

Type II Blow-up in the 5-dimensional Energy Critical Heat Equation

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We consider the Cauchy problem for the energy critical heat equation

$$u_t = \Delta u + |u|^{4n-2}u \text{ in } \mathbb{R}^n \times (0, T), u(x, 0) = u_0 \text{ in } \mathbb{R}^n$$
 in dimension $n = 5$. More precisely we find that for given points q_1, q_2, \dots, q_k and any sufficiently small $T > 0$ there is an initial condition u_0 such that the solution $u(x, t)$ of (0.1) blows-up at exactly those k points with rates type II, namely with absolute size $\sim (T-t)^{-\beta}$ for $\beta > 3/4$. The blow-up profile around each point is of bubbling type, in the form of sharply scaled Aubin-Talenti bubbles.