness to pay for a location while the presence of industry has a negative effect for high-income households. Car accessibility has a positive effect for households with one or more cars while access to public transport attracts households with no car. Households with two or more cars have a lower willingness to pay for locations with high access to public transport, probably due to the street priority of the former over private modes.

## *Simulation design and details*

We use the ODD protocol (Grimm et al., 2006, 2010) to describe important concepts and definitions of the simulation algorithm (in *italics*). Since the proposed approach is not formally an ABM, this description is not exhaustive and should be considered only as a complement of the definitions and concepts of the general framework already proposed.

The *objective* of the simulation is to predict future real estate prices and the spatial distribution of households in a city. The main *agents* in this simulation are households looking for locations; they are described by the *attributes* shown in Table 2. All agents make decisions based on their expectations and their preferences (described by the parameters of Table 3), their behavior is *adaptive* since their expectations (and bidding behavior) react to observed market conditions. The *spatial units* are zones and communes (which are aggregations of zones), *state variables* for these units are the amount, type and price of supply in them, the number and characteristics of households located in them, and other attributes like accessibility levels. Of all the spatial state variables, the only *endogenous* ones are the number and characteristics of located households, number of vacant units in a zone, and real estate prices. All other state variables are either kept constant or evolve from their initial state in a proportional way, following exogenous control-totals. *Stochasticity* is present only in the simulation of the auction for each location, which follows the probability distribution described by (12) using parameters found in Table 3. The model is *initialized* with all state variables set to observed values in the year 2001.

## *Simulation results*

Simulations are run for a period of eight years, from 2001 to 2008. The reason to select this period is the availability of validation data regarding transaction prices for residential units and the population at the commune level for this period of time.

First, the stability of the simulation is tested by running 100 full simulations with different random seeds and computing the standard deviation of the simulated prices in the last simulation year. Figure 4 shows the distribution of the standard deviation over the average value of the price for all 19,780 combinations of building types and zones. Most cases have a remarkably low standard deviation, with more than 94% of prices presenting a standard deviation of less than 5% of the average value. This confirms the stability of the simulation and indicates that results are not path dependent.

*Price dynamics.* Figure 5 shows the evolution of prices at the zone level for each of the simulation years. The values are shown as the difference between the simulated prices and

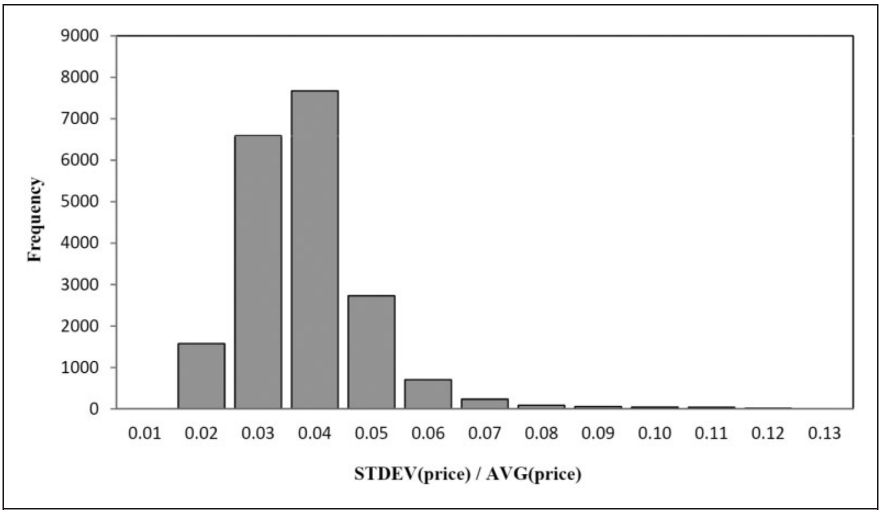


**Figure 4.** Standard deviation of predicted prices in 2008 (100 simulations).

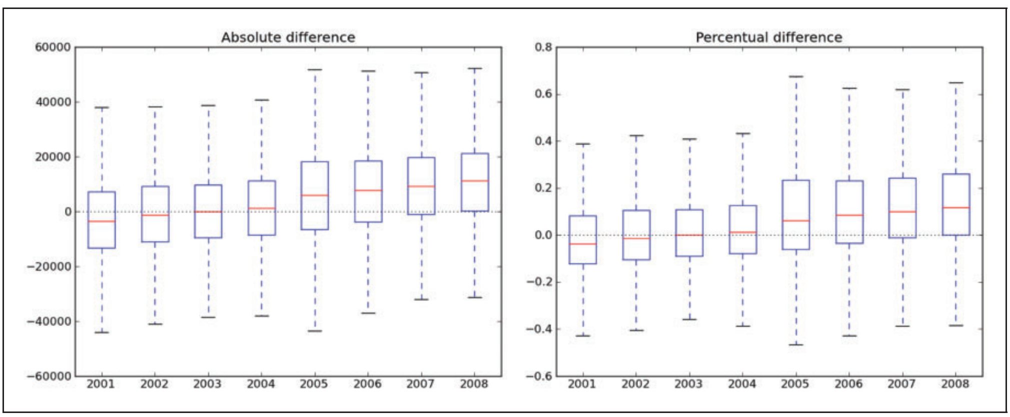


**Figure 5.** Error in price forecast, 2001–2008.

the observed average prices in absolute and relative terms, respectively. The boxplot graphic describes the span for each quartile of the values with the central box containing the 50% of values that are closer to the median error. Since observed prices are also affected by elements that are not considered by the model presented in this paper, like inflation or interest rate effects, we normalize all prices to the average of the observed price in the base year. This allows us to analyze the relative variation of prices.

The simulation begins with a small average underestimation of prices that turns into an average overestimation for later periods. This is explained by the fact that simulation prices will always increase, due to the increase of population and its positive effect on the expected maximum bid (see equation (13)). The quality of the forecast for prices is relatively stable across time, with the increase in the extreme values of year the 2005 explained by a large increase in the population for that particular year. The relatively large errors are explained by the fact that the plot shows the difference between average price by zone with the observed average price by commune.

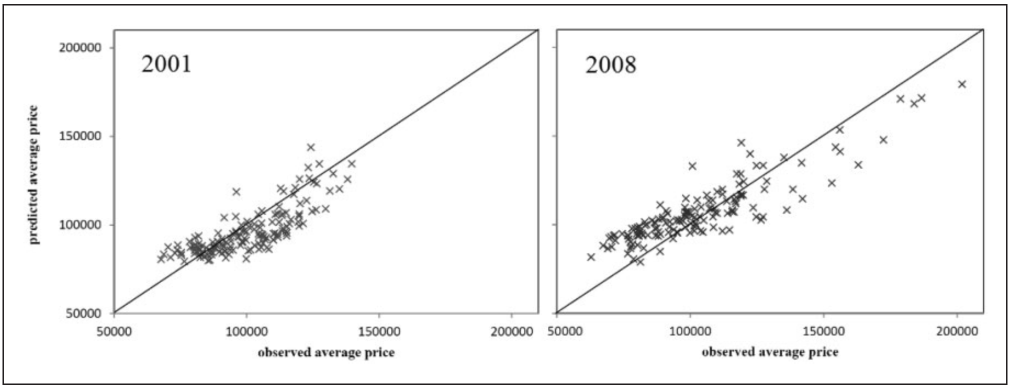


**Figure 6.** Forecasted vs. observed price by commune (2001–2008).

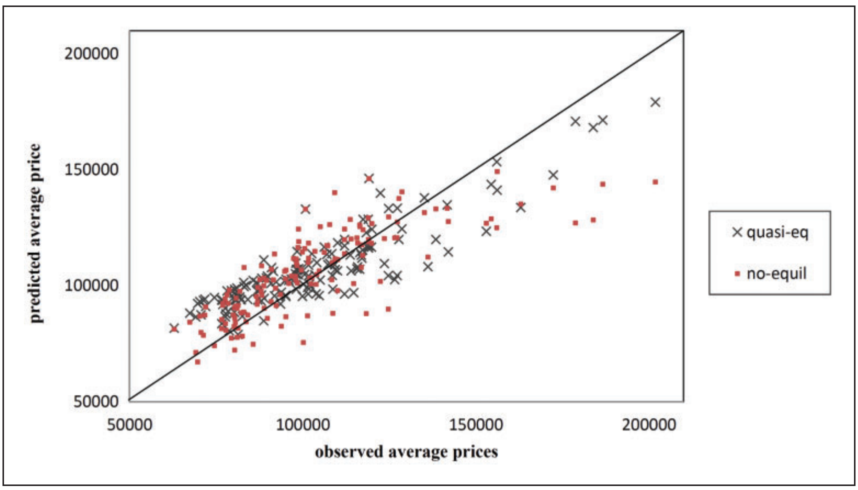


**Figure 7.** Comparison of results with and without bid adjustment.

In terms of quality of the price forecast, a more disaggregate analysis is shown in Figure 6, comparing commune-level forecasted (simulated) prices and observed prices for years 2001 and 2008, in the left- and right-hand side, respectively.

Prices in 2001 are concentrated in the range between 70,000 and 140,000 Euros. However in year 2008 a group of communes goes beyond the 150,000 Euros threshold, with the simulated prices following this trend, although systematically underestimating the magnitude of the price increase. The higher prices appear in the communes of Woluwe-Saint-Pierre, Woluwe-Saint-Lambert, Lasne and Ukkel, some of the richest communes in Belgium, measured by the average taxable income of their inhabitants. The systematic underestimation of prices in 2008 for these communes may be due to the fact that the uniform increase in supply mis-predicts the number of new units by type, possibly under-predicting the new supply of the more expensive types.

It is important to notice that the model was estimated over a reduced number of observations (1,346) where some of the communes are not included. Therefore, despite results having a fit that is far from good, the trend-following pattern is an indicator of the good quality of the approach. Figure 7 compares predicted prices for 2008 (quasi-equilibrium, correlation coefficient $\rho = 0.79$) with those obtained using the same model, but without the bid adjustment process (no-equilibrium, $\rho = 0.63$). This confirms that adjusting the bids during the simulation process helps to better predict prices, because it captures the competition between bidders, especially those of high income who will increase their willingness to pay in order to increase their chances to be the highest bidders in already expensive neighborhoods.

*Population distribution.* The location of new households follows the spatial distribution of the new supply. Since the generation of new supply follows the observed distribution, most new households are located in communes that originally presented high density, specially in the urban areas. The simulation does not take into account land use regulations or development constraints and results could be clearly improved by doing so. However, results are consistent with the observed trend of population increase in rural areas of the Flanders regions.

Since the simulation locates households that can have different sizes, the quality of the forecast number of people by commune is an indicator of the capacity of the model to forecast the spatial distribution of agents according to socioeconomic attributes. Figure 8 shows the comparison between simulated and observed data for the number of people by commune in 2008. The simulation predicts with very good fit for smaller communes and underestimates the population in 13% for the largest commune (Brussels). This is, however, most likely due to the oversimplified supply model.

*Income distribution.* There are no available data for validation regarding the number of households by income level and zone in 2008, but information on the average income by commune is available from tax declarations. Predicted average income per commune can be roughly estimated from the simulation results using the observed average income (in Euros) per income level from the MOBEL database. The relevant variable to analyze is the variation

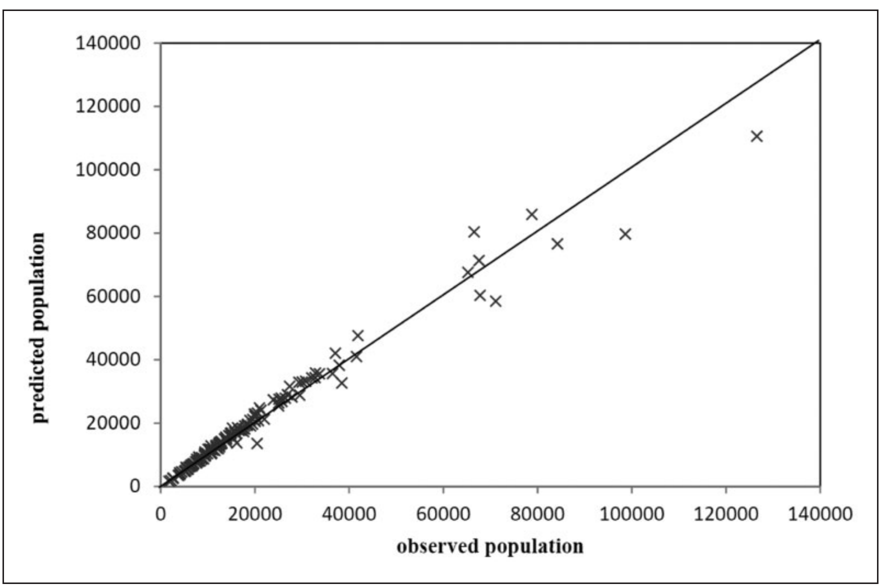


**Figure 8.** Forecasted vs. observed population by commune (2008).

in the average income by commune, as a proxy of change in the income distribution. Figure 9 shows a comparison between observed and predicted variation in the average income by commune. Although the simulation tends to overestimate the increase in the average income, it is clear that results follow the trend observed in reality, predicting large and positive variations for the communes with the greatest increase in observed income.

The correct prediction of the trend in change of the income distribution explains the quality of the price forecast results shown in Figures 6 and 7.

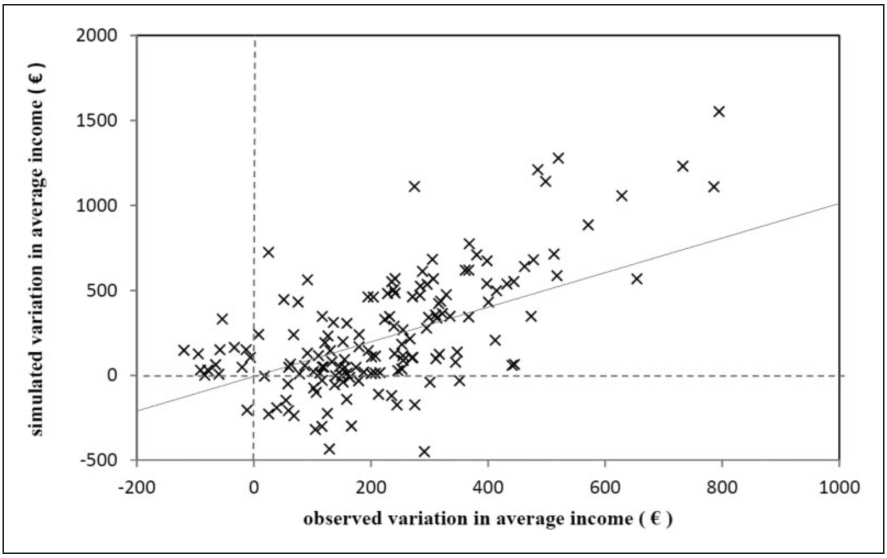


**Figure 9.** Observed vs. simulated variation in average income per commune.

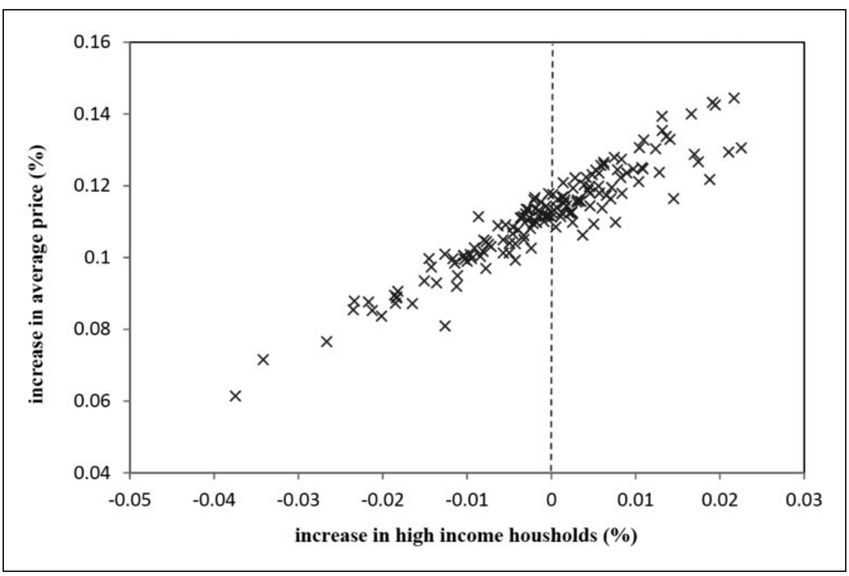


**Figure 10.** Predicted price vs. income increase by commune, 2001–2008.

There is a clear pattern of relative increase in income in communes where a relative increase in price also takes place, as seen in Figure 10. This is due to the fact that the willingness to pay for a dwelling increases with a high income in the location and decreases with the presence of low-income households (see Table 3).

## Conclusions

A method for market clearing in location choice microsimulations is proposed. The method is based on the Bid-auction approach and assumes that agents adjust their perceived expected utility by observing market prices before participating in real estate auctions. The adjustment translates in a correction of each agent's bid level as they attempt to ensure their location. Auctions for each real estate good are simulated and prices are computed as the expected maximum bid of all agents in the market.

The method is feasible to be implemented in large-scale microsimulations and ABMs, because it does not require solving an equilibrium and hence computational requirements are low. Through the adjustment of the bids, the simulation follows the direction of an equilibrium, without necessarily reaching it. Results are stable across simulations with different random seeds, suggesting that the process is not path dependent. The proposed method allows us to compute market prices using (if necessary) an absolutely heterogeneous population of households and location units. Comparison of results with those of an aggregate equilibrium model is difficult given the significant differences in aggregation levels. Further work will explore these differences in order to compare the predictive capabilities of each approach.

The market clearing method is embedded in RUSH-LoUD, a general land use simulation framework, and is applied to a real case study for the city of Brussels considering the year 2001 as the base year. Simulation results for the year 2008 are compared with observed data. The proposed model is able to forecast trends in price increases and changes in the income distribution by commune that are consistent with observed reality, outperforming simulations without the quasi-equilibrium adjustment. The correct prediction of the trend takes place as a result of the adjustment of bids under which each active agent goes in each simulation period. Higher bids from high-income households make them more likely to win auctions in expensive neighborhoods, hence increasing the zonal income and making those neighborhoods even more attractive for other high income households.

The model implementation described in this paper considers many simplifications for the supply-generation and non-residential location components of the general framework. The amount of error that can be explained by these simplifying assumptions is not clear and should be analyzed in future work. These components could, however, be improved by using better behavioral assumptions and real data. Further research will estimate all the models within the RUSH-LoUD framework, including the auction model, for a different city, probably Santiago de Chile, in order to test the framework in its full capacity. Exploring other auction configurations, such as those described at the end of the "Clearing" section, are also part of future research.

This framework can be applied to other markets or choice situations, where expectations and competition between decision makers play an important role like, for example, when public services or contracts are assigned through a call for bids or tender process. Another example of a possible extension is the labor market, where individuals compete for different jobs by "bidding" their experience and skills (their CV) and their salary expectations, with the job being assigned to the best "bidder". In general, any market where the assignment mechanism is an auction could be modeled with the proposed approach or extensions of it.

## Declaration of conflicting interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

## Funding

The author(s) disclosed receipt of the following financial support for the research, authorship, and/or publication of this article: The research presented in this paper was performed in the context the SustainCity project (FP7-244557), financed by the Seventh Framework Programme (FP7) of the European Commission Research Area. The authors would also like to thank the support provided by the Complex Engineering Systems Institute, ISCI (ICM-FIC: P05-004-F, CONICYT: FB0816), and the Center for Sustainable Urban Development CEDEUS (CONICYT/FONDAP 15110020).

## Note

1. www.sustaincity.org

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