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Strategic Public Transport Design Using Autonomous Vehicles and Other New Technologies

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Abstract
We examine potential improvements to public transport systems induced by the autonomous vehicle technology (AVT). To do so, we study a feeder system that operates on-demand in an idealized local zone, and the design of a trunk system that operates over a more general city model and with traditional lines. It is shown that the AVT encourages larger fleets of smaller vehicles that follow more direct routes, when compared with the traditional technology (TT). In both sub-systems, the total savings induced by the AVT reach up to one third of TT's costs. Congestion could increase by a marginal amount.

Keywords Public transport · Autonomous vehicles · Line structures

1 Introduction

Autonomous vehicles (AVs) running on the streets are expected to induce radical transformations in the way people move across cities. Immediate effects are expected on the average speed and number of trips performed by each individual (which is related to the decrease in the discomfort induced by driving). In a longer term, new mobility services should emerge (AV-taxis, for instance) which can be expected to be frontrunners in using AVs, and thus, the modal share will be affected together with decisions regarding owning vehicles and the spatial distribution of activities, among other factors. The tension between a possible better traffic management with an expected increase in the number of vehicle-kilometers traveled lead to uncertain scenarios regarding congestion and speed (as shown in [1, 2], among others).

In this context, the adaptation of public transport systems is imperative. If this is not achieved, many passengers may replace them with new private systems, increasing the risk of falling into the public transport vicious cycle [3]. Recalling that many users rely on public transport systems to move within the city (to their jobs or places of study, for instance), this could increase the inequalities in the whole transport system of a city, with some users enjoying a better system than that in the current situation, and other users being in a much worse condition. As Braess' and Down-Thomson's paradoxes have shown, the overall effect of new technologies or infrastructure might worsen the transport system; recent experiences with ride-hailing platforms as Uber or Cabify are a good example, as they have been shown to increase congestion during peak periods [4]. The opportunity cost of not improving transit systems is already a sufficient reason to study and implement these possible changes.

There are at least two ways in which AV technology (combined with other new technologies) may benefit transit systems. On the one hand, the ability of coordinating large numbers of passengers (as ride-hailing platforms, including Uber or Cabify, do) might enable massive public transport systems that work on-demand rather than with fixed lines. On the other hand, driverless vehicles are much cheaper to operate (because of no wages); thus, fleets could be heavily augmented, reducing waiting and/or access times and impacting how lines are arranged in space (line structures design). This last aspect needs special care during transition processes with current drivers, but this point (part of the wide-ranging issue about the future of work under new automatization processes) is beyond the scope of this article.

The expected impact of AV technology has triggered an intense debate on the topic during the last years. Nevertheless, most of the research has been focused on the general impacts of this technology over transport systems or over private transport modes (such as taxis). Those who have

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studied transit systems have done it mostly with agent-based models (like in [5, 6]), which provide interesting numerical results but are not so useful to obtain explicit expressions for the equations and variables that govern the model. There are some analytical models about specific aspects of plausible AV public transport systems (like in [7, 8]), but this is still a quite incipient area that needs to be deepened.

In this study, we address the potential of these new technologies to improve the design of public transportation. For this purpose, we propose a novel way to structure a public transport system that takes advantage of the AV technology, mixing on-demand systems for local trips (feeder) with a more traditional system for longer trips (trunk), which form an AV feeder–trunk system (AVFT).

The paper is structured as follows. Section 2 describes the urban model in which the AV public transport system is going to be optimized. Section 3 studies the feeder on-demand system for local trips. Section 4 studies the trunk system and how the line structure design is affected by the technology. Section 5 calculates the congestion effects of the whole AVFT system. Finally, Section 6 concludes and proposes some future lines of research.

2 Brief Description of the Urban Model

We will use an extension of the symmetric version of the city model proposed in [9]. That model is composed by a graph — represented in Fig. 1a, with g and T_0 being some spatial parameters — containing n zones, and an origin-destination (OD) pattern represented in Fig. 1b. Each zone has a periphery (P) and a subcenter (SC), which together with the CBD make $2n + 1$ nodes. As this is modeled for the morning peak, the CBD only attracts trips, and the peripheries only generate trips. Subcenters attract and generate trips. The number of trips per hour is Y, which is evenly distributed across zones according to the following parameters: a portion a of the passengers depart from the peripheries, and $b = 1 - a$ do it from the subcenters. Passengers starting from the peripheries go either to the Central Business District (CBD, a percentage α), to their own subcenter (β) or to a different subcenter ($\gamma = 1 - \alpha - \beta$, with $\frac{\gamma}{n-1}$ to each foreign subcenter). Passengers departing from the subcenters are divided into those going to the CBD ($\tilde{\alpha}$) and those going to the rest of the subcenters ($\tilde{\gamma}$), with $\alpha/\gamma = \tilde{\alpha}/\tilde{\gamma}$.

This model can represent monocentric, polycentric or dispersed cities, depending on the parameters α, β, γ. It is structured around its “primary road system,” i.e., its set of main avenues. As proposed in [10], a city can be conceived as a set of neighborhoods linked by these large avenues. In this model, arcs linking the subcenters with the CBD, together with the arcs linking subcenters, represent the relevant streets of the city. This

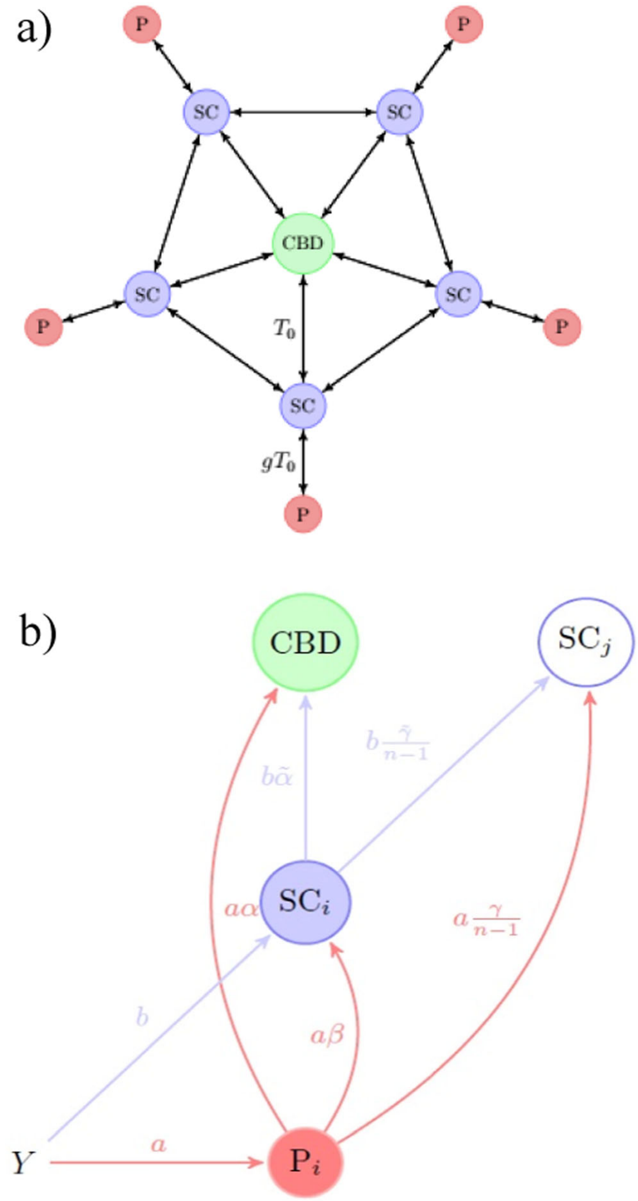

Fig. 1 The city model (**a**) and its OD pattern (**b**)

is the main argument why a grid — a graph that is commonly used in transport models — is an inefficient means of representation: all the arcs are evenly relevant and all the nodes are also similar, and thus, neither the big avenues nor the centers can be well modeled. A hierarchical representation is required.

This model is useful for a macroscopic model of the city. Nevertheless, the mobility requirements within each zone are overlooked. Local trips are relevant for the purpose of this research, as the ability of coordinating massive numbers of users has been claimed as one of the comparative advantages of the new technologies: aggregating passengers does not permit taking into account their spatial distribution, a factor that can be decisive when analyzing a determinate (novel) way of operating a public transport system.

With this in mind, local trips are going to be modeled making a “zoom” on each of the peripheries and subcenters of the city model. Every node is going to be considered as continuous in space, with the passengers homogeneously distributed. The trunk stations are assumed to be at the center of the node, and the goal of the local system is to carry every passenger from their original point of departure to that trunk station. As in the morning peak destinations tend to be spatially concentrated, they are assumed to be at the center of the destination node. Intrazonal trips are not going to be modeled.

3 The Feeder Model

3.1 The Model

We propose a feeder system that responds on-demand to take each passenger to the center of the circular zone (radius R). When a passenger X requests a trip, one of the two following options happens:

a) If another passenger made a recent request and has an assigned vehicle, which is not full of passengers yet, and the distance that X needs to walk is less than d (which is a design variable), then X is assigned to that vehicle.
b) Otherwise, a new vehicle is assigned to pick up X.

Users first request the trips and then walk to the gathering point where they meet the other passengers and the vehicle in order to be transferred to the center of the zone. Regarding the vehicle movements: it picks up passengers at some point inside the zone, it carries them to the center of the zone, and then it goes back to the same point to pick up new passengers. The system is designed in such a way that spatial and temporal homogeneity assures that an equilibrium will be reached, i.e., that enough passengers are going to be assigned to it and that each passenger will be assigned to a vehicle; this is the rebalancing process for this system, which is quite simple and allows for analytical solutions, as will be shown below.

The capacity of the vehicle is K, which is adjusted in such a way that the precise number of passengers have arrived when the vehicle arrives. As there are no intermediate stops and vehicles are expected to be small, the time for passengers' boarding and alighting should be small, and thus, is neglected for design purposes. This is a considerable advantage in comparison with traditional public transport systems, as time at the stops strongly decreases the commercial speed of vehicles. Traveling times are also reduced by requesting the passengers to go to the gathering point instead of picking each user up in their domicile, which avoids detours.

The number of passengers per hour will be denoted as y. Design variables are fleet B, vehicle capacity K, and maximum walkable distance d, all modeled as continuous variables. These variables are going to be optimized, thus minimizing the value of the resources consumed, which is defined following [11, 12], among others:

$$VRC = (c_0 + c_1K)B + yp_w\overline{t_w} + yp_v\overline{t_v} + yp_a\overline{t_a} \tag{1}$$

Where p_w, p_v, and p_a are the users' values of waiting, in-vehicle, and access times, respectively, and t_w, t_v and t_a are the respective average times observed in the system. Parameters c_0 and c_1 are of special interest for this study, as they represent the unitary operator costs, which vary strongly with AV technology; we will show these differences in detail when we optimize the model numerically.

Let us study these expressions. Regarding users, the in-vehicle average time is just the average distance from any point to the center of the circle, divided by the commercial speed of the vehicles V:

$$\overline{t_v} = \frac{2R}{3V} \tag{2}$$

Something analogous happens with the access time. As the vehicle covers a demand located in a circle of radius d, the average access time can be reasonably estimated as the average distance between any point and the center of that circle, divided by the walking speed v_a:

$$\overline{t_a} = \frac{2d}{3v_a} \tag{3}$$

Passengers need to wait for other passengers to emerge, as a vehicle will only depart with K passengers. As all passengers walk in average the same time, and the system is rebalanced to coordinate the vehicle's and the last passenger's arrival, the average waiting time is estimated as the difference between request times. Denoting $s = \frac{y}{\pi R^2}$ as the flow of passengers per time and area unit, the average time needed for K passengers to emerge is $\frac{K}{s\pi d^2}$, which would be the waiting time for a passenger that requests the vehicle exactly when the previous vehicle (at a distance lower than d) gets full and departs. On average, users wait half that time:

$$\overline{t_w} = \frac{K}{2s\pi d^2} \tag{4}$$

Before analyzing operators' costs, it is interesting to obtain a better insight into expressions (2–4) and compare them with analogous expressions in studies regarding classical public transport systems. In-vehicle time is constant, which differs from all those models in which vehicles have many stops. Access time depends linearly on a design parameter d, which can be interpreted as analogous to the line separation in models in [13, 14]. Finally, waiting times do not depend in this case on how long it takes for the vehicle to arrive, but on the time needed for gathering all passengers. This seems to be quite different from the waiting time expressions in other models (such as [11] or [12]).

To analyze the operators' performance, let us define ρ as the percentage of the time that a vehicle is carrying passengers. Recalling that the vehicle drops its passengers and then returns to the initial point, it is direct to observe that

$$\rho = \frac{1}{2} \tag{5}$$

Nevertheless, as this value does not consider the time that the vehicle spent stopped, and it relies on assumptions over homogeneity and the fact of having only origin-trips, it is useful to consider it as a benchmark. This is why results are going to be expressed as a function of ρ in order to enlighten the role of the rebalancing process, and equation (5) will only be used for numerical analysis. A relationship between the fleet size and vehicle capacity can be obtained by equating two expressions for the total passenger-kilometers traveled in a unit of time:

$$BVK\rho = y\frac{2}{3}R \tag{6}$$

The left side of equation (6) comes from the supply side, as it represents how many pax-km are toured in the vehicles during one period of time. The right side comes directly from the demand. Recalling that in (4) it was shown that the average waiting time depends inversely on K, (6) shows that the larger the fleet is, the less time the passengers need to wait, in a similar way to the referred previous models.

Replacing equations (2–6) in (1) poses the following optimization problem:

$$(\mathrm{P})\ \min_{B,l}\ \left(c_0 + c_1\frac{2}{3}\frac{yR}{BV\rho}\right)B + p_w\frac{R^2}{2d^2}\frac{2yR}{3BV\rho} + yp_v\frac{2R}{3V} + yp_a\frac{2d}{3v_a}\ \mathrm{s.t.}\ B, d \geq 0 \tag{7}$$

Making the derivatives with respect to B and d equal to zero yields

$$B^* = \frac{R^{3/4}{p_a}^{1/2}{p_w}^{1/4}}{\rho^{1/4}V^{1/4}{v_a}^{1/2}}y^{3/4}\left(\frac{1}{3c_0}\right)^{3/4} \quad d^* = y^{-1/4}\left(\frac{3R^3c_0p_w{v_a}^2}{\rho Vp_a^2}\right)^{1/4} \tag{8}$$

Which implies

$$K^* = \frac{2}{3^{1/4}}\frac{R^{1/4}{v_a}^{1/2}{c_0}^{3/4}}{V^{3/4}{p_w}^{1/4}{p_a}^{1/2}}y^{1/4} \tag{9}$$

Some interesting aspects of these results are the following:

- There are scale economies for both operators and users. Indeed, in each of the summands, y is to the power of some exponent lower than (or equal to) 1, meaning that the average costs decrease with y. This is the case for operators in the summand involving c_0 because the fleet grows less than linearly (the capacity grows too). For users, scale economies come from decreasing waiting times (because the capacity grows less than linearly) and access times (because d^* decreases with y).
- The decrease rate for the distance walked by passengers is lower in this model than in [13] or [14], as for them the separation between parallel lines resulted proportional to the cubic root of the inverse of the demand, and in this case it is the fourth root. The main difference between those models and the one studied here is that in this case the waiting time depends directly —decreasing with it — on the distance walked by users.
- Nevertheless, the ratio between the average access and waiting times is proportional to B^*d^{*3}, which clearly does not depend on y. This ratio is constant, independently of how many passengers are using the system. Further, it is proportional to the quotient between the respective time values p_w and p_a. This interesting property is preserved from the results of [13, 14].

3.2 Results

Numerical simulations of this model were performed. The parameters are based on Santiago, Chile, and are shown in Table 3 in the Appendix. Only cost-related parameters are listed in Table 1, because the AV technology plays a crucial role on them. These parameters are the result of linear regressions over the values that we adapted to Santiago from the costs estimated in [15]. Results are going to be calculated for AVE (autonomous vehicles powered with electricity), AVF

Table 1 Parameters that define operator costs for different types of vehicles

	AVE	AVF	Trad
c_0[US$/ hour]	4.02	4.38	29.96
c_1[US$/ pax-hour]	0.29	0.32	0.33
R^2	0.85	0.8	0.87

(autonomous vehicles powered with fuel), and traditional vehicles (neither autonomous nor electric). The linear correlation R^2 is also presented for each of these vehicle types. The most relevant difference occurs in c_0 because drivers' salaries are expressed in that parameter.

Figure 2 shows the average cost per trip — i.e., the value of the resources consumed including users' and operators' costs — for different levels of patronage. Decreasing curves are expected, as scale economies were previously identified. The difference between AV costs and not-AV costs is particularly important, whereas fuel versus electricity does not seem relevant from the cost point of view. The reasonable range of the average costs for autonomous vehicles shows that operating on-demand is feasible.

>Figures 3 show the operational characteristics of this system: waiting (a) and access (b) times for users, and vehicle capacity (c) and fleet size (d) for operators. In all these figures, the differences between AVE and AVF are quite low, while the traditional vehicles exhibit relevant differences; AV systems use much larger fleets of much smaller vehicles (a direct consequence of the lower fixed value per vehicle), allowing for smaller waiting and access times.

As a final remark for this section, let us explain that an analogous model for intrazonal trips, which do not share their final destination, would lead to unrealistic results (with vehicles' capacities lower than 0.25); that is to say, having a common final point is a key need in order for such an on-demand system to be suitable.

4 The Trunk Model

The trunk system is aimed to transport the passengers across the graph shown in Fig. 1a, according to the OD pattern described in Fig. 1b. It operates with independence of the feeder system, but as the feeder is dropping its passengers just at the connections, there is no access time. As now the OD pattern is more complex than that in the previous section, several lines are going to be used. The first question is then which lines to

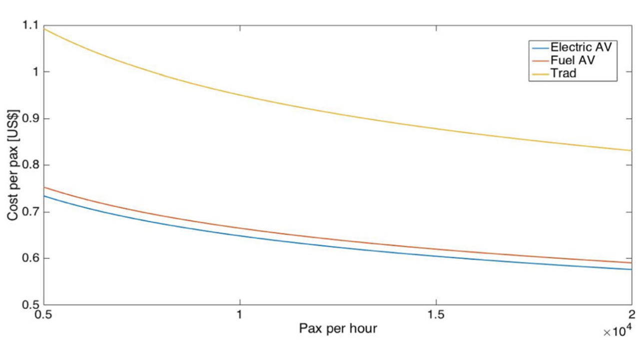

Fig. 2 Average cost per passenger for different types of vehicles in the feeder system

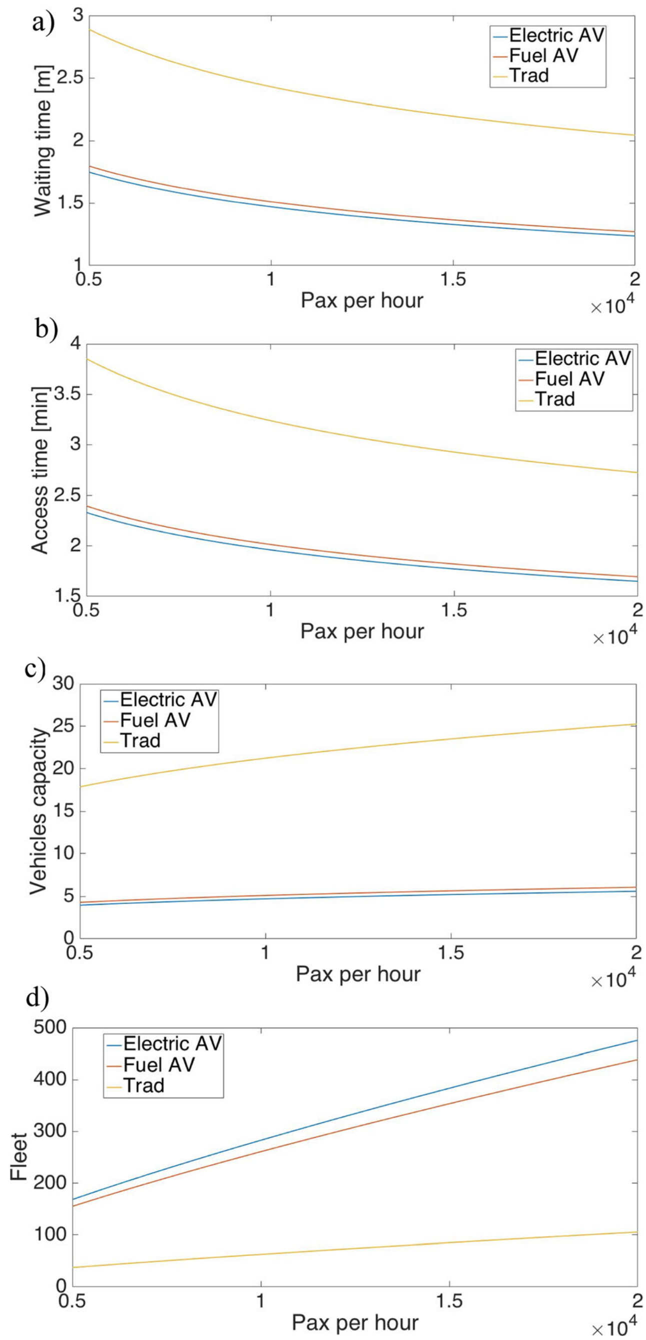

Fig. 3 Operational characteristics of the system for different types of vehicles

use, i.e., which is the *line structure* that best serves this demand. The problem of finding the optimal set of public transport lines has been shown to be NP-hard for many of its specifications [16], and thus, we are going to search for the best structure within a predefined set of structures (as in [17]). For each of those structures, a *VRC* function similar to equation (1) is going to be defined, with two relevant novelties: the design variables are now the fleet size for each line (the vehicle capacity is determined by a constraint that assures that all passengers fit into the vehicles), and user costs now take into account how many transfers each user is making within the trunk system, with a fixed cost per transfer p_T. We will not deepen on these aspects as they are all explained in more detail in [17].

Four structures are going to be considered, namely, hub and spoke, local–global,[1] no transfers, and no stops. These are the same structures studied in [17], but the cost parameters are now different because of the AV technology, and thus, different results are expected. In other words, in this section we study specifically which is the effect of the AV technology when studying the traditional public transport design, including the line structure choice. These line structures are shown in Fig. 4 (where only lines emerging from the "south" zone are drawn), and can be briefly described as:

- Hub and spoke (HS): most trips go to the CBD, and if they do not finish there, they take a second bus to their final destination.
- Local–global (LG): all passengers that depart from a periphery take first a "local" line to their own subcenter where they alight. There exist "global" lines connecting each pair of subcenters and each subcenter with the CBD.
- Direct (DIR): it is a direct-type structure, where each OD pair is connected by some line without need of transferring.
- Exclusive (EXC): it is also a direct-type structure, but each OD pair is connected by an exclusive line that does not have intermediate stops.

Let us analyze the numerical simulations performed varying the total number of passengers per hour in the city Y. The rest of the parameters are shown in Table 3 in the Appendix, and we are again comparing results for the operator costs' parameters from Table 1.

To analyze the results of this model, let us first look at Fig. 5, in which the average costs per trip are shown per vehicle type and line structure, with parameters $\alpha = 1$, $\beta = \gamma = 1/2$. The fuel AV is not considered because of its similarity with the electric AV, and LG is not drawn because its results were always by far the worst. Several conclusions are reached from Fig. 5:

– Using the AV technology can reduce up to almost one third of the average costs. This analysis reinforces what was argued in the Introduction: the potential for improving transport public systems is huge.
– The line structure optimization depends strongly on the technology. Without the AV, the best line structure

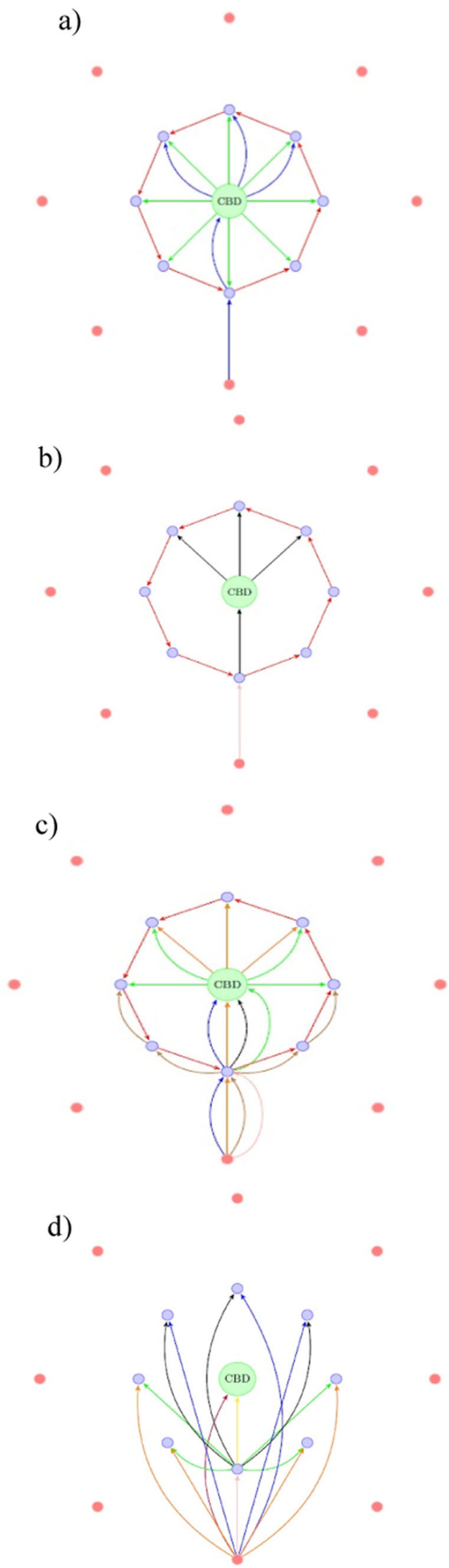

Fig. 4 Line structures optimized and compared for the trunk system

[1] In [17] this is defined as feeder–trunk. In this study, we change its name to avoid confusions with the general feeder–trunk system.

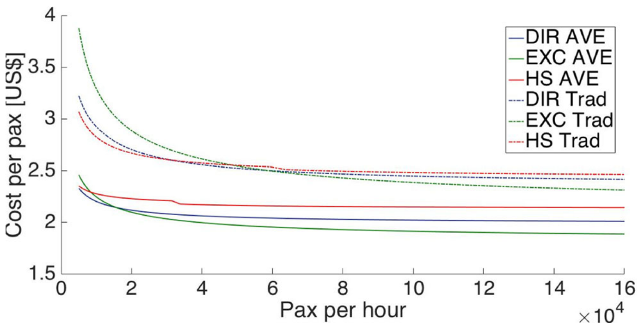

Fig. 5 Average cost per passenger for different lines structures and types of vehicles in the trunk system

evolves — as patronage grows — from HS to DIR and then to EXC. With the AV, HS is never optimal, and NS becomes the best line structure for lower numbers of passengers. That is to say, this new technology might induce trunk systems to increase what is defined in [18] as "directness," i.e., to diminish time at stops, length of the routes traveled by passengers, and number of transfers.

– There are also scale economies, which appear within each line structure, but that are significantly amplified each time a change in line structure occurs (as in [18]).

Operational characteristics are well synthesized by the operators' and users' costs per trip, which are shown in Fig. 6. As the direct impact of the AV technology is on operator costs, it is expected that the largest differences between the two curves is expressed there (6a). Nevertheless, Fig. 6b shows that users also benefited from the new technology, as lower values for c_0 and c_1 allow for higher frequencies and more direct structures. The "discontinuities" on the curves represent the exact levels of patronage where the line structure changes.

Until now, we have studied the results of this model for different levels of patronage. Nevertheless, the literature (as in [17, 19]) shows that the internal distribution of the trips also affects transit design, including the line structure. An analogous analysis but using α, β and γ as the variables, reveals that different line structures dominates for different types of cities (as in [17]), but this comparison is affected by the AV technology, which again reduces total costs significantly.

5 A Gross Analysis of Impact on Congestion

A relevant issue is how these changes would affect congestion. To analyze that, we are not considering changes in modal share, as they are quite unclear; instead, we just determine if these models increase or decrease the amount of road space required by the public transport vehicles for a fixed patronage. To do so, we calculate the equivalent-cars (EC) used by the AVFT system, and we will make a gross comparison with a known value for similar parameters. EC is a function of the

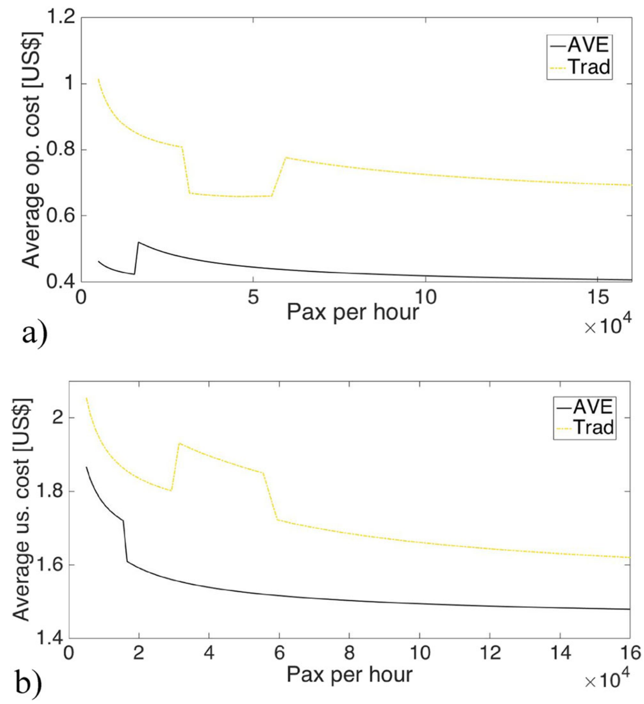

Fig. 6 Resulting average (**a**) operators' and (**b**) users' cost in the trunk system

capacity of each vehicle, but the exact expression is not clear. Here, we will use as a benchmark the expression proposed by [20] for traditional vehicles:

$$EC(K) = \frac{K}{100} + 1 \qquad (10)$$

The parameters used in the model were based on Santiago, Chile. According to the last origin–destination survey [21], the number of passengers that use the public transport system Transantiago in one morning peak hour is 553,581. To make a correct comparison, intrazonal trips shall not be considered (because our model is not carrying them), yielding a patronage of $Y = 393{,}013$. The area of Santiago without CBD is about 620 km^2 and 12 zones, each of them composed by a periphery and a subcenter with radius of 2 km, constitute a reasonable approximation to cover the whole area. Each periphery generates $y_P = 26201$ trips per hour, and each subcenter generates $y_{SC} = 6550$. Transantiago operates with 6154 buses and 184 metro trains during peak morning; as each metro train's length is equivalent to about ten average buses, the Transantiago's fleet can be considered as composed by 7994 vehicles offering 642,141seats [22]. The values for both systems are presented in Table 2.

For a proper comparison, it needs to be considered that Transantiago's values should be lower as intrazonal trips are not being modeled. Results show that congestion might increase; but this increase would be marginal (about five thousand equivalent cars, and in Santiago there are more than one million private cars on the streets during a peak hour).

Table 2 Comparison of equivalent car fleets for the feeder–trunk model and current operation of Transantiago

	Fleet	Number of seats	*EC*
Feeder system — peripheries	$583 \cdot 12 = 6996$	$6 \cdot 583 \cdot 12 = 41976$	$6996 + \frac{41976}{100} = 7415$
Feeder system — subcenters	$206 \cdot 12 = 2472$	$5 \cdot 206 \cdot 12 = 12360$	$2472 + \frac{12360}{100} = 2596$
Trunk system	12202	562133	$12202 + \frac{562133}{100} = 19643$
AVFT Total	21470	616469	22,281
Transantiago	7994	642141	$7994 + \frac{642141}{100} = 14415$

6 Synthesis, Conclusions and Future Research

In this study, we analyzed how massive public transport design should evolve when incorporating AV and other new technologies. We first recognized two main potential benefits for transit: 1) flexibility and coordination capacity may enable massive systems working on-demand instead of the traditional fixed lines, and 2) the optimal design of frequencies, bus sizes, and lines' routes should be affected by driverless technology, as one key source of expenditure (wages) can now be saved.

To address these issues, we proposed a feeder–trunk AV public transport model, where feeder vehicles respond on-demand to carry passengers within a local area (modeled as an uniform circle), from their origins to the center of that zone, to connect with a trunk vehicle. The feeder system admits explicit solutions, such that its fleet is proportional to $y^{3/4}$, while the maximum distance walkable by passengers is proportional to the inverse of that expression. The vehicle capacity grows proportionally to $y^{1/4}$. There are scale economies for the operators (because the fleet grows less than proportionally) and for the users, in waiting and access times. Numerical results were obtained considering three types of technology that affect the parameters related to operators' costs: electric autonomous vehicles, fuel autonomous vehicles, and traditional vehicles (fuel-powered and human-driven). Results showed that the differences between fuel and electric vehicles are small (emissions are not considered in this model), but traditional vehicles could cost approximately 50% more than AVs. Waiting times, access times, number of vehicles required, and vehicle size were all within reasonable values. This last aspect was particularly affected by the AV technology: 5 pax per AV vs about 20 pax per traditional vehicle.

The trunk system operates under a traditional public transport scheme with fixed lines and bus stops. We optimized the line structure (i.e., the spatial arrangement of the lines) and, for each line, it's fleet and bus capacity. Results showed that the traditional technology costs approximately 50% more than the AV technology, which is expressed both in operators' costs (owing to direct savings) and users' costs (as the operation of each bus is less costly, frequencies increase and lines become more direct). The comparison between the line structures also changes, as the more direct line structures become better.

Finally, we performed a gross analysis about the possible congestion effects induced by changing the public transport design according to this model, with data corresponding to Santiago, Chile. We showed that, disregarding the potential effects of the AV technology over modal share and vehicle speed, a marginal increase in congestion is expected.

Most relevant conclusions can be synthetized as follows:

– Public transport can operate on-demand, provided that at least two conditions are satisfied: capacity for mass and simultaneous coordination of passengers (which seems to be easily fulfilled with the new technologies, but requires that all passengers have access to these technologies); and a large number of passengers sharing their origin and destination. This last condition can be relaxed to share only the origin, as shown in this study, by adding transfers to a second system that carry each passenger to their final destination.
– The adaptation of traditional public transport lines to the AV technology moves the system towards larger fleets of smaller vehicles and to line structures that prioritize direct routes, reducing the number of intermediate stops.

As this is an emerging topic, there are countless directions for future research. Regarding the two models studied in this article, both the urban scheme and the OD pattern may be made more complex to represent more realistic situations, introducing asymmetry and heterogeneity within passengers at the local and global scales. Losing the symmetry of the model would require more detailed designs, while spatial and temporal heterogeneity would induce some randomness that hinders the perfect adjustment of fleets and vehicle sizes to demand. Idle capacity and/or overcrowding could emerge as problems. The combination of the two systems is also something that needs research, as there is room for coordination and for an optimal design of the connection stations. Moreover, operation schemes other than feeder–trunk should also be studied, and their results compared with these ones.

In this model, intrazonal trips were not considered. This is a major issue, as they represent a high percentage of transit trips, and the feeder system designed here took serious advantage on the common destination of passengers. Non-shared trips and/or local traditional lines might be necessary. Besides, the possibility of avoiding transfers for those passengers that go to very attractive places (as the CBD) should also be explored: would it be efficient to have some of the feeder vehicles crossing zones in order to carry passengers directly to their final destination?

For all these results, changes in the model share are determinant. Although there are some emerging studies about how the AV technology could impact mode choice, they do not incorporate how public transport could be improved. This is another relevant direction for future research. Long-term effects related with urban development, the transition that mixes this type of vehicles with traditional ones and the crucial issue of replacing human jobs with machines should also be investigated.

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Appendix

Table 3 Numeric value of the parameters

Parameter	V	T_0	g	n
Value	20 [km/h]	30 [min]	1/3	8
Parameter	a	b	R	p_v
Value	0.8	0.2	2 [km]	2.32 [US$/h]
Parameter	p_w	p_a	p_T	v_a
Value	4.64 [US$/h]	6.96 [US$/h]	0.58 [US$]	4 [km/h]

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