Weighted Averages in the Ordered Weighted Average Inflation

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Abstract. This paper presents the ordered weighted average weighted average inflation (OWAWAI). The OWAWAI operator is a new formulation for calculating inflation that provides different criteria for the association between the arguments and weights. OWAWAI presents the possibility to generate new approaches that under- or overestimate the results according to the knowledge and expertise of the decision maker. The works present an approach in Chile inflation.

Keywords: OWA operator · Inflation · Decision making

1 Introduction

Inflation has become an important financial indicator not only for the monetary policies but also for individuals' and enterprises' financial decisions (Malmendier and Nagel 2015). In the case of policy makers, it is important for improving their forecasting and policy choices, and in the case of the enterprises and individuals, their choices in the housing market, real expenditure decisions and macroeconomic outcomes are influenced by expected inflation (Woodford and Walsh 2005).

Many models have been developed in order to find the relations in inflation, such as the relationship between openness and inflation (Romer 1993), between inflation and unemployment (Ruge-Murcia 2004) and between the behaviour of the central bank and inflation (Hayat *et al.* 2018). An important aspect of the central bank of each country is that these banks use different techniques in order to calculate inflation. A common element is that these banks divide the different elements of the economy, assign a

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specific weight to each with respect to the total inflation and calculate the change in the consumer price index monthly.

To accomplish this task, this paper proposes using aggregation operators. The specific aggregation operator that was used is the ordered weighted average (OWA) operator developed by Yager (1988). This operator was selected because it can provide different scenarios between the maximum and the minimum results, and this information is helpful for the enterprises that seek to make decisions about different aspects of the enterprise, such as income, expenses and profits.

2 Preliminaries

2.1 Inflation Formulas for Latin America Countries

2.1.1 Chile

In Chile, the calculation of the inflation is based on the determination of the Consumer Price Index of 12 different divisions that have different associated weights. This is presented in Table 1.

Table 1. Divisions and weights used to calculate inflation in Chile

Division	Weight
Food and non-alcoholic beverages	19.05855
Alcoholic beverages and tobacco	3.31194
Clothing and footwear	4.48204
Housing and basic services	13.82810
Equipment and maintenance of the home	7.02041
Health	6.44131
Transportation	14.47381
Communications	5.00064
Recreation and culture	6.76121
Education	8.08996
Restaurants and hotels	4.37454
Miscellaneous goods and services	7.15749

See: http://www.ine.cl/estad%C3%ADsticas/precios/ipc

To calculate the inflation, each of the divisions is compared with their previous month's value with the formula $\left(\frac{CPI_n - CPI_{n-1}}{CPI_{n-1}}\right)(100)$ and, at the end, each value is multiplied by its weight and summed in order to obtain the inflation rate.

2.2 Basics and Extensions of the OWAWA Operator

To better understand the operator that will be used to improve inflation, it is important to understand the aggregation operator that is being used in the traditional formulation and the weighted average (WA) operator (Beliakov *et al.* 2007; Torra and Narukawa 2007). They are defined as follows.

Definition 1. A WA operator of dimension n is a mapping $WA : R^n \rightarrow R$ that has an associated weighting vector V, with $v_j \in [0,1]$ and $\sum_{i=1}^{n} v_i = 1$, such that the following exists:

$$WA(a_1, \ldots, a_n) = \sum_{i=1}^{n} v_i a_i, \quad (1)$$

where a_i represents the argument variable.

This idea was developed further by Yager (1988) by adding a reordering step in the WA operator, this new operator was called ordered weighted average (OWA) operator and one of its main characteristic is that it is possible to obtain the maximum and minimum operator. The formulation is.

Definition 2. An OWA operator of dimension n is a mapping $F : R^n \rightarrow R$ with a weight vector $w = [w_1, w_2, \ldots, w_n]^T$, where $w_j \in [0,1]$, $1 \le i \le n$ and $\sum_{j=1}^{n} w_j = 1$, such that:

$$OWA(a_1, a_2, \ldots, a_n) = \sum_{j=1}^{n} w_j b_j, \quad (2)$$

where b_j is the jth element that is the largest of the collection a_n.

The OWA operator was developed further by Merigó (2011) with the inclusion of another weighting average vector into the formulation (Merigó *et al.* 2017). By doing this, it is possible to include two different weighting vectors with some degree of importance for each that can help us better understand a problem or a situation. The formulation is as follows.

Definition 3. An OWAWA operator of dimension n is a mapping $OWAWA : R^n \rightarrow R$ that has an associated weighing vector W of dimension n such that $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$ and is calculated according to the following formula:

$$OWAWA(a_1, \ldots, a_n) = \sum_{j=1}^{n} \hat{v}_j b_j, \quad (3)$$

where b_j is the jth largest of the a_i, each argument a_i has an associated weight (WA) v_i with $\sum_{i=1}^{n} v_i = 1$ where $v_i \in [0,1]$, $\hat{v}_j = \beta w_j + (1-\beta) v_j$ where $\beta \in [0,1]$ and v_j is ordered according to b_j, that is, according to the jth largest a_i. Also note that if the reordering step is omitted, then the OWAWA operator becomes the weighted average weighted average (WAWA) operator.

3 The Ordered Weighted Average Weighted Average Inflation

3.1 The OWAWA Inflation

A particular case of the inflation is that each country has different ways to provide a result based on different factors that are important to each one. Another characteristic is that not all the country uses the same weights for the same factor, in this sense, a way to provide a better understand of the phenomenon of the inflation is by adding another weighting vector that can be specific to the characteristic of the enterprise or that market that will be evaluated and by doing that it is possible to take into the same formulation the idea of the country inflation and the specific needs of the decision maker.

This new operator is called the ordered weighted average weighted average inflation (OWAWAI) and its definition is as follows

Definition 4. The OWAWAI operator of dimension n is a mapping $F : R^n \rightarrow R$ with a weight vector $w = [w_1, \ldots, w_n]^T$, where $w_j \in [0, 1]$, $1 \leq i \leq n$ and $\sum_{i=1}^{n} w_j = w_1 + \ldots + w_n = 1$ and can be defined as:

$$OWAWAI(i_1, i_2, \ldots, i_n) = \sum_{k=1}^{n} \widehat{v}_j h_j, \tag{4}$$

where h_j is the jth element, which is the largest of the collection $i_1, i_2, \ldots, i_n$. Each element of the collection represents the factors that are considered and used in order to obtain the average inflation. Each argument i_i has an associated weight (WA) v_i with $\sum_{i=1}^{n} v_i = 1$ where $v_i \in [0, 1], \widehat{v}_j = \beta w_j + (1 - \beta) v_j$, where $\beta \in [0, 1]$ and v_j is ordered according to h_j, that is, according to the jth largest i_i.

An extension can be obtained if an induced reordering step is done. This operator is the induced ordered weighted average weighted average inflation (IOWAWAI) and is defined as follows.

Definition 5. The IOWAWAI operator of dimension n is a mapping $IOWAWAI : R^n \times R^n \rightarrow R$ that has an associated weighting vector W of dimension n, where the sum of the weights is 1 and $w_j \in [0, 1]$, where an induced set of ordering variables is included (u_i) so the formula is

$$IOWAWAI(\langle u_1, i_1 \rangle, \ldots, \langle u_n, i_n \rangle) = \sum_{k=1}^{n} \widehat{v}_j h_j, \tag{5}$$

where h_j is the i_i value of the OWA pair $< u_i, i_i >$ that has the jth largest u_i. u_i is the order-inducing variable, and i_i are the inflation factors. Each argument i_i has an associated weight (WA) v_i with $\sum_{i=1}^{n} v_i = 1$ where $v_i \in [0, 1], \widehat{v}_j = \beta w_j + (1 - \beta) v_j$, where $\beta \in [0, 1]$ and v_j is ordered according to h_j, that is, according to the jth largest i_i

It is important to note that another extension can be made if the weighting vector is unbounded in this sense that the heavy ordered weighted average weighted average inflation (HOWAWAI) is obtained and can be defined as follows.

Definition 6. The HOWAWAI operator is a map $R^n \rightarrow R$ that is associated with a weight vector w, where $w_j \in [0,1]$ and $1 \leq \sum_{j=1}^{n} w_j \leq n$, such that

$$HOWAWAI(i_1, i_2, \ldots, i_n) = \sum_{k=1}^{n} \hat{v}_j h_j, \tag{6}$$

where h_j is the jth largest element of the collection $i_1, i_2, \ldots, i_n$, each argument i_i has an associated weight (WA) v_i with $1 < \sum_{i=1}^{n} v_i < n$ or even $-\infty < \sum_{i=1}^{n} v_i < \infty$ and $v_i \in [0,1], \hat{v}_j = \beta w_j + (1-\beta) v_j$ and $\beta \in [0,1]$ and v_j ordered according to h_j, that is, according to the jth largest of the i_i

Finally, if both of the main characteristic of the Definition 8 and 9 are included in one formulation the induced heavy ordered weighted average weighted average inflation (IHOWAWAI) operator is done. Its definition is.

Definition 7. The IHOWAWAI operator of dimension n is a mapping $IOWAWAI$: $R^n \times R^n \rightarrow R$ that has an associated weighting vector W of dimension n, with $w_j \in [0,1]$ and $1 \leq \sum_{j=1}^{n} w_j \leq n$ and an induced set of ordering variables is included (u_i) so the formula is

$$IHOWAWAI(u_1, i_1, \ldots, u_n, i_n) = \sum_{k=1}^{n} \hat{v}_j h_j, \tag{7}$$

where h_j is the i_i value of the OWA pair $< u_i, i_i >$ having the jth largest u_i. u_i is the order-inducing variable, and i_i are the inflation factors. Each argument i_i has an associated weight (WA) v_i with $1 < \sum_{i=1}^{n} v_i < n$ or even $-\infty < \sum_{i=1}^{n} v_i < \infty$ where $v_i \in [0,1], \hat{v}_j = \beta w_j + (1-\beta) v_j$, where $\beta \in [0,1]$ and v_j is ordered according to h_j, that is, according to the jth largest i_i.

4 Numerical Example

To explain the new inflation formulations, the same information that was presented in Table 2 will be used, but in this case another weighting vector, a heavy vector and an induced vector will be used. This information is presented in Table 2.

Table 2. Information to calculate the inflation aggregation operator for Chile in August 2018

Division	Inflation	Official weighting vector	Expert weighting vector
Food and non-alcoholic beverages	0.47	0.19059	0.20
Alcoholic beverages and tobacco	0.18	0.03312	0.05
Clothing and footwear	0.56	0.04482	0.10
Housing and basic services	0.63	0.13828	0.10
Equipment and maintenance of the home	0.13	0.07020	0.05
Health	−0.05	0.06441	0.10
Transportation	−0.54	0.14474	0.15
Communications	0.35	0.05001	0.03
Recreation and culture	−0.52	0.06761	0.05
Education	–	0.08090	0.10
Restaurants and hotels	0.13	0.04375	0.03
Miscellaneous goods and services	0.48	0.07157	0.04

With the information in Table 2, the different results are presented in Tables 3, 4, 5 and 6. (Note that $\beta = 40\%$ for the original weighting vector and $\beta = 60\%$ for the expert weighting vector.)

Table 3. OWAWAI operator results

Inflation	Official weighting vector	Expert weighting vector	Weighted inflation
−0.54	0.03312	0.03	−0.017
−0.52	0.04375	0.03	−0.018
−0.05	0.04482	0.04	−0.002
0	0.05001	0.2	0
0.13	0.06441	0.05	0.007
0.13	0.06761	0.05	0.007
0.18	0.0702	0.05	0.010
0.35	0.07157	0.10	0.031
0.47	0.0809	0.10	0.043
0.48	0.13828	0.10	0.055
0.56	0.14474	0.10	0.066
0.63	0.19059	0.15	0.105
		OWAWAI result	**0.288**

Table 4. IOWAWAI operator results

Inflation	Official weighting vector	Expert weighting vector	Weighted inflation
0.48	0.07157	0.04	0.025
0.18	0.03312	0.05	0.008
0.13	0.04375	0.03	0.005
−0.52	0.06761	0.05	−0.030
0.13	0.0702	0.05	0.008
0.35	0.05001	0.03	0.013
0.63	0.13828	0.10	0.073
0	0.0809	0.10	0
−0.54	0.14474	0.15	−0.080
−0.05	0.06441	0.10	−0.004
0.56	0.04482	0.10	0.044
0.47	0.19059	0.20	0.092
		IOWAWAI result	**0.153**

Table 5. HOWAWAI operator results

Inflation	Official weighting vector	Expert heavy weighting vector	Weighted inflation
−0.54	0.03312	0.04	−0.020
−0.52	0.04375	0.04	−0.022
−0.05	0.04482	0.05	−0.002
0	0.05001	0.05	0
0.13	0.06441	0.05	0.007
0.13	0.06761	0.07	0.009
0.18	0.0702	0.10	0.016
0.35	0.07157	0.10	0.031
0.47	0.0809	0.12	0.049
0.48	0.13828	0.15	0.070
0.56	0.14474	0.15	0.083
0.63	0.19059	0.22	0.131
		HOWAWAI result	**0.352**

Table 6. IHOWAWAI operator results

Inflation	Official weighting vector	Expert heavy weighting vector	Weighted inflation
0.48	0.07157	0.04	0.025
0.18	0.03312	0.07	0.010
0.13	0.04375	0.04	0.005
−0.52	0.06761	0.05	−0.030
0.13	0.0702	0.05	0.008
0.35	0.05001	0.05	0.018
0.63	0.13828	0.10	0.073
0	0.0809	0.10	0
−0.54	0.14474	0.15	−0.080
−0.05	0.06441	0.15	−0.006
0.56	0.04482	0.12	0.050
0.47	0.19059	0.22	0.098
		IHOWAWAI result	**0.171**

As seen in the numerical example, the inflation was originally 0.158 using only the official weighting vector, but with the use of the different operators, the different scenarios increased the inflation from 0.153 to 0.352, which was more than double the original estimate for the month. This is important when the experts make decisions because it is possible to generate specific results depending on the area of the enterprise.

Finally, it is important to note that with the use of the aggregation operators an important amount of data have been included in the decision making process, in this sense, analyzing each of the scenarios generated is necessary to understand the inflation better and how it will impact on the finance of the company. Also, when the information between the different aggregation operators and the traditional inflation are in conflict we suggest that the most complex operator (in this case the IHOWAWAI operator) should be taken into account, this because is the operator that includes more information about the problem in the result and must be the closest to the reality. Another important thing to consider, is that when the decision must be made quickly and not all the information needed for the use of the IHOWAWAI operator cannot be obtained, the analysis between the traditional formula and the most simple aggregation operator (in this case the OWAWAI) must be done to visualize how much the difference can be and determine if the decision must be done hasty or not.

5 Conclusions

The main purpose of the paper is to provide a new aggregation operator called the ordered weighted average weighted average inflation (OWAWAI) operator. The main characteristics of these new formulations is that they can provide new inflation scenarios that can be calculated using the expectations, knowledge and characteristics of the market of the enterprise and, depending on the complexity of the situation or the problem, different formulations can be calculated

For future research, new extensions of the OWA operator and applications can be derived by using the Bonferroni, moving averages and its application in other areas of engineering, business, economics and finance.

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