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Vagueness in multidimensional proposals

Qiaoxi Zhang

Department of Industrial Engineering, University of Chile, Av. Republica 701, Santiago, Chile

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ABSTRACT

This paper studies how agents choose to be vague in their proposals in a delegation environment. Two agents compete for the approval of a decision maker to implement a multidimensional action. Based on their knowledge of the consequences of actions, agents propose future actions but can be vague about any dimension. The decision maker, uncertain about the consequences of actions, chooses one agent to act. I show that vagueness on the dimension where one stands closer to the decision maker than his opponent preserves such an advantage, while preciseness undermines it. Vagueness therefore tends to occur on agents' advantageous dimensions.

1. Introduction

In many situations, a decision maker (DM) needs to choose between competing agents based on information provided by those agents. How should an uninformed DM evaluate the statements made by the agents? From the agents' perspective, when they can only propose actions and the DM is not sure what the best action is, how can they convince her that one proposal is better than the other? If, in addition to proposing actions, agents also have the choice to be vague in their proposals, will they choose to do so? How does the choice of vagueness shape such communication? I discuss how the answers to these questions relate to the agents' interest alignment with the DM. As it turns out, not only can vagueness happen, if carefully deployed it could even help an agent achieve a better outcome.

As an example, consider two political candidates competing for the vote of a representative voter so as to influence policies on multiple issues. A candidate may be aligned with the voter on certain issues, but biased on others. Each player would like the policies to bring about her (his) ideal consequences for the issues, but there is uncertainty about what those policies would be. The candidates are experts who understand the consequences of policies and make proposals based on their expertise during the campaign. For each issue, they can choose to propose a specific policy, or to be vague and not propose any policy at all. The voter, who understands the candidates' biases, then elects her most preferred candidate.

I consider a model with two informed agents and one uninformed DM. Nature determines a state of the world. Agents observe the state, and simultaneously make proposals to the DM. The DM, who does not observe the state, updates her belief about it given the proposals, and chooses one of the agents. The chosen agent implements his proposal, which determines the payoff of all players. More specifically, the state of the world is a pair of two numbers, the first for issue 1 and the second for issue 2. The DM would like the actions to match the state, while each agent would like the actions to differ from the state by his bias. The biases are commonly known. A proposal is a pair of messages, one for each issue. Each message

E-mail address: jdzhangqx@gmail.com.

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could either be specific, in which case the agent announces a specific action that he commits to, or vague, in which case the agent is free to implement any action if chosen.[1]

First, let us suppose that the state space is one-dimensional. Given any state θ, the DM's ideal action is θ, Agent 1's ideal action is $\theta - 3$, and Agent 2's ideal action is $\theta + 5$. Each player would like to minimize the distance between the implemented action and their ideal action. Consider the strategy profile where at each θ, Agent 1 proposes $\theta - 3$, while Agent 2 is vague. Since Agent 2 will implement his own ideal action if chosen, the DM strictly prefers Agent 1. Now suppose that Agent 2 deviates to a specific proposal $y = \theta + 2.5$. Since the DM learns the true state from Agent 1's proposal, she realizes that this action is better and chooses Agent 2. Therefore, by being specific, Agent 1 reveals the state and allows undercutting by his opponent.

Now consider the strategy profile where both agents are always vague. Therefore, if chosen, each agent will implement his own ideal action. Since Agent 1 has a smaller bias, the DM again chooses Agent 1. Now suppose that Agent 2 deviates to the same specific proposal $y = \theta + 2.5$. Although this action is indeed better than Agent 1's action, since Agent 1 does not reveal the state, the DM is free to form any belief about how good y is; in particular, she is free to believe that y is Agent 2's ideal action and still choose Agent 1. Therefore, vagueness allows Agent 1 to take advantage of the DM's ignorance about the state, prevent credible compromises by his opponent, and preserve his advantage.

The insight above can be generalized to a two-dimensional state space where Agent i has a smaller bias on dimension i. That is, dimension i is Agent i's advantageous dimension and Agent $(-i)$'s disadvantageous dimension. In the main result of the paper, I pin down where vagueness occurs depending on the advantages of the agents. Within a class of equilibria, when one agent's bias on his disadvantageous dimension is sufficiently large, vagueness occurs on the advantageous dimensions (Proposition 1). When biases are rather comparable, with additional restrictions on the equilibria, a similar result can be obtained (Proposition 2). To characterize equilibria in which vagueness occurs on agents' advantageous dimensions, I first show that for any bias of the agent, vagueness which occurs on both dimensions can always be sustained in equilibrium (Proposition 3). I then characterize a class of equilibria in which vagueness occurs on the advantageous dimensions only and specific actions are proposed for all states (Proposition 4). The existence of equilibria in my model is not surprising because in signaling games many equilibria can be supported with unreasonable beliefs. I then introduce an appropriate extension of an equilibrium refinement for signaling games and show that all equilibria identified are robust to this refinement (Proposition 5).

The result has implications on how informed parties communicate to uninformed decision makers. In the example of political competition, it shows that parties have incentives to inform voters of their policies on their opponents' advantageous issues. In 2000, the U.S. Republican presidential candidate George W. Bush focused on the education issue, which is traditionally perceived as an issue in which the Democratic party is more competent. On the other hand, the 1996 U.S. Democratic presidential candidate Bill Clinton focused on the issue of criminality, turning a Democratic weakness into a strength (Aragonès et al., 2015). Apart from these anecdotal evidence, Damore (2004) presents campaign advertisement data from major party presidential candidates between 1976 and 1996 to show that in fact, such issue trespassing is not uncommon. It appears that all candidates issue trespass. This ranges from a high of 34% for Walter Mondale in 1988 to a low of 4% by George H. W. Bush in 1988. On average, 15% of the campaign advertisements focused on issues dominated by opposing parties. Petrocik et al. (2003) analyze the content of presidential candidates' acceptance speeches and television commercials throughout 1952-2000. They categorize issues into "Democratic," "Republican" and "Performance" and count the number of times an issue belonging to each category is mentioned. Whenever an issue is mentioned during Democratic TV commercials, only 22% of the time does it belong to Democratic issues. 44% of the time, it belongs to a Republican issue. The Republicans devote roughly equal number of mentions to Democratic and Republican issues - both are around 40%.

1.1. Related literature

To my knowledge, this is the first paper to address competition in delegation in a multidimensional action space. Ambrus et al. (2015) study competition in delegation in a one-dimensional setup. In their model, two imperfectly informed agents propose actions to a DM who cannot measure the difference between two actions. In my model, the DM has no such impairment and the agents are perfectly informed; moreover, the agents have the choice to be vague. For one-agent delegation, Alonso and Matouschek (2008) study the DM's problem of how to optimally restrict the set of actions that an agent can take; Li and Suen (2004) discuss when a DM should delegate, and how biased the agent should be in order to be delegated. In contrast I focus on the agents' communication to the DM and assume that the only choice that the DM can make is choosing between two agents.

The question of what drives parties' campaign issue choices has been studied extensively. Early evidence and theory suggest that parties focus on issues in which they have an advantage and ignore those in which their opponents have an advantage (Riker, 1993; Petrocik, 1996). Historical counterexamples and more recent empirical evidence show that parties spend considerable effort on the opposing parties' advantageous issues (Sigelman and Buell, 2004). Some papers seek to

[1] Here I assume that agents have commitment power to implement the specific actions they announced. The political science literature has not been consistent. Some papers assume campaign messages are cheap talk (for example, Alesina (1988) and Harrington (1992)), while others assume they are commitments (for example, Donald (1983) and Calvert (1985)).

reconcile this conflict by focusing on how campaigns shift voters' preferences (Amorós and Puy, 2013; Aragonès et al., 2015; Dragu and Fan, 2016). I take a more traditional approach by assuming that preferences are fixed. As it turns out, incomplete information alone can get a sharper result. Krasa and Polborn (2010) study a game in which specialized candidates choose future efforts on each policy area if elected. They give precise conditions on voters' preferences that determine when parties should focus on the same or different issues. Again, the driving force of my results comes from information revelation instead of the voter's preference.

Ash et al. (2017), Egorov (2015), and I all attribute issue choice to information asymmetry. Ash et al. (2017) study how an incumbent's desire to signal his policy preference makes him pursue issues on which voters disagree. I study a competitive setting in which candidates simultaneously make proposals. Moreover, the uncertainty in my model is a payoff-relevant state of the world. Lastly, I focus on how candidates' issue advantages, rather than issue divisiveness, affect issue choices. Egorov (2015) studies how the desire to signal competence makes a challenger signal on the same issue following an incumbent's issue choice. He assumes that voters get better information when both signal on the same issue. As a result, a challenger may have incentives to signal on an issue in which he is less competent. In my paper, the information revealed is endogenously determined through equilibrium. Therefore, the underlying intuition in my model is equilibrium reasoning instead of candidates' trade-off between favorableness and informativeness of signals.

Others model the campaign issue choice problem through channels other than information asymmetry. The mechanisms they offer include social agreement on an issue as well as social discontent (Colomer and Llavador, 2012), non-expected utility (Berliant and Konishi, 2005), shifting re-election campaign issue focus (Glazer and Lohmann, 1999), etc. In my paper, there is only one voter so there is always social consensus. All players are expected utility maximizers. The game is one-shot instead of sequential. Therefore, I offer significantly different explanations.

I will mention papers supporting vagueness[2] in a one-dimensional issue space briefly. To formalize the concept of vagueness, Meirowitz (2005) uses the same setup as mine and models vagueness as a complete lack of policy commitment. Others model vagueness as a commitment to a set of policies instead of a single policy (Alesina and Cukierman, 1990; Aragonès and Neeman, 2000; Aragonès and Postlewaite, 2002; Alesina and Holden, 2008; Callander and Wilson, 2008; Kartik et al., 2017; Baghdasaryan and Manzoni, 2019).[3] Similar to my model, both models by Kartik et al. (2017) and Baghdasaryan and Manzoni (2019) have an uncertain state of the world which determines players' payoffs. The difference is that in their models, the state is unknown to the candidates during the campaign stage and therefore vagueness benefits the electorate in that it allows the winning candidate to adapt the policy to the state. In my model, the candidates observe the state from the outset and therefore the role of vagueness is different. There are also a few papers on the role of vagueness in coordination and signaling games (De Jaegher, 2003; Lipman, 2009; Serra-Garcia et al., 2011; Agranov and Schotter, 2012; Blume and Board, 2014). Similar to these papers, I show that vagueness can be supported in equilibrium. In addition, the multidimensional setup allows me to pinpoint on which dimension vagueness is likely to occur.

Broadly, this paper belongs to the literature that studies communication of private information in a competitive setting. Communication can take different forms. Many papers, this paper included, study communication through binding policy proposals (Roemer, 1994; Schultz, 1996; Martinelli, 2001; Martinelli and Matsui, 2002; Heidhues and Lagerlöf, 2003; Laslier and Van der Straeten, 2004; Loertscher, 2012; Morelli and Van Weelden, 2013; Jensen, 2013; Kartik et al., 2015; Ambrus et al., 2015). Others study communication through cheap talk (Austen-Smith, 1990; Krishna and Morgan, 2001; Battaglini, 2002, 2004; Schnakenberg, 2016; Kartik and Van Weelden, 2014). Banks (1990) and Callander and Wilkie (2007) analyze models in which misrepresenting one's policy intentions is costly. Gul and Pesendorfer (2012) and Gentzkow and Kamenica (2016) study information provision with competing persuaders. It is worth noting that many models show competition can increase the amount of information revealed (Milgrom and Roberts, 1986; Battaglini, 2002; Gentzkow and Kamenica, 2016). Here, when communication takes the form of proposals and uncertainty involves the state of the world as well as action choices, competition may lead to less certainty.

2. The model

There are three players: one DM and two agents, Agent 1 and Agent 2. Agents observe a state of the world $\theta = (\theta^1, \theta^2) \in \Theta \equiv \mathbb{R}^2$, chosen by Nature according to the probability measure F with support Θ.[4] A (pure) strategy of Agent i is $s_i : \Theta \to M \equiv (\mathbb{R} \cup \{\varnothing\})^2$, where $\varnothing$ denotes vagueness. Given s_i, $s_i^k(\theta) \in \mathbb{R} \cup \{\varnothing\}$ denotes Agent i's dimension-k proposal at state θ. Given Agent i's proposal m_i, $y_i = y_i(m_i) \in \mathbb{R}^2$ denotes the corresponding action and $x_i = x_i(\theta, y_i) = \theta - y_i$ denotes the corresponding outcome. Throughout the paper, I focus on equilibria in which agents play pure strategies. Given a proposal profile $(m_1, m_2) \in M \times M$, the DM updates her belief about the state $\mu(\cdot \mid m_1, m_2) \in \Delta(\Theta)$ and chooses the probability that Agent 1 implements his proposal $\beta(m_1, m_2) \in [0, 1]$.

[2] Vagueness in political science literature is usually labeled as ambiguity. To avoid confusion with ambiguity in the decision theory literature, I use the term vagueness.

[3] My main result trivially extends to the setup treating vagueness as commitment to a set of policies: an equilibrium in which at each state, each agent is precise about his advantageous dimension and committing to a subset containing his ideal action on his disadvantageous dimension does not exist.

[4] I discuss the implications of a restricted state space in Appendix E.

Players' payoffs are determined by the state and the chosen agent's proposal. In particular, the agents do not receive additional payoffs from being chosen. For $j = 0, 1, 2$, $b_j = (b_j^1, b_j^2) \in \mathbb{R}^2$ denotes the ideal points of the DM, Agent 1, and Agent 2, respectively. Player j's payoff function is denoted by $u_j(\theta, m_1, m_2, \beta)$:

$$u_j(\theta, m_1, m_2, \beta) = \beta(m_1, m_2) v_j(\theta, 1, m_1) + (1 - \beta(m_1, m_2)) v_j(\theta, 2, m_2),$$

where

$$v_j(\theta, i, m_i) = \begin{cases} -\|(\theta - m_i) - b_j\|^2 & \text{if } m_i \in \mathbb{R}^2, \\ -\|(\theta^1 - m_i^1, b_i^2) - b_j\|^2 & \text{if } m_i \in \mathbb{R} \times \{\emptyset\}, \\ -\|(b_i^1, \theta^2 - m_i^2) - b_j\|^2 & \text{if } m_i \in \{\emptyset\} \times \mathbb{R}, \\ -\|b_i - b_j\|^2 & \text{if } m_i = (\emptyset, \emptyset). \end{cases}$$

Whenever an agent is vague on a dimension, he implements his own ideal action for that dimension; otherwise he commits to the specific action proposed. I normalize $b_0 \equiv (0, 0)$ and assume that $|b_1^1| < |b_2^1|$ and $|b_2^2| < |b_1^2|$ throughout the paper. Moreover, without loss of generality I assume that $\|b_1\| \le \|b_2\|$. Dimension 1 is called Agent 1's advantageous dimension and Agent 2's disadvantageous dimension. The opposite is true for dimension 2. The state distribution, rules of the game, and each player's payoff function are common knowledge.

One of the challenges in characterizing the equilibrium choices of vagueness is that the set of possible equilibrium strategies is large. Each agent at each state could choose to be vague on 0, 1 or 2 dimensions. If he chooses to be specific on a dimension, he is free to choose any number on the real line. Moreover, in order to rule out a certain proposal profile, sufficient knowledge about the entire equilibrium is necessary, the DM's belief function $\mu(\cdot \mid m_1, m_2)$ in particular. To focus on the choice of vagueness without the knowledge of the entire set of equilibria, I introduce two properties, *symmetry* and *efficiency*. *Symmetry* ensures that an undercutting deviation makes the DM realize that a deviation has occurred. It is however still possible that the DM identifies the wrong agent as the deviator. *Efficiency* ensures that any undercutting deviation is still preferred by the DM even in this case. Both of them place restrictions on the agents' equilibrium strategies only and are silent about what kind of deviations are allowed.

Definition 1 *(Symmetry).* A weak PBE (s_1, s_2, β, μ) is *symmetric* if for each $\theta \in \Theta$,

(1) $s_1(\theta) = (\emptyset, \emptyset)$ iff $s_2(\theta) = (\emptyset, \emptyset)$;
(2) $s_1(\theta) \in \{\emptyset\} \times \mathbb{R}$ iff $s_2(\theta) \in \mathbb{R} \times \{\emptyset\}$;
(3) $s_1(\theta) \in \mathbb{R} \times \{\emptyset\}$ iff $s_2(\theta) \in \{\emptyset\} \times \mathbb{R}$.

At any given state, if Agent i is vague about his disadvantageous (or advantageous) dimension, then Agent $(-i)$ is also vague about his disadvantageous (or advantageous) dimension. Therefore, the two agents' choices of vagueness depend on their advantages in the same way at any given state. *Symmetry* dramatically simplifies the procedure of characterizing putative equilibrium strategies. Moreover, whenever a deviation leads to an asymmetric proposal profile, the DM is sure to realize that a deviation has occurred (see also the discussion of Definition 3).

In order to focus on the equilibrium choice of vagueness, I further restrict the specific proposals that the agents make in equilibrium. In particular, a specific policy proposal on dimension k must lie between the proposer's ideal action on dimension k and the DM's ideal action on dimension k. In other words, the agents' strategic choices are, for each dimension, (1) whether to be specific or vague and (2) how much to compromise if they choose to be specific. By eliminating "bad" equilibrium policy proposals, *efficiency* ensures that for any undercutting deviation that could be profitable, if the DM fail to identify correctly the deviator, she still prefers the deviator.

Definition 2 *(Efficiency).* A weak PBE (s_1, s_2, β, μ) is *efficient* if for each $\theta \in \Theta$, $i \in \{1, 2\}$ and $k \in \{1, 2\}$,

(1) $s_i^k(\theta) \in [\theta^k - b_i^k, \theta^k] \cup \{\emptyset\}$ whenever $b_i^k > 0$,
(2) $s_i^k(\theta) \in [\theta^k, \theta^k - b_i^k] \cup \{\emptyset\}$ whenever $b_i^k \le 0$.

I now introduce a new consistency notion called *single-deviation consistency*. This notion limits how the DM updates her belief about agents' strategies when she is surprised. After observing a surprising move by the agents, she updates her belief under the assumption that agents make strategic choices independently. As a result, knowing that one agent has deviated does not impact her belief about the other agent's strategy.

Given a weak PBE (s_1, s_2, β, μ), Agent i's proposal m_i is *consistent with equilibrium* if $s_i^{-1}(m_i)$ is non-empty and *inconsistent with equilibrium* if otherwise. A proposal profile (m_1, m_2) is *on-path* if $s_1^{-1}(m_1) \cap s_2^{-1}(m_2)$ is non-empty and *off-path* if otherwise.

Definition 3 *(Single-deviation consistency).* Let (s_1, s_2, β, μ) be a weak PBE. The DM's belief μ satisfies *single-deviation consistency* if, for each (m_1, m_2) such that $s_1^{-1}(m_1) \cap s_2^{-1}(m_2)$ is empty:

$$\mu(\cdot \mid m_1, m_2) \in \Delta\big(s_1^{-1}(m_1) \cup s_2^{-1}(m_2)\big)$$

whenever $s_1^{-1}(m_1) \cup s_2^{-1}(m_2)$ is non-empty.

In other words, a DM faced with an off-path proposal profile believes that at least one of the agents has not deviated, whenever believing so is possible.

By definition, an off-path (m_1, m_2) falls into one of the following cases:

1. Neither m_1 nor m_2 is consistent with equilibrium;
2. Exactly one of m_1 and m_2 is consistent with equilibrium;
3. Both m_1 and m_2 are consistent with equilibrium.

According to *single-deviation consistency*, whenever the DM is surprised by an off-path proposal profile and some agent is consistent with equilibrium (i.e., Case 2 and 3), she believes that this agent is indeed playing according to equilibrium. In Case 3, she considers both cases. So for all states in the support of her belief, either Agent 1 or Agent 2 has deviated.

To see how Case 3 is possible, consider a putative equilibrium in which both agents always propose the action equal to the state, i.e., $s_1(\theta) = s_2(\theta) = \theta$ for all θ. Now suppose that at some state $\overline{\theta}$, Agent 1 deviates to the proposal $y \in \mathbb{R}^2$ such that $y \neq \overline{\theta}$. The DM then observes an off-path proposal profile $(y, \overline{\theta})$. Although each agent's proposal is consistent with equilibrium on its own, the proposal profile is off-path. Note that in this case, *single-deviation consistency* does not specify which agent the DM believes; all that is required is that she believes that exactly one of the agents has deviated.

Notice that *symmetry* ensures that any asymmetrical proposal profile is off-path. Combined with *single-deviation consistency*, after one agent's deviation which leads to an asymmetrical proposal profile and is inconsistent with equilibrium, the DM is able to believe that the other agent has not deviated and therefore updates her belief accordingly.

The intuition behind *single-deviation consistency* is the independence of agents' strategic choices. When agents act independently, their proposals contain only information about their own strategic choices and not their opponents'. Therefore, Agent 1's deviation should not be an excuse for the DM to change her beliefs about Agent 2. When the DM cannot tell who the deviator(s) are, she applies minimal departure from rationality and believes only one agent has deviated.

The notion that strategic choices are independent is not new. Battigalli (1996) defines the independence property for conditional belief systems over strategy profiles. An implication of the independence property is that the marginal conditional probabilities about player i's strategies are independent of information which exclusively concerns player j's strategies. It is shown that the independence property of conditional belief systems is necessary for an equivalent assessment to satisfy the consistency notion á la Kreps and Wilson (1982).[5]

Watson (2015) first formally defines perfect Bayesian equilibrium for infinite games without Nature moves. The definition retains sequential rationality and puts forward a new notion for consistency called "plain consistency." One of the implications of plain consistency is that when surprised, players only alter their beliefs about the strategies of those players who appear to have deviated. Vida and Honryo (2015) focus on equilibria in multi-sender signaling games and support rationalizing deviations with the smallest number of deviators possible as a sensible refinement.

Now I am ready to state the equilibrium concept used throughout the paper, which I call the *SES-equilibrium*.

Definition 4. A tuple (s_1, s_2, β, μ) is called an *SES-equilibrium* if it is a weak PBE satisfying symmetry, efficiency and single-deviation consistency.

3. Main results

What should agents be vague about? Conventional wisdom suggests that they should focus on their advantageous dimension and ignore the other dimension. Here, the model suggests otherwise. On one hand, if Agent 1 has an advantage on dimension 1, then vagueness on dimension 1 does no harm since the DM already trusts him. More importantly, for any action proposed by Agent 2 that results in an off-path proposal profile, the DM is free to believe that it is his most preferred action. On the other hand, if Agent 1 is specific about dimension 1 and reveals θ^1 to the DM, Agent 2 can then anchor on his revelation and deviate by offering a compromise. If the DM believes that only Agent 2 has deviated and continues to trust the information about θ^1 revealed by Agent 1, Agent 2 is able to credibly compromise on his disadvantageous dimension.

Proposition 1 and 2 demonstrate how this intuition helps predict how agents choose to be vague. I first characterize a putative equilibrium proposal profile where agents are vague about their disadvantageous dimensions and specific about their advantageous dimensions. Then I rule out this profile by showing that one agent can profitably undercut the other. In order to characterize such putative equilibria, I need to introduce a useful lemma: whenever both agents are simultaneously specific about their advantageous dimensions and vague about their disadvantageous dimensions, no agent wins with probability 1.

[5] Note that although the strategic independence idea in *single-deviation consistency* is inherited from Kreps and Wilson (1982), *single-deviation consistency* is not implied by their consistency notion. Although players' strategies are independent, proposals are correlated because they are functions of the same state variable. An example is provided in Appendix D.

Lemma 1. *Suppose that the equilibrium proposal profile at $\theta = \overline{\theta}$ takes the following form:*

$$m = ((w, \varnothing), (\varnothing, z))$$

where $w, z \in \mathbb{R}$. Then $\beta(m) \in (0, 1)$. Consequently, the DM is indifferent.

Proof. Suppose that $\beta(m) = 0$. Then Agent 1 has a profitable deviation $m_1^{dev} = (\varnothing, z)$. To see this, first note that Agent 1's equilibrium payoff at $\overline{\theta}$ is

$$u_1(\overline{\theta}, m, \beta(m)) = v_1(\overline{\theta}, 2, (\varnothing, z)) = -\|(b_2^1, \overline{\theta}^2 - z) - b_1\|^2.$$

Now let Agent 1 deviate to $(\varnothing, z)$. Since $|b_1^1| < |b_2^1|$, given $m^{dev} = ((\varnothing, z), (\varnothing, z))$ and any $\mu \in \Delta(\Theta)$, Agent 1 is strictly preferred to Agent 2. Therefore $\beta(m^{dev}) = 1$ and Agent 1's deviation payoff is

$$\begin{aligned} u_1(\overline{\theta}, m^{dev}, \beta(m^{dev})) =& v_1(\overline{\theta}, 1, (\varnothing, z)) \\ =& -\|(b_1^1, \overline{\theta}^2 - z) - b_1\|^2 \\ >& -\|(b_2^1, \overline{\theta}^2 - z) - b_1\|^2 \\ =& u_1(\overline{\theta}, m, \beta(m)). \end{aligned}$$

Similarly, if $\beta(m) = 1$, then Agent 2 has a profitable deviation $m_2^{dev} = (w, \varnothing)$. Therefore, we have $\beta(m) \in (0, 1)$. □

If an agent is vague about his disadvantageous dimension, his opponent can easily defeat him on that dimension by being vague about it also. Consequently, if Agent 1 proposes $(w, \varnothing)$ at some $\overline{\theta}$ and wins with probability 1, Agent 2 can mimic his proposal and win with probability 1. This deviation is profitable for Agent 2 since on dimension 2 he gets his own ideal action instead of Agent 1's ideal action, and on dimension 1 he gets the same action as in equilibrium. A similar argument applies if Agent 2 proposes $(\varnothing, z)$ at some $\overline{\theta}$ and wins with probability 1. This in turn implies that whenever agents are only vague about their disadvantageous dimensions, the DM must be indifferent between the agents. In Proposition 1, I further characterize such equilibria under the condition that one agent's bias on his disadvantageous dimension is sufficiently large and then rule them out.

When Agent 2's bias on his disadvantageous dimension $|b_2^1|$ is relatively large, as long as equilibrium proposals satisfy the *efficiency* condition, Agent 2's equilibrium proposal $(\varnothing, z)$ would be worse than Agent 1's equilibrium proposal $(w, \varnothing)$ for the DM. Therefore, indifference cannot be maintained in equilibrium. When the biases are such that indifference can be maintained (i.e. when $|b_2^1| = \|b_1\|$ or $|b_1^2| = \|b_2\|$), agents must be revealing the state, and we can apply the undercutting intuition.

Proposition 1. *Suppose that $|b_2^1| \geq \|b_1\|$ or $|b_1^2| = \|b_2\|$. In all SES-equilibria in which vagueness occurs at some state, it occurs on agents' advantageous dimensions.*

The proof is relegated to Appendix A and here is a sketch. By definition, I only need to rule out equilibria in which vagueness only occurs on agents' disadvantageous dimensions at some state; that is, equilibria with a proposal profile $(w, \varnothing), (\varnothing, z)$ at some state $\overline{\theta}$. By Lemma 1, given this proposal profile, the DM is indifferent between the two agents. By *efficiency*, for any given belief of the DM, Agent 1's proposal is at least as good as the outcome b_1 and Agent 2's proposal is at most as good as the outcome $(b_2^1, 0)$. Therefore, if $|b_2^1| > \|b_1\|$, the DM could not be indifferent (Fig. 1).

Therefore, indifference can only happen if $|b_2^1| = \|b_1\|$ (or symmetrically, $|b_1^2| = \|b_2\|$), Agent 1 proposes the outcome b_1, and Agent 2 proposes the outcome $(b_2^1, 0)$ (or symmetrically, Agent 1 proposes the outcome $(0, b_1^2)$ and Agent 2 proposes the outcome b_2). The DM then learns $\overline{\theta}^i$ from Agent i and chooses $\overline{\beta} \in (0, 1)$. Now I show that there is a profitable deviation $(\varnothing, z')$ for Agent 1 at $\overline{\theta}$.

To see this, first notice that since Agent 1 proposes b_1 in equilibrium, for any $\overline{\beta} \in (0, 1)$ one can find z' such that $(\varnothing, z')$ is potentially profitable. That is, if Agent 1 wins with probability 1 after the deviation, then $(\varnothing, z')$ is profitable. Second, let $\tilde{\theta}$ be in the support of the DM's belief after such a deviation. By *single-deviation consistency*, either $\tilde{\theta} \in s_2^{-1}(\varnothing, z)$, or $\tilde{\theta} \in s_1^{-1}(\varnothing, z')$. If $\tilde{\theta} \in s_2^{-1}(\varnothing, z)$, then the DM learns the true $\overline{\theta}^2$ from Agent 2 after the deviation and realizes Agent 1 is making a compromise, so she chooses $\beta' = 1$ (Fig. 2). If $\tilde{\theta} \in s_1^{-1}(\varnothing, z')$ and, by way of contradiction, $\beta' < 1$, this implies the DM believes that Agent 2 is the deviator and weakly prefers him. Now, given the proposal profile $((\varnothing, z'), (\varnothing, z))$, since on dimension 1 Agent 1's proposal is strictly preferred to Agent 2's, for the DM to weakly prefer Agent 2 given the state $\tilde{\theta}$, it has to be that z is much closer to $\tilde{\theta}^2$ than z' is. This means that at $\tilde{\theta}$, Agent 1's equilibrium proposal $(\varnothing, z')$ is such that z' is sufficiently far from $\tilde{\theta}^2$, which violates *efficiency*. *Efficiency* therefore ensures that after an undercutting deviation by Agent

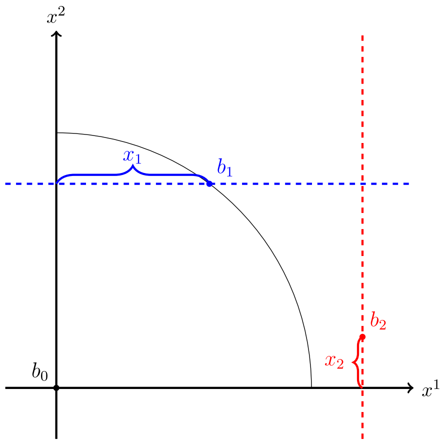

Fig. 1. $m_1 = (w, \varnothing)$, $m_2 = (\varnothing, z)$ at $\overline{\theta}$ cannot be sustained when $|b_2^1| > \|b_1\|$ since DM cannot be indifferent. Here I depict the outcome space, where the origin represents the DM's ideal consequence and the quadrant represents an indifference curve of the DM. The disks represent the agents' ideal consequences: $b_1 = (3, 4)$, $b_2 = (6, 1)$. The curly brackets show the possible outcomes from the agents' equilibrium proposals x_1 and x_2. Since Agent 1's outcome is always better than Agent 2's, the DM cannot be indifferent.

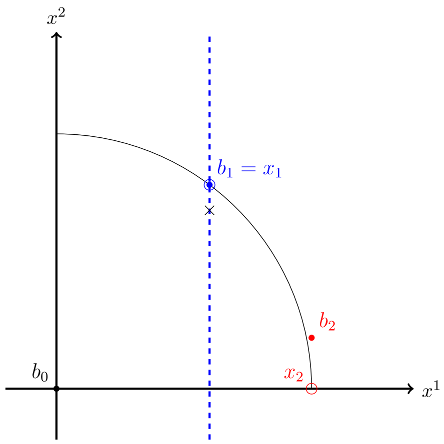

Fig. 2. $m_1 = (w, \varnothing)$, $m_2 = (\varnothing, z)$ at $\overline{\theta}$ cannot be sustained when $|b_2^1| = \|b_1\|$ since Agent 1 has a profitable deviation. Here I depict the outcome space, where the origin represents the DM's ideal consequence and the quadrant represents an indifference curve of the DM. The disks represent the agents' ideal consequences: $b_1 = (3, 4)$, $b_2 = (5, 1)$. The circles represent the outcomes from the agents' equilibrium proposals: $x_1 = (3, 4)$, $x_2 = (5, 0)$. The DM is indifferent between x_1 and x_2 and chooses $\beta \in (0, 1)$. The cross at $(3, 3.5)$ represents Agent 1's deviation outcome. Indifference of the DM indicates that Agent i reveals θ^i. Now, let Agent 1 deviate to $(\varnothing, \overline{\theta}^2 - 3.5)$. Since the DM learns $\overline{\theta}^2$ from Agent 2, she now prefers Agent 1. For low enough β, $(\varnothing, \overline{\theta}^2 - 3.5)$ is profitable.

1, even when the DM recognizes the wrong deviator, she still prefers Agent 1.[6] Under the quadratic utility assumption, any such potentially profitable compromise by Agent 1 results in an *efficiency* violation.

In the remaining cases where $|b_2^1| < \|b_1\|$ and $|b_1^2| < \|b_2\|$, neither agent's bias on his disadvantageous dimension alone is as large as his opponent's total bias. In this case, indifference no longer implies information revelation in equilibrium. However, once the agents are assumed to make constant amount of compromises across the states, we can guarantee that after undercutting occurs, the DM recognizes the information content in the non-deviator. The undercutting intuition can then be directly applied (the proof is in Appendix A).

Proposition 2. *For any b_1, b_2, $\delta_1 \in [0, 1]$ and $\delta_2 \in [0, 1]$, $s_1(\theta) = (\theta^1 - \delta_1 b_1^1, \varnothing)$, $s_2(\theta) = (\varnothing, \theta^2 - \delta_2 b_2^2)$, $\forall\theta$ cannot be sustained in an SES-equilibrium.*

[6] Without *efficiency*, if the DM wrongly believes that Agent 2 is the deviator, then depending on the equilibrium (which now for some state $\overline{\theta}$ prescribes the proposal profile $((w, \varnothing), (\varnothing, z))$ while for some other state $\underline{\theta}$ prescribes the proposal profile $((\varnothing, z'), (w', \varnothing))$ for some $w' \in \mathbb{R}$), it is technically possible that she would prefer Agent 2 given the information conveyed by Agent 1's equilibrium proposal $(\varnothing, z')$.

In this class of equilibria, agents' equilibrium proposals could potentially be far from their own ideal actions. Depending on the biases and the amount of compromise they make in equilibrium, it is possible that an undercutting deviation is not profitable even if the deviator wins with probability 1. For this not to be the case, we need to ensure that the deviation proposal is preferred by the deviator to both agents' equilibrium proposals.[7] Although it is not always true that both agents have potentially profitable undercutting deviations, one can show that at least one of them does. By making sure that it is this agent who undercuts, we can bypass this problem.

Although agents cannot be vague only about their disadvantageous dimensions, they could be vague about their advantageous dimensions, as can be seen in Propositions 3 and 4. The proofs are in Appendix B. Proposition 3 identifies an equilibrium in which both agents are vague about all dimensions in all states: $s_1(\theta) = s_2(\theta) = (\varnothing, \varnothing)$, $\forall\theta$.

Proposition 3. *For any b_1, b_2, an SES-equilibrium exists in which both agents are vague about both dimensions at all states.*

Suppose at some $\overline{\theta}$, Agent 1 deviates to be specific about any given dimension(s). The deviation leads to an off-path proposal profile. The DM then maintains the belief that Agent 2 has not deviated but is free to choose any beliefs regarding Agent 1's strategy. In particular, she can choose the belief that Agent 1 has deviated to his most preferred action and best responding by her equilibrium action. Such a deviation is therefore unprofitable.

Proposition 4 characterizes equilibria in which both agents are vague on their advantageous dimensions and specific on their disadvantageous dimensions at all states. To contrast with Proposition 2, I focus on the same class of equilibria; that is, compromises are constant across the states. As can be seen, the existence of equilibria in this class depends on the agents' biases.

Proposition 4. *Consider the following proposal profile:*

$$s_1(\theta) = (\varnothing, \theta^2 - \delta_1 b_1^2),$$
$$s_2(\theta) = (\theta^1 - \delta_2 b_2^1, \varnothing),$$

where $\delta_1, \delta_2 \in [0, 1]$. Define $\delta^* := \dfrac{\sqrt{\|b_1\|^2 - (b_2^2)^2}}{|b_2^1|}$.

(1) The following cannot be sustained in an SES-equilibrium under any preferences:

(a) $\delta_1 = 1, \delta_2 \in [0, \delta^*)$, or
(b) $\delta_1 < 1$.

(2) Suppose that $|b_2^1| > \|b_1\|$ or $|b_2^1| = \|b_1\| < \|b_2\|$. Then any SES-equilibrium in which $\delta_1 = 1$, $\delta_2 \in [\delta^*, 1]$ can be sustained.
(3) Suppose that $|b_2^1| = \|b_1\| = \|b_2\|$, or symmetrically, $|b_1^2| = \|b_1\| = \|b_2\|$. Then the SES-equilibrium in which $\delta_1 = 1$, $\delta_2 = \delta^* = 1$ can be sustained.
(4) Suppose that $|b_2^1| < \|b_1\| < \|b_2\|$. The SES-equilibrium in which $\delta_1 = 1$, $\delta_2 \in [\delta^*, 1]$ can be sustained iff Agent 2 weakly prefers the outcome b_1 to $(b_2^1, \sqrt{\|b_1\|^2 - |b_2^1|^2} \cdot \dfrac{b_2^2}{|b_2^2|})$.
(5) Suppose that $|b_2^1| < \|b_1\|$, $\|b_1\| = \|b_2\|$, and $|b_1^2| < \|b_2\|$. Then no SES-equilibrium can be sustained.

In this class of equilibria, the DM's payoff from an agent is the same across the states. Therefore, if Agent i is strictly preferred at some $\overline{\theta}$, he is strictly preferred at all states; if the DM is indifferent at some $\overline{\theta}$, she is indifferent across all states (though she may choose different actions across the states). Moreover, when $\delta_2 = \delta^*$, the DM is exactly indifferent between Agent 2's proposal and b_1. Lastly, when $\|b_1\| = \|b_2\|$ (which is the case for 3 and 5), $\delta^* = 1$.

I first discuss nonexistence (Case 1). When $\delta_1 = 1$ and, $\delta_2 \in [0, \delta^*)$, Agent 1 proposes own ideal proposal while Agent 2 compromises such that the DM strictly prefers Agent 2 and therefore chooses $\beta = 0$ at all states. Agent 2 then has incentives to deviate to his own ideal proposal $(\theta^1 - b_2^1, \varnothing)$: since this leads to a on-path proposal profile, the DM continues to choose $\beta = 0$ and Agent 2 gets his own ideal outcome.

Now, let us consider the case in which $\delta_1 < 1$. If $\beta(s(\theta)) = 0$ for all θ, then the DM must weakly prefers Agent 2. Since Agent 1 is compromising and $\|b_1\| \leq \|b_2\|$, Agent 2 must also be compromising in equilibrium. The same argument as the one in the previous paragraph then shows that Agent 2 has incentives to deviate. So let us suppose that $\beta(s(\overline{\theta})) > 0$ for some $\overline{\theta}$. That is, Agent 1 wins with positive probability at $\overline{\theta}$ with a compromise proposal. Since $\Theta = \mathbb{R}^2$, one can find $\tilde{\theta}$ such

[7] If the deviation proposal is only better than his opponent's equilibrium proposal and the deviator's equilibrium probability of winning is sufficiently high, the deviation may not be profitable. Similarly, if the deviation proposal is only better than his own equilibrium proposal and the deviator's equilibrium probability of winning is sufficiently low, the deviation may not be profitable either.

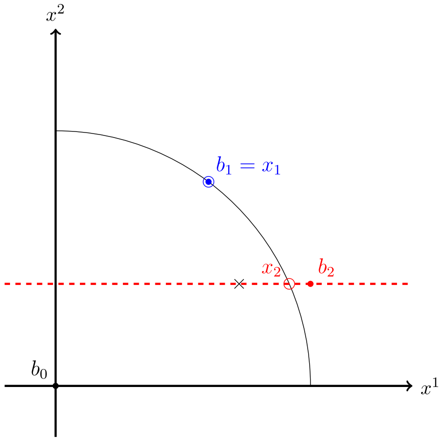

Fig. 3. $s_1(\theta) = (\varnothing, \theta^2 - b_1^2)$, $s_2(\theta) = (\theta^1 - \delta^* b_2^1, \varnothing)$ can be sustained in equilibrium when $|b_2^1| = \|b_1\| < \|b_2\|$. Agent 2 has deviated to $(\overline{\theta}^1 - 3.6, \varnothing)$ at some $\overline{\theta}$. Here I depict the outcome space, where the origin represents the DM's ideal consequence and the quadrant represents an indifference curve of the DM. The disks represent the agents' ideal consequences: $b_1 = (3, 4)$, $b_2 = (5, 2)$. The circles represent the outcomes from the agents' equilibrium proposals: $x_1 = (3, 4)$, $x_2 = (\sqrt{21}, 2)$. The DM is indifferent between x_1 and x_2 and chooses $\beta = 1$. The cross at $(3.6, 2)$ represents Agent 2's deviation outcome. Even though the deviation is in fact preferred by the DM, it is not profitable since the DM is free to believe that it gives Agent 2 his ideal outcome.

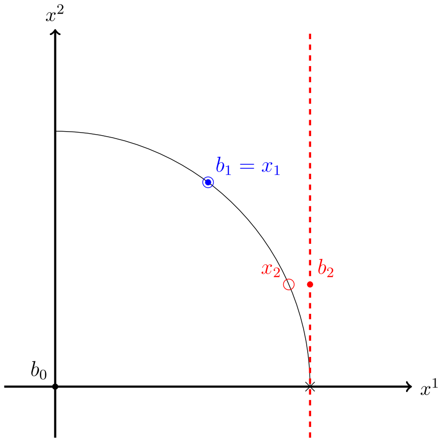

Fig. 4. $s_1(\theta) = (\varnothing, \theta^2 - b_1^2)$, $s_2(\theta) = (\theta^1 - \delta^* b_2^1, \varnothing)$ can be sustained in equilibrium when $|b_2^1| = \|b_1\| < \|b_2\|$. Agent 2 has deviated to $(\varnothing, \overline{\theta}^2)$ at some $\overline{\theta}$. Here I depict the outcome space, where the origin represents the DM's ideal consequence and the quadrant represents an indifference curve of the DM. The disks represent the agents' ideal consequences: $b_1 = (3, 4)$, $b_2 = (5, 2)$. The circles represent the outcomes from the agents' equilibrium proposals: $x_1 = (3, 4)$, $x_2 = (\sqrt{21}, 2)$. The DM is indifferent between x_1 and x_2 and chooses $\beta = 1$. The cross at $(5, 0)$ represents Agent 2's deviation outcome. Agent 2 has deviated to the DM's ideal policy on dimension 2, but since the DM is still indifferent, this is not sufficient to achieve a higher probability of winning.

that: (i) $s_2(\tilde{\theta}) = s_2(\overline{\theta})$ and (ii) $s_1(\overline{\theta})$ is Agent 1's ideal proposal at $\tilde{\theta}$. In order for Agent 1 to not have incentives to deviate to $s_1(\overline{\theta})$ at $\tilde{\theta}$, it must be that his probability of winning after the deviation is strictly lower. In particular, $\beta(s(\tilde{\theta})) = C\beta(s(\overline{\theta}))$, where $C > 1$ depends on b_1 and b_2 only. Now we have that Agent 1 wins with positive probability at $\tilde{\theta}$ with a compromise proposal. By iterating the same argument, one can eventually find some state θ^* for which $\beta(\theta^*) > 1$, a contradiction.

We have thus ruled out equilibria in which $\delta_1 = 1$, $\delta_2 \in [0, \delta^*)$ as well as $\delta_1 < 1$ for all b_1, b_2. What is left to be checked is the set of equilibria in which $\delta_1 = 1$, $\delta_2 \in [\delta^*, 1]$. That is, Agent 1 does not compromise and the DM weakly prefers Agent 1. The existence depends on the preferences of the agents, which I elaborate below.

Any such equilibria can be sustained for the preferences in 2 and 3. When $|b_2^1| > \|b_1\|$ or $|b_2^1| = \|b_1\| \le \|b_2\|$, let $\beta(s(\theta)) = 1$ for all θ. Although Agent 1 reveals θ^2 which allows Agent 2 to compromise on dimension 2, $|b_2^1|$ is sufficiently large that no compromise is sufficient to increase Agent 2's chance of winning. Figs. 3 and 4 illustrate two possible deviations for Agent 2, neither of which is profitable. When $|b_1^2| = \|b_1\| < \|b_2\|$, let $\beta = 0$ for all states and the argument is identical.

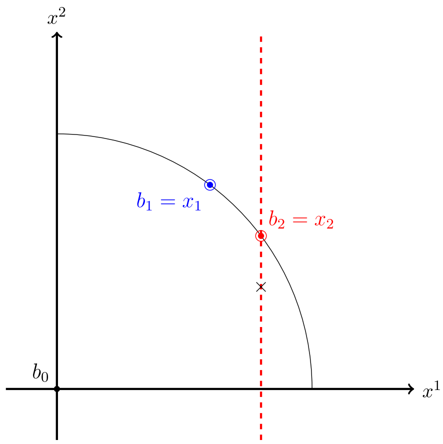

Fig. 5. $s_1(\theta) = (\varnothing, \theta^2 - b_1^2)$, $s_2(\theta) = (\theta^1 - b_2^1, \varnothing)$ cannot be sustained in equilibrium when $|b_2^1| < \|b_1\| = \|b_2\| > |b_1^2|$. Agent 2 has deviated to $(\varnothing, \overline{\theta}^2 - 2)$ at some $\overline{\theta}$. Here I depict the outcome space, where the origin represents the DM's ideal consequence and the quadrant represents an indifference curve of the DM. The disks represent the agents' ideal consequences: $b_1 = (3,4)$, $b_2 = (4,3)$. The circles represent the outcomes from the agents' equilibrium proposals: $x_1 = (3,4)$, $x_2 = (4,3)$. The DM is indifferent between x_1 and x_2 and chooses $\beta = 1$. The cross at $(4,2)$ represents Agent 2's deviation outcome. Agent 2 has deviated to a better policy on dimension 2, and since the DM learns θ^2 from Agent 1, the compromise is credible and therefore profitable.

For the preferences in 4, we first note that by previous arguments, $\beta(s(\theta)) = 1$ for all θ.[8] Therefore the equilibrium outcome is b_1. Now, since in this class of preference $|b_2^1|$ is sufficiently small, undercutting on dimension 2 could lead to a win for Agent 2. However, depending on the distance of b_1 and b_2, undercutting may or may not be profitable for Agent 2. If Agent 2 weakly prefers the equilibrium outcome b_1 to his deviation outcome, then undercutting is not profitable and the equilibrium can be sustained. The outcome $(b_2^1, \sqrt{\|b_1\|^2 - |b_2^1|^2} \cdot \frac{b_2^2}{|b_2^2|})$ is the deviation where the DM would be exactly indifferent between b_1 and Agent 2's deviation proposal. The two examples below illustrate the two cases: in Example 1 the equilibrium can be sustained and in Example 2 it cannot.

Example 1. Consider the equilibrium proposal profile given by $\delta_1 = 1$, $\delta_2 = \delta^*$ when $b_1 = (3,4)$, $b_2 = (3.2, 3.9)$. Let Agent 2 deviate to $(\varnothing, \overline{\theta}^2 - \sqrt{14.76})$ at any $\overline{\theta}$. Note that $|b_2^1| < \|b_1\| < \|b_2\|$. The outcomes from the agents' equilibrium proposals are $x_1 = (3,4)$, $x_2 = (\sqrt{9.79}, 3.9)$. The DM is indifferent between x_1 and x_2 and chooses $\beta = 1$. Agent 2's best deviation outcome is $(3.2, \sqrt{14.76})$. Since Agent 2 prefers the equilibrium outcome x_1, this deviation is not profitable.

Example 2. Consider the equilibrium proposal profile given by $\delta_1 = 1$, $\delta_2 = \delta^*$ when $b_1 = (3,4)$, $b_2 = (4, 3.2)$. Let Agent 2 deviate to $(\varnothing, \overline{\theta}^2 - 3)$ at any $\overline{\theta}$. Note that $|b_2^1| < \|b_1\| < \|b_2\|$. The outcomes from the agents' equilibrium proposals are $x_1 = (3,4)$, $x_2 = (\sqrt{14.76}, 3.2)$. The DM is indifferent between x_1 and x_2 and chooses $\beta = 1$. Agent 2's best deviation outcome is $(4,3)$. Since Agent 2 prefers his deviation to the equilibrium outcome x_1, this deviation is profitable.

For the last class of preferences 5, first note that the DM can only be indifferent between the agents since any strictly preferred agent must be compromising and therefore have incentives to deviate. Moreover, since Agent i reveals θ^{-i}, the agent who does not win with probability 1 at any state $\overline{\theta}$ has an incentive to undercut and this deviation would be profitable. Fig. 5 illustrates.

4. Refinement: extended intuitive criterion

In the previous section, I showed the existence of equilibria in which agents are vague about their advantageous dimensions. This result should not be surprising because in signaling games, there are typically multiple equilibria, some of them supported by unreasonable beliefs. In this section, I develop an equilibrium refinement for two-sender signaling games similar to the Intuitive Criterion (Cho and Kreps, 1987) and show that the previous equilibria are robust under this refinement.

[8] This is automatically true if the DM strictly prefers Agent 1 in equilibrium. If the DM strictly prefers Agent 2 and therefore chooses $\beta = 0$ at all states, given that Agent 1 makes his own ideal proposal, it must be the case that Agent 2 compromises. But then Agent 2 has incentives to deviate to his own ideal proposal. If the DM is indifferent (and therefore Agent 2 compromises) and chooses $\beta < 1$ at some state, then we can apply the previous iteration argument we used to rule out equilibria in which $\delta_1 < 1$ to rule this out.

4.1. One sender: intuitive criterion

First, I will review the Intuitive Criterion. As a refinement for one-sender signaling games, it requires the receiver to believe that, after a deviation by the sender, the deviation is potentially profitable. In particular, the receiver is required to believe that the state of the world is such that the highest payoff that the sender can get by deviating to the observed message, given that the receiver does not play never-best responses, is weakly higher than the sender's equilibrium payoff. The idea is that if the highest possible payoff the sender gets by deviating is lower than his equilibrium payoff, then he should not bother to deviate in the first place.

I will keep using the terminologies and notations from Section 2 to describe both one-sender and two-sender signaling games and the equilibrium refinements. In a one-sender signaling game, given the state $\theta \in \Theta$, the Agent's proposal $m \in M$, and the DM's action $\beta \in B$, the DM's payoff is denoted by $u_0(\theta, m, \beta)$, the Agent's payoff by $u_1(\theta, m, \beta)$, and the DM's belief by $\mu(\cdot \mid m)$. The set of the DM's best responses to a proposal m against the belief μ is:

$$\widetilde{\mathrm{BR}}\,(\mu, m) = \arg\max_{\beta \in B} \int_{\Theta} u_0(\theta, m, \beta)\, d\mu.$$

For any non-empty $T \subset \Theta$ and $m \in M$,

$$\mathrm{BR}\,(T, m) = \bigcup_{\mu:\mu(T|m)=1} \widetilde{\mathrm{BR}}\,(\mu, m)$$

denotes the set of the DM's best responses according to beliefs with the support T. When T is empty, let $\mathrm{BR}\,(T, m) = \mathrm{BR}\,(\Theta, m)$.

Given a weak PBE (s, β, μ), the Agent's equilibrium payoff at θ is denoted by $u_1^*(\theta)$. For a proposal m^{dev} that is inconsistent with equilibrium, the set of states at which the Agent's highest payoff from deviating to m^{dev} given that the DM plays a best response is higher than the Agent's equilibrium payoff is

$$\Theta(m^{dev}) = \left\{ \theta \mid \max_{\beta \in \mathrm{BR}\,(\Theta, m^{dev})} u_1(\theta, m^{dev}, \beta) \geq u_1^*(\theta) \right\}.$$

Finally, an equilibrium (s, β, μ) fails the Intuitive Criterion if there exists $\theta \in \Theta$ and $m^{dev} \in M$ inconsistent with equilibrium such that

$$u_1^*(\theta) < \min_{\beta \in \mathrm{BR}\,(\Theta(m^{dev}), m^{dev})} u_1(\theta, m^{dev}, \beta).$$

When the DM observes an unexpected proposal m^{dev}, the support of her belief is restricted to states at which m^{dev} is potentially profitable. That is, the highest payoff that the Agent can get given that the DM does not play never-best responses is higher than his equilibrium payoff. The Agent then contemplates deviations given that the DM best responds to beliefs thus restricted. If for some θ and m^{dev}, any such best response makes the Agent strictly better off compared to an equilibrium when the state is θ, then the equilibrium fails the Intuitive Criterion.

4.2. Two senders: extended intuitive criterion

I extend the Intuitive Criterion to the two-sender case by combining single-deviation consistency with the Intuitive Criterion. Single-deviation consistency identifies the deviator. The Intuitive Criterion identifies the deviations in which the receiver is allowed to believe.

In a two-sender signaling game, given the state $\theta \in \Theta$, agents' proposal profile $m = (m_1, m_2)$, and the DM's action $\beta \in B$, Agent i's payoff is denoted by $u_i(\theta, m, \beta)$, the DM's payoff by $u_0(\theta, m, \beta)$, and the DM's belief over Θ by $\mu(\cdot \mid m)$. Given a weak PBE (s_1, s_2, β, μ), Agent i's equilibrium payoff at θ is denoted by $u_i^*(\theta)$. Agent i's opponent is denoted by Agent $-i$.

Definition 5 *(The Extended Intuitive Criterion).* Let (s_1, s_2, β, μ) be a weak PBE. For each $m = (m_1, m_2)$ such that $s_1^{-1}(m_1) \cap s_2^{-1}(m_2)$ is empty and each $i = 1, 2$, form the set

$$\Theta_i(m) = \left\{ \theta \mid s_{-i}(\theta) = m_{-i},\, u_i^*(\theta) \leq \max_{\beta \in BR\,(\Theta, m)} u_i(\theta, m, \beta) \right\}.$$

(s_1, s_2, β, μ) fails *the Extended Intuitive Criterion* if there exists $\theta \in \Theta$, $i \in \{1, 2\}$, $m_i^{dev} \in M$ such that

$$u_i^*(\theta) < \min_{\beta \in BR\,(\Theta_1(m^{dev}) \cup \Theta_2(m^{dev}), m^{dev})} u_i(\theta, m^{dev}, \beta), \tag{1}$$

where m^{dev} is the proposal profile formed by $s_{-i}(\theta)$ and m_i^{dev}. An equilibrium satisfying the Extended Intuitive Criterion is called an *intuitive equilibrium*.

Following Battigalli and Siniscalchi (2002), I now informally describe the assumptions about players' strategic reasoning imposed by the Extended Intuitive Criterion (EIC) (see Battigalli and Siniscalchi (2002) for more details). In the following, I assume that (s_1, s_2, β, μ) is an intuitive equilibrium and the DM observes $m = (m_1, m_2)$ where m_1 is inconsistent with equilibrium while m_2 is consistent with equilibrium.

First, EIC assumes that the agents believe that the DM's belief agrees with the equilibrium outcome after she observes a proposal profile. Moreover, the agents believe that the DM believes that agents act independently. Therefore, the DM knows that m_2 is consistent with equilibrium while m_1 is not, and she believes that m_2 is the result of Agent 2 playing his equilibrium strategy. This is captured by $s_2(\theta) = m_2$ in $\Theta_1(m)$.

Second, EIC assumes that the agents believe that upon observing any proposal profile, the DM believes that (1) the agents are rational, (2) the agents' initial beliefs agree with the equilibrium, (3) the agents do not expect the DM to play dominated actions, and (4) each agent expects the other agent to play his equilibrium strategy. This is the forward-induction assumption that, upon observing an off-path proposal profile, the DM's beliefs are concentrated on states for which m_1 is not equilibrium-dominated. This is captured by $u_1^*(\theta) \leq \max_{\beta \in \mathrm{BR}(\Theta,m)} u_1(\theta, m, \beta)$. Combined with the first point, we now have that the DM's belief is restricted to $\Theta_1(m)$. Third, the agents believe that the DM is rational. Therefore, they believe that she plays $\beta \in \mathrm{BR}(\Theta_1(m), m)$ upon observing m.

I have thus described the agents' beliefs about the DM. Since the agents are themselves rational and their beliefs agree with the equilibrium, they play best responses to their beliefs about the DM. Therefore, for some belief of Agent 1 about β, m_1 must give a weakly lower payoff than Agent 1's equilibrium payoff, which is captured by inequality (1).

4.3. Robustness of equilibria

In Section 3, I identified some equilibria in which agents are vague about their advantageous dimensions.

$$s_1(\theta) = s_2(\theta) = (\varnothing, \varnothing), \forall\theta$$

can be sustained in equilibrium for all preferences. For other various classes of preferences, the following equilibria may also be sustained:

$$s_1(\theta) = (\varnothing, \theta^2 - b_1^2), s_2(\theta) = (\theta^1 - \delta b_2^1, \varnothing), \forall\theta, \delta \in [\delta^*, 1].$$

All these equilibria are robust to the EIC. The proof is in Appendix C.

Proposition 5. *All SES-equilibria identified in Proposition 3 and 4 are intuitive.*

By definition of an equilibrium, we know that for any proposal profile m, $\beta(m)$ is a best response to belief $\mu(\cdot \mid m)$ and that $\beta(\cdot)$ deters any deviation. Therefore, it suffices to show that for any off-path proposal profile m^{dev} resulting from a unilateral deviation, $\mu(\cdot \mid m^{dev}) \in \Delta(\Theta_1(m^{dev}) \cup \Theta_2(m^{dev}))$ whenever $\Theta_1(m^{dev}) \cup \Theta_2(m^{dev})$ is nonempty. I call a belief $\mu(\cdot \mid m)$ satisfying this property *reasonable*.

For the all-vague equilibrium, notice that any unilateral deviation is believed to be the deviator's ideal action. This belief is reasonable because, if the DM chooses the same action as in equilibrium (which is not dominated since it is a best response to this very belief), this deviation does not lead to a worse-than-equilibrium payoff for the deviator.

For the equilibria in which agents are only vague on their advantageous dimensions, let us take Agent 1 as the deviator (the argument for when Agent 2 deviates is completely symmetric).[9] Suppose he deviates to $(\varnothing, \varnothing)$. According to the equilibrium specification, $\mu(\cdot \mid m^{dev})$ is the updated belief given Agent 2's proposal only. If the DM responds by choosing $\beta = 1$, the deviation leads to a payoff which is no less than Agent 1's equilibrium payoff. Moreover, choosing $\beta = 1$ after the deviation is not dominated, since it is a best response if the DM believes that Agent 2's proposal on dimension 1 is far from θ^1.

Suppose Agent 1 deviates to $(w, \varnothing)$. Given that $\mu(\cdot \mid m^{dev})$ assigns probability 1 to the event that Agent 2 has not deviated, we can pin down the payoff of Agent 1 from Agent 1's deviation proposal and Agent 2's equilibrium proposal. Therefore, either $\mu(m^{dev}) \in \Delta(\Theta_1(m^{dev}))$, or $\Theta_1(m^{dev}) \cup \Theta_2(m^{dev})$ is empty.

Suppose Agent 1 deviates to (q, w). $\mu(\cdot \mid m^{dev})$ is concentrated on a single state at which Agent 2 has not deviated and Agent 1's deviation on dimension 2 is to his own ideal action. Since this is the most favorable belief for Agent 1 possible that assigns probability 1 to the event that Agent 2 has not deviated, either $\mu(\cdot \mid m^{dev}) \in \Delta(\Theta_1(m^{dev}))$ or $\Theta_1(m^{dev}) \cup \Theta_2(m^{dev})$ is empty.

5. Conclusion

This paper discusses the role of vague proposals in a setting with two competing agents. When uncertainty involves both the policy-relevant state and the action choice, preciseness allows the opponent to anchor on the information revealed

[9] We only need to consider deviations of the form $(\varnothing, \varnothing)$, $(w, \varnothing)$, and (q, w). Any deviation of the form $(\varnothing, z)$ leads to an on-path proposal profile so EIC does not apply.

about both aspects of uncertainty and offer a compromise action, while vagueness conceals the information about the state and makes compromises impossible. To demonstrate this intuition, I identify equilibria in which agents are only vague about their advantageous dimensions, and show such equilibria as being unique within a class.

Two interesting questions require a fuller characterization of equilibria. The first is whether vagueness should be allowed, especially in the political campaign context. Whether or not banning vagueness is possible, conceivably one can deal with this question by studying how vagueness impacts the voter's welfare. To do this, one needs to characterize the voter's equilibrium welfare when vagueness is allowed and when it is not. The former requires characterizing equilibria not just when vagueness occurs, but also when it does not.

To get a vague idea of how banning vagueness could negatively affect the voter's welfare, consider the case in which both candidates propose a constant, precise policy for each dimension in all states. It is easy to see that this strategy profile can be sustained in equilibrium even though the proposal could be arbitrarily far from the true state realization. In contrast, consider the equilibrium in which both candidates are always vague about both dimensions, in which case the voter gets the less biased candidate's ideal policies. Although this is not a fair comparison, these two equilibria have qualitative similarities: both involve constant proposals, do not transmit any information about the state, and survive the refinement. But under at least some distributions of the state, vagueness improves the voter's welfare.

Another question is: when does competition lead to less certainty? As mentioned in a previous section, in cheap talk, persuasion, and disclosure models, competition can increase the amount of information revealed. In this model, where uncertainty involves both the state and action and communication takes the form of proposals, I demonstrate that competition does not necessarily lead to less uncertainty. However, I have not shown if all equilibria of the game involve vagueness and this is left for future research.

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Appendix A. Proof for $((\boldsymbol{w}, \varnothing), (\varnothing, \boldsymbol{z}))$

For convenience, I use the following notation. For $j = 0, 1, 2$, I rewrite $v_j(\theta, i, m_i)$ as $v_j(\theta, y_i(m_i))$ to make Agent i's action explicit. For $\mu \in \Delta(\Theta)$ and $m \in M \times M$, $\pi_i(\mu, m)$ denotes the DM's expected payoff from choosing Agent i given her belief μ and agents' proposal profile m. Given an equilibrium, $u_i^*(\theta)$ denotes Agent i's equilibrium payoff when the state is θ. After some agent's deviation, the resulting proposal profile is m^{dev} and Agent i's payoff given m^{dev} is $u_i(\theta, m^{dev}, \beta(m^{dev}))$.

A.1. Proof of Proposition 1

The proof rules out an equilibrium proposal profile which takes the form

$$m = ((w, \varnothing), (\varnothing, z))$$

where $w, z \in \mathbb{R}$ at some $\theta = \overline{\theta}$.

Case 1. $|b_2^1| > \|b_1\|$.

From Lemma 1, $\beta(m) \in (0, 1)$. Since $b_2^1 \neq b_1^1$, this implies that $u_1^*(\overline{\theta}) \neq 0$. I show that Agent 1 has a profitable deviation $m_1^{dev} = (\varnothing, \varnothing)$ at $\overline{\theta}$. To see this, let $m^{dev} = ((\varnothing, \varnothing), (\varnothing, z))$ and $\mu \in \Delta(\Theta)$. Then we have

$$\begin{aligned}
\pi_1(\mu, m^{dev}) &= -\|b_1\|^2, \\
\pi_2(\mu, m^{dev}) &= \int_\Theta v_0(\theta, (\theta^1 - b_2^1, z))\mu d\theta \\
&\leq -|b_2^1|^2 < -\|b_1\|^2.
\end{aligned}$$

Therefore $\beta(m^{dev}) = 1$. Moreover, because for any θ and z,

$$v_1(\theta, y_1(\varnothing, \varnothing)) > v_1(\theta, y_2(\varnothing, z)),$$

we have

$$u_1(\overline{\theta}, m^{dev}, \beta(m^{dev})) = v_1(\overline{\theta}, y_1(\varnothing, \varnothing))$$
$$> \beta(m)v_1(\overline{\theta}, y_1(w, \varnothing)) + (1-\beta(m))v_1(\overline{\theta}, y_2(\varnothing, z))$$
$$= u_1^*(\overline{\theta}).$$

$(\varnothing, \varnothing)$ is then a profitable deviation for Agent 1. Therefore m cannot be an equilibrium proposal profile at $\overline{\theta}$.

Case 2. $|b_2^1| = \|b_1\|$.

Step 1. There exists $\tilde{\theta}$ ($\tilde{\theta}$ may equal $\overline{\theta}$) s.t.

$$s_1(\tilde{\theta}) = (\tilde{\theta}^1 - b_1^1, \varnothing), s_2(\tilde{\theta}) = (\varnothing, \tilde{\theta}^2).$$

It follows from Lemma 1 that $\beta(m) \in (0,1)$. Therefore the DM is indifferent. Now, given *efficiency*, the worst outcome the DM can get from Agent 1 is (b_1^1, b_1^2), while the best she can get from Agent 2 is $(b_2^1, 0)$. Since the DM is exactly indifferent between (b_1^1, b_1^2) and $(b_2^1, 0)$, she must assign probability 1 to the state $(b_1^1 + w, z)$. Since the DM's belief must be consistent with agents' strategies, there exists $\tilde{\theta}$ such that $s_1(\tilde{\theta}) = (\tilde{\theta}^1 - b_1^1, \varnothing)$ and $s_2(\tilde{\theta}) = (\varnothing, \tilde{\theta}^2)$. If $\tilde{\theta} \neq \overline{\theta}$, then replace $\overline{\theta}$ with $\tilde{\theta}$ for the rest of the proof.

Step 2.

Let Agent 1 deviate to $m_1^{dev} = (\varnothing, \overline{\theta}^2 - (1-\varepsilon)b_1^2)$ at $\overline{\theta}$, where $\varepsilon \in (0,1)$, close enough to 0 such that

$$v_1(\overline{\theta}, (\overline{\theta}^1 - b_1^1, \overline{\theta}^2 - (1-\varepsilon)b_1^2)) > u_1^*(\overline{\theta}) = \beta(m) \times 0 + (1-\beta(m)) \times v_1(\overline{\theta}, (\overline{\theta}^1 - b_2^1, \overline{\theta}^2)).$$

Note that since $\beta(m) \in (0,1)$, this can always be done.

Step 2a.

If $\beta(m^{dev}) = 1$, then m_1^{dev} is a profitable deviation for Agent 1.

Step 2b.

If $\beta(m^{dev}) < 1$, then there exists some $\underline{\theta}$ in the support of the DM's belief, at which Agent 1 is playing according to equilibrium. This is because, since m_2 is consistent with equilibrium and m^{dev} is not a symmetric proposal profile, m^{dev} could either be:

1. off-path with m_1^{dev} inconsistent with equilibrium and m_2 consistent with equilibrium;
2. off-path with both m_1^{dev} and m_2 consistent with equilibrium;

If it is the first case, then by *single-deviation consistency*, the DM believes that Agent 2 has not deviated and therefore the state is one in which the corresponding equilibrium proposal profile takes the form

$$(\tilde{w}, \varnothing), (\varnothing, \tilde{z})$$

where $\tilde{w}, \tilde{z} \in \mathbb{R}$. Moreover, the DM learns $\theta^2 = z$ and therefore strictly prefers Agent 1. This contradicts to $\beta(m^{dev}) < 1$. The same argument applies if it is the second case and the DM assigns probability 1 to Agent 2 has not deviated. Therefore, there must exist a state $\underline{\theta}$ at which Agent 2 has deviated. Again by *single-deviation consistency*, this means that Agent 1 has not. That is, there exists $\underline{\theta} = (\underline{\theta}^1, x)$ such that

$$s_1(\underline{\theta}) = m_1^{dev}$$
$$-(b_1^1)^2 - [x - \overline{\theta}^2 + (1-\varepsilon)b_1^2]^2 \leq -(b_2^1)^2 - (x - \overline{\theta}^2)^2.$$

In other words, there must exist a state at which m_1^{dev} is the corresponding equilibrium proposal for Agent 1 and at this state, Agent 2 is weakly preferred.

Regrouping the items in the inequality above, we have that

$$2(x - \overline{\theta}^2)(1-\varepsilon)b_1^2 \geq (b_2^1)^2 - (b_1^1)^2 - (1-\varepsilon)^2(b_1^2)^2.$$

Since the preferences of the agents satisfy

$$(b_2^1)^2 = (b_1^1)^2 + (b_1^2)^2,$$

we can replace term $(b_2^1)^2$ on the RHS and get

$$2(x - \overline{\theta}^2)(1-\varepsilon)b_1^2 \geq (2-\varepsilon)\varepsilon(b_1^2)^2.$$

If $b_1^2 > 0$, we then have

$$x-\overline{\theta}^2 \geq \frac{(2-\varepsilon)\varepsilon b_1^2}{2(1-\varepsilon)} > \varepsilon b_1^2.$$

On the other hand, efficiency requires that

$$x-b_1^2 \leq \overline{\theta}^2-(1-\varepsilon)b_1^2 \leq x,$$

which implies

$$x-\overline{\theta}^2 \leq \varepsilon b_1^2,$$

a contradiction.

If $b_1^2<0$, we then have

$$x-\overline{\theta}^2 \leq \frac{(2-\varepsilon)\varepsilon b_1^2}{2(1-\varepsilon)} < \varepsilon b_1^2.$$

Efficiency now requires that

$$x \leq \overline{\theta}^2-(1-\varepsilon)b_1^2 \leq x-b_1^2,$$

which implies

$$x-\overline{\theta}^2 \geq \varepsilon b_1^2,$$

also a contradiction.

To summarize, $\beta(m^{dev})=1$ and m_1^{dev} is a profitable deviation.

Case 3. $|b_1^2|=\|b_2\|$.

Since $\|b_1\| \leq \|b_2\|$ (assumed throughout the paper), $|b_1^1|=0$. If $b_2^2=0$, then $|b_2^1|=\|b_1\|$, which is covered in Case 2. If $b_2^2 \neq 0$, this is symmetric to the case where $b_2^2=0$ and $|b_2^1|=\|b_1\|$, which is also covered in Case 2. □

A.2. Proof of Proposition 2

When $|b_2^1| \geq \|b_1\|$ or $|b_1^2| \geq \|b_2\|$, Proposition 1 applies. So here I restrict to the case where $|b_2^1|<\|b_1\|$ and $|b_1^2|<\|b_2\|$. Suppose by way of contradiction that $s_1(\theta)=(\theta^1-\delta_1 b_1^1,\varnothing)$, $s_2(\theta)=(\varnothing,\theta^2-\delta_2 b_2^2)$ are the equilibrium strategies for the agents for some δ_1 and δ_2.

Case 1. $|b_1^1|=0$.

By *efficiency*, $\delta_1=0$. Therefore Agent 1 proposes his ideal outcome b_1.

Fix $\overline{\theta}$. By Lemma 1, $\overline{\beta}=\beta(s_1(\overline{\theta}),s_2(\overline{\theta})) \in (0,1)$ and the DM is indifferent. Therefore $\delta_2=\sqrt{\|b_1\|^2-(b_2^1)^2}\cdot\frac{b_2^2}{|b_2^2|}$. For any $\overline{\beta} \in (0,1)$, we can find some $\varepsilon>0$ such that $(0,(1-\varepsilon)b_1^2)$ gives Agent 1 strictly higher payoff than his equilibrium payoff $\overline{\beta}\cdot 0+(1-\overline{\beta})\cdot v_1(\overline{\theta},y_2(\varnothing,\overline{\theta}^2-\delta_2 b_2^2))$. Let Agent 1 deviate to $(\varnothing,\overline{\theta}^2-(1-\varepsilon)b_1^2)$, and the DM would strictly prefer Agent 1, so it's a profitable deviation.

Case 2. $|b_1^1| \neq 0$.

First, we note that either $|b_1^1|<|b_1^2|$, or $|b_2^1|>|b_2^2|$. If not, then we have $|b_1^1| \geq |b_1^2|$ and $|b_2^1| \leq |b_2^2|$. Since $|b_1^1|<|b_2^1|$ and $|b_2^2|<|b_1^2|$ (assumed throughout the paper), we then have $|b_1^1|<|b_2^1| \leq |b_2^2|<|b_1^2|$, a contradiction.

If $|b_1^1|<|b_1^2|$, then by Claim 1 below, at any $\overline{\theta}$, there exists $\varepsilon>0$ such that the outcome $(b_1^1,\overline{\theta}^2-\tilde{z}(\varepsilon))$ is strictly preferred by Agent 1 to both agents' proposals. Let Agent 1 deviate to $(\varnothing,\tilde{z}(\varepsilon))$. Since $(\varnothing,\tilde{z})$ is inconsistent with equilibrium, the DM believes that Agent 2 has not deviated, learns the true $\overline{\theta}^2$ from Agent 2 and prefers Agent 1's deviation proposal.

Claim 1. *If $|b_1^1|<|b_1^2|$, then at any $\overline{\theta}$, there exists $\varepsilon>0$ such that $(b_1^1,\tilde{z}(\varepsilon))$ is preferred by Agent 1 to the outcome $(\delta_1 b_1^1, b_1^2)$ and $(b_2^1,\delta_2 b_2^2)$.*

Proof. For any $\varepsilon>0$, let

$$\tilde{z}(\varepsilon)=\overline{\theta}^2+\frac{b_1^2}{|b_1^2|}\left(\varepsilon-\sqrt{(\delta_1 b_1^1)^2+(b_1^2)^2-(b_1^1)^2}\right).$$

I first show that for ε small enough, Agent 1 strictly prefers the outcome from a deviation proposal $(\varnothing,\tilde{z}(\varepsilon))$ to the one from his own equilibrium proposal $(\overline{\theta}^1-\delta_1 b_1^1,\varnothing)$. (See Fig. 6.) I show this by first showing that $OA>OB$. To see this, notice that since O is above the 45-degree line, $CE>DE$. This implies $\sin\angle CAE>\sin\angle EBD$. Therefore $\angle CAE>\angle EBD$. Moreover,

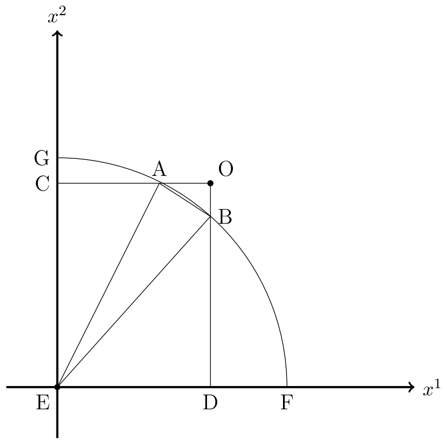

Fig. 6. E denotes the outcome $(0, 0)$. O denotes the outcome $(|b_1^1|, |b_1^2|)$, which is above the 45-degree line. A denotes the outcome $(\delta_1|b_1^1|, |b_1^2|)$. B denotes the outcome $(|b_1^1|, \sqrt{(\delta_1 b_1^1)^2 + (b_1^2)^2 - (b_1^1)^2})$. The arc FG is an indifference curve of the DM. OC and OD are parallel to the axes.

since $EA = EB$, $\angle EAB = \angle EBA$. Since $\angle CAE + \angle EAB + \angle OAB = \angle EBD + \angle EBA + \angle OBA = 180°$, we have $\angle OBA > \angle OAB$. Therefore $OA > OB$. That is,

$$\|(|b_1^1|, |b_1^2|) - (\delta_1|b_1^1|, |b_1^2|)\| > \|(|b_1^1|, |b_1^2|) - (|b_1^1|, \sqrt{(\delta_1 b_1^1)^2 + (b_1^2)^2 - (b_1^1)^2})\|,$$

which implies

$$\|(b_1^1, b_1^2) - (\delta_1 b_1^1, b_1^2)\| > \|(b_1^1, b_1^2) - (b_1^1, \frac{b_1^2}{|b_1^2|}\sqrt{(\delta_1 b_1^1)^2 + (b_1^2)^2 - (b_1^1)^2})\|.$$

Therefore, for ε sufficiently small,

$$\|(b_1^1, b_1^2) - (\delta_1 b_1^1, b_1^2)\| > \|(b_1^1, b_1^2) - (b_1^1, \frac{b_1^2}{|b_1^2|}(\sqrt{(\delta_1 b_1^1)^2 + (b_1^2)^2 - (b_1^1)^2} - \varepsilon))\|.$$

That is, Agent 1 strictly prefers the outcome from $(\varnothing, \tilde{z}(\varepsilon))$ to that from $(\overline{\theta}^1 - \delta_1 b_1^1, \varnothing)$.

Now I show that for ε small enough, Agent 1 strictly prefers the outcome from a deviation proposal $(\varnothing, \tilde{z}(\varepsilon))$ to the one from Agent 2's equilibrium proposal $(\varnothing, \overline{\theta}^2 - \delta_2 b_2^2)$. First, note that since $|b_2^1| > |b_1^1|$ (assumed throughout the paper) and the DM is indifferent between the agents' equilibrium proposals, the outcome $(|b_2^1|, \delta_2|b_2^2|)$ lies on the arc BF. Second, note that on dimension 1 alone, Agent 1 strictly prefers his deviation proposal. Therefore it suffices to show that this is also true on dimension 2 alone. It then suffices to show that

$$|b_1^2 - \frac{b_1^2}{|b_1^2|}\sqrt{(\delta_1 b_1^1)^2 + (b_1^2)^2 - (b_1^1)^2}| < |b_1^2 - \delta_2 b_2^2|.$$

Notice that LHS $\leq |b_1^2|$. Therefore if $b_2^2 \cdot b_1^2 < 0$, RHS $> |b_1^2|$. Let us then assume that $b_2^2 \cdot b_1^2 > 0$. It then suffices to show

$$||b_1^2| - \sqrt{(\delta_1 b_1^1)^2 + (b_1^2)^2 - (b_1^1)^2}| < ||b_1^2| - \delta_2|b_2^2||.$$

This is true since any point G on the arc BF is automatically below B. $\square$

Since it is obvious that for ε small, the DM strictly prefers Agent 1's deviation $(\varnothing, \tilde{z}(\varepsilon))$, we have found a profitable deviation.

Similarly, when $|b_2^1| > |b_2^2|$, Agent 2 has a profitable deviation $(\tilde{w}(\varepsilon), \varnothing)$ where ε is sufficiently small and

$$\tilde{w}(\varepsilon) = \overline{\theta}^1 + \frac{b_2^1}{|b_2^1|}\left(\varepsilon - \sqrt{(\delta_2 b_2^2)^2 + (b_2^1)^2 - (b_2^2)^2}\right). \quad \square$$

Appendix B. Proof for $((\varnothing,\cdot),(\cdot,\varnothing))$

B.1. Proof of Proposition 3

The proof shows that the following is an equilibrium proposal profile:

$$s_1(\theta)=s_2(\theta)=(\varnothing,\varnothing),\forall\theta.$$

The DM's belief and strategy is as follows: for any m such that there is exactly one agent who is precise for any dimension(s), the DM believes that he is proposing his own ideal policy and chooses the agents with the same probability as in equilibrium. In other words, if m is such that $m_i\in(\mathbb{R}\times\{\varnothing\})\cup(\{\varnothing\}\times\mathbb{R})\cup(\mathbb{R}^2)$ and $m_{-i}=(\varnothing,\varnothing)$, $\mu(\{\tilde{\theta}\mid\tilde{\theta}^j=m_i^j+b_i^j,\forall j \text{ s.t. } m_i^j\in\mathbb{R}\}\mid m)=1$. Any other m is the result of bilateral deviation and $\mu(\cdot\mid m)$ is not restricted by single-deviation consistency or consequential in sustaining the equilibrium. Therefore $\mu(m)\in\Delta(\Theta)$ and $\beta(m)\in\widetilde{\mathrm{BR}}(\mu,m)$.

To see that no agent has an incentive to deviate, suppose that at $\overline{\theta}$ Agent i deviates to m_i^{dev}. Given $m^{dev}=(m_i^{dev},m_{-i})$, note that

$$\pi_i(\mu(\cdot\mid m^{dev}),m^{dev})=\int\limits_{\Theta}v_0(\theta,\theta-b_i)\mu d\theta.$$

Therefore deviating does not change how favorable the DM perceives the deviator's proposal. So $\beta(m^{dev})=\beta(s_1(\overline{\theta}),s_2(\overline{\theta}))$ and m_i^{dev} is unprofitable. □

B.2. Proof of Proposition 4

First I prove the non-equilibria. There are three possible subcases:

Case 1 $\delta_1<1$ and $\|(b_1,\delta_1 b_1^2)\|=\|(\delta_2 b_2^1,b_2^2)\|$.

In this case, the DM is indifferent in equilibrium. Therefore $\beta\in[0,1]$. Now, if $\beta(s(\theta))=0$ $\forall\theta$, then Agent 2 wins with probability 1 at any state. Note that $\delta_1<1$, so Agent 1 is compromising. Since $\|b_2\|\geq\|b_1\|$, for the DM to be indifferent, Agent 2 must be compromising as well. Therefore $\delta_2<1$. Now we can see that Agent 2 has incentive to deviate to $(\overline{\theta}^1-b_2^1,\varnothing)$ at any $\overline{\theta}$ and get his own ideal proposal with probability 1, since the deviation is undetectable by the DM.

Therefore $\beta(s(\theta))=\beta>0$ for some θ. Also notice that for some other $\overline{\theta}$, we have

$$\overline{\theta}^2-b_1^2=\theta^2-\delta_1 b_1^2,\overline{\theta}^1=\theta^1$$

That is, Agent 1's ideal proposal at $\overline{\theta}$ is his equilibrium proposal at θ. His equilibrium payoff at $\overline{\theta}$ is

$$u_1^*(\overline{\theta})=\overline{\beta}[-(1-\delta_1)^2\cdot(b_1^2)^2]+(1-\overline{\beta})k,$$

where $\overline{\beta}=\beta(s(\overline{\theta}))$ and $k<0$ is Agent 1's payoff from Agent 2's equilibrium proposal. Note that k does not depend on the state.

Now suppose that at $\overline{\theta}$ Agent 1 deviates to his ideal proposal; that is, $m_1^{dev}=(\varnothing,\overline{\theta}^2-b_1^2)=s_1(\theta)$. Note that this deviation leads to the equilibrium proposal profile $s(\theta)$ and the deviation payoff is as follows:

$$\beta\cdot 0+(1-\beta)\cdot k.$$

For Agent 1 to have no incentive to deviate at $\overline{\theta}$ to $s_1(\theta)$, we need

$$-\overline{\beta}(1-\delta_1)^2(b_1^2)^2+(1-\overline{\beta})k\geq(1-\beta)k$$

which means

$$[-k-(1-\delta_1)^2(b_1^2)^2]\overline{\beta}\geq -k\beta.$$

1. If $-k-(1-\delta_1)^2(b_1^2)^2>0$, we then have
$$\overline{\beta}\geq\frac{-k}{-k-(1-\delta_1)^2(b_1^2)^2}\cdot\beta,$$
where $\dfrac{-k}{-k-(1-\delta_1)^2(b_1^2)^2}>1$. Therefore, eventually we would have that for some $\tilde{\theta}$, $\beta(s(\tilde{\theta}))>1$, a contradiction.
2. If $-k-(1-\delta_1)^2(b_1^2)^2=0$, we then have that Agent 1 gets same payoff from his and Agent 2's equilibrium proposal. Then his equilibrium payoff at $\overline{\theta}$ equals k regardless of the value of $\overline{\beta}$, whereas his deviation payoff equals $\beta\cdot 0+(1-\beta)\cdot k=(1-\beta)\cdot k$. Since $\beta>0$, this is strictly greater than k. Therefore deviation must be profitable.

3. If $-k-(1-\delta_1)^2(b_1^2)^2<0$, we then have that

$$\overline{\beta} \leq \frac{-k}{-k-(1-\delta_1)^2(b_1^2)^2} \cdot \beta < 0,$$

also a contradiction.

Case 2 $\delta_1 < 1$ and $\|(b_1, \delta_1 b_1^2)\| < \|(\delta_2 b_2^1, b_2^2)\|$.

In this case, Agent 1 wins with probability 1. For the same reason as that stated in the first paragraph in Case 1, Agent 1 could deviate to his own ideal proposal and still wins with probability 1.

Case 3 $\delta_1 \leq 1$ and $\|(b_1, \delta_1 b_1^2)\| > \|(\delta_2 b_2^1, b_2^2)\|$.

In this case, Agent 2 wins with probability 1. Since $\delta_1 \leq 1$, we must have that $\delta_2 < 1$ also. Therefore for the same reason as in Case 2, Agent 2 could deviate to his own ideal proposal and still wins with probability 1.

Now I prove the existence of equilibria when $\delta_1 = 1$ and $\|(b_1, \delta_1 b_1^2)\| \leq \|(\delta_2 b_2^1, b_2^2)\|$.

Case 4 $\delta_1 = 1$ and $\|(b_1, \delta_1 b_1^2)\| < \|(\delta_2 b_2^1, b_2^2)\|$.

This equilibrium can be sustained when

a. $|b_2^1| > \|b_1\|$, or
b. $|b_2^1| = \|b_1\| < \|b_2\|$, or
c. $|b_2^1| < \|b_1\| < \|b_2\|$ and Agent 2 weakly prefers b_1 to $(b_2^1, \sqrt{\|b_1\|^2 - |b_2^1|^2} \cdot \frac{b_2^2}{|b_2^2|})$,

but not when

d. $|b_2^1| < \|b_1\| < \|b_2\|$ and Agent 2 strictly prefers $(b_2^1, \sqrt{\|b_1\|^2 - |b_2^1|^2} \cdot \frac{b_2^2}{|b_2^2|})$ to b_1.

The DM's belief and strategy are as follows:

1. For any $m = ((\varnothing, w), (z, \varnothing))$ where $w, z \in \mathbb{R}$, $\mu(\{(z+\delta b_2^1, w+b_1^2)\} \mid m) = 1$, $\beta(m) = 1$.
2. For any $m = ((\varnothing, \varnothing), (z, \varnothing))$ where $z \in \mathbb{R}$, $\mu(\cdot \mid m) = F(\cdot \mid \{\tilde{\theta} \mid \tilde{\theta}^1 = z + \delta b_2^1\})$ and $\beta(m) = 1$.
3. For any $m = ((w, \varnothing), (z, \varnothing))$ where $w, z \in \mathbb{R}$, $\mu(\cdot \mid m) = F(\cdot \mid \{\tilde{\theta} \mid \tilde{\theta}^1 = z + \delta b_2^1\})$.

$$\beta(m) = \begin{cases} \frac{1}{2} & \text{if } \|(z+\delta b_2^1 - w, b_1^2)\| = \|(\delta b_2^1, b_2^2)\|, \\ 0 & \text{if } \|(z+\delta b_2^1 - w, b_1^2)\| > \|(\delta b_2^1, b_2^2)\|, \\ 1 & \text{if } \|(z+\delta b_2^1 - w, b_1^2)\| < \|(\delta b_2^1, b_2^2)\|. \end{cases}$$

4. For any $m = ((w, q), (z, \varnothing))$ where $q, w, z \in \mathbb{R}$, $\mu(\cdot \mid m) = F(\cdot \mid \{\tilde{\theta} \mid \tilde{\theta}^1 = z + \delta b_2^1, \tilde{\theta}^2 = q + b_1^2\})$.

$$\beta(m) = \begin{cases} \frac{1}{2} & \text{if } \|(z+\delta b_2^1 - w, b_1^2)\| = \|(\delta b_2^1, b_2^2)\|, \\ 0 & \text{if } \|(z+\delta b_2^1 - w, b_1^2)\| > \|(\delta b_2^1, b_2^2)\|, \\ 1 & \text{if } \|(z+\delta b_2^1 - w, b_1^2)\| < \|(\delta b_2^1, b_2^2)\|. \end{cases}$$

5. For any $m = ((\varnothing, w), (\varnothing, \varnothing))$ where $w \in \mathbb{R}$, $\mu(\cdot \mid m) = F(\cdot \mid \{\tilde{\theta} \mid \tilde{\theta}^2 = w + b_1^2\})$ and $\beta(m) = 1$.
6. For any $m = ((\varnothing, w), (\varnothing, z))$ where $w, z \in \mathbb{R}$, $\mu(\cdot \mid m) = F(\cdot \mid \{\tilde{\theta} \mid \tilde{\theta}^2 = w + b_1^2\})$.

$$\beta(m) = \begin{cases} 0 & \text{if } \|b_1\| > \|(b_2^1, w + b_1^2 - z)\|, \\ 1 & \text{if } \|b_1\| \leq \|(b_2^1, w + b_1^2 - z)\|. \end{cases}$$

7. For any $m = ((\varnothing, w), (q, z))$ where $w, q, z \in \mathbb{R}$, $\mu(\{(q + b_2^1, w + b_1^2)\} \mid m) = 1$.

$$\beta(m) = \begin{cases} 0 & \text{if } \|b_1\| > \|(b_2^1, w + b_1^2 - z)\|, \\ 1 & \text{if } \|b_1\| \leq \|(b_2^1, w + b_1^2 - z)\|. \end{cases}$$

8. For any other m, $\mu \in \Delta(\Theta)$ and $\beta(m) \in \widetilde{\mathrm{BR}}(\mu, m)$.

$\mu(\cdot \mid m)$ satisfies single-deviation consistency for all m. To see this, first notice that any off-path proposal profile which is the result of a unilateral deviation must contain a proposal m_i^{dev} which is inconsistent with equilibrium. Second, whenever Agent i deviates to m_i^{dev} which is inconsistent with equilibrium, the DM believes that Agent $-i$ has not deviated.

Now I show that no agent has an incentive to deviate. For any m on-path, $\beta(m) = 1$. Since Agent 1 is proposing his own ideal action, he has no incentive to deviate. To see that Agent 2 has no incentive to deviate either:

(a) any deviation of the form $(z, \varnothing)$ where $z \in \mathbb{R}$ is undetectable by the DM and therefore does not increase Agent 2's probability of winning from 0;
(b) any deviation $(\varnothing, \varnothing)$ leads the DM to believe that Agent 1 is playing equilibrium. Since $\|b_1\| < \|b_2\|$ Agent 2 continues to lose;
(c) any deviation of the form (z, w) where $z, w \in \mathbb{R}$ leads the DM to continue believing that Agent 1 is playing equilibrium and Agent 2 is proposing his own ideal policy on dimension 1. Whenever $|b_2^1| > \|b_1\|$ or $|b_2^1| = \|b_1\| < \|b_2\|$, for any $w \in \mathbb{R}$, the DM still weakly prefers Agent 1 and chooses $\beta = 1$. When $|b_2^1| < \|b_1\| < \|b_2\|$ and Agent 2 weakly prefers b_1 to $(b_2^1, \sqrt{\|b_1\|^2 - |b_2^1|^2} \cdot \frac{b_2^2}{|b_2^2|})$, Agent wins by deviating to $(\overline{\theta}^1 - b_2^1, \overline{\theta}^2 - \sqrt{\|b_1\|^2 - |b_2^1|^2} \cdot \frac{b_2^2}{|b_2^2|})$ at some $\overline{\theta}$. However, since this deviation is less preferred to the equilibrium outcome b_1, the deviation is not profitable.
(d) Same argument as (c).

When $|b_2^1| < \|b_1\| < \|b_2\|$ and Agent 2 strictly prefers $(b_2^1, \sqrt{\|b_1\|^2 - |b_2^1|^2} \cdot \frac{b_2^2}{|b_2^2|})$ to b_1, however, at any $\overline{\theta}$ Agent 2 has an incentive to deviate to $(\overline{\theta}^1 - b_2^1, \overline{\theta}^2 - \sqrt{\|b_1\|^2 - |b_2^1|^2} \cdot \frac{b_2^2}{|b_2^2|})$.

Case 5 $\delta_1 = 1$ and $\|(b_1, \delta_1 b_1^2)\| = \|(\delta_2 b_2^1, b_2^2)\|$.
This can be sustained when

a. $|b_2^1| > \|b_1\|$, or
b. $|b_2^1| = \|b_1\| < \|b_2\|$, or
c. $|b_2^1| = \|b_1\| = \|b_2\|$, or
d. $|b_1^2| = \|b_1\| = \|b_2\|$, or
e. $|b_2^1| < \|b_1\| < \|b_2\|$ and Agent 2 weakly prefers b_1 to $(b_2^1, \sqrt{\|b_1\|^2 - |b_2^1|^2} \cdot \frac{b_2^2}{|b_2^2|})$

but not when

f. $|b_2^1| < \|b_1\| < \|b_2\|$ and Agent 2 strictly prefers $(b_2^1, \sqrt{\|b_1\|^2 - |b_2^1|^2} \cdot \frac{b_2^2}{|b_2^2|})$ to b_1, or
g. $|b_2^1| < \|b_1\| = \|b_2\|$ and $|b_1^2| < \|b_2\|$.

The DM's belief and strategy that sustain the equilibrium under preferences a.-e. are same as in Case 4. Under f., the same reason in Case 4 implies that Agent 2 has a profitable deviation. Under g., at any $\overline{\theta}$ for which Agent 1 is not winning with probability 1, he has an incentive to deviate to $(\overline{\theta}^1 - (1-\varepsilon)b_1^1, \varnothing)$ for some ε. Similarly, at any $\overline{\theta}$ for which Agent 2 is not winning with probability 1, he has an incentive to deviate to $(\varnothing, \overline{\theta}^2 - (1-\varepsilon)b_2^2)$ for some ε. $\square$

Appendix C. Proof of Proposition 5

I first show that the equilibrium in Proposition 3 in which

$$s_1(\theta) = s_2(\theta) = (\varnothing, \varnothing), \forall \theta$$

satisfies the Extended Intuitive Criterion. For any $m^{dev} \neq ((\varnothing, \varnothing), (\varnothing, \varnothing))$ that results from a unilateral deviation, $\mu(\cdot \mid m^{dev})$ is as follows: if $m_{-i}^{dev} = (\varnothing, \varnothing)$ and $m_i^{dev} \in (\mathbb{R} \times \{\varnothing\}) \cup (\{\varnothing\} \times \mathbb{R}) \cup (\mathbb{R}^2)$, then

$$\mu(\{\tilde{\theta} \mid \tilde{\theta}^j = m_i^{dev,j} + b_i^j, \forall j \in \{1,2\} \text{ s.t. } m_i^{dev,j} \in \mathbb{R}\} \mid m^{dev}) = 1$$

and $\beta(m^{dev}) = \beta((\varnothing, \varnothing), (\varnothing, \varnothing))$. Note that $\beta(m^{dev}) = \beta((\varnothing, \varnothing), (\varnothing, \varnothing))$ is a best response to $\mu(\cdot \mid m^{dev})$. Moreover, for any $\tilde{\theta}$ in the support of $\mu(\cdot \mid m^{dev})$, $u_i(\tilde{\theta}, m^{dev}, \beta(m^{dev})) = u_i^*(\tilde{\theta})$, so obviously $u_i^*(\tilde{\theta}) \leq \max_{\beta \in BR(\Theta, m^{dev})} u_i(\tilde{\theta}, m^{dev}, \beta)$. Therefore

$$s_1(\theta) = s_2(\theta) = (\varnothing, \varnothing), \forall \theta$$

can be sustained in an intuitive equilibrium.

Now I show that the equilibria in Proposition 4 are intuitive by showing that all $\mu(\cdot \mid m^{dev})$ are supported in $\Theta_i(m^{dev})$ for some i, whenever $\Theta_1(m^{dev}) \cup \Theta_2(m^{dev})$ is nonempty.

First, consider $m^{dev} = ((\varnothing, \varnothing), (z, \varnothing))$. The equilibrium states that $\mu(\cdot \mid m^{dev})$ is updated such that the DM believes that Agent 2 has not deviated. To see it is reasonable, first note that $\Theta_2(m^{dev})$ is empty because m_1^{dev} is inconsistent with equilibrium. To characterize $\Theta_1(m^{dev})$, first notice that Agent 1's highest deviation payoff is $u_1(b_1)$, since the DM is free to believe that the actual θ^1 is arbitrarily far from z and choose $\beta = 1$ (remember that $\|b_1\| \leq \|b_2\|$). Since b_1 is Agent 1's most preferred outcome, $(\varnothing, \varnothing)$ must be potentially profitable. That is, the inequality

$$u_1^*(\theta) \le \max_{\beta \in BR(\Theta, m^{dev})} u_1(\theta, m^{dev}, \beta)$$

is satisfied by any θ. Therefore,

$$\Theta_1(m^{dev}) = \{\theta \mid s_2(\theta) = m_2^{dev}\}.$$

Therefore $\mu(\cdot \mid m^{dev})$ is reasonable.

Second, consider $m^{dev} = ((w, \varnothing), (z, \varnothing))$. The equilibrium states that $\mu(\cdot \mid m^{dev})$ is updated such that the DM believes that Agent 2 has not deviated. Again, $\Theta_2(m^{dev})$ is empty because m_1^{dev} is inconsistent with equilibrium. To characterize $\Theta_1(m^{dev})$, notice that the condition $s_2(\theta) = m_2^{dev}$ pins down Agent 1's payoff from m_2^{dev} as well as his payoff from m_1^{dev}, which means

$$\max_{\beta \in BR(\Theta, m^{dev})} u_1(\theta, m^{dev}, \beta)$$

is also pinned down. Therefore, either $\Theta_1(m^{dev})$ is empty, or

$$\Theta_1(m^{dev}) = \{\theta \mid s_2(\theta) = m_2^{dev}\}.$$

In either case, $\mu(\cdot \mid m^{dev})$ is reasonable.

Lastly, consider $m^{dev} = ((q, w), (z, \varnothing))$. The equilibrium states that $\mu(\cdot \mid m^{dev})$ is updated such that the DM believes that Agent 2 has not deviated (that is, $\theta^1 = z + \delta b_2^1$ or $\theta^1 = z + b_2^1$, depending on the equilibrium) and $\theta^2 = q + b_1^2$. Again, $\Theta_2(m^{dev})$ is empty. I show that if the state $\overline{\theta}$ in the support of μ does not belong to $\Theta_1(m^{dev})$, then $\Theta_1(m^{dev})$ is empty as well.

First note that given any q, w, z, $0 \in \mathrm{BR}(\Theta, m^{dev})$ since the DM is free to believe that w is arbitrarily far from θ^2. $1 \in \mathrm{BR}(\Theta, m^{dev})$ since the DM is free to believe that $\theta = (q, w)$. Given the hypothesis that $\overline{\theta} \notin \Theta_1(m^{dev})$, we have

$$\max_{\beta \in \mathrm{BR}(\Theta, m^{dev})} \beta v_1(\overline{\theta}, (q, w)) + (1-\beta) v_1(\overline{\theta}, (z, w + b_1^2 - b_2^2)) < u_1^*(\overline{\theta}),$$

then

$$\begin{aligned} v_1(\overline{\theta}, (q, w)) &< u_1^*(\overline{\theta}) \\ v_1(\overline{\theta}, (z, w + b_1^2 - b_2^2)) &< u_1^*(\overline{\theta}). \end{aligned}$$

Now, suppose $\tilde{\theta} \in \Theta_1(m^{dev})$ and $\tilde{\theta} \neq \overline{\theta}$. Then we must have $\tilde{\theta}^1 = \overline{\theta}^1$ and $\tilde{\theta}^2 \neq \overline{\theta}^2$. That is, at state $\tilde{\theta}$, Agent 1's deviation (q, w) must be such that

$$v_1(\tilde{\theta}, (q, w)) < v_1(\overline{\theta}, (q, w)) < u_1^*(\overline{\theta}) = u_1^*(\tilde{\theta})$$

and

$$v_1(\tilde{\theta}, (z, w + b_1^2 - b_2^1)) < u_1^*(\overline{\theta}) = u_1^*(\tilde{\theta}).$$

Therefore, $\tilde{\theta} \notin \Theta_1(m^{dev})$.

Although we have only considered m^{dev} in which Agent 1 is the sole deviator, the same argument applies to any m^{dev} in which Agent 2 is the sole deviator. □

Appendix D. A counterexample

Let $\Theta = \{\overline{\theta}, \hat{\theta}\}$ with $\Pr(\theta = \overline{\theta}) = \alpha = 1 - \Pr(\theta = \hat{\theta})$, where $\alpha \in (0, 1)$. Agent i's set of proposals is $\{\overline{m_i}, \hat{m}_i\}$. Agent 1's strategy is $s_1(\hat{m}_1 \mid \theta) = 1$, $\forall \theta$. Agent 2's strategy is $s_2(\overline{m_2} \mid \overline{\theta}) = 1 = s_2(\hat{m}_2 \mid \hat{\theta})$. One can then construct s_1^n such that $s_1^n(\hat{m}_1 \mid \overline{\theta}) = 1 - \frac{1}{n} = 1 - s_1^n(\overline{m_1} \mid \overline{\theta})$, $s_1^n(\hat{m}_1 \mid \hat{\theta}) = 1 - \frac{1}{n^2} = 1 - s_1^n(\overline{m_1} \mid \hat{\theta})$ and s_2^n such that $s_2^n(\overline{m_2} \mid \overline{\theta}) = 1 - \frac{1}{n} = 1 - s_2^n(\overline{m_2} \mid \overline{\theta})$ and $s_2^n(\hat{m}_2 \mid \hat{\theta}) = 1 - \frac{1}{n} = 1 - s_2^n(\overline{m_2} \mid \overline{\theta})$. One can easily check that $s_1^n \to s_1$ and $s_2^n \to s_2$.

Suppose now the DM observes the following proposal profile $(\overline{m_1}, \hat{m}_2)$. By Bayes's rule, the DM's belief μ^n derived through (s_1^n, s_2^n) satisfies

$$\begin{aligned} \mu^n(\hat{\theta} \mid \overline{m_1}, \hat{m}_2) &= \frac{\Pr(\hat{\theta}) \cdot \Pr(\overline{m_1}, \hat{m}_2 \mid \hat{\theta})}{\Pr(\hat{\theta}) \cdot \Pr(\overline{m_1}, \hat{m}_2 \mid \hat{\theta}) + \Pr(\overline{\theta}) \cdot \Pr(\overline{m_1}, \hat{m}_2 \mid \overline{\theta})} \\ &= \frac{(1-\alpha)\frac{1}{n^2}(1 - \frac{1}{n})}{(1-\alpha)\frac{1}{n^2}(1 - \frac{1}{n}) + \alpha \frac{1}{n}\frac{1}{n}} \\ &\to 1 - \alpha. \end{aligned}$$

Therefore, (s_1, s_2, μ) where $\mu(\hat{\theta} \mid \overline{m_1}, \hat{m_2}) = 1 - \alpha$ is part of a consistent assessment according to Kreps and Wilson (1982).

However, since Agent 1's proposal is inconsistent with equilibrium while Agent 2's proposal is consistent with equilibrium, *single-deviation consistency* states that the DM should believe that Agent 2 has not deviated, and therefore $\theta = \hat{\theta}$ with probability 1.

Appendix E. Restricted state space

The reason why the equilibrium in which $s_1(\theta) = (\varnothing, \theta^2 - b_1^2)$, $s_2(\theta) = (\theta^1 - b_2^1, \varnothing)$ identified in Proposition 4 can be sustained has to do with the assumption that $\Theta = \mathbb{R}^2$. Indeed, when Θ is a proper subset of $\mathbb{R}^2$, it is possible that a deviation $(\varnothing, \overline{\theta}^2 - (1-\varepsilon)b_1^2)$ by Agent 1 leads to an off-path proposal profile. If the DM correctly identifies the deviator and the state space is such that the information from the non-deviator is sufficient for her to evaluate the proposal of the deviator, the deviation may be profitable. The example below illustrates such a situation.

Example 3. Let $\Theta := \{(a, b) \in \mathbb{R}^2 : a = b\}$, $b_1 = (0, 1)$, and $b_2 = (1, 0)$ and consider the putative equilibrium in which

$$s_1(\theta) = (\varnothing, \theta^2 - 1), \forall\theta,$$
$$s_2(\theta) = (\theta^1 - 1, \varnothing), \forall\theta.$$

Suppose that $\beta < 1$ (if $\beta = 1$, then the same argument can be applied to Agent 2). Let Agent 1 deviate to $(\varnothing, \overline{\theta}^2 - 1 + \varepsilon)$ at $\theta = \overline{\theta}$, where ε is sufficiently small that if Agent 1 wins with probability 1, then the deviation is profitable. Note that if $\Theta = \mathbb{R}^2$, then this proposal will not lead to an off-path proposal profile. Here, however, since both proposals are consistent with equilibrium but the resulting proposal profile is off-path, the DM believes that exactly one agent has deviated. If she believes that Agent 1 is the deviator, then she learns $\theta^2 = \overline{\theta}^2$, realizes that Agent 1 has deviated to a compromise, and therefore strictly prefers Agent 1. If she believes Agent 2 is the deviator, then she wrongly learns that $\overline{\theta} = (\overline{\theta}^2 + \varepsilon, \overline{\theta}^2 + \varepsilon)$, but still strictly prefers Agent 1. Consequently, for any belief concentrating on the event that only one agent has deviated, she prefers Agent 1. Therefore, $(\varnothing, \overline{\theta}^2 - 1 + \varepsilon)$ is a profitable deviation for Agent 1.

In the example above, the two dimensions are perfectly correlated. If the DM correctly identify the deviator, she obtains the correct information about the state from the non-deviator and prefers the compromise by the deviator. If the DM does not correctly identify the deviator, the relative direction of the two agents' biases implies that she still prefers the deviator.

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