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METHODOLOGIES AND APPLICATION

Multi-person and multi-criteria decision making with the induced probabilistic ordered weighted average distance

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Abstract

This paper presents a new approach for selecting suppliers of products or services, specifically with respect to complex decisions that require evaluating different business characteristics to ensure their suitability and to meet the conditions defined in the recruitment process. To address this type of problem, this study presents the multi-person multi-criteria induced ordered weighted average distance (MP-MC-IOWAD) operator, which is an extension of the OWA operators that includes the notion of distances to multiple criteria and expert valuations. Thus, this work introduces new distance measures that can aggregate the information with probabilistic information and consider the attitudinal character of the decision maker. Further extensions are developed using probabilities to form the induced probabilistic ordered weighted average distance (IPOWAD) operator. An example in the management of insurance policies is presented, where the selection of insurance companies is very complex and requires the consideration of subjective criteria by experts in decision making.

Keywords Fuzzy logic · Multi-criteria decision making · OWA operator · Fuzzy distances

1 Introduction

In many cases, the selection of services provided to businesses is a complex decision. The provision of insurance is of particular importance, in particular in large volumes, as in the case of customers or employees. Therefore, it is necessary to assess not only the characteristics of insurance companies but also the best guarantees of contract compliance and improved stability over the medium and long term (Belles-Sampera et al. 2013; Casanovas et al. 2015).

In the field of operations research, several authors have developed mathematical models that help individuals make better decisions when it is necessary to balance different objectives or criteria that are in conflict with each other and that, to some extent, penalize the others. These models are an aspect of decision analysis with multiple criteria, which is known as multiple criteria decision making (MCDM) (Figueira et al. 2005); they are useful in problems where there is typically no optimal solution, and it is necessary to consider the decision maker's preferences for the given alternatives.

In recent years, several studies have introduced a variety of MCDM methods and theories with very good results in the field of fuzzy logic (Blanco-Mesa et al. 2017; Qi et al. 2015; Tsao 2006), including the following: a preference ranking organization method for enrichment evaluation (PROMETHEE) (Brans and Vincke 1985), an analytic hierarchy process (AHP) (Önut et al. 2009), and a technique for order preference by similarity to the ideal solution (TOPSIS) (Bai et al. 2014; Dursun and Karsak 2010;

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Liang 1999) or simple additive weighting (Noor-E-Alam et al. 2011) as well as others (Figueira et al. 2005).

An important element in some MCDM methods is the notion of distances (Gil-Aluja 1999; Kaufmann 1975). A significant issue is the application of the ordered weighted averaging (OWA) operators (Emrouznejad and Marra 2014; He et al. 2017; Yager 1988) in problems that are evaluated and analyzed based on the aggregation of the individual distances of the decision criteria from an ideal value, which is typically defined by experts (Karayiannis 2000). The distances that are common in OWA operators are Euclidean, Hamming, and Minkowski. Extensions or methods of these operators are the ordered weighted averaging distance (OWAD) operator (Merigó and Gil-Lafuente 2010; Xu and Chen 2008), the induced ordered weighted averaging distance (IOWAD) operator (Merigó and Casanovas 2011a; Vizuete et al. 2015), and the induced Minkowski ordered weighted averaging distance (IMOWAD) operator (Casanovas et al. 2016; Merigó and Casanovas 2011b). Recently, several authors have developed extensions for complex information using Bonferroni means (Blanco-Mesa et al. 2016, 2018), moving averages (León-Castro et al. 2018a), prioritized information (Avilés-Ochoa et al. 2018), logarithmic means (Alfaro-García et al. 2018), weighted averages (Merigó et al. 2017), interval numbers (Su et al. 2015; Zeng 2013; Zeng et al. 2013), linguistic variables (Liu et al. 2014; Xian and Sun 2014; Zeng and Su 2012), fuzzy numbers (Ulutagay and Kantarci 2015; Xu 2012) and intuitionistic fuzzy sets (Chen et al. 2015; Zeng et al. 2017; Zhou et al. 2016).

The aim of this paper is to develop a new decision making approach using induced aggregation operators and distance measures. We introduce the induced probabilistic ordered weighted average distance (IPOWAD) operator, an aggregation operator that unifies the IOWAD operator with the weighted Hamming distance, taking into account the importance that each concept may have in the aggregation. Some of its main properties and particular cases are studied. These operators have been selected because of their suitability for environments in which risk and uncertainty exist within the same formulation. In the real world, this is quite common because complex environments typically offer partial information based on probabilities, but there are also other sources of information, including attitudes and opinions. Thus, aggregation operators that consider these issues, such as the IPOWAD operator, are necessary. Moreover, the advantage of the IPOWAD operator is that it can use distance measures in decision environments under risk and uncertainty. The analysis is focused on the use of multi-criteria and multi-person techniques in decision making. The study develops an application for group decision making in insurance management.

This article is structured as follows. In Sect. 2, the basic concepts of distance measures, OWA, OWAD, IMOWAD, and IOWA operators are presented. Section 3 introduces a method for calculating inducing variables, and the operator MP-MC-IOWAD is proposed. Section 4 addresses the IPOWAD operator. Section 5 analyses the steps of the proposed method. An illustrative example is developed for selecting insurers in Sect. 6, and finally, Sect. 7 summarizes the conclusions.

2 Preliminaries

2.1 Distance measures

The notion of distance is used to measure the difference or calculate the degree of distance between two elements or sets (Gil-Aluja 1999; Kaufmann 1975). Among the most commonly used distances are Hamming, Euclidean and Minkowski. A measure of the distance can be considered provided that the following properties are satisfied. Assume three sets, $A = \{a_1, a_2, \ldots, a_n\}, B = \{b_1, b_2, \ldots, b_n\}$ and $C = \{c_1, c_2, \ldots, c_n\}$.

1. Nonnegativity: $D(A,B) \geq 0$.
2. Commutativity: $D(A,B) = D(B,A)$.
3. Reflexivity: $D(A,A) = 0$.
4. Triangle inequality: $D(A,B) + D(B,C) \geq D(A,C)$.

The Hamming, Euclidean and Minkowski distances can be formulated as follows for two fuzzy sets $A = \{u_{\underline{A}}(x_1), u_{\underline{A}}(x_2), \ldots, u_{\underline{A}}(x_j), \ldots, u_{\underline{A}}(x_n)\}$ and $B = \{u_{\underline{B}}(x_1), u_{\underline{B}}(x_2), \ldots, u_{\underline{B}}(x_j), \ldots, u_{\underline{B}}(x_n)\}$, where $u_{\underline{A}}(x_j)$ and $u_{\underline{B}}(x_j)$ represent the values of the membership functions.

Definition 1 Let E be a finite referential with $A, B \subset E$, so that the Hamming distance is defined as:

$$d(A,B) = \sum_{i=1}^{n} \left| u_{\underline{A}}(x_j) - u_{\underline{B}}(x_j) \right|, \tag{1}$$

with; $x_j \in E \;\; \forall j = 1, 2, \ldots n;\; u_{\underline{A}}(x_j),\; u_{\underline{B}}(x_j) \in [0, 1]$.

Definition 2 Let E be a finite referential with $A, B \subset E$, so that the Euclidean distance is defined as:

$$e(A,B) = \sqrt{\sum_{i=1}^{n} \left(u_{\underline{A}}(x_j) - u_{\underline{B}}(x_j) \right)^2}, \tag{2}$$

with; $x_j \in E \;\; \forall j = 1, 2, \ldots n;\; u_{\underline{A}}(x_j),\; u_{\underline{B}}(x_j) \in [0, 1]$.

Definition 3 Let E be a finite referential with $A, B \subset E$, so that the Minkowski distance is defined as:

$$r(A,B) = \left(\sum_{i=1}^{n} \left| u_{\underline{A}}(x_j) - u_{\underline{B}}(x_j) \right|^{\lambda} \right)^{1/\lambda}, \tag{3}$$

with $x_j \in E\ \forall j = 1,2,\ldots n$; $u_{\underline{A}}(x_j),\ u_{\underline{B}}(x_j) \in [0,1]$ and $\lambda \in (-\infty, \infty) - \{0\}$.

In the Minkowski distance, note that:

- $\lambda = 1$ allows the Hamming distance to be obtained.
- $\lambda = 2$ allows the Euclidean distance to be obtained.

2.2 The OWA operator

The OWA operator is an aggregation operator (Beliakov et al. 2007; Grabisch et al. 2011) that provides a parameterized family of aggregation operators between the minimum and the maximum and includes the arithmetic mean as a particular case (Kacprzyk et al. 2019). It is defined as follows.

Definition 4 An OWA operator is defined as a function of dimension n $F: R^n \to R$, which has a vector W of dimension n,$W = [w_1, w_2, \ldots, w_n]^T$ that satisfies the following associated conditions:

– $w_j \in [0,1]$.

– $\sum_{j=1}^{n} w_j = 1.$

– $\mathrm{OWA}(a_1, a_2, \ldots, a_n) = \sum_{j=1}^{n} w_j b_j.$

The essence of the OWA (Yager 1988, 1993) is the rearrangement of the elements or arguments so that the arguments a_j are not associated with a weight w_j, but are associated with a position in the order of aggregation. The OWA can be sorted in a descending or ascending order (Yager 1993).

2.3 Distance measures with OWA operators

2.3.1 The OWAD operator

The ordered weighted averaging distance (OWAD) operator (Merigó and Gil-Lafuente 2010; Xu and Chen 2008) is an operator that is based on the measurement of a single distance or a Hamming distance (Hamming 1950) to obtain an aggregate distance from the minimum and maximum distances. The OWAD is defined as follows for two sets, $A = \{a_1, a_2, \ldots, a_n\}$ and $B = \{b_1, b_2, \ldots, b_n\}$.

Definition 5 An OWAD operator of dimension n is a mapping $\mathrm{OWAD}: R^n x R^n \to R$ that has an associated weighting vector W of dimension n, such that $\sum_{j=1}^{n} w_i = 1$ and $w_j \in [0,1]$. Thus,

$$\mathrm{OWAD}(d_1, d_2, \ldots, d_n) = \sum_{j=1}^{n} w_j D_j, \tag{4}$$

where D_j is the jth greatest d_j, and d_j is the individual distance between A and B, such that $d_j = |a_j - b_j|$.

2.3.2 The IOWAD operator

The induced ordered weighted averaging distance (IOWAD) operator (Merigó and Casanovas 2011a) is an aggregation operator that uses the IOWA operator (Yager 2003; Yager and Filev 1999) with distance measures on the arguments, which are aggregated taking into account the induction variables in the order. The IOWAD is defined as follows for two sets, $A = \{a_1, a_2, \ldots, a_n\}$ and $B = \{b_1, b_2, \ldots, b_n\}$.

Definition 6 The IOWAD operator of dimension n is a function $\mathrm{IOWAD}: R^n x R^n \to R$ that has an associated weight vector W of dimension n, such that $\sum_{i=1}^{n} w_j = 1$ and $w_j \in [0,1]$. Thus,

$$\mathrm{IOWAD}(\langle u_1, a_1, b_1 \rangle, \langle u_{2,} a_2, b_2 \rangle, \ldots, \langle u_n, a_n, b_n \rangle) = \sum_{j=1}^{n} w_j D_j, \tag{5}$$

where D_j is the value $d_j = |a_j - b_j|$ of the triplet IOWAD $\langle u_j, a_j, b_j \rangle$ with the largest jth u_j, u_j is an induced ordering variable, and $|a_j - b_j|$ is the variable argument represented by the individual distances.

2.3.3 The IMOWAD operator

The induced Minkowski OWA distance (IMOWAD) operator (Merigó and Casanovas 2011b) is an operator that generalizes the IOWAD operator by using the Minkowski distance.

Definition 7 The IMOWAD operator of dimension n is a function $\mathrm{IMOWAD}: R^n x R^n \to R$, which has an associated weight vector W of dimension n, such that $\sum_{i=1}^{n} w_j = 1$ and $w_j \in [0,1]$. Thus,

$$\mathrm{IMOWAD}(\langle u_1, a_1, b_1 \rangle, \langle u_{2,} a_2, b_2 \rangle, \ldots, \langle u_n, a_n, b_n \rangle) = \left(\sum_{j=1}^{n} w_j D_j^{\lambda} \right)^{1/\lambda}, \tag{6}$$

where D_j is the value $d_j = |a_j - b_j|$, d_j is the value of the triplet IMOWAD $\langle u_j, a_j, b_j \rangle$ with the largest jth u_j, u_j is the induced ordering variable, $\left|a_j - b_j\right|$ is the variable argument that is represented by individual distances, and λ is a parameter such that $\lambda \in (-\infty, \infty) - \{0\}$.

Example 1 To understand numerically the IOWAD and IMOWAD, let us look into a simple numerical example. Assume two sets {6, 4, 9, 5} and {3, 2, 5, 4}. We want to calculate the distance between them and the decision maker considers the following weighting vector W = (0.5, 0.2, 0.2, 0.1) to represent his complex attitudinal character. Additionally, he uses the following order-inducing variables U = (5, 9, 3, 7). With this data, the aggregation with the IOWAD operator using Eq. (5) is as follows:

$$\begin{aligned}\text{IOWAD} &= 0.5 * |4-2| + 0.2 * |5-4| + 0.2 * |6-3| + 0.1 \\ &\quad * |9-5| \\ &= 1 + 0.2 + 0.6 + 0.4 = 2.2.\end{aligned}$$

Note that the IMOWAD provides a more general framework that also includes the IOWAD as a particular case. For example, if $\lambda = 2$, the IMOWAD becomes the Euclidean IOWAD and would provide the following results using Eq. (6):

$$\begin{aligned}\text{EIOWAD} = \Big[&0.5 * |4-2|^2 + 0.2 * |5-4|^2 + 0.2 * |6-3|^2 \\ &+0.1 * |9-5|^2\Big] = 2 + 0.2 + 1.8 + 1.6 = 5.6.\end{aligned}$$

2.3.4 Characterization of the weighting vector

Another important aspect to consider in the OWA operators is the characterization measures of the weight vector, as proposed by Yager (1988). The objective of these tools is to explain the meaning of using a specific weighting vector in order to define if it is optimistic pessimistic and so on.

The first measure refers to the attitudinal character of the decision maker as well:

$$\alpha(W) = \sum_{j=1}^{n} w_j \left(\frac{n-j}{n-1}\right) \tag{7}$$

where $\alpha \in [0, 1]$.

The second measure relates to the entropy of dispersion (Shannon 1948); it is used to identify the information being used in the aggregation and is defined as:

$$H(W) = \sum_{j=1}^{n} w_j \ln(w_j) \tag{8}$$

The third measure indicates the degree of favoritism toward pessimistic or optimistic values and is known as the operator balance. It is defined as:

$$\text{Bal}(W) = \sum_{j=1}^{n} w_j \left(\frac{n+1-2j}{n-1}\right) \tag{9}$$

where $\text{Bal}(W) \in [-1, 1]$, remain $\text{Bal}(W) = 1$ for the optimistic criterion or the maximum operator, $\text{Bal}(W) = -1$ for the pessimistic criterion or the minimum operator and $\text{Bal}(W) = 0$ for the Laplace criterion or the arithmetic average.

The fourth measure indicates the degree of divergence and is generally used when the measure of dispersion and attitudinal distances are incomplete. It is defined as:

$$\text{Div}(W) \sum_{j=1}^{n} w_j \left(\frac{n-j}{n-1} - \alpha(W)\right)^2 \tag{10}$$

where $\text{Div}(W) = 0$ for the pessimistic or optimistic case.

3 New method to obtain the inducing variables with multi-person and multi-criteria decision making

This section presents a new method to define the array of inducing variables using a group of experts and to evaluate the multiple criteria of the alternatives using the objective data that is obtained. The advantage of this method is that it can take into account the particular situations of each criterion, which are assigned values by different experts. For example, if a criterion refers to the volume of business or market share, of which certain companies may have a greater share of and others may be losing relative to competitors, experts can underestimate or overestimate the information to make it conform to reality or future expectations. Recall that the OWA weights represent the attitude of the decision maker with respect to the problem being considered. Therefore, it is important to correctly represent this attitude in the weighting vector.

Definition 8 Let $(U = u_1, u_2, \ldots, u_n)$ be a multi-person induction array, with a set of alternatives $A = (a_1, a_2, \ldots, a_i)$ and a set of criteria $C = (c_1, c_2, \ldots, c_j)$ with which experts evaluate the alternatives; the criteria also have a weighting vector P of dimension n, such that $\sum_{k=1}^{n} p_k = 1$ and $p_k \in [0, 1]$:

$$U = \sum_{k=1}^{n} u_k p_k, \tag{11}$$

where expert reviews u_k are added in weighted form through vector P that represents the importance of the experts.

This method can improve the results of multi-person and multi-criteria operators using induced variables. Table 1 shows some of these operators.

Table 1 Multi-person and multi-criteria aggregation operators

	Multi-criteria				
	WA	OWA	OWAD	IOWA	IOWAD
Multi-person					
WA	MP-WA	MP-WA	MP-WA	MP-WA	MP-WA
	MC-WA	MC-OWA	MC-OWAD	MC-IOWA	MC-IOWAD
OWA	MP-OWA	MP-OWA	MP-OWA	MP-OWA	MP-OWA
	MC-WA	MC-OWA	MC-OWAD	MC-IOWA	MC-IOWAD
OWAD	MP-OWAD	MP-OWAD	MP-OWAD	MP-OWAD	MP-OWAD
	MC-WA	MC-OWA	MC-OWAD	MC-IOWA	MC-IOWAD
IOWA	MP-IOWA	MP-IOWA	MP-IOWA	MP-IOWA	MP-IOWA
	MC-WA	MC-OWA	MC-OWAD	MC-IOWA	MC-IOWAD
IOWAD	MP-IOWAD	MP-IOWAD	MP-IOWAD	MP-IOWAD	MP-IOWAD
	MC-WA	MC-OWA	MC-OWAD	MC-IOWA	MC-IOWAD

MP multi-person, *MC* multi-criteria, *WA* weighted averaging, *OWA* ordered weighted averaging, *OWAD* ordered weighted averaging distance, *IOWA* induced ordered weighted averaging, *IOWAD* induced ordered weighted averaging distance

3.1 Multi-person multi-criteria induced OWA distance (MP-MC-IOWAD) operator

This operator has been improved by estimating induction variables based on the views of several experts and thus combines objective information with the subjective evaluation criteria of the experts, taking into account the individual circumstances of each case that are not reflected in the arguments for each criterion.

Definition 9 The MP-MC-IOWAD operator is a function of dimension n *MP-MC-IOWAD*: $R^n x R^n x R^n \to R$, which has a weight vector W such that $\sum_{j=1}^{n} w_j = 1$ and $w_j \in [0, 1]$. This is calculated by the following formula:

$$\text{MP-MC-IOWAD}(\langle u_1, x_1, y_1\rangle, \langle u_2, x_2, y_2\rangle, \ldots, \langle u_n, x_n, y_n\rangle) = \sum_{j=1}^{n} w_j b_j, \tag{12}$$

where b_j is the value of the individual distances $|x_j - y_j|$ of the triplet $\langle u_j, x_j, y_j\rangle$ containing the jth largest induction variable u_j, and u_j is calculated from definition 8.

4 The induced probabilistic ordered weighted average distance operator

4.1 Main concepts

The induced probabilistic ordered weighted averaging distance (IPOWAD) operator is a distance measure that uses the probability and the OWA operator in the normalization process of the Hamming distance by using the IPOWA operator. Thus, the reordering of the individual distances is developed according to order-inducing variables that represent a complex reordering process of the individual distances formed by comparing two sets. The main advantage of this new approach is that it is able to address situations where there is some objective information about the possibility that different results will occur. In addition, the attitudinal character of the decision maker is assessed based on order-inducing variables that measure a wide range of attributes such as the degree of optimism, psychological aspects and time pressure. It can be defined as follows for two sets, $X = \{x_1, x_2, \ldots x_n\}$ and $Y = \{y_1, y_2, \ldots, y_n\}$.

Definition 10 An IPOWAD operator of dimension n is a mapping IPOWAD: $R^n \times R^n \times R^n \to R$ that has an associated weighting vector W such that $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$, according to the following formula:

$$\text{IPOWAD}(\langle u_1, x_1, y_1\rangle, \ldots, \langle u_n, x_n, y_n\rangle) = \sum_{j=1}^{n} \hat{v}_j b_j \tag{13}$$

where b_j is the $|x_j - y_j|$ value of the IPOWAD triplet $\langle u_j, x_j, y_j\rangle$ having the jth largest u_j, u_j is the order-inducing variable, x_j is the jth argument of the set $X = \{x_1, x_2, \ldots x_n\}$, y_j is the jth argument of the set $Y = \{y_1, y_2, \ldots, y_n\}$, each argument or individual distance $|x_j - y_j|$ has an associated probability v_j with $\sum_{j=1}^{n} v_j = 1$ and $v_j \in [0, 1]$, $\hat{v}_j = \beta w_j + (1 - \beta) v_j$ with $\beta \in [0, 1]$ and v_j is the probability that v_j is ordered according to the jth largest u_j.

Note that it is also possible to formulate the IPOWAD operator separating the part that strictly affects the IOWAD operator and the part that affects the probability.

Definition 11 An IPOWAD operator is a mapping IPOWAD: $R^n \times R^n \times R^n \to R$ of dimension n if it has an associated weighting vector W, with $\sum_{j=1}^{n} w_j = 1$ and $w_j \in [0,1]$, and a weighting vector V that affects the probability, with $\sum_{j=1}^{n} v_j = 1$ and $v_j \in [0,1]$, such that:

$$\mathrm{IPOWAD}(\langle u_1, x_1, y_1\rangle, \ldots, \langle u_n, x_n, y_n\rangle) = \beta \sum_{j=1}^{n} w_j b_j + (1-\beta) \sum_{j=1}^{n} v_j |x_j - y_j|, \quad (14)$$

where b_j is the $|x_j - y_j|$ value of the IPOWAD triplet $\langle u_j, x_j, y_j\rangle$ having the jth largest u_j, u_j is the order-inducing variable, x_j is the jth argument of the set $X = \{x_1, \ldots, x_n\}$, y_j is the jth argument of the set $Y = \{y_1, \ldots, y_n\}$, and $\beta \in [0, 1]$.

Note that if the weighting vector is not normalized, i.e., $\hat{V} = \sum_{j=1}^{n} \hat{v}_j \neq 1$, the IPOWAD operator can be expressed as:

$$\mathrm{IPOWAD}(\langle u_1, x_1, y_1\rangle, \ldots \langle u_n, x_n, y_n\rangle) = \frac{1}{\hat{V}} \sum_{J=1}^{n} \hat{v}_j b_j, \quad (15)$$

If D is a vector corresponding to the ordered arguments b_j, we shall call this the ordered argument vector, and $\hat{V}^T$ is the transpose of the weighting vector; then, the IPOWAD operator can be represented as follows:

$$\mathrm{IPOWAD}(\langle u_1, x_1, y_1\rangle, \ldots \langle u_n, x_n, y_n\rangle) = \hat{V}^T D, \quad (16)$$

Note that it is possible to distinguish between descending (DIPOWAD) and ascending (AIPOWAD) orders. The weights of these operators are related by $\hat{v}_j = \hat{v}_{n-j+1}$, where $\hat{v}_j$ is the jth weight of the DIPOWAD, and $\hat{v}_{n-j+1}$ is the jth weight of the AIPOWAD operator.

Example 2 Recall the data of Example 1. Assume the importance of each argument in terms of the probabilities is defined with the following weighting vector $P = (0.4, 0.2, 0.1, 0.3)$. Also, assume that the probabilistic information has an importance of 70% while the attitude is 30%. With this data, the aggregation with the IPOWAD operator using Eq. (14) is as follows:

$$\begin{aligned}\mathrm{IPOWAD} = {} & 0.7 * (0.4 * |6-3| + 0.2 * |4-2| + 0.1 * |9-5| \\ & + 0.3 * |5-4|) + 0.3 * (0.5 * |4-2| + 0.2 * |5-4| \\ & + 0.2 * |6-3| + 0.1 * |9-5|) = 2.27.\end{aligned}$$

From the point of view of a distance measure, note that the IPOWAD is reflexive and commutative. That is, $\mathrm{IPOWAD}(\langle u_1, x_1, y_1\rangle, \ldots \langle u_n, x_n, y_n\rangle) = 0$ if and only if $x_j = y_j$ for all $j \in [1, n]$. It is commutative because $\mathrm{IPOWAD}(\langle u_1, x_1, y_1\rangle, \ldots \langle u_n, x_n, y_n\rangle) = \mathrm{IPOWAD}(\langle u_1, x_1, y_1\rangle, \ldots \langle u_n, x_n, y_n\rangle)$.

The IPOWAD operator is monotonic, bounded and idempotent. It is monotonic because if $|x_j - y_j| \geq |s_j - t_j|$, for all $|x_j - y_j|$, then, $\mathrm{IPOWAD}(\langle u_1, x_1, y_1\rangle, \ldots, \langle u_n, x_n, y_n\rangle) \geq \mathrm{IPOWAD}(\langle u_1, s_1, t_1\rangle, \ldots \langle u_n, s_n, t_n\rangle)$. It is bounded because the IPOWAD aggregation is delimitated by the minimum and the maximum. That is, $\mathrm{Min}\{|x_j - y_j|\} \leq \mathrm{IPOWAD}(\langle u_1, x_1, y_1\rangle, \ldots \langle u_n, x_n, y_n\rangle) \leq \mathrm{Max}\{|x_j - y_j|\}$. It is idempotent because if $|x_j - y_j| = |x - y|$ for all $|x_j - y_j|$, then $\mathrm{IPOWAD}(\langle u_1, x_1, y_1\rangle, \ldots \langle u_n, x_n, y_n\rangle) = |x - y|$.

4.2 Families of IPOWAD operators

A further interesting issue to consider is the different families of IPOWAD operators that are found in the weighting vector W and the coefficient β.

- If $\beta = 0$, we obtain the probabilistic distance.
- If $\beta = 1$, we obtain the IOWAD operator.
- The arithmetic probabilistic distance (if $w_j = 1/n$ for all j).
- The arithmetic IOWAD operator (if $v_j = 1/n$ for all j).
- The normalized probabilistic distance (if $v_j = 1/n$ for all j, and $w_j = 1/n$ for all j).
- The maximum probabilistic distance ($w_p = 1$ and $w_j = 0$ for all $j \neq p$, and $u_p = \mathrm{Max}\{|x_j - y_j|\}$).
- The minimum probabilistic distance ($w_p = 1$ and $w_j = 0$ for all $j \neq p$, and $u_p = \mathrm{Min}\{|x_j - y_j|\}$).
- The Hurwicz probabilistic distance criteria ($w_p = \alpha$, with $u_p = \mathrm{Max}\{|x_j - y_j|\}$; $w_q = 1 - \alpha$, $u_q = \mathrm{Min}\{|x_j - y_j|\}$; and $w_j = 0$ for all $j \neq p, q$).
- The step-IPOWAD ($w_k = 1$ and $w_j = 0$ for all $j \neq k$).
- The IPOWA operator (if one of the sets is empty).
- The POWA (the ordered position of the u_i is the same as the ordered position b_j, and one of the sets is empty).
- The IOWA ($\beta = 1$, and one of the sets is empty).
- The OWA ($\beta = 1$, one of the sets is empty, and the ordered position of the u_i is the same as the ordered position b_j).
- The probabilistic aggregation or expected value ($\beta = 0$, and one of the sets is empty).
- The centered IPOWAD (if it is symmetric, strongly decaying from the center to the maximum and the minimum, and inclusive).
- The S-IPOWAD ($w_1 = (1/n)(1 - (\alpha + \beta)) + \alpha$, $w_n = (1/n)(1 - (\alpha + \beta)) + \beta$, and $w_j = (1/n)(1 - (\alpha + \beta))$ for $j = 2$ to $n - 1$, where $\alpha, \beta \in [0, 1]$ and $\alpha + \beta \leq 1$).
- The olympic-IPOWAD operator ($w_1 = w_n = 0$, and $w_j = 1/(n - 2)$ for all others).

- The general olympic-IPOWAD operator ($w_j = 0$ for $j = 1, 2, \ldots, k, n, n-1, \ldots, n-k+1$; and for all others $w_{j*} = 1/(n - 2k)$, where $k < n/2$).

Note that other families of IPOWAD operators may be used following a similar methodology as has been developed for the OWA operator and its extensions (Merigó et al. 2013; Yager 1993; Yager et al. 2011). Moreover, we could extend this analysis to other types of distances, such as the Euclidean (or quadratic) distance (Merigó and Casanovas 2011c), the Minkowski (or generalized) distance, the quasi-arithmetic distance, continuous distance (Zhou et al. 2013, 2014), heavy distances (Leon-Castro et al. 2018a, b; Merigó et al. 2014), Bonferroni distances (Blanco-Mesa et al. 2016, 2018), logarithmic distances (Alfaro-García et al. 2018), prioritized distances (Avilés-Ochoa et al. 2018) and the Choquet distances (Meng and Zhang 2014; Merigó and Casanovas 2011a).

It is worth noting that some previous models already considered the possibility of using OWA operators and probabilities in the same formulation. The main model is the concept of immediate probabilities (Engemann et al. 1996; Yager et al. 1995). In this case, following these methodologies, we could develop the induced immediate probabilistic distance (IIP-OWAD) operator in a similar way as has been done in the IPOWAD operator. Another approach that could be analyzed is the weighted OWA (WOWA) (Torra 1997) and hybrid averaging (HA) (Xu and Da 2003) for situations where probabilities are used instead of weighted averages. In these cases, we could form the induced WOWA distance (IWOWAD) and the induced hybrid averaging distance (IHAD) operator. Note that other models could also be considered in this framework where different extensions and generalizations are being developed (Aggarwal 2015; Merigó and Yager 2013; Yusoff et al. 2017).

4.3 Generalized and quasi-arithmetic means with the IPOWAD operator

The IPOWAD operator can be generalized by using generalized and quasi-arithmetic means. Thus, we obtain a more general formulation that can consider a wide range of particular cases, including the IPOWAD operator. By using generalized means, the IPOWAD operator becomes the induced generalized probabilistic OWA distance (IGPOWAD). Following Eq. (14), it can be formulated as follows:

$$\mathrm{IGPOWAD}(\langle u_1, x_1, y_1\rangle, \ldots, \langle u_n, x_n, y_n\rangle) = \beta\left(\sum_{j=1}^{n} w_j b_j^{\lambda}\right)^{1/\lambda} + (1-\beta)\left(\sum_{i=1}^{n} v_i |x_i - y_i|^{\delta}\right)^{1/\delta}, \tag{17}$$

where λ and δ are parameters such that λ and $\delta \in \{-\infty, \infty\} - \{0\}$.

Note that by using different values in λ and δ, we obtain a wide range of particular distance operators. For example:

- IPOWAD operator: If $\lambda = 1$ and $\delta = 1$.
- Harmonic IPOWAD: If $\lambda = -1$ and $\delta = -1$.
- Euclidean IPOWAD: If $\lambda = 2$ and $\delta = 2$.
- Geometric IPOWAD: If $\lambda \to 0$ and $\delta \to 0$.
- Cubic IPOWAD: If $\lambda = 3$ and $\delta = 3$.
- Maximum distance: If $\lambda \to \infty$ and $\delta \to \infty$.
- Minimum distance: If $\lambda \to -\infty$ and $\delta \to -\infty$.
- Euclidean OWA probabilistic distance: If $\lambda = 2$ and $\delta = 1$.

Furthermore, the IGPOWAD operator can be generalized by using quasi-arithmetic means to form the Quasi-IPOWAD operator. Following Eqs. (14) and (17), it is formulated as follows:

$$\mathrm{Quasi\text{-}IPOWAD}(\langle u_1, x_1, y_1\rangle, \ldots, \langle u_n, x_n, y_n\rangle) = \beta \times g^{-1}\left(\sum_{j=1}^{n} w_j g(b_j)\right) + (1-\beta) \times h^{-1}\left(\sum_{i=1}^{n} v_i h(d_i)\right), \tag{18}$$

where $g(b)$ and $h(d)$ are strictly continuous monotonic functions and $d_j = |x_j - y_j|$.

Note that the Quasi-IPOWAD operator includes many particular types of operators, including the IGPOWAD operator. This occurs when the functions $g(b) = b^{\lambda}$ and $h(d) = d^{\delta}$.

5 Decision making with the MP-MC-IOWAD operator

When you choose to purchase a high-cost high-volume product or service, the price-quality relationship of each of the alternatives is also necessary to consider, along with other criteria to ensure the adequacy of the company for hire. The following method is valid in the process of selecting a company for the purchase of products and contracting services when using the IPOWAD operators. Note that by using IPOWAD operators, the decision maker can consider probabilistic information together with a complex attitudinal character in the analysis of distances between two sets that describe the information:

Step 1. Selection of indicators There are many criteria to assess a company. However, not everyone has the same importance in the decision making process, as it depends on the situation and the preferences of the decision makers. Therefore, among the many criteria that can be used, evaluators should choose those that are the most appropriate for the selection of the supplier. Usually, these indicators are related to market share, the economic environment and operational management, among others. For this article, the criteria of the insurance market published quarterly by the Superintendent of Insurance of the Nation of Argentina are taken into account.

Step 2. Matrix of indicators Based on the criteria selected in the previous step, an array $U = AxC$ containing the set of alternatives (companies) $A = \{A_1, A_2, \ldots A_i\}$ and the set of selected criteria $C = \left\{C_1, C_2, \ldots C_j\right\}$ is built.

Step 3. Ideal profile criteria The experts define the vector corresponding to the ideal values for each criterion $I = i_1, i_2, \ldots i_j$. One way to do this is by taking the best result in each criterion as shown in the illustrative application.

Step 4. Matrix of distances The Hamming distances between feature vector C and ideal company I for each strategy A are calculated by obtaining a new array of distance matrix $U' = AxC$ with selected criteria for $C = \left\{C_1, C_2, \ldots C_j\right\}$ to assess companies $A = \{A_1, A_2, \ldots A_i\}$. You can also use the Euclidean distance where required.

Step 5. Induction matrix As there are many features that affect the value of the criteria for each company, a group of experts evaluate each criterion for the companies on a scale of 0–10, obtaining an induction matrix of selected criteria for the companies being evaluated. Later, the weighted average will be added to obtain an array of aggregate induction.

Step 6. Aggregation results Operators such as the arithmetic mean, weighted average, OWAD, AOWAD, IOWAD, IAOWAD, POWAD and IPOWAD, are added to evaluate the companies.

Step 7. Ranking of alternatives The optimal alternative is chosen, establishing a ranking of alternatives for each aggregation operator used in ascending order because the shortest distance will be the best result.

6 Illustrative example

The following example is presented for a company that is hiring an insurance company for the transport sector in Argentina. This case takes into account the insurance market indicators published by the Superintendent of Insurance of the Nation. Five criteria were chosen for the analysis:

- *Criterion 1. Total production* Refers to the size of the insurance company taking into account premiums in the transport sector. The result varies between 0 and 1 or from 0 to 100 in percentage terms. A higher value of this indicator is better for the company in the sector.
- *Criterion 2. Credits/Assets* This ratio indicates the degree of dependence of the insurer with respect to its creditors. The result ranges from 0 to 1 or from 0 to 100 in percentage terms. It is the best asset of the company when this indicator is lower.
- *Criterion 3. (Investment + Property)/Assets* This ratio shows the backing of the company over the medium and long term. The result varies from 0 to 1 or from 0 to 100 in percentage terms. A higher value of this indicator indicates a better position in the medium and long term, although this is at the expense of immediate liquidity.
- *Criterion 4. Ceded premiums/Issued premiums* This ratio indicates the proportion of premiums that are reserved for the payment of reinsurance. The result varies from 0 to 1 or from 0 to 100 in percentage terms. The higher the ratio, the lower the risk of claim defaults. However, one must take into account other factors in assessing this indicator.
- *Criterion 5. Total expenses/Issued premiums* This ratio refers to the proportion of collected premiums that are to be allocated to cover the full costs. The lower bound of the result is 0, and it has no upper limit but usually does not exceed 100%. The lower the ratio, the lower the business costs, demonstrating greater efficiency and a greater likelihood of staying in the market.

Table 2 shows the indicators for five insurance companies in the transport sector in Argentina from December 31, 2013:

As shown in the table above, any one of the insurers is superior to the others in every category, which means that a superior indicator may come at the expense of a lower result in another category. Given the above, the ideal level for each indicator was determined from the best result among the five companies, as shown in Table 3.

Table 2 Indicators of the insurance market. *Source*: Argentina Insurance Superintendency (www.ssn.gov.ar)

Business	*C*1	*C*2	*C*3	*C*4	*C*5
Insurer 1	0.082	0.036	0.957	0.086	0.166
Insurer 2	0.029	0.120	0.867	0.073	1.003
Insurer 3	0.051	0.426	0.558	0.121	1.854
Insurer 4	0.312	0.260	0.709	0.032	0.089
Insurer 5	0.526	0.487	0.440	0.028	0.542

Table 3 Indicators of the ideal profile

Ideal Profile	*C*1	*C*2	*C*3	*C*4	*C*5
(*I*)	0.526	0.036	0.957	0.121	0.089

Table 4 Matrix of distances

Business	*C*1	*C*2	*C*3	*C*4	*C*5
Insurer 1	0.444	0.000	0.000	0.058	0.077
Insurer 2	0.497	0.084	0.090	0.045	0.914
Insurer 3	0.475	0.390	0.399	0.093	1.765
Insurer 4	0.214	0.224	0.248	0.004	0.000
Insurer 5	0.000	0.451	0.517	0.000	0.453

Table 5 Matrix of induced variables-Expert 1

Business	*C*1	*C*2	*C*3	*C*4	*C*5
Insurer 1	7	8	7	7	7
Insurer 2	7	8	8	7	4
Insurer 3	7	7	5	8	5
Insurer 4	6	5	6	6	8
Insurer 5	9	5	6	6	9

From the data in Table 2, the Hamming distances of the indicators of each of the insurers with respect to the ideal profile are calculated. The matrix of distances is shown in Table 4.

Evaluating the results of the above table is important to consider the situation of the indicators of each company. For example, when the total output of a company with a large market share that is in decline is compared to a smaller company with better prospects for growth, the assessment by the decision maker may be different than what is reflected by the data. Another case might be to compare the level of reinsurance, where the highest level indicates a better guarantee of solvency, but it is important to consider accidents that can justify the proportion of reinsurance. To better evaluate the results, a committee of experts is asked to assess the indicators of each insurer, taking advantage of the knowledge and experience that each may have about this case. The expert valuations are shown in Tables 5, 6 and 7 on a scale of 0–10.

The opinions of the experts have been added, with the averaged opinions of the three experts to obtain the results in Table 8.

Table 6 Matrix of induced variables-Expert 2

Business	*C*1	*C*2	*C*3	*C*4	*C*5
Insurer 1	7	7	7	6	8
Insurer 2	8	7	8	8	4
Insurer 3	6	6	6	8	3
Insurer 4	5	7	7	7	9
Insurer 5	8	6	5	7	9

Table 7 Matrix of induced variables-Expert 3

Business	*C*1	*C*2	*C*3	*C*4	*C*5
Insurer 1	8	9	6	8	8
Insurer 2	7	6	8	8	2
Insurer 3	7	6	6	9	2
Insurer 4	5	7	7	9	9
Insurer 5	10	6	5	8	8

Table 8 Matrix of induced variables aggregated

Business	*C*1	*C*2	*C*3	*C*4	*C*5
Insurer 1	7.33	8.00	6.67	7.00	7.67
Insurer 2	7.33	7.00	8.00	7.67	3.33
Insurer 3	6.67	6.33	5.67	8.33	3.33
Insurer 4	5.33	6.33	6.67	7.33	8.67
Insurer 5	9.00	5.67	5.33	7.00	8.67

The results were aggregated with the vectors $P = (0.15,0.20,0.25,0.15,0.25)$ and $W = (0.30,0.25,0.20,0.15,0.10)$ using the following criteria:

– *Arithmetic Mean (AM)* The average distances of the indicators for each insurer in Table 4.
– *Weighted Average (WA)* The aggregation of the indicator distances of each insurer in Table 4 using the weight vector P.
– *OWAD* The aggregation in descending order of the indicator distances shown in Table 4 from the weight vector *W*.
– *AOWAD* The aggregation in ascending order of the distance indicators shown in Table 4, from the weight vector *W*.
– *IOWAD* The aggregation in descending order of the distance indicators shown in Table 4 from the weight vector *W*, considering the aggregated matrix of induction (Table 8) for sorting.

Table 9 Aggregated results

Business	AM	WA	OWAD	AOWAD	IOWAD	IAOWAD	POWAD	IPOWAD
Insurer 1	0.111	0.091	0.148	0.074	0.113	0.109	0.114	0.100
Insurer 2	0.327	0.349	0.434	0.219	0.242	0.411	0.383	0.306
Insurer 3	0.606	0.690	0.786	0.426	0.433	0.778	0.729	0.587
Insurer 4	0.155	0.152	0.136	0.174	0.127	0.183	0.146	0.142
Insurer 5	0.303	0.347	0.366	0.237	0.251	0.355	0.355	0.309

Table 10 Ranking of the alternatives

Criteria	Ranking
AM	A1 < A4 < A5 < A2 < A3
WA	A1 < A4 < A5 < A2 < A3
OWAD	A4 < A1 < A5 < A2 < A3
AOWAD	A1 < A4 < A2 < A5 < A3
IOWAD	A1 < A4 < A2 < A5 < A3
IAOWAD	A1 < A4 < A5 < A2 < A3
POWAD	A1 < A4 < A5 < A2 < A3
IPOWAD	A1 < A4 < A2 < A5 < A3

A1 = Insurer 1, A2 = Insurer 2, A3 = Insurer 3, A4 = Insurer 4 and A5 = Insurer 5

- *IAOWAD* The aggregation in ascending order of the distance indicators shown in Table 4 from the weight vector *W*, considering the aggregated matrix of induction (Table 8) for sorting.
- *POWAD* The unification between the WAD and the OWAD with $\beta = 0.4$.
- *IPOWAD* The integration between the WAD and the IOWAD with $\beta = 0.4$.

The results of the aggregation with the above criteria are shown in Table 9.

Next, let us present in Table 10, the ranking of the alternatives with the results from Table 9.

Nearly all criteria except the OWAD operator indicate that the best alternative is A1 and the worst is A3. It is worth mentioning that occasionally, each type of distance may lead to different results. This depends on the initial information available, which produces a different representative result with each approach. Note that the IPOWAD approach is a more general model that includes a wide range of particular cases by using probabilities and complex attitudinal characters. Observe that several particular aggregation criteria are presented because they represent the particular positions that the decision maker may adopt with the IPOWAD operator. Thus, we offer a more complete picture of the analysis that can be developed using the IPOWAD operator.

7 Conclusions

This study introduced a new MCDM method for the selection of companies using the multi-person multi-criteria induced ordered weighted distance (MP-MC-IOWAD) operator. This approach includes a new tool to calculate the array of induction based on the opinions of experts and assesses the criteria by taking into account specific situations that are not reflected in the data but are instead derived from the knowledge and experience of experts. Thus, factual information is supplemented by indicators that underestimate or overestimate the information in different situations, mainly in uncertain environments.

A new distance aggregation operator is suggested in this paper: the IPOWAD operator. This is a new distance measure that uses probabilities in the aggregation process. Moreover, it also uses the OWA operator in an environment where the attitudinal character of the decision maker is very complex and can be assessed with induced aggregation operators. The main advantage of this approach is that it provides a unified framework between the probability and the IOWA operator when using distance measures. Thus, this model includes a wide range of distance measures that are also new in the literature, such as arithmetic probabilistic distance, probabilistic distance, maximum probabilistic distance and minimum probabilistic distance.

The applicability of the MP-MC-IOWAD operator and the proposed method is demonstrated with the development of an illustrative application that intends to select an insurance company based on the expert analysis of indicators and the use of distance measures to improve the processes of decision making when the characteristics of the indicator analysis are difficult. Note that the main advantage of these operators is that they represent a wide range of scenarios and can be adapted to the particular needs of the decision maker and the available information in the specific problem considered.

Future research of other MCDM methods using tools such as the adequacy ratio, extension OWA operators and distance measures will be developed. We will also consider implementation in other problems of economics and business management (León-Castro et al. 2018b; Wei et al.

2013) and distance measures (Scherger et al. 2017) may be considered in the analysis.

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Compliance with ethical standards

Conflict of interest The authors declare that they do not have any conflicts of interest.

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