

# Shear strength estimation of masonry walls using a panel model

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## ABSTRACT

Masonry walls are structural elements generally used in housing or small buildings. Given their structural configuration, they commonly present shear failure due to seismic actions, characterized by a fragile response. Thus, it is important to have simple, yet reliable tools that correctly estimate the shear capacity of walls. For that, a simple existing model developed for reinforced concrete elements and based on a panel model is used and adapted to masonry walls, providing a novel formulation that can be applicable to both materials. For compression and tension behavior, the prismatic resistance of the panel is used, which, due to the anisotropy of the material, degrades with the angle formed by the load with the vertical mortar joint. Strain values are set for compression and tension failure modes, and a degradation coefficient in compression due to the biaxial strain loading is included. Additionally, bond failure is also incorporated into the model. A database of 41 tests of reinforced masonry walls and 12 tests of confined masonry walls is used for model validation. The strength ratio between the shear strength obtained by the model and the test is compared, giving an average and a coefficient of variation (COV) of 1.0 and 0.15, respectively for reinforced walls, and 1.08 and 0.14 for confined walls, showing a satisfactory performance and better behavior than simple models from the literature. The analysis of general trends of the strength ratio reveals that there is a low dependence between the strength ratio and the studied parameters, implying that the model captures the physical behavior of masonry walls.

## 1. Introduction

Masonry is a material used among others in walls for multi-family housing from 1 to 4 stories high or for private single-family houses. In Chile and many other places, these are basically built in two ways: reinforced with vertical and horizontal reinforcing bars in the panel, or confined by a reinforced concrete frame, similar to infill walls, but in this case the frame is built after the masonry wall, such that the masonry panel and the frame are better connected. Masonry is a material characterized by its anisotropy, which affects properties such as compression and tension strengths that change with the loading angle, yielding a complex shear strength mechanism. Seismic behavior of masonry walls, commonly with low aspect ratio, is usually controlled by shear strength and having reliable and yet simple analytical tools to quantify the shear strength and failure mode of masonry walls is required in design.

A panel-type model used to estimate the shear capacity of reinforced concrete walls is described and herein adapted to masonry walls, providing a novel formulation that can be applicable to both materials. The model was originally developed by Kaseem and Elsheikh [1], as an iterative panel model for short reinforced concrete walls. This isolated

element is subject to a lateral and axial force and has reinforcement in the longitudinal (L) and transverse (t) directions, which coincide with the vertical and horizontal directions of the wall, respectively (Fig. 1). The base panel model uses average stress and average strain states for the wall panel, imposing equilibrium, strain compatibility and constitutive material laws that govern concrete and reinforcing steel behavior. The concrete material model considers a biaxial behavior, where the principal tensile axial strain, perpendicular to the principal compression direction (forming an angle  $\alpha$  with the longitudinal direction), causes a degradation of the compressive response. The vertical and horizontal reinforcement bars contribute to strength in their longitudinal direction, without a dowel action. Two coordinate systems are generated, one given by the reinforcement layout (system “L-t”) and another by the concrete principal directions (called system “d-r”), as shown in Fig. 1.

Equilibrium is imposed in the L-t coordinate system determining the concrete stresses in the principal directions (d-r) based on the concrete strains in such directions and the steel stresses in the L-t coordinate. Considering that principal concrete stresses coincide with principal strains, the principal concrete stresses acting in an angle  $\alpha$  are  $\sigma_d$  and  $\sigma_r$ . Eqs. (1) and (2) show the longitudinal and shear equilibrium:

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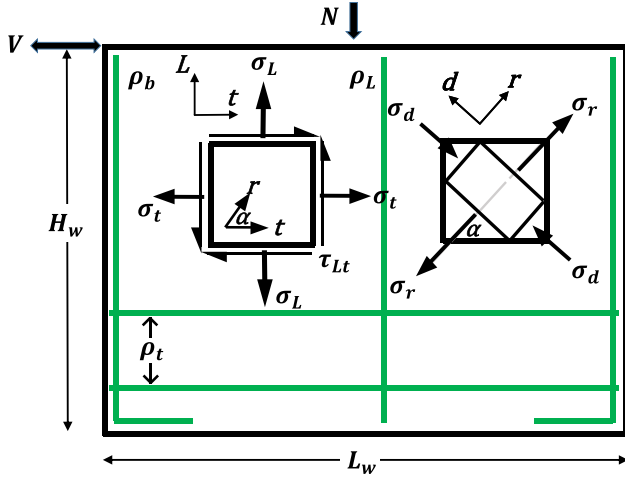


Fig. 1. Masonry wall with stress resultants in L-t coordinates, and in principal direction d - r coordinates.

$$\sigma_L = \sigma_d \cos^2 \alpha + \sigma_r \sin^2 \alpha + \rho_L f_L \quad (1)$$

$$\tau_{Lt} = (-\sigma_d + \sigma_r) \cos \alpha \sin \alpha \quad (2)$$

where  $\sigma_d$  and  $\sigma_r$  are the axial concrete stress in the d and r directions,  $\sigma_L$  is the longitudinal panel equilibrium stress,  $\tau_{Lt}$  is the shear stress resultant in the "L-t" system,  $\rho_L f_L$  is the steel force per concrete area (steel ratio times stress) in the L direction.

The shear resultant force (V) is expressed as:

$$V = \tau_{Lt} t_w d_w \quad (3)$$

where  $t_w$  is the wall thickness and  $d_w$  is the length of the wall between the centroids of the boundary elements ( $0.8L_w$ , if no boundary element exists, with  $L_w$  being the wall length).

Strain compatibility in the L-t system is established as,

$$\varepsilon_L = \varepsilon_d \cos^2 \alpha + \varepsilon_r \sin^2 \alpha \quad (4)$$

$$\varepsilon_t = \varepsilon_d \sin^2 \alpha + \varepsilon_r \cos^2 \alpha \quad (5)$$

$$\gamma_{Lt} = 2(-\varepsilon_d + \varepsilon_r) \cos \alpha \sin \alpha \quad (6)$$

where  $\varepsilon_L$  and  $\varepsilon_t$  are the normal strain in the L and t directions, respectively;  $\gamma_{Lt}$  is the shear strain in the direction L-t; and  $\varepsilon_d$  and  $\varepsilon_r$  are the normal principal strain in the directions d and r, respectively. For walls controlled by shear deformations, the lateral wall displacement can be estimated as  $\Delta = \gamma_{Lt} H_w$ , where  $H_w$  is the wall height.

Once the strains are known, a rotating-angle material model is used to evaluate the concrete stresses in the principal directions [2]. The rotating-angle approach is a material model formulation for panel elements (plane stresses) that treats the concrete component as an orthotropic material that is characterized by estimating concrete stresses in two principal directions provided by uniaxial material constitutive laws evaluated with the strains in the correspondent two principal directions (which might rotate). For a simple model, uniaxial compression behavior for concrete follows a parabolic stress-strain relationship that includes a degradation caused by the tensile strains in the orthogonal direction. For tension, the stress-strain relationship is linear until cracking and then degrades linearly to zero. For the reinforcement, perfect bonding to the concrete is assumed and an elastic perfectly-plastic constitutive material law is considered for both the L and t directions.

## 2. Modifications to the original model

Massone and Álvarez [3] incorporated the effect of the wall boundary reinforcement in the longitudinal equilibrium equation (Eq.

(1)), through a parameter  $\beta$ , which represents the contribution of the boundary reinforcement to shear strength. The optimum value of  $\beta$  was 0.3. The equilibrium in the longitudinal direction L is modified as,

$$\sigma_L = \sigma_d \cos^2 \alpha + \sigma_r \sin^2 \alpha + \rho_L f_L + \beta \rho_b f_b \quad (7)$$

where  $\rho_b f_b$  is the boundary longitudinal force per concrete area (steel ratio times the stress).

Massone and Orrego [4] re-calibrated the principal strain angle ( $\alpha$ ), established by Massone and Ulloa [5], developing a unique expression for walls, deep beams and corbels and another expression for beam-column connections based on strain estimations from finite element analysis [6]. In Eq. (8), the expression for the principal strain angle for simple curvature is shown. Eq. (8) is also applicable to masonry walls experiencing single curvature, i.e., cantilever walls subjected to lateral load at the top of the wall.

$$\alpha = 13.87 \left( \frac{H_w}{L_w} + 0.5 \right)^{-0.13} \left( \frac{N}{f_c t_w L_w} + 0.1 \right)^{-0.67} \quad (8)$$

Massone and Melo [7] incorporated the transverse reinforcement component to the nominal tensile capacity of the concrete as it was done by Wang et al. [8]. The component associated with the longitudinal reinforcement is not added because its contribution is already incorporated in the longitudinal equilibrium of the panel element (Eqs. (1) and (7)). The final expression for the nominal tensile capacity of the element with these modifications is shown in Eq. (9).

$$f_{ct}^* = f_{cto}^* + \rho_t f_t \cos^2 \alpha \quad (9)$$

where  $f_{cto}^*$  is the basic tensile strength of concrete and  $\rho_t f_t$  is the horizontal web force per concrete area (steel ratio times the stress); the horizontal reinforcement is assumed to have yielded.

The model, including its modifications, using equilibrium, compatibility and non-linear material constitutive laws, allows determining the wall response (force versus displacement) using numerical methods to solve the non-linear equation of longitudinal equilibrium (Eq. (1)), later replaced by Eq. (7)). In order to simplify the methodology, Massone and Melo [7] instead of performing an incremental analysis to obtain the complete wall response (shear force-displacement curve) focused on calculating shear strength values for each type of potential failure in the model by setting the material strain at the material capacity, which for reinforced concrete elements corresponds to: (i) concrete in compression ( $\varepsilon_d$  associated with  $\sigma_d$ ), (ii) concrete in tension ( $\varepsilon_r$  associated with  $\sigma_r$ ), and (iii) yielding of web and boundary reinforcement ( $\varepsilon_L$  associated with  $f_L$  and  $f_b$  since it uses the same strain), whose terms appear in Eq. (7). The solution of the nonlinear Eq. (7) is replaced by calibrated expressions of strain ( $\varepsilon_d^*$  for tension failure of concrete or reinforcement yielding, and  $\varepsilon_r^*$  for compression failure of concrete) that closely reproduces the results of such an equation [7]. Thus, given a failure mode, a strain value is fixed ( $\varepsilon_d$  for (i),  $\varepsilon_r$  for (ii) or  $\varepsilon_L$  for (iii)), which together with a calibrated strain expression ( $\varepsilon_d^*$  for (ii) and (iii) or  $\varepsilon_r^*$  for (i)) and the known principal strain/stress direction angle ( $\alpha$ ), the strain field is identified allowing determining the stress values in both materials and therefore the shear strength (V). A calibration was performed for  $\varepsilon_d^*$  and  $\varepsilon_r^*$  in order to better reproduce the results obtained with the iterative model that solves the nonlinear Eq. (7). The expressions for the different failure modes, i.e., tension (Eq. (10)), compression (Eq. (11)), and reinforcement yielding (Eq. (12)), are shown in Eqs. (10)–(12).

$$\varepsilon_d^* = -1.292 \times 10^{-3} (\cos \alpha)^{-2.56} \left( \frac{N}{f_c A_g} + 0.1 \right)^{1.40} \quad (10)$$

$$\varepsilon_r^* = 3.610 \times 10^{-4} \left( \frac{\rho_L f_{yL}}{f_c} + 0.05 \right)^{-0.59} \left( \frac{\beta \rho_b f_{yb}}{f_c} + 0.05 \right)^{-0.60} (\cos \alpha)^{3.46} \left( \frac{N}{f_c A_g} + 0.1 \right)^{-0.86} \quad (11)$$

$$\varepsilon_d^* = -0.635 \left( \frac{\rho_L f_{yL}}{f_c} + 0.05 \right)^{1.24} \left( \frac{\beta \rho_b f_{yb}}{f_c} + 0.05 \right)^{1.22} (\cos \alpha)^{-2.45} \left( \frac{N}{f_c A_g} + 0.1 \right)^{1.36} \quad (12)$$

where  $N/f_c A_g$  is the axial stress normalized by the concrete compressive strength ( $f_c$ ), and  $f_{yb}$  and  $f_{yL}$  are the boundary and longitudinal web yield stress, respectively.

Once strength values for all failure models are determined, it is necessary to check if they can be reached. If the shear strain required to reach the compressive strength of the concrete occurs before the shear strain required for yielding of the longitudinal web or boundary reinforcement is reached, then the failure mode is associated with concrete failure. Conversely, if yielding of the reinforcement is reached at an earlier shear strain than that for the concrete compression failure, then the failure mode is associated with reinforcement yielding.

### 3. Adaptation to masonry walls

The present work aims to adapt this closed-form model developed for reinforced concrete elements to masonry walls, for both reinforced and confined masonry walls. Moreover, the model for strength estimation is validated against a database of masonry wall.

Reinforced masonry walls are composed by a masonry panel with vertical reinforcement installed and grouted in the holes of the masonry units and horizontal reinforcement embedded in grout between lines of masonry units (Fig. 2a), while confined masonry walls are constructed with the masonry panel and a reinforced concrete frame surrounding the panel (Fig. 2b). Considering that the masonry panel can be assimilated to a concrete panel and the reinforcement is present in masonry and reinforced concrete solutions, some failure modes can be also assimilated. Masonry walls have different failure mechanisms when subjected to a lateral load, but there are four main mechanisms related to shear strength: (i) diagonal tension, (ii) diagonal compression, (iii) yielding of reinforcement and (iv) bond, which corresponds to cracking either in the mortar joints, in a row of masonry units or with a staggered pattern. Also, flexural failure can occur.

The following sections define the modifications required to adapt such failure modes in the model according to the material characteristics. Tensile and compression failure are considered similar as that of a reinforced concrete wall, such that, shear failure due to diagonal tension and diagonal compression failure are similar in reinforced

concrete and masonry, provided that adherence of masonry joint is strong. Besides, an additional model is included to capture bond failure. The constitutive material laws for masonry in compression and tension, analogous to the case of concrete, are shown in Fig. 3a and b, respectively; whereas steel is shown in Fig. 3c.

#### 3.1. Reinforcing steel

Similar to other formulations, in the case of steel, a uniaxial elastic perfectly-plastic constitutive material law (Fig. 3c) is implemented for all reinforcing steel. Thus, when yielding is reached  $\varepsilon_L = \varepsilon_y$  and consistently  $f_L = f_b = f_y$ .

#### 3.2. Masonry under compression

A difference with reinforced concrete in compression is that the compressive strength of masonry is determined by testing a prism, consisting of a series of units stacked on top of each other and joined with mortar. The prismatic resistance ( $f'_m$ ) is obtained under compression testing. However, such value does not account for slenderness effect or anisotropy of the material.

Page and Marshall [9] conducted a series of uniaxial compression tests on prisms with different aspect ratios (ratio between the height and width of the unit) to evaluate the influence of this parameter on the prismatic strength. For low aspect ratios, the compressive strength requires a correction due to artificial strength increase caused by constrain at element ends that affects the overall element behavior. A common prism slenderness of 4 is used in tests (including the database used in this work), resulting in a compression strength correction of  $K_c = 0.93$ .

Masonry is an anisotropic material, such that there is a variation of uniaxial compression strength of the element with the direction of the axial load relative to the unit orientation. Hamid and Drysdale [10] tested 17 specimens, without grouted holes, at different angles  $\alpha$ , between the applied compression direction and the vertical prism joint (Fig. 4a). Three prisms were tested for each angle orientation of  $\alpha = 15^\circ, 45^\circ, 60^\circ, 75^\circ$  and  $90^\circ$ , and 6 prisms for  $\alpha = 0^\circ$ . The strength reduction factor is then defined as the ratio between the strength at a specific orientation ( $\alpha$ ) and the strength at  $\alpha = 0^\circ$ , as  $C_\alpha = f'_m(\alpha)/f'_m(0^\circ)$  (Fig. 4b).

The coefficient  $C_\alpha$  is calibrated as,

$$C_\alpha = 2.26 \cdot 10^{-7} \alpha^4 - 3.42 \cdot 10^{-5} \alpha^3 + 1.48 \cdot 10^{-3} \alpha^2 - 0.022 \alpha + 1 \quad (13)$$

The panel model requires the strain value associated with the maximum compression stress in the element. Naraine and Sinha [11], for biaxial compression, suggest  $\varepsilon_0 = 0.0035$  for ceramic bricks, and Hidalgo [12] a value of  $\varepsilon_0 = 0.003$  for concrete blocks. Considering that the compressive strength is reached, the compressive strain at peak strength is  $\varepsilon_d = -\xi \varepsilon_0$  and the strength is  $\sigma_d = -\xi f'_m$  (Fig. 3a), where  $\xi$  represents a strength and strain softening coefficient (reduction) in the compressive direction due to tensile strains in the opposite direction

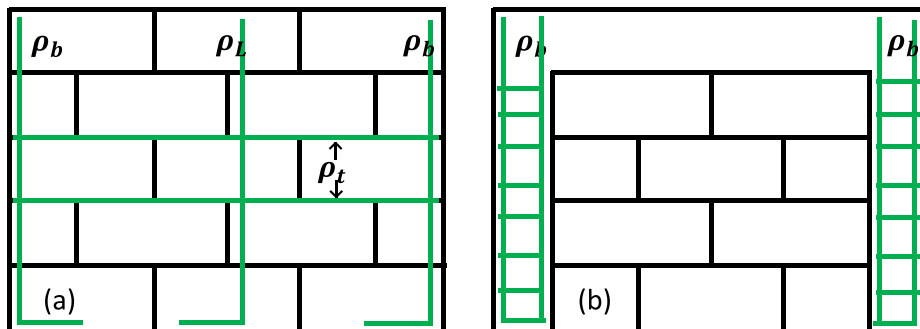


Fig. 2. (a) reinforce masonry wall, and (b) confined masonry wall.

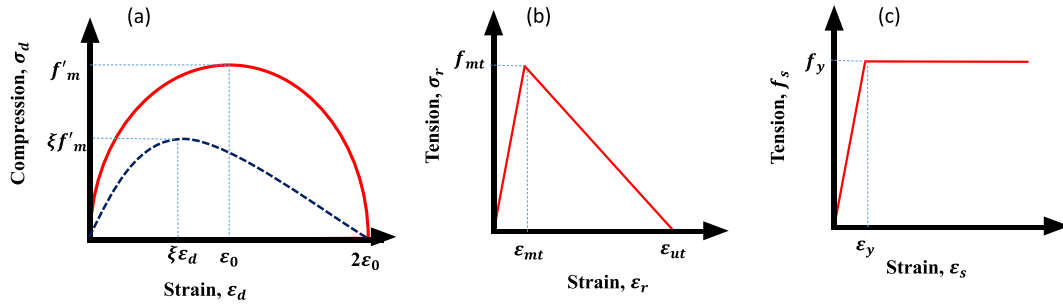


Fig. 3. Material constitutive laws – (a) masonry in compression, (b) in tension, and (c) steel.

( $\epsilon_r$ ). Considering a  $\xi$  factor consistent with the concrete behavior, the expression is defined as [13],

$$\xi = \frac{5.8}{\sqrt{f'_m}} \cdot \frac{1}{\sqrt{1 + \gamma\epsilon_r}} \leq \frac{0.9}{\sqrt{1 + \gamma\epsilon_r}} \quad (14)$$

where  $\gamma$  is a parameter that depends on the type of material subjected under biaxial stress state (400 for reinforced concrete [13]). For this work, a value equal to 2500 is used for all specimens, which yields the best results for the model. For simplicity, a large and refined set of values of  $\gamma$  was implemented, selecting the optimum value.

### 3.3. Masonry under tension

Tomazevic [14], states that the tensile strength of masonry,  $f'_{mt}$ , can be calculated as 3% of the prismatic resistance ( $f'_m$ ), for units with similar strength to those used in this work. Similar to compression, Drysdale and Hamid [15] tested axial tensile prisms to evaluate the variation of stress for different loading direction angles. Three tests for each inclination angle were performed for  $\alpha = 45^\circ$  and  $90^\circ$ , and four specimens for  $\alpha = 0^\circ$ . Thus, the tension reduction factor  $C_{t\alpha} = f_m(\alpha)/f_m(90^\circ)$ , is calculated based on the average data reported by Drysdale and Hamid [15] as shown in Fig. 4c.

$$C_{t\alpha} = 1.47 \cdot 10^{-4} \alpha^2 - 0.0058 \alpha + 0.33 \quad (15)$$

The tensile strain at peak tensile strength is generally low [16], which is estimated as  $\epsilon_{mt} = 0.0001$  parallel to the horizontal joint as suggested by Drysdale and Hamid [15]. After reaching the peak strength, the tensile response is assumed to reduce linearly until a zero-stress value for a strain of  $\epsilon_{ut} = 0.00035$  (Fig. 3b).

### 3.4. Masonry bond

Bond failure occurs when there is a stepped cracking pattern through the joints in the masonry panel due to mortar-unit failure.

Dialer [17] proposed a bond model for masonry panels, which applies a Mohr-Coulomb strength model for elements prone to bond failure between the unit and mortar. Units of height  $b$  and length  $d$  are subjected to normal ( $f_n$  and  $f_p$ ) and shear stresses ( $\tau_{xy}$  and  $\tau_{yx}$ ), which are part of panel elements under global uniaxial normal principal stresses  $f_1$  and  $f_2$  as shown in Fig. 5a. According to Dialer [17], the normal stresses acting on the unit can be related by a factor  $\chi$ , as  $\chi = \frac{f_n}{f_p}$ .

According to Chary [18], there is a moment decompensation that must be balanced by the addition and subtraction of a normal stress  $\Delta f_n$  that tend to cause lifting of half part of the unit (reduced normal stress). The work by Crisafulli [19], for an elastic model of the masonry panel derives an estimation of the additional stress as  $\Delta f_n = \frac{1.5 \cdot b \cdot (\tau_{yx} - \tau_{xy})}{d}$ . Based on a Mohr-Coulomb strength criterion and considering a unit basic material shear strength as  $\tau_{0xy}$  and  $\tau_{0yx}$  (shear strength for zero normal stress), and coefficients of friction between the unit and the mortar as  $\mu_{xy}$  and  $\mu_{yx}$ , the bond shear model is defined as  $\tau_{yx} = \tau_{0yx} + \mu_{yx} \cdot f_n$  and  $\tau_{xy} = \tau_{0xy} + \mu_{xy} \cdot \chi \cdot f_n$ . According to Dialer [17], there is a quality factor,  $F$ , given the difference between the properties of the horizontal and vertical joints, which is represented by  $F = \frac{\tau_{0yx}}{\tau_{0xy}} = \frac{\mu_{xy}}{\mu_{yx}}$ . According to the stress state presented in Fig. 5b, a bond failure occurs when  $f_n$  is reduced by the effect of  $\Delta f_n$ , that is, when the unit vertical normal stress is  $f_n - \Delta f_n$ . Then, reorganizing the expressions, it yields

$$\tau_{yx} = \tau^* + \mu^* \cdot f_n \quad (16)$$

$$\text{where } \tau^* = \frac{\tau_{0yx} \cdot (1 + \mu_{yx} \cdot 1.5 \cdot \frac{b}{d} \cdot F)}{1 + \mu_{yx} \cdot 1.5 \cdot \frac{b}{d}} \text{ and } \mu^* = \frac{\mu_{xy} \cdot (1 + \mu_{yx} \cdot 1.5 \cdot \frac{b}{d} \cdot F)}{1 + \mu_{yx} \cdot 1.5 \cdot \frac{b}{d}}$$

Assuming that the compression is larger than tension ( $f_1$  larger than  $f_2$ ), then  $\tau = f_1 \cdot \cos(\theta) \cdot \sin(\theta) = \tau^* + \mu^* \cdot f_1 \cdot \sin^2(\theta)$ , where  $\theta$  is the complement of  $\alpha$  ( $\theta = \pi/2 - \alpha$ ), which yields,

$$\tau = \frac{\tau^*}{(1 - \mu^* \cdot \tan(\theta))} \quad (17)$$

According to a review of tests on mechanical properties of masonry,

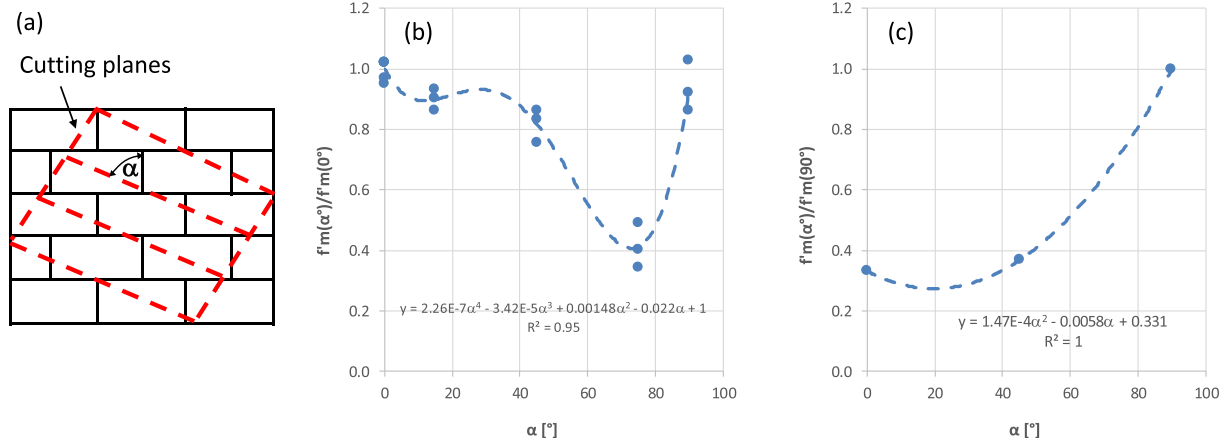


Fig. 4. Prism under uniaxial loading in different directions – (a) prism cutting, (b) under compression, and (c) under tension.

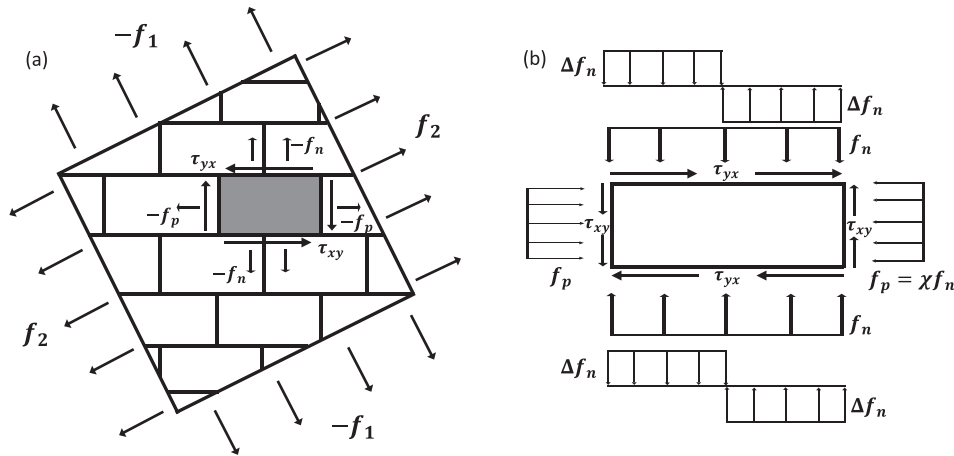


Fig. 5. Bond model – (a) prism under principal stress state, and (b) unit stress state (after [17]).

Cabezas [20] determined that the values of coefficient of friction are  $\mu = 0.7$  for ceramic bricks, and  $\mu = 0.8$  for concrete blocks. The determination of the basic adhesion strength is carried out by triplet tests. Out of the triplet tests carried out by Fernández [21], there is a series of test elements that represent the mortar used in the wall masonry of the database, yielding a basic adhesion strength of  $\tau_0 = 0.56$  MPa. According to Delfin and Bullemore [22], for walls built with concrete blocks and with a mortar paste similar to the walls of the database used in this article, the value of the basic resistance to adhesion corresponds to  $\tau_0 = 0.38$  MPa.

The quality factor  $F$  relates the basic adhesion and the friction factors of the vertical joints with the horizontal joints. Fernández [21] performed tests of triplets built with ceramic bricks, subjected to direct shear with a smooth and rough face, representing the horizontal and the vertical joint. The ceramic bricks units with similar characteristics to the specimens collected in the database yielded a quality factor equal to  $F = 0.5$ . Cruz [23] performed tests on masonry walls built with concrete blocks with and without the presence of vertical joints to study their effect on strength. Based on Cruz [23] results, Maldonado [24] determined the quality factor for walls built with concrete blocks as  $F = 0.35$ .

The ratio of normal stresses acting on the unit  $\chi = \frac{f_n}{f_p}$  was determined by Maldonado [24] through a finite element analysis for an isotropic linear-elastic material as a simplified model for masonry. The factor was calibrated for aspect ratios between  $\lambda = 0.5$  and 2, as

$$\chi = -0.83\lambda + 1.6 \geq 0 \tag{18}$$

### 3.5. Flexure

For reinforced masonry walls, the equations proposed by Hidalgo [12] and developed by Silva [25] are used. In this case, a parabolic distribution of stresses in the compression zone of the masonry, with an equivalent block of width equal to two thirds of the distance to the neutral axis is generated. For confined masonry walls, there is a contribution of reinforced concrete columns in the flexural strength. An ultimate concrete compressive strain of 0.003 is taken into account and an equivalent strength block as described in ACI 318-19 [26] is used.

### 3.6. Shear strength estimation

Once all potential failure modes are established, the shear strength of the masonry wall can be determined. Based on the closed-form solution, Fig. 6 shows a flowchart that starts with the estimation of the strain field that begins with the principal strain direction ( $\alpha$ , Eq. (8)) and the strain values associated to the first 3 failure modes: (1) masonry

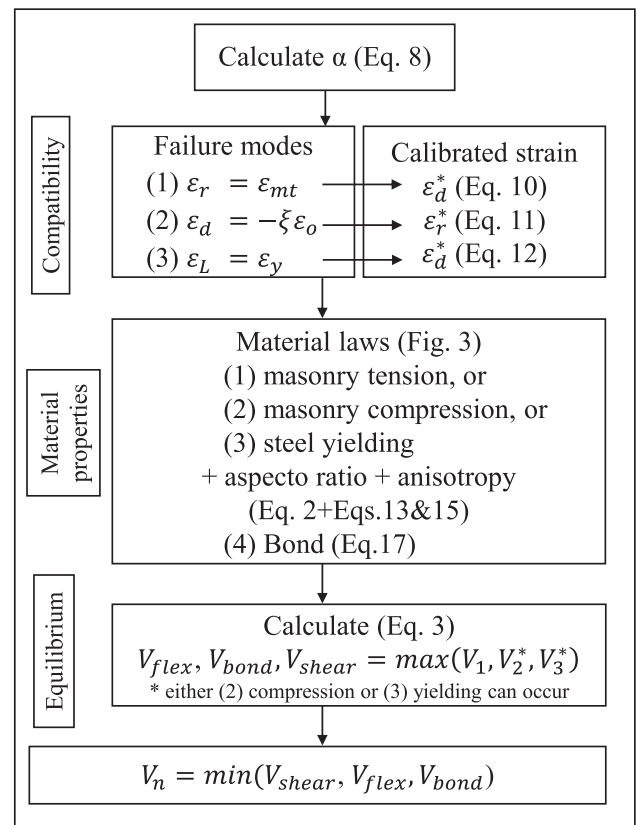


Fig. 6. Flowchart for the proposed closed-form shear model for masonry walls.

in tension, (2) masonry in compression and (3) yielding of reinforcement. Each failure mode has a calibrated expression for either  $\epsilon_d^*$  or  $\epsilon_r^*$  (Eqs. (10)–(12)), such that the strain in all materials (masonry and steel) can be determined. Material properties are used to determine stresses in masonry and steel (Fig. 3) for all 3 failure modes, but in this case, aspect ratio ( $K_c = 0.93$ ) and anisotropy needs to be taken into account by reducing the compressive and tensile stresses of the masonry panel. In the case of anisotropy, the coefficients  $C_\alpha$  (Eq. (13)) and  $C_{t\alpha}$  (Eq. (15)) are used for compression and tension, respectively. Once stresses are determined, the shear stress in the wall is determined according to Eq. (2) (the largest stress from all 3 failure modes is selected, recalling that either compression failure or reinforcement yielding is considered based on the one that is reached at smaller shear strain). In the case of adherence failure, Eq. (17) provides the estimation of shear







**Table 3**  
Statistical analysis of strength estimate ratio for the proposed model distinguishing different failure modes for reinforced and confined masonry walls.

Failure	Reinforced masonry walls			Confined masonry walls		
	No	Avg.	COV	No	Avg.	COV
All	41	1.00	0.15	12	1.08	0.14
Tension	20	0.96	0.14	8	1.03	0.12
Compression	5	1.10	0.14	2	1.01	0.11
Bond	8	0.95	0.17	2	1.32	0.03
Flexure	8	1.05	0.13			

from the test photos at failure. From about 50% of the specimens, the failure mode can be deduced, with a correct prediction of the failure type for about 80% of them.

#### 4.2. Analysis of general trends of the model

In this section, the general trends of the model with respect to relevant parameters of masonry walls are shown. The input parameters of the model are chosen, which correspond to the characteristics that define the wall properties, such as the aspect ratio ( $H_w/L_w$ ), the prismatic resistance of the element, the axial compression stress ( $N/(f'_m A_g)$ ) and the transverse, longitudinal and boundary steel quantities, as well as the principal direction angle. The dependence of the model to the variations of each parameter is evaluated, considering that the lower the dependency the better its physical behavior is incorporated in the analysis. This is shown graphically by the ratio between the predicted shear strength of the model and the experimental shear with respect to the variation of the studied parameters. Trend lines are also shown for all cases. The trend lines are the best-fit linear curves of the data, which shows how the strength ratio (for the proposed model) changes with a specific parameter (e.g., axial load, aspect ratio). Therefore, if a trend line shows a constant value (no slope) equal to one that means that the strength is well captured independent of the value of the specific parameter. Having moderate slopes in the trend lines indicate that the dependency of the strength model to the specific parameter is correctly incorporated.

Fig. 8 shows the strength ratio versus selected parameters. The data is separated between reinforced masonry walls (blue) and confined masonry (red) walls, together with the trend lines (consistent colors). The trend lines are shown for cases where the range of parameters covers at least about 50% of the total range for the walls. The overall trend lines for all walls is also included (black). Regarding the aspect ratio (Fig. 8a), the data concentrates the values close to aspect ratios of 1 or 0.5, where the strength ratio for all specimens almost does not

present dependency to the aspect ratio (trend line), indicating that the strength model correctly captures such parameter. Similar situation is observed for other parameters (Fig. 8) such as: prismatic compressive strength ( $f'_m$ ), principal strain/stress direction ( $\alpha$ ), axial stress level ( $N/(f'_m A_g)$ ), transverse yield force per area ( $\rho_t f_{yt}$ ), longitudinal yield force per area ( $0.3\rho_b f_{yb} + \rho_l f_{yl}$ ). In these cases, there is a low dependency to the different parameters, where for the entire range of each parameter, the trend line of the strength ratio for all specimens varies less than 10% in all cases. When the specimens are separated between reinforced and confined masonry walls, the dependency is still small, with similar values as for the entire database, which confirms that the model captures correctly the response for both types of walls.

## 5. Comparison with other models

In this section, other formulations available in the literature are presented in order to compare their performance against the proposed model (Tables 1 and 2). All comparison formulations presented here are related to design of masonry walls. They provide shear strength equations with code design applications. However, all of them propose expressions with either a safety factor or a strength reduction factor, which is set as one for the current analysis. Such that the expressions are not as conservative as it would be for design. The comparison, however, is intended to show the capability of other approaches to provide good predictions of masonry wall strength, and as shown with the different formulations and different terms, a unique expression cannot easily predict the strength for all specimens (e.g., separated between reinforced and confined masonry walls). The unique expression proposed in this research can deal with different configurations including all relevant parameters.

### 5.1. Reinforced masonry walls

#### 5.1.1. Model by Silva [25]

Silva [25], for walls with partially grouted holes and low horizontal reinforcement ratios ( $\leq 0.06\%$ ), recommends calculating the nominal shear strength ( $V_n$ ) as,

$$V_n = V_m + V_s \quad (19)$$

$$V_m = 0.4\tau_o A_g + 0.25N \leq 0.7\tau_o A_g \quad (20)$$

$$V_s = 0.5\rho_t f_{yt} t_w \min(L_w, H_w) \leq V_m \quad (21)$$

where  $V_m$  and  $V_s$  are the masonry panel and reinforcement contribution, respectively,  $\tau_o$  is the basic shear stress,  $N$  the axial load, and  $t_w$  is the masonry panel thickness.

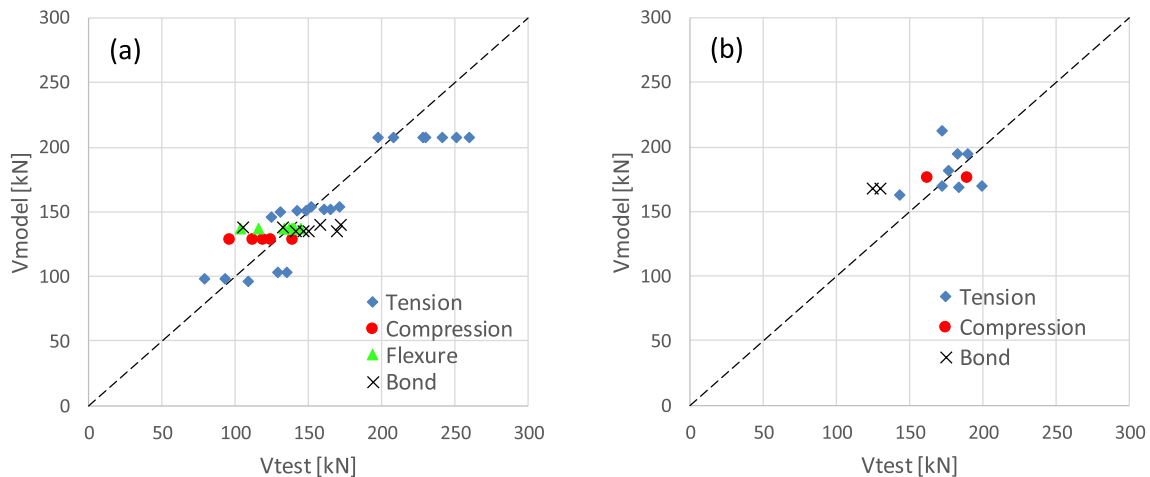
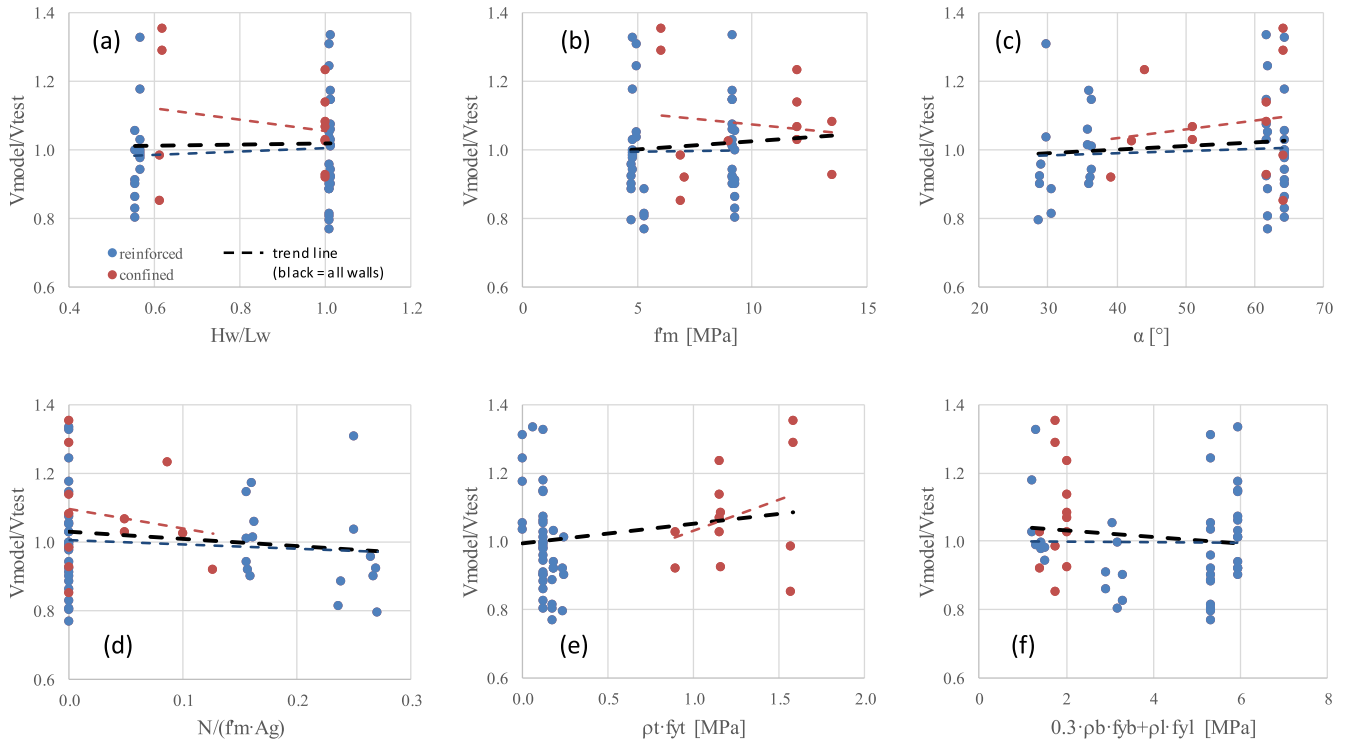


Fig. 7. Strength ratio  $V_{model}/V_{test}$  for different failure modes – (a) reinforced masonry walls, and (b) confined masonry walls.





**Fig. 8.** Sensitivity analysis for masonry walls – (a) aspect ratio, (b) prismatic strength, (c) principal stress/strain angle, (d) axial load level, (e) transverse reinforcement force per unit area, and (f) longitudinal reinforcement force per unit area.

### 5.1.2. Model by Tomazevic [34]

Tomazevic [34] proposes a masonry shear strength model that incorporates the contribution of the masonry panel and reinforcement, as

$$H_{sd,r} = H_{sd,w} + C_{rh} \cdot H_{sd,rh} + H_{dd,rv} \quad (22)$$

$$H_{sd,w} = A_g \frac{f'_{mt}}{b} \sqrt{\frac{f_n}{f'_{mt}}} + 1 \quad (23)$$

$$H_{sd,rh} = 0.9d \frac{A_t f_{yt}}{s} \quad (24)$$

$$H_{dd,rv} = 0.806 \cdot n \cdot d_{bl}^2 \sqrt{f'_m f_{yl}} \leq 0.25 d_{bl}^2 f_{yl} \quad (25)$$

where  $H_{sd,w}$  is the masonry contribution to shear strength,  $H_{sd,rh}$  is the horizontal web reinforcement contribution to shear strength,  $H_{dd,rv}$  is the vertical web reinforcement contribution to shear strength,  $C_{rh}$  is a reduction factor of the transverse reinforcement, set as 0.3,  $f'_{mt}$  is the tensile masonry panel strength,  $b$  is a shear distribution factor (1.1 for wall with aspect ratio less or equal to 1 and 1.5 for aspect ratio of 1.5),  $f_n$  is the wall axial stress ( $N/A_g$ ),  $d$  is the effective wall depth,  $A_t$  is the total transverse steel area,  $s$  is the transverse steel spacing,  $n$  is the number of longitudinal bars, and  $d_{bl}$  is the longitudinal bar diameter.

## 5.2. Confined masonry walls

### 5.2.1. Model by Stafford Smith and Riddington [35]

The work by Stafford Smith and Riddington [35] proposes a set of three independent equations for shear strength of masonry walls divided into the potential failure modes: bond, tension and compression by modeling the masonry panel as a strut, as

$$V_s = \frac{\tau_0 L_m t_w}{[1.43 - \mu \left(0.8 \frac{h_m}{L_m} - 0.2\right)]} \quad (26)$$

$$V_t = 1.72 A_g f'_{tm} \quad (27)$$

$$V_c = 4f'_m \cos^2(\theta) \sqrt[4]{I_c h_m t_w^3} \quad (28)$$

where  $L_m$  is the wall panel length,  $\mu$  is the coefficient of friction between the mortar and the unit,  $h_m$  is the wall panel height,  $\theta$  is the complement of  $\alpha$  ( $\theta = \pi/2 - \alpha$ ), and  $I_c$  is the column inertia.

### 5.2.2. Model by Raymondi [36]

The Chilean code, NCh 2123 (2003) [37], uses an admissible force that corresponds to about 50% of the actual strength (safety factor of 2), based on the expression by Raymondi [36]. The strength expression by Raymondi [36] is,

$$V_n = (0.45\tau_0 + 0.24f'_n)A_g \quad (29)$$

## 5.3. Comparison

A summary of the results obtained for each type of wall and a comparison of the proposed panel type model with the models from the literature is shown in Fig. 9 and Table 4. The models by Silva [25] and Tomazevic [34] underestimate the capacity of the walls by about 30% and present a high COV, given that they are very sensitive to the area of the wall panel and the contribution of the transverse reinforcement. In the case of the model of Stafford Smith and Riddington [35], although the axial load and the reinforcement are not taking into account in the model, the strength is well predicted, but with larger COV than the proposed model. The equation by Raymondi [36] also does not take into account the effect of the reinforcement, but considers the axial load. However, the model shows an underestimation of average strength of 20% with an adequate COV. In all cases, the proposed model presents a better combination of strength prediction and low COV.

## 6. Conclusions

In this work, a panel model is proposed for masonry walls either reinforced or confined based on a model originally developed for short walls and other reinforced concrete elements, providing a novel

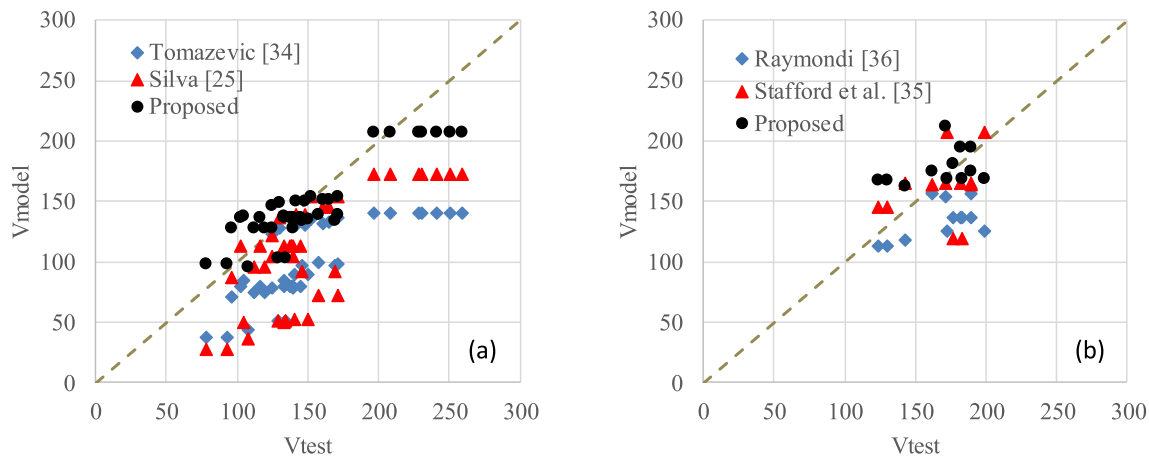


Fig. 9. Strength ratio  $V_{model}/V_{test}$  including models from literature – (a) reinforced masonry walls, and (b) confined masonry walls.

Table 4

Statistical analysis of strength estimate ratio of models from the literature for reinforced and confined masonry walls.

Reinforced masonry walls			Confined masonry walls		
Model	Avg.	COV	Model	Avg.	COV
Silva [25]	0.72	0.32	Stafford et al. [35]	0.97	0.19
Tomazevic [34]	0.65	0.23	Raymondí [36]	0.8	0.12
Proposed	1.00	0.15	Proposed	1.08	0.14

formulation that can be applicable to both materials. The modifications required for adaptation to masonry walls include its anisotropy and bond failure (based on a Mohr-Coulomb model) not present in reinforced concrete elements. The effect of the degradation of the compressive and tensile strength due to the inclination between the applied axial load and the vertical mortar joint is also analyzed.

The accuracy of the model is revised comparing the shear strength predictions of the model with a database of 53 specimens comprising reinforced (41) and confined (12) masonry walls. The ratio between the predicted shear and the experimental shear strength gives an average and a coefficient of variation (COV) of 1.0 and 0.15, respectively for reinforced walls, and 1.08 and 0.14 for confined walls, which indicates that the model satisfactorily predicts the shear capacity of masonry walls. The failure mode that predominates is tension (53% of the cases) with correct estimation of the capacity on average. In most cases, the strength ratio presents an average error less than 10% for both types of walls, except for the case of bond failure in the case of confined walls where the average error increases to 32%.

In order to study the dependency of the predictions to common model parameters, the shear strength ratio between the model and the experiments is compared with selected parameters. All selected parameters (aspect ratio, longitudinal reinforcement force per area, transverse reinforcement for per area, axial load level, compressive strength of masonry, and principal strain/stress direction) show little dependency for the strength ratio, which indicates a good incorporation of the parameter in the physical behavior of the shear strength mechanism. In general, the strength ratio varied less than 10% for the overall range of parameters.

The proposed model is also compared with models from the literature. Considering that several models from the literature are intended for shear design, the comparison is intended to show the capability of the current approach to capture the capacity for different types of walls, which is commonly not possible for formulations from the literature. For reinforced masonry walls, the models from the literature underestimate the capacity of the walls, while the proposed model yields an average of 1.0. For confined walls, there is also an underestimation of

the capacity, but to a lesser extent. In general, the proposed model presents a better combination of strength prediction and low COV, being capable of capturing the strength for all types of walls and for all failure modes.

## Appendix A. Supplementary material

Supplementary data to this article can be found online at <https://doi.org/10.1016/j.engstruct.2019.109900>.

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