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Sustainable development: Structural transformation and the consumer demand

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ABSTRACT

This paper examines the feasibility of environmentally sustainable growth in a competitive market economy assuming various types of technological changes affecting pollution emissions and ultimately climate change. We consider two final outputs and two factors of production, accounting for both pollution flow and stock effects. If the initial level of pollution emissions satisfies certain boundary conditions, a Pigouvian pollution tax may assure sustainable growth without any further government intervention. This is true even if exogenous technological change is assumed to benefit exclusively the pollution-intensive industries (the "dirty" sector). A consumers' composition effect (often neglected in the literature), driven by an endogenous change in the relative prices between clean and dirty final goods under an optimal pollution tax, plays a critical role in the structural transformation process to achieve long-run sustainable economic growth.

1. Introduction

Concern over climate change has led to more studies to focus on the feasibility of environmentally sustainable long-run growth (e.g., Stokey, 1998; Barbier, 1999, Peretto, 2009; Brock and Taylor, 2010), and on the role of innovation and technological change mechanisms to achieve sustainable economic growth (e.g., Barbier, 1999; Bretschger and Smulders, 2012; Acemoglu et al., 2012). Reflecting on the evidence of low substitutability between man-made capital inputs and natural resources, many economic growth studies show the possibilities of sustainable economic growth via endogenous pro-environment innovation (e.g., Bovenberg and Smulders 1995; Peretto, 2009; Bretschger and Smulders, 2012).[1]

While the role of innovation and endogenous technological change is widely discussed in the literature as a mechanism to solve limits to economic growth, few studies of economic growth focus on an alternative mechanism via a structural change in the consumption of final goods in a model of renewable resources.[2] In a context of non-renewable resources and endogenous growth, Bretschger and Smulders (2012) examine a role of the structural transformation process in a free market system when faced with a sustained rise in the price of non-renewable resources. Poor input substitution promotes sectoral changes and investment activities. In our model of economic growth with renewable resources, sustainable growth requires government intervention to trigger a change in relative prices between clean and dirty consumer goods which may eventually induce a structural transformation over the long run.

The main objective of this paper is to show the critical role of changes in the structure of consumption as a mechanism for sustainable development. We consider various types of technological change in a dynamic general equilibrium framework of a closed economy with two final outputs and two factors of production.

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[1] Empirical evidence suggests that even substitution among fuels is much less than 1. For example, the consensus estimates for the elasticity of substitution between coal (the dirtiest fuel) and natural gas (one of the cleanest fuels) in the electricity generation industry in the United States is about 0.4 (Energy Information Administration, 2012).

[2] Recently, López and Yoon (2014) and López and Yoon (2016) explicitly took the consumer's role into account in achieving sustainable growth. However, they do not systematically consider various technological change that may work unfavorably toward achieving environmental sustainability.

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We allow for various types of exogenous technological change, including neutral and pollution-augmenting in the dirty or pollution-intensive sector that increases the use of pollution-intensive inputs in such sector. In fact, it has been often argued that improving the efficiency of fossil energy in a knowledge-based economy leads to greater use of energy, causing energy rebound effects (e.g., Greening et al., 2000; Herring, 2006).[3] We consider the long-run impact of a pollution tax in the presence of exogenous technological changes, either neutral or pollution-augmenting, that make sustainable growth more difficult, and examine the technological and consumption characteristics of the economy in which environmental sustainability is still feasible.[4]

An important application of the model analyzed is to show that climate change can be arrested merely using an appropriate carbon tax without any further intervention, even under unfavorable technological conditions and consumer preferences for rapid economic growth.

One essential feature of consumers' preferences for economic growth is the elasticity of marginal utility of income (*EMU* or the inverse of the intertemporal elasticity of substitution). A low (high) *EMU* is associated with high (low) consumers' preferences for rapid economic growth, which means that the capacity of an economy to achieve sustainable development is highly influenced by its magnitude. In fact, and perhaps not uncoincidentally, many studies of sustainable development have assumed values of *EMU* greater than or equal to 1 (e.g., Bovenberg and Smulders, 1996; Stokey,1998; Bye, 2000; Hartman and Kwon, 2005).

However, the empirical evidence regarding the size of *EMU* is mixed, with some studies obtaining values of *EMU* above and others well below 1 (e.g., Attanasio and Browning, 1995; Ogaki and Reinhart 1998; Vissing-Jørgensen, 2002; Layard et al., 2008; Colacito and Croce, 2013). Thus, we consider the mixed empirical evidence raised in the literature and examine how an economy with *EMU* above or below 1 may achieve sustainable economic growth in the context of an optimal Pigouvian pollution emissions tax.

Recent microeconomic studies increasingly recognize the importance of accounting for the complexities of the consumption effect on the environment that growth studies typically omit (e.g., Ahmad and Wyckoff, 2003; Peters, 2008; Weber et al., 2008; Liu et al., 2013). The importance of the consumption composition effect for achieving sustainable growth can also be observed from empirical studies (e.g., Grossman and Krueger, 1995; Cole and Elliot, 2003). While empirical studies on production generally report a weak substitution between clean and dirty inputs (e.g., Kemfert and Welsch, 2000; Van der Werf, 2008; Hassler et al., 2012), studies on consumer demand report stronger substitution between clean and dirty consumer goods, often obtaining elasticity of substitution estimates much greater than 1 (e.g., Lin et al., 2008; Galarraga et al., 2011, obtain high elasticities between organic and conventional consumer goods). Moreover, studies have shown that consumers' substitution between clean and dirty goods is highly responsive to increased information and public education on the pollution content of consumer goods and eco-labeling (e.g., Kotchen and Moore, 2007).[5] Consequently, it appears that the scope for consumer substitution between clean and dirty goods is greater than the substitution potential among inputs by producers, a feature that we consider in this study.[6]

Our main finding is that a consumer's composition effect, driven by an endogenous change in the relative prices between clean and dirty final goods under optimal pollution tax, may induce long-run sustainable economic growth even if: (1) the elasticity of marginal utility of income is less than 1 (e.g., economy opts to grow faster via sacrificing future consumption for the current), and/or (2) the production elasticity of substitution between clean and dirty inputs is inelastic, and (3) technical change occurs in a way that favors the pollution-intensive industries and hence contributes to increase pollution.

The main mechanism underlying this result is the degree of consumers' flexibility regarding substituting their consumption of pollution-intensive final goods. In this case the relative price of the "dirty" pollution-intensive final goods over the "clean" final goods increases because of a pollution emissions tax, which causes consumers to progressively reduce their demand for dirty goods along the growth path if consumers are willing to substitute. Moreover, we show that while the pollution tax needs to be enough to induce a strong consumption substitution effect, the cost of achieving sustainability in terms of foregone economic growth is likely to be limited. Also, under certain plausible conditions, economic growth may partially recover over time to levels like those that would prevail in the absence of the tax. However, without enough consumers' flexibility, an optimal pollution tax is not enough to induce structural change over time, and sustainability cannot be obtained in a fast-growing economy or an economy with poor input substitution.

The analysis proceeds first in the context of the flow effects of pollution under the assumption that the pollution stock lies below the threshold level beyond which an environmental catastrophe occurs. This part of the analysis is self-contained, and can be used to examine the environmental effects, including climate change, of short-lived pollutants, such as soot, methane, hydro-fluorocarbons, surface ozone, and nitrogen dioxide that dissipates quickly and consequently has no stock effects.[7] Next, we allow for pollution accumulation and hence we consider the implications for the analysis of a stock constraint. We extend López and Yoon's (2016) results to confirm that a family of suboptimal growth path exists, each guaranteeing sustainable growth once an initial pollution level starts from a level below a critical level that we characterize in the paper, even after considering the various types of technological progress. We conclude with a simulation analysis to illustrate the importance of the stock effects in the analysis of climate change.

The paper is organized as follows. Section 2 presents the theoretical multi-sector growth model, while Section 3 shows optimality and market clearing conditions. Section 4 provides the conditions for dynamic equilibrium and Section 5 considers the effect of pollution tax on long run real consumption growth. Section 6 examines various conditions for sustainable growth. Section 7 considers the stock effect of pollution and shows conditions for

[3] Empirical evidence regarding the long-run effects of technological change that improves the efficiency of fossil energy has been found in the literature. See also, Sorrell and Dimitropoulos (2008) and Gillingham et al. (2015).

[4] An exogenous technological change, which is essentially factor augmenting for the clean input, is also allowed in some studies (e.g., Stokey, 1998; Brock and Taylor, 2010). This assumption dramatically raises the likelihood of sustainability. An important exception is Acemoglu et al. (2012), which allows for the endogenous factor-augmenting technological change in a model of constant elasticity of substitution technology between the clean and the dirty inputs. Acemoglu et al. (2012) show that a (temporary) subsidy to research and development for the clean input sector (sufficient to transform pollution-augmenting technological change into clean input-augmenting technological change) may cause sustainable economic growth as long as the production elasticity of substitution between the clean and the dirty inputs is greater than 1. Otherwise, targeted research subsidies are not guaranteed to affect the structure of technological change.

[5] These studies have shown that not only a price-based mechanism but also soft instruments, including information campaign or educational programs, can have influential impact on changing consumers' behavior (e.g., Thøgersen and Ölander, 2002; Arkesteijn and Oerlmans, 2005; Kotchen and Moore, 2007)

[6] This is in contrast with the earlier report on the lack of responsiveness to these interventions by manufacturing firms (Banerjee and Solomon, 2003).

[7] Recent studies have shown that the combined effect of soot and other short-lived pollutants is almost as important a source of climate change as carbon dioxide (CO_2), which contributes close to 40% of the total human-induced warming effect (Bond et al., 2013).

avoiding an environmental disaster. Section 8 presents numerical illustrations for both flow and stock effects of pollution, and Section 9 presents the conclusion.

2. Framework for the analysis

The framework builds upon the endogenous growth literature by adopting the modeling of endogenous growth of two final goods and two inputs (López and Yoon, 2014).

2.1. Production

Let k denote the total man-made composite input available at the time t in the economy. Consistent with the literature showing that clean or intangible forms of capital that impose little or no environmental pressures, such as new knowledge and human capital, are the key sources of economic growth, we define this composite input as clean capital.[8] Henceforth, we refer to k as "capital," which is distributed at each point in time between the clean industry and the dirty industry. Let k_d denote the amount of capital employed in the dirty industry that uses fossil fuels as a source of energy. The flow of pollution from the dirty sector is represented by x. Following Cropper and Oates (1992), López (1994), and Copeland and Taylor (2004), we consider pollution as a factor of production directly.[9] The output of the dirty good is:

$$y_d = A_d F(k_d, bx). \tag{1}$$

The parameter A_d denotes total factor productivity with a proportional growth rate $\dot{A}_d/A_d \equiv g_d \geq 0$, and $b > 1$ represents a pace of factor-augmenting technological progress with $\dot{b}/b \equiv \zeta \geq 0$.[10]

The dirty sector produces only a final consumer good and does not produce an investment good.[11] The production function F is a constant elasticity of substitution (CES) function given as follows:

$$F(k_d, bx) = \left[\alpha k_d^{-\frac{1-\omega}{\omega}} + (1-\alpha)(bx)^{-\frac{1-\omega}{\omega}}\right]^{-\frac{\omega}{1-\omega}},$$

where ω is the elasticity of substitution between capital and pollution, and α is a fixed distribution coefficient.

The output of the clean good is assumed to depend only on the capital input, and is governed by the linear production technology as follows:

$$y_c = A_c(k - k_d). \tag{2}$$

where the parameter A_c is the return to capital in the clean sector and k is the total stock of (clean) capital in the economy at a point in time. The clean sector produces a final consumer good as well as new clean capital, which is one of two sources of economic growth considered here: (clean) capital accumulation and technological change. To reduce notational clutter, we focus primarily on pollution-augmenting and neutral technological change.[12]

If we normalize the price of the clean good to unity (e.g. $p_c = 1$), the economy's budget constraint can be written as:

$$\dot{k} = A_c(k - k_d) + pA_d F(k_d, bx) - c - \delta k, \tag{3}$$

where $p \equiv p_d/p_c$ is the relative price of the dirty good, $c \equiv c_c + pc_d$ is the total consumption expenditure expressed in units of the clean good, δ is the rate of capital depreciation, and $\dot{k} \equiv dk/dt$ is the net capital accumulation. The sum of the first two terms on the right-hand side of Eq. (3) represents the income of the economy expressed in units of clean goods. The gross capital accumulation, $\dot{k} + \delta k$, is equal to net savings (income less consumption), which is also expressed in units of the clean good.[13]

2.2. Stock of clean air[14]

Economic activity releases pollution emissions flows into the atmosphere. A portion of pollution emissions is removed by nature's revitalization processes, but some emissions may remain as a stock that accumulates in the upper atmosphere. Pollution emissions, whether they accumulate in the atmosphere or rapidly dissipate, have instantaneous negative effects on welfare. In addition, a portion of the emissions accumulating in the atmosphere cause very gradual and subtle changes in climate, which may have gradual effects on welfare until accumulation reaches a threshold level where catastrophic events may be triggered, causing massive welfare losses.

Thus, pollution may reduce the stock of clean air so that the changes to clean air stock are the net result of two forces: the natural purification rate of pollution and the flow emission of pollution. Following the literature, we assume a constant rate of environmental regeneration (e.g., Aghion et al., 1998; Acemoglu et al., 2012; Lopez and Yoon, 2016). We denote the stock of clean air in the upper atmosphere as E, the threshold of minimal stock of clean air below which an environmental catastrophe occurs as $\underline{E}$, the pristine stock level by $\bar{E}$, and let $0 < \psi < 1$ be the constant rate of natural atmospheric purification. We assume that such regeneration rate does not depend on the stock, an assumption that is probably appropriate for physical resources, such as the atmosphere. Then we have:

$$\begin{aligned} \dot{E} &= \psi E - x \quad && \text{for } \underline{E} \leq E < \bar{E}. \\ &= -x && \text{for } E < \underline{E}. \end{aligned} \tag{4}$$

For future reference, we note that by integrating (4) within the specified boundaries we obtain:

$$E(t) = \exp(\psi t)\left(E_0 - \int_0^t x(\upsilon)\exp(-\psi\upsilon)d\upsilon\right) \tag{4'}$$

for $E(t) \geq \underline{E}$; E_0 is the initial, predetermined level of the stock of clean air.

2.3. Consumption and welfare

The welfare function of the representative consumer is comprised of two parts: Utility derived from the consumption of goods and disutility generated by pollution. We represent the utility

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[8] For example, Corrado et al. (2009) explain the dominant contribution of intangible forms of capital to economic growth in the U.S. economy. Galor and Moav (2004) present a model of economic growth to explain increasing importance of human capital. For empirical measurements of human and other intangible forms of capital, see for example, Barro (2001), Cohen and Soto (2007), and Barro and Lee (2013).

[9] As in Bovenberg and Smulders (1995), an increase in pollution is interpreted as an increase in the rate of harvest of natural capital. A more detailed description is given later in the next section.

[10] Technological change in the dirty sector is regarded as exogenous to the private entrepreneurs in the dirty sector. It may originate from the publicly funded investment as the social price of clean environment or pollution input increases over time. This paper, however, does not deal with Hicksian-induced innovation.

[11] It can be shown that the sustainable growth is not possible when the capital consists of a dirty good only since the real return to capital declines as the environmental standard becomes upgraded over time. There should be a positive minimal level of the proportion of a clean capital. We assume that capital consists of a clean good only to emphasize the role of clean capital such as human and knowledge capital in sustainable development.

[12] In the Appendix, however, we show that the results remain mostly unchanged by considering the capital-augmenting technological change.

[13] We assume that the investment in capital is irreversible. Once the economy builds capital, it cannot be transformed back into consumption goods: capital can be reduced over time only by allowing it to depreciate.

[14] Throughout this paper, the stock of clean air represents a stock of natural capital. The term 'clean air' is used for expositional convenience.

derived from the consumption of goods by an indirect utility function as follows:

$$u = \frac{1}{1-a}\left(\frac{c}{e(1,p)}\right)^{1-a},$$

where c denotes the total consumption expenditure, $e(1, p)$ is the unit (dual) expenditure function or cost-of-living index, and $a > 0$ is a parameter representing the *EMU*. If $a < 1$, we adopt a positive utility scale such that $0 < u < \infty$, while we scale the utility index to $-\infty < u < 0$ when $a > 1$. Of course, a special case of the above specification occurs when $a = 1$, in which case we obtain the often-used logarithmic specification, $u = \ln[c/e(1,p)]$. The indirect utility function is assumed to be increasing and strictly concave in the real consumption level, $c/e(1, p)$.

We assume that the consumers' underlying preferences for goods are described by a CES utility function so that the unit expenditure function is:

$$e(1,p) = \left[\gamma_c + \gamma_d p^{1-\sigma}\right]^{\frac{1}{1-\sigma}},$$

where σ is the consumption elasticity of substitution between the dirty and clean goods, and $\gamma_c > 0$ and $\gamma_d > 0$ are fixed parameters. The indirect utility function defined above presumes homothetic preferences. Consumer demand for the clean good c_c and dirty good c_d can be retrieved from the indirect utility function using Roy's identity. The optimal level of c is determined by the intertemporal optimization (as detailed below).

The second part of the welfare function corresponds to the disutility generated by pollution. Let $v(x; E)$ denote the environmental damage function, which is assumed to be increasing and convex in the level of pollution, x. We assume that the environmental damage function is:

$$v(x;E) = \frac{x^{1+\eta}}{1+\eta} \text{ if } E \geq \underline{E},$$

where $\eta > 0$ denotes the elasticity of marginal damage caused by pollution and is assumed to be a fixed parameter. When $E < \bar{E}$, the economy is in catastrophic calamity, and the consumption falls to 0 (Cropper, 1976; Weitzman, 2007; Nordhaus, 2012). We specify the consumers' total welfare function as :[15]

$$\begin{aligned} U(c,x;E) &\equiv \tfrac{1}{1-a}\left(\tfrac{c}{e(1,p)}\right)^{1-a} - \tfrac{x^{1+\eta}}{1+\eta} \; \textit{when } E \geq \underline{E} \\ &\equiv -\infty \quad \textit{when} \quad E < \underline{E}. \end{aligned}$$

Assuming a fixed pure time discount rate (ρ) and a socially optimal intervention, the competitive economy is modeled as if it maximizes the present discounted value of the utility function:

$$\int_0^\infty U(c,x;E)\exp(-\rho t)dt,$$

subject to the budget constraint (e.g. Eq. (3)), clean air stock level constraint $E \geq \underline{E}$ (Eq. (4)), and the initial conditions $k = k_0$ and $E = E_0$. In other words, the competitive behavior of the representative consumer and producer under optimal pollution tax and lump-sum reimbursement is described by the choices of the optimal levels of c and x at each point in time.

We assume that both goods are always produced, which implies $k_d(t) < k(t)$ for all t. Thus, the current value Hamiltonian function assuming an interior solution is:

$$\begin{aligned} H_E &= U(c,x,E) + \lambda[A_C(k-k_d) + pA_dF(k_d,bx) - c - \delta k] \\ &\quad + \mu[\psi E - x] + \phi[E - \underline{E}] \end{aligned}$$

where λ and μ denote co-state variables, each representing the shadow price of man-made capital and natural capital, respectively, while $\phi \geq 0$ is a time-varying Lagrange multiplier associated with the stock constraint.

2.4. Analytical strategy

We assume that the economy maximizes H_E subject to market equilibrium conditions for the final goods introduced below in the next section. So, in addition to the usual endogenous variables of the optimal control problem, we need to solve for the endogenous market prices. Using the system of necessary conditions for dynamic optimization (maximum principle and Kuhn-Tucker conditions) and market clearing conditions, we may in principle solve for seven endogenous variables ($c, k_d, x, p, \lambda, \mu, \phi$) at each point in time. While the analysis of the original problem is extremely complex given that the utility function is discontinuous at $E = \underline{E}$, the dynamic optimization process can be examined in a more tractable way if the shadow price of the stock of pollutant, ϕ, is 0, that is, if the stock constraint is not binding.

Therefore, we use the following strategy: first, we solve the model of dynamic optimization and market equilibrium using as a maintained assumption that $\phi = 0$, the stock of clean air remains above $\underline{E}$ throughout all time. Next, we analyze the conditions under which, given the solution derived from the first step, the constraint $E(t) \geq \underline{E}$ is satisfied for all t given initial stock levels of the natural and man-made assets, E_0 and K_0. Thus, the first part of the solution is obtained by maximizing H_E (subject to the relevant market clearing conditions) with $\phi = 0$, and the second part examines whether this solution satisfies the stock constraint.

Under our stated assumptions on preferences and production technology, H_E is strictly concave with respect to state and control variables, and the necessary conditions also become sufficient. In fact, a unique solution for the optimal control problem exists.[16] In subsequent sections, we also characterize the conditions for the clean air stock to remain above the threshold level. If the optimal path of emissions obtained by maximizing H_E does not permit the stock of clean air to fall below the critical threshold at any point in time, it constitutes an optimal solution for the original problem of dynamic market equilibrium with stock constraint.

We now define "sustainable economic growth."

Definition. : *We assert that sustainable economic growth is possible if, at some point along the growth process, the economy can continue growing indefinitely while pollution emissions permanently decline and the stock of natural capital never falls below the critical threshold level.*

Therefore, sustainability requires a finite time, $T \geq 0$, such that at any time $t > T$, $\hat{x} < 0$, which implies $\lim_{t\to\infty} \hat{x} \leq 0$ and $E(t) \geq \underline{E}$ for all t .[17] This paper is primarily concerned with the defining characteristics of the economy under which sustainable economic growth is possible by a competitive market system with a pollution tax. If sustainable growth is possible, the socially optimal growth rate should be positive in our model economy.

[15] As Nordhaus (2012) and Weitzman (2009) argue, the consumption utility becomes $-\infty$when $a > 1$. Since individual optimization when the economy falls toward the catastrophic calamity cannot be described, we assume that the environmental damage itself is greater than any finite conceivable magnitude, and hence minus infinity, even when $a < 1$.

[16] We note also that the Inada condition is satisfied. In other words, for any $a > 0$, our utility scale guarantees that $\lim_{c\to 0} U_x(c,x,E) = \infty$ for any finite x and$\underline{E} \leq E$.

[17] A similar notion of sustainable growth (e.g., growth of welfare of consumer with falling pollution level) has been adopted by several authors, including Barbier and Markandya (1990), Stokey (1998), Brock and Taylor (2010), and, most recently, Barbier (2016). Arrow et al. (2012) indicate that economic development is sustainable if the intergenerational welfare does not decline over time. We show later in Section 5 that our definition of sustainable growth conforms to their definition.

2.5. Additional considerations

Here we establish some basic properties of consumption and factor shares essential for the ensuing analysis. The budget share of the dirty consumer good in the consumption expenditure for the CES utility function is $s(p) = \frac{\gamma_d}{\gamma_c p^{\sigma-1}+\gamma_d}$ and the factor share of clean input in the cost of production of the dirty good for a CES production function is $S_k(k_d/bx) = \alpha[(1-\alpha)(\frac{k_d}{bx})^{\frac{1-\omega}{\omega}} + \alpha]^{-1}$. Of course, the share of dirty input in the cost of production of the dirty final good is $1 - S_k$. Then, we have the following remark:

Remark 1. The share $s(p)$ is an increasing (decreasing) function of p if $\sigma < 1 (\sigma > 1)$. The share $S_k(k_d/bx)$ is increasing (decreasing) in k_d/bx if $\omega > 1 (\omega < 1)$.

Remark 1 is important for subsequent analysis because it allows us to predict the evolution of $s(p)$ and $S_k(k_d/bx)$ over time if we know the dynamics of p and k_d/bx, on the basis of the size of the elasticity of substitution. As shown below, the dynamics of these shares are key factors determining the sustainability (or lack of sustainability) of the economy.

Finally, the next remark states the conditions under which pollution-augmenting technological change (increases in b in Eq. (1)) increases the marginal product of the dirty input. Under these circumstances, pollution-augmenting technological change can seriously undermine the possibility of sustainable growth.

Remark 2. Pollution-augmenting technological change increases the marginal product of pollution if and only if $S_k < \omega$.

Proof: See Appendix.

This condition is certainly satisfied when $\omega \geq 1$. If $\omega < 1$ then, as shown by Remark 1, S_k falls with k_d/bx and since, as we show below in this paper, k_d/bx continuously increases over time, we have that if $\omega < 1$ then $\lim_{t\to\infty} S_k = 0$. That is, pollution-augmenting technological change increases the marginal product of pollution at all times if $\omega \geq 1$, or in the long-run if $\omega < 1$. In this sense, we can say that pollution-augmenting technological progress is complementary with pollution. Thus, pollution-augmenting technological progress only concentrated in the dirty sector might not be favorable to sustainable development.

2.6. Assumptions

We make the following assumptions:

Assumption 1. The clean sector of the economy is sufficiently productive so that the marginal return to capital (A_c) is higher than the marginal opportunity cost of capital ($\rho + \delta$), hence $M \equiv A_c - \rho - \delta > 0$.

Assumption 2. Technological change can be pollution-augmenting occurring at an exogenous rate $\zeta \geq 0$ and/or neutral, raising the total factor productivity of the dirty sector at an exogenous rate $g_d \geq 0$. However, the rate of technological change in the dirty sector is bounded from above as follows: $\zeta + g_d \leq min\{M, M/a\}$.

Assumption 1 is a necessary condition for the economy to accumulate capital over time. Assumption 2 implies that exogenous technological change is concentrated on augmenting the dirty input in the dirty industry only while the clean input does not augment its productive capacity in either industry.

Assumption 2 also places a limit on the speed of technological progress in the dirty sector. This limit allows the price of the dirty good to increase over the growth process because if the productivity of the dirty good increases too fast the pollution tax increasing its cost of production may not be enough to induce a rise in its price. This, in turn, would cause the consumption substitution effect to disappear and, hence, a key mechanism for sustainability would not be in effect. There are two contradictory effects on the price of the dirty good over time; one is the increasing cost associated with the continuous pollution tax increase which raises the price of the dirty good. The other effect is the increasing productivity in the sector due to technological change. Assumption 2 allows the former effect to dominate the latter one and hence that the net cost of production of the dirty good increase. This causes its relative price to rise (Brock and Taylor, 2010).

3. Optimality and market clearing conditions

3.1. Optimality conditions

The first-order necessary conditions for maximization of the Hamiltonian function imply that the marginal utility of consumption must be equal to the shadow price of capital, λ:

$$e(1, p)^{a-1} c^{-a} = \lambda. \tag{5}$$

Along the optimal path the well-known no arbitrage condition must be satisfied:

$$\frac{\dot{\lambda}}{\lambda} = -[A_c - \rho - \delta] \equiv -M. \tag{6}$$

There are two additional conditions for optimality for an interior solution:

$$pA_d \frac{\partial F(k_d, bx)}{\partial k_d} - A_c = 0, \tag{7}$$

$$pA_d \frac{\partial F(k_d, bx)}{\partial x} - v'(x)/\lambda = 0. \tag{8}$$

Eq. (7) indicates that in equilibrium, the marginal value product of capital should be equalized across the two sectors. Eq. (8) states that the optimal pollution tax, which is equal to the marginal rate of substitution between pollution and consumption, $\tau \equiv v'(x)/\lambda$, is equalized to the marginal value product of pollution. Finally, savings should be equal to the net investment at each moment of time, so that we have Eq. (3) as an additional first order condition. Finally, we have the standard transversality condition, $\lim_{t\to\infty} \lambda k(t) e^{-\rho t} = 0$.

3.2. Market clearing conditions

In the Appendix, we show that the rate of growth of the consumer demand for dirty goods is:

$$\hat{c}_d = \frac{1}{a} M - \left[\frac{s(p)}{a} + (1 - s(p))\sigma\right] \hat{p}. \tag{9}$$

A circumflex above the symbol reflects its corresponding rate of growth. In addition, the rate of growth of production of the dirty goods is:

$$\hat{y}_d = g_d + \hat{F}(k_d, bx) = g_d + S_k \left(\widehat{\frac{k_d}{bx}}\right) + \left(\hat{bx}\right). \tag{10}$$

Because the dirty goods are used for consumption only, market equilibrium requires that $y_d = c_d$ at all points in time. Furthermore, once the dirty goods market is cleared, the market for the clean goods automatically clears because the current savings are equal to the current investment, as stipulated in Eq. (3). Therefore, the relative price of dirty goods must adjust endogenously over time to allow for the equilibrium to persist. Along the equilibrium path, the growth rate of production and demand for the dirty good must be equal so that $\hat{y}_d = \hat{c}_d$.

4. Dynamic equilibrium

4.1. Equilibrium conditions

In this section, we derive the system of dynamical equations that can be solved for the dynamic equilibrium path for $\hat{p}$, $(\frac{\hat{k}_d}{bx})$ and $\hat{x}$. Using Eqs. (9) and (10), we obtain:

$$z\hat{p} + S_k\left(\frac{\hat{k}_d}{bx}\right) + \hat{x} = \frac{M}{a} - g_d - \zeta \quad (11)$$

where $z \equiv s(p)\frac{1}{a} + (1 - s(p))\sigma > 0$ (also recall that $\zeta \equiv \dot{b}/b$ and $g_d \equiv \dot{A}_d/A_d$). The function z corresponds to the weighted average of the inter-temporal elasticity of substitution $(1/a)$ and the temporal elasticity of substitution, using the budget shares as weighting factors. We note that Assumption 2 assures that the right-hand-side of (11) is positive.

From Eq. (7), we have $\hat{p} + \hat{A}_D + \hat{F}_1(k_d, bx) = 0$, which, given the CES production function, implies that:

$$\hat{p} - \frac{1}{\omega}(1 - S_K)\left(\frac{\hat{k}_d}{bx}\right) = -g_d. \quad (12)$$

Finally, in the Appendix we show that using Eq. (8), the following expression follows:

$$\hat{p} + \frac{1}{\omega}S_K\left(\frac{\hat{k}_d}{bx}\right) - \eta\hat{x} = M - g_d - \zeta. \quad (13)$$

It states that the rate of increase of the private marginal revenue of the dirty input, $\hat{p} + \frac{1}{\omega}S_K(\frac{\hat{k}_d}{bx}) + \zeta + g_D$, is equal to the rate of increase of the input price which in turn equals the rate of change of the pollution tax, $\hat{\tau} = \eta\hat{x} + M$.

4.2. Solution of the dynamical system and the optimal pollution tax

In the Appendix, we show that the dynamical system of Eqs. (11)–(13) solves for the equilibrium growth rates of $\hat{p}$, $(\frac{\hat{k}_d}{bx})$ and $\hat{x}$, as follows:

$$\hat{p} = \frac{1}{|W|\omega}\Bigg(\left[M(1 - S_k)\left(\frac{\eta}{a} + 1\right)\right] - g_d\left[(1 - S_k)(\eta + 1) + \omega S_k\left(\eta + \frac{1}{\omega}\right)\right] - \zeta[(1 - S_k)(\eta + 1)]\Bigg), \quad (14)$$

$$\left(\frac{\hat{k}_d}{bx}\right) = \frac{1}{|W|}\left[M\left(\frac{\eta}{a} + 1\right) + g_d\eta(z - 1) - \zeta(\eta + 1)\right] > 0, \quad (15)$$

$$\hat{x} = \frac{1}{|W|\omega}\left\{M\left(\frac{1}{a} - z(1 - S_k) - \omega S_k\right) + g_d(z - 1) + \zeta(z(1 - S_k) + \omega S_k - 1)\right\}, \quad (16)$$

where $|W| \equiv \frac{1}{\omega}[(1 - S_k)(1 + z\eta) + S_k] + \eta S_k > 0$.

We can derive the dynamics of the optimal pollution tax consistent with the system using Eqs. (14)–(16). Noting that since $\tau = v'(x)/\lambda$, we have $\hat{\tau} = \eta\hat{x} + M$. Therefore, using Eqs. (8), (13), and (15) we can derive the rate of change of the pollution tax over time:

$$\hat{\tau} = \frac{\eta}{|W|\omega}\Bigg(\left(\left(\frac{1}{a} + \frac{1}{\eta}\right)M - (\zeta + g_d) + zg_d + ((1 - S_k)z + \omega S_k)\zeta\right) > 0.$$

By Assumption 2, $M/a \geq \zeta + g_d$ means that the pollution tax increases continuously along the optimal path. While the tax increases over time, the share of pollution tax costs on the total value of consumption, $\tau x/c$, may eventually decline along the optimal path.[18]

Expanding income due to capital accumulation induces an increase of the pollution tax since the marginal utility of consumption, λ, falls as $M > 0$. This means that the relative price of the dirty input (pollution) increases over time and triggers a technique or input substitution effect with a pollution-reducing effect. Under Assumption 2, Eq. (15) shows that the capital-to-effective pollution ratio (k_d/bx) increases over time.

In general, the relative price of the final dirty good does not necessarily increase over the growth path, and hence in principle the consumption composition is no assured. The fact that, as shown by (15), the k_d/bx ratio continuously rises over the full growth path means that the share of dirty inputs in the dirty industry is not stable along such path. If $\omega > 1$ then S_K converges towards 1 meaning that the dirty industry becomes progressively cleaner becoming a clean sector in the long-run. Therefore, in this case there no real need of a composition effect in order to attain sustainable development.

If $\omega < 1$ then S_K converges towards 0 meaning that the dirty sector becomes dirtier over time. In this case sustainable development does need the consumption composition effect which can only be triggered by an increase in the relative price of the dirty final good. It is in this case when Assumption 2 becomes essential to allow the price of the dirty good to continuously increase beyond a certain point in time. Over the long-run as S_K becomes smaller and smaller the relative price of the dirty final good eventually starts increasing inducing the consumption composition effect and thus making sustainable development possible.

Pollution-augmenting technological change weakens both the technique and composition effects. The increasing productivity of pollution counters the effect of the pollution tax, weakening the incentives to substitute pollution with clean inputs. Similarly, the increased productivity associated with technological change reduces the burden of pollution tax in the production of dirty goods. This, in turn, reduces the price increase of the dirty goods and weakens consumers' incentives to substitute dirty goods with clean ones. More on this in Section 7.

4.4. Suboptimal pollution paths

Since we can obtain an explicit and tractable solution for the optimal rates of change of pollution and the other relevant variables with enough information regarding the key parameters considered, this part of the solution is relatively easy to obtain for a government or planner. However, this is not a complete solution. To obtain a complete solution, we need to solve for the *initial* values of the endogenous variables (p,k_d/bx, x and, therefore, τ) in addition to their optimal rates of change as provided by Eqs. (14)–(16). In fact, determining such initial values is extremely complex for governments and analysts. Fortunately, as shown through an inspection of Eqs. (14)–(16), the optimal rates of change of the variables are not dependent on the variables' initial values.

This characteristic of the dynamical solution is important because, as we shall see below, it allows us to determine the maximal critical initial level of pollution that assures that the stock of clean air will never fall below the catastrophic threshold. If the government can determine such a critical level, its job would be reduced to ensure that the initial pollution level is below the critical point, and from then on follow the myopic growth rule dictated by Eq. (16). The result would be a suboptimal rule, implying higher pollution levels than the optimum at all points in time, but one

[18] Later in the paper we prove this assertion when the conditions for sustainable growth are satisfied.

that assures sustainable and positive economic growth thus preventing environmental disaster. Section 6 deals with these issues.

5. Economic growth (the growth cost of the pollution tax)

An important issue is whether the dynamic path described by Eqs. (14)–(16) in a market economy under a Pigovian pollution tax implies a positive rate of consumption growth despite that the pollution tax is continuously increasing. Otherwise, the growth path does not constitute a sustainable growth path (e.g., Barbier, 2016). The following proposition shows that this is indeed the case:

Proposition 1. *(i) The growth rate of real consumption expenditure is:*$(\hat{\frac{c}{e}}) = \frac{1}{a}[M - s(p)\hat{p}]$, *where* $\hat{p}$ *is given by (14). (ii) The rate of growth of real consumption remains positive throughout the equilibrium dynamic path for any positive* ω *and* σ. *(iii) If either input substitution or consumption substitution is elastic (if* $\omega > 1$ *or* $\sigma > 1$*), but not both, the rate of growth of real consumption converges from below toward a rate* M/a. *If both* $\omega > 1$ *and* $\sigma > 1$, *then the growth rate of real consumption converges to* $(1/a)(M + g_d)$. *(iv) If* $\omega < 1$ *and* $\sigma < 1$, *then the rate of growth of real consumption converges from above toward a rate* $((1+\eta)/(a+\eta))(\zeta + g_d) < M/a$.

Proof. See the Appendix.

Proposition 1 demonstrates that the dynamic equilibrium path described by Eqs. (14)–(16) is associated with a positive rate of growth of real consumption regardless of the size of the elasticity of substitution. Yet, the economy's growth rate is below its potential because the optimal pollution tax forces the relative price of dirty goods to increase continuously over time. This, in turn, increases the cost of living for consumers, implying that economic growth must be partially sacrificed. However, as shown in Remark 1, if $\sigma > 1$, the share of the dirty goods in the consumption bundle declines, and if $\omega > 1$, the share of the clean input in production increases. In either case, the sacrifice of the growth rate vis-à-vis its potential level becomes progressively smaller beyond a certain point. The growth rate of the economy approaches its maximum potential rate in the long run, which, in this case, is equal to M/a in the absence of neutral technological progress in the dirty sector.

When $\sigma > 1$ or $\omega > 1$, the convergence (or long run rate of growth) of the economy is not affected by the rate of pollution-augmenting technological change because in this case, the consumer budget share of pollution and/or the share of pollution in the cost of production approaches 0.[19] Pollution-augmenting technological change becomes irrelevant for economic growth over the long-run because the share of the dirty input in the production of the dirty goods and/or the share of dirty final goods constitute a negligible fraction of the economy. In this case the cost of the tax in terms of economic growth is purely temporary and declining, and the economy is able to reach its maximum growth potential over the long run.

Furthermore, from Remark 1 it follows that if $\omega < 1$ and $\sigma < 1$, the share of the dirty input (pollution) in the cost of production increases over time, and the share of dirty goods in the consumer budget increases over time, both converging to 1. Therefore, in such a case technological change becomes the key determinant of the convergence rate of economic growth. Conversely, because the share of the clean goods approaches 0, the capacity of the economy to expand such goods becomes increasingly irrelevant for economic growth. Thus, in the inelastic case, the economy's growth rate declines and becomes increasingly dependent on the rate of technological change and less dependent on the rate of capital accumulation as the shares of the dirty input and dirty final output increase over time. Moreover, Assumption 2 implies that the growth rate of the economy converges to a lower level than in the elastic case.

The following corollary to Proposition 1 summarizes the results discussed in the previous two paragraphs:

Corollary 1. *Economies characterized by elastic producer and/or consumer choices tend to grow more rapidly and converge towards higher secular growth rates than economies exhibiting inelastic producer and consumer choices.*

6. Conditions for sustainable growth (assuming $\phi = 0$)

When *EMU* is greater than 1, as is assumed by standard sustainable growth models, society's willingness to pay for a marginal reduction of pollution increases rapidly with income. The growth effect then becomes relatively weak vis-à-vis the case where $EMU < 1$. Even when both consumption and input elasticity of substitution are less than 1, sustainable development arises.

The following proposition emerges for the case when *EMU>1*:

Proposition 2. *Suppose* $a > 1$, *assumptions 1 and 2 hold, and either* σ *and/or* ω *is positive, then an optimal pollution tax is sufficient to induce sustainable development for any type of exogenous technological progress (pollution-augmenting and/or neutral).*

Proof. See the Appendix.

6.3. The case when EMU *is less than 1*

Some recent studies have shown that, contrary to previous assumptions, the *EMU* may reach levels below 1 (e.g., Ortu et al., 2013; Colacito and Croce, 2011; Vissing-Jørgensen, 2002). When $a < 1$**,** the conditions for sustainable economic growth are more demanding than in the previous case. This section will characterize the output composition effect, and briefly discuss the input substitution (or technique) effect.

6.3.1. The output composition effect

The composition effect works when consumers substitute dirty goods with clean goods in the face of the rising relative price of dirty goods. Here, we consider the case when the consumption elasticity of substitution is strictly greater than 1, but the production elasticity of substitution is less than 1. In this case, the feasibility of sustainable growth relies exclusively on consumer flexibility because the dirty sector becomes progressively dirtier along the growth process. Using Remark 1, it follows that the factor share of the clean input in the output value of the dirty final goods,S_k, converges to 0 (and concomitantly, the share of the dirty input converges to 1). The relative price of dirty goods continuously increases over time meaning that consumers substitute dirty goods with clean ones.

Therefore, assuming $\sigma > 1$ and $\omega < 1$, the limit to Eq. (16) is:

$$\lim_{t\to\infty} \hat{x} = \frac{M\left(\frac{1}{a} - \sigma\right) - (\zeta + g_d)(1-\sigma)}{(1+\sigma\eta)}. \tag{17}$$

From Eq. (17), it follows that $\lim_{t\to\infty} \hat{x} < 0$, if and only if

$$\sigma > \frac{\frac{M}{a} - (\zeta + g_d)}{M - (\zeta + g_d)} \equiv d(M, a; \zeta, g_d) > 1.$$

The threshold level $d(M,\ a;\ \zeta,\ g_d)$, above which sustainable growth becomes possible, is increasing in ζ and g_d, respectively. Technological change in the dirty sector makes sustainable growth

[19] This is true if $\sigma > 1$, but $\omega < 1$ because in this case the consumption share of the dirty goods approaches 0 and hence the participation of the dirty goods in the economy becomes negligible in the very long run. Furthermore, if $\sigma < 1$ but $\omega > 1$, the share of pollution in the production of the dirty goods approaches 0, meaning that in the very long run the participation of pollution as an input becomes negligible.

more difficult. The threshold level reduces to $1/a$ in the absence of any form of technological progress. The following lemma summarizes the previous results:

Lemma 1 (on the role of the composition effect). *Suppose that technological progress is pollution-augmenting and/or is neutral or non-existent and that assumptions 1 and 2 hold. If $a<1$, then an inelastic technical substitution ($\omega<1$) does not preclude sustainable economic growth if and only if σ is greater than a threshold level exceeding 1 (e.g.,$\sigma>d(M,\ a;\ \zeta,\ g_d)>1$).*

Lemma 1 underlines the importance of the composition effect in circumventing the case of an inelastic production technology. Previous analyses that have often assumed a single final good ignored the output composition effect, concluding that a flexible production technology ($\omega\geq 1$) is a necessary condition to allow for sustainable development. Lemma 1 shows that this is not true if consumer preferences are sufficiently flexible ($\sigma>d(M,\ a;\ \zeta,\ g_d)>1$). Remarkably, sustainable growth under an optimal pollution tax may occur even if the production function of dirty goods is Leontief ($\omega=0$), that is, even if clean and dirty inputs are complements rather than substitutes.

We note that even if the share of dirty goods approaches 0, it does not necessarily imply that the rate of growth of the demand for (and hence supply of) the dirty final goods will become negative. In fact, the growth rate of dirty goods continues to be positive over the long-run if the economy's growth rate is sufficiently rapid and may even surpass the rate of pollution-augmenting technological change, in which case pollution will continue to increase in the long run. Lemma 1 shows that only when $\sigma>d(M,\ a;\ \zeta,\ g_d)>1$ will the consumption of dirty goods (and hence the production of dirty goods) grow at a rate below the pollution-augmenting technological change, thus leading to a reduction of pollution levels.[20]

6.3.2. The input substitution or technique effect

We will now consider the case when the technical elasticity of substitution between the two inputs is strictly greater than 1, while the consumption elasticity of substitution is less than 1, but still positive. In this case, the cost share of the clean input approaches 1, while the share of the dirty good in the consumer budget also approaches 1. In this case the feasibility of sustainable growth depends solely on technique effect. From Eq. (16) we have:

$$\lim_{t\to\infty}\hat{x}=\frac{\left(\frac{M}{a}-\zeta\right)-\omega(M-\zeta)+g_d(\sigma-1)}{1+\omega\eta}. \qquad (18)$$

The numerator's first term (which is positive) of Eq. (18) captures the growth effects that measures the scale effects effect of economic growth and the direct effect of technical change. The numerator's second term represents the technique effect resulting from a change in the relative factor costs of production. The optimal pollution tax causes the pollution input to become increasingly expensive. In addition, if the elasticity of substitution between the clean and the dirty input is greater than 1, the pollution input is gradually substituted with capital, causing its share to converge to 0. The third term represents the effect of growth of total factor productivity in the dirty sector, which reduces pollution growth when $\sigma<1$. It follows that sustainable growth only becomes possible if the technique or substitution effect outweighs the technological change effect. This condition is satisfied if $\omega>d(M,\ a;\ \zeta,\ 0)>1$ where

$$d(M,a;\zeta,0)=\frac{\left(\frac{M}{a}-\zeta\right)}{M-\zeta}.$$

Consequently, if $a<1$, a Cobb–Douglas production function ($\omega=1$) is not consistent with sustainable development when $g_d=0$. As we demonstrate below, the standard growth models almost always assumed Cobb-Douglas production functions, and was able to conclude that growth is sustainable only because such models assumed that *EMU>1*. The following lemma summarizes these findings.

Lemma 2 (on the technique or input composition effect). *Suppose that technological progress is pollution-augmenting and that assumptions 1 and 2 hold. If $a<1$, then $\sigma<1$ does not preclude sustainable economic growth if an optimal pollution tax is implemented, and ω is greater than a threshold level $d(M,\ a;\ \zeta,\ 0)$ that exceeds 1.*

In our model, unlike Acemoglu et al. (2012), for example, capital (e.g., the clean input) is expanding in a growing economy, and the rate of economic growth is endogenous. Hence, even if, as we assume that technological change is pollution-augmenting and concentrated in the dirty sector, the capital-to-effective pollution ratio (k_d/bx) may increase without requiring so rapid an increase of the pollution tax as to smother economic growth. This follows because the technique effect does not rely exclusively on the pollution tax but is reinforced by the capital growth effect. Therefore, if the elasticity of substitution between capital and pollution is greater than the threshold level, then the substitution effect may dominate the expansion effect within the dirty sector, and pollution will begin decreasing at some finite time along the growth path. Combining Lemmas 1 and 2, we obtain the following proposition:

Proposition 3. *Suppose that technological change is pollution-augmenting and assumptions 1 and 2 hold. If $a<1$, then sustainable growth is feasible if an optimal pollution tax is implemented, and either ω or σ is greater than the threshold level, $d(M,\ a;\ \zeta,\ 0)$, which exceeds 1.*

Proof. See the Appendix.

Proposition 3 demonstrates that even if technological progress only benefits the dirty sector, and is biased toward the dirty input in a pollution-augmenting fashion, and if the *EMU* is less than 1, then an optimal pollution tax may be sufficient to induce environmental sustainability if either the consumers' preferences or the producer's technologies exhibit sufficient flexibility. From Proposition 1, it follows that this occurs while the economy's growth rate is positive throughout the full adjustment path. Moreover, since environmental sustainability requires either $\sigma>1$ or $\omega>1$, Proposition 1 shows that economic growth is lowered in the short run, but the economy's growth rate gradually recovers towards its potential rate over the long run. Therefore, under the conditions established in Proposition 2, an optimal pollution tax alone may lead to sustainable growth without requiring further policy interventions.

7. Stock effects: conditions for avoiding an environmental disaster

While the previous analysis has depicted a relatively optimistic case for sustainable economic growth in the long run by using a Pigouvian tax as the only policy instrument, this solution does not preclude the possibility that pollution continues increasing over a certain period of time. An important issue is whether the pollution stock along the growth path may reach a level that causes a catastrophic and irreversible damage. This is particularly relevant

[20] Given that $\omega<1$, which implies that $\lim_{t\to\infty}S_k=0$, it follows from Eq. (10) that the rate of growth of the dirty good production over the long run is equal to the growth rate of effective pollution, $\hat{y}_d^{\infty}=\hat{x}+\zeta$. Hence, if $\hat{y}_d^{\infty}=\hat{c}_d^{\infty}>\zeta$ then $\hat{x}>0$, where a superscript ∞ denotes long run levels.

for the case of climate change: If the stock of atmospheric clean air falls below a certain threshold level the earth climate could change causing an environmental disaster which is also irreversible.

Here we discuss the conditions under which the solution for the dynamical system as developed in the previous sections are indeed consistent with avoidance of environmental disaster at any point in time.[21] Assuming a dynamic path of pollution emissions as defined by Eq. (16), then for any given initial level of clean air stock a corresponding critical level of initial emission flow exists such that if the initial value of pollution emission is less than a critical level, the clean air stock remains at all times above a minimal threshold level that prevents environmental disaster. Otherwise, if the initial pollution level is above the critical level, the clean air stock falls below the threshold level, and catastrophic environmental disaster will eventually ensue. The intuition behind this result is that since Eq. (16) gives the (optimal) rate of change of pollution for all times (a rate which is independent of the initial pollution level), the full path of pollution is entirely determined by its initial level. The question is whether along this path the stock of clean air ever reaches the catastrophic level. If we find the initial (critical) level of pollution which, in conjunction with Eq. (16), causes a pollution path that exactly avoids reaching such a catastrophic stock level, then any other pollution path following the same rate of change established by Eq. (16), but starting from a lower pollution level, will also avoid catastrophe.

To identify such a critical level of initial emissions, we first note that for any given initial level of man-made capital, the system of Eqs. (14)–(16) yields a unique optimal growth path for p,(k_d/bx) and x. In fact, the system of Eqs. (14)–(16) can be represented as a system of autonomous differential equations: $\dot{p} = \Theta(S_k, s(p), p)$,$\dot{(k_d/bx)} = \Gamma(S_k, s(p), (k_d/bx))$, and $\dot{x} = \Phi(S_k, s(p), (k_d/bx), x)$. Since $\Theta(\cdot)$, $\Gamma(\cdot)$, and $\Phi(\cdot)$ are all continuously differentiable functions, a unique solution exists for each set of initial values. We also note that the solution for emission, x, constitutes an optimal control for dynamic optimization in the absence of stock constraints. The initial level of emission is determined endogenously within the system. Likewise, initial values of k_d, and therefore p, are all endogenously determined within the system. We can then define the unique path of pollution emission flows and stock of clean air as conditional functions of the (endogenous) initial value of pollution as well as the (predetermined) initial stocks of clean air and natural capital as follows:

$$x(t) = Q(t, x_0; k_0, \chi, \zeta, g_d) \text{ and } E(t) = L(t, x_0; k_0, E_0, \chi, \zeta, g_d),$$

where the function $L(t,\ x_0;\ k_0,\ E_0,\ \chi,\ \zeta,\ g_d)$ is defined by Eq. (4') and $\chi = (a, \sigma, \omega)$ denotes a vector of structural parameters. From Eq. (4'), unless the pollution emissions $x(t)$ eventually starts falling over time, the stock constraint, $E(t) \geq \underline{E}$ for all $t \geq 0$, may not be satisfied.

Let χ^* denote the set of $\chi = (a, \sigma, \omega)$ which guarantees eventual decline of pollution emissions, satisfying the conditions established by either Proposition 2 or Proposition 3. Then for any χ in χ^*, and man-made stock of capital, we can define the *admissible* set, $J(\chi,\ k_0,\ E_0;\ \zeta,\ g_d)$ of initial values of clean air stock and flow level of pollution, which assures sustainable growth. Thus,

$$J(\chi, k_0, E_0; \zeta, g_d) = \{ (x_0, E_0) | L(t, x_0; k_0, E_0, \chi, \zeta, g_d) \geq \underline{E}, \text{ for all } t > 0 \}.$$

Given the initial level of clean air, E_0, the set $J(\chi,\ k_0,\ E_0;\ \zeta,\ g_d)$ of initial levels of flow pollution an economy can emit while maintaining the stock of clean air above the threshold level is bounded above and closed because the function $L(t,\ x_0;\ k_0,\ E_0,\ \chi,\ \zeta,\ g_d)$ is continuous, as shown by Eq. (4'), and is also bounded from above. Following López and Yoon (2016) we confirm that the maximal element,$x_0^c(E_0; \zeta, g_d)$ of the set $J(\chi,\ k_0,\ E_0;\ \zeta,\ g_d)$ exists, above which an environmental disaster occurs sometime in the future. We define $C(\chi, k_0; \zeta, g_d) = \{E_0, x_0^c(E_0; \zeta, g_d) | \underline{E} \leq E_0\}$, which constitutes the boundary or envelope of the set $J(\chi,\ k_0,\ E_0;\ \zeta,\ g_d)$. Since a time $T \geq 0$ exists, after which pollution emissions decrease in a monotonic way for any eventually declining pollution emissions path, there is a critical turnaround time $t^* > T$ such that

$$x(t*) = Q(t*, x_0^c; k_0, \chi, \zeta, g_d) = \psi \underline{E}, \tag{19}$$

$$E(t*) = L(t*, x_0^c; k_0, E_0, \chi, \zeta, g_d) = \underline{E}, \tag{20}$$

where x_0^c is the *maximum* initial level of pollution emissions that corresponds to any given $E_0 > \underline{E}$, is consistent with avoiding environmental disaster, and t^* is the critical turnaround time when the stock of clean air reaches the minimum level necessary to avoid a catastrophe. Eqs. (19) and (20) solve for the two endogenous variables $x_0^c = x_0^c(E_0; k_0, \underline{E}, \chi, \psi, \zeta, g_d)$ and $t* = N(E_0; k_0, \underline{E}, \chi, \psi, \zeta, g_d)$. We now turn to a numerical solution of this.

Fig. 1 illustrates the previous analysis. The thick curve, denoted as C, is the envelope of set J as defined above. Therefore, C provides an envelope for all trajectories of x as a function of E_0 that satisfy the constraint $E(t) \geq \underline{E}$ at all times, which is called set J in Fig. 1. By contrast, any trajectory that is outside (above) the envelope C, denoted as the complement of set J (set J^c) in Fig. 1 (which is shaded), reaches an environmental catastrophe. Fig. 1 shows the particular case where pollution emissions follow an inverted U-shaped pattern where the envelope C reaches $\underline{E}$ at the turnaround time t^*. The uniqueness property of the adjustment paths guarantees that any two different trajectories starting from different initial positions move in parallel and never cross each other. Hence, any trajectory starting below $x_0^c(E_0)$ never reaches the catastrophic stock level, while any trajectory starting above C is bound to eventually violate the stock constraint.

The curve labeled OO and the curve labeled SS in Fig. 1 represent the optimal trajectory and an arbitrary suboptimal but sustainable trajectory associated with a suboptimal tax, respectively. Fig. 1 does not illustrate time profiles of pollution emissions for the two trajectories. It can be shown, however, that each level of E is reached at an earlier time along the trajectory OO than SS. Although it appears in the figure that the level of pollution emissions is higher in OO than SS beyond the turnaround level, this is due to the fact that the visual comparison considers indeed different points in time. At each point of time the level of E is higher within trajectory OO than SS.

8. Numerical illustration

Here we develop a numerical example to obtain further insights into this paper's propositions. To highlight the role of the consumption composition effect, we assume that the clean and dirty inputs are complements (e.g.,$\omega = 0$), so that sustainability in this case depends mainly on the consumption composition effect. For simplicity, we only focus on pollution-augmenting technological progress. First, we calibrate our model with only flow emissions of pollution using parameters based on data from the U.S. economy, and later we check the sustainability condition for the stock constraint.

8.1. Parameter choices

In recent literature, the long-run annual growth rate of the U.S. economy is often assumed to be 2% (e.g., Nordhaus, 2007; Weitzman, 2007; Acemoglu et al., 2012). As shown in Proposition 1 above, this corresponds to M/a where $a = EMU$.

[21] See Lopez and Yoon (2016) for a similar treatment of this problem.

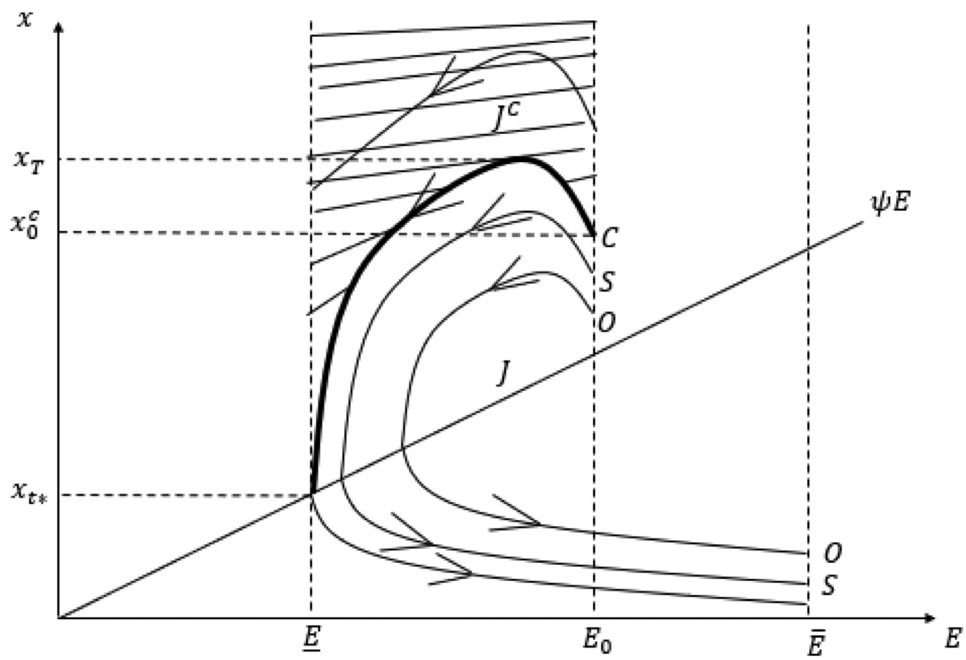

Fig. 1. The admissible set J and the Envelop C in E-x space.

In the literature *EMU* is often assumed to be approximately 2; this would imply that the net return to the capital input, *M*, is approximately 0.04 (or 4%).[22] We examine the feasibility of sustainable growth under varying assumptions of *EMU* and elasticity of substitution in consumption, σ.

Based on recent econometric estimates, we alternatively consider *EMU* values of 2 and 0.8. (e.g., Ogaki and Reinhart, 1998; Vissing-Jørgensen, 2002). For $M = 0.04$, the economy's long-run growth rate becomes 5% when $EMU = 0.8$, which is much greater than the commonly accepted rate of 2%. Nonetheless, we perform this simulation to highlight that when *EMU* is low, the scale effect is much larger, and, therefore, makes sustainable growth more difficult to achieve.

A number of empirical studies report a high degree of substitution between environmentally- mild consumer goods (such as organic products or high efficiency appliances) and conventional ones, often reporting elasticity of substitution estimates well above 3.[23] We thus consider three different values for σ: 4, 2, and 0.8. Finally, based on the data from World Bank (2012) of the global capital formation growth rate between 1990 and 2012, we assume that the annual rate of pollution-augmenting technological progress is approximately 0.5%, so that $\zeta = 0.005$.[24]

8.2. The pollution emissions path

Fig. 2 provides the growth of pollution emissions over time for various *EMU* values.[25] Panel (a) shows the case when $EMU = 0.8$. If the elasticity of substitution is greater than the threshold level, $(\frac{M/a-\zeta}{M-\zeta}) \approx 1.28$, implied by Proposition 3, a critical time exists where pollution increases monotonically, and then starts declining. This turning point depends on the level of σ. If $\sigma = 4$, the turning point takes place in the year 2069, and if $\sigma = 2$, in 2185, because the consumption composition effect becomes more effective when σ is larger.

Panel (b) depicts the case when $EMU = 2$: if $\sigma = 4$ then pollution begins falling very quickly by the year 2025, but if $\sigma = 2$ or $\sigma = 0.8$, then the turning point occurs during a much later year (2057 and 2178, respectively). Panel (c) illustrates the pollution emissions path for the case when both *EMU* and σ are less than 1, in which case pollution increases in all periods. When $\sigma < 1.28$ pollution emissions continue to increase over time for all periods as indicated by Lemma 1.

In summary, if $EMU < 1$, sustainable growth requires that the consumption elasticity of substitution be greater than the threshold level. However, as shown in Panel (b), if $EMU > 1$, then economic growth is sustainable even if σ is low (and $\omega = 0$ as we assumed here). In this case, as predicted by Proposition 2, even highly inelastic consumer preferences and producer technology do not prevent pollution from beginning to decline along the optimal path.

8.3. Growth sacrifice caused by the pollution tax

Finally, Panel (a) in Fig. 3 shows the rates of growth of real consumption when $EMU = 0.8$. The rate of economic growth always is positive, although it falls below the potential growth rate over the short run. However, if $EMU > 1$, it recovers toward the potential growth rate over the long run. The growth sacrifices over the short- and medium-terms are rather small, and growth recovers more quickly if the elasticity of substitution is larger. Even when σ is relatively low (e.g., 2), the growth sacrifice is not very large, reaching a maximum value on the order of 0.6 percentage points per year, although the growth rate begins recovering at a much later date than when $\sigma = 4$. The growth sacrifice is large if σ is less than 1 (e.g.,$\sigma = 0.8$), and more importantly, and as predicted by Proposition 1, the economy's growth rate converges to a lower, but still positive rate of growth, over the long-run.

[22] See, for example, Kydland and Prescott (1982), Jones et al. (2000), and Bansal and Yaron (2004), among others.

[23] See for example, Glaser and Thompson (2000), Thompson and Glaser (2001), Lin et al. (2008), and Galarraga et al. (2011).

[24] We calibrate the parameters such that $\eta = 1$ in the pollution damage function, and the ratio, γ_c/γ_d =0.7 in the unit expenditure function equals. The value of ηdoes not change the long run feasibility of sustainability if the damage function is convex. The values for γ do not affect the behavior of pollution emissions growth if it satisfies convexity conditions for the expenditure function (e.g., Eq. (16)).

[25] For illustration purposes, we use a time scale obtained by calibrating the changes in the share of the clean input (labor) of the US manufacturing industry over the past decade.

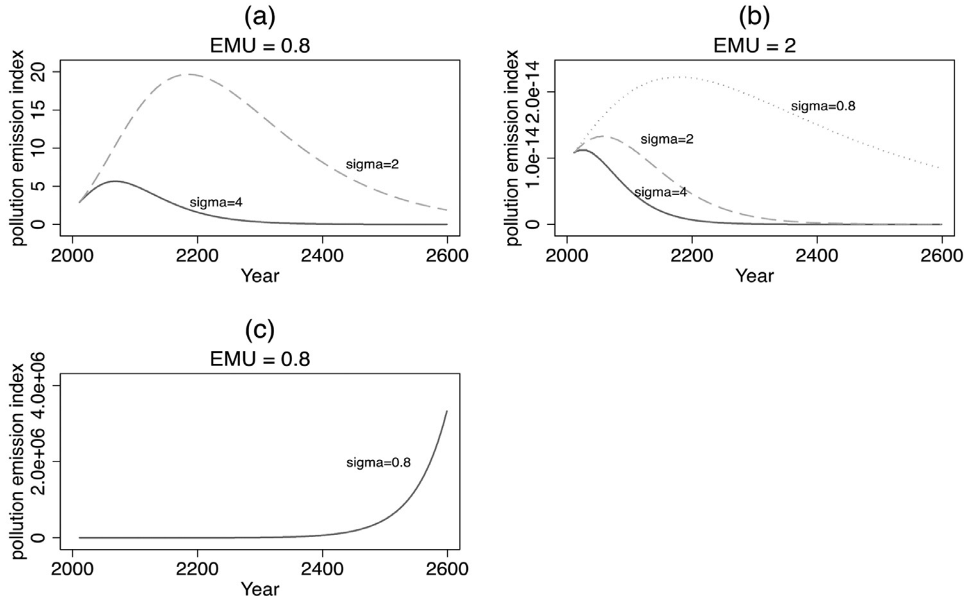

Fig. 2. Pollution emissions for different values of σ and *EMU*.

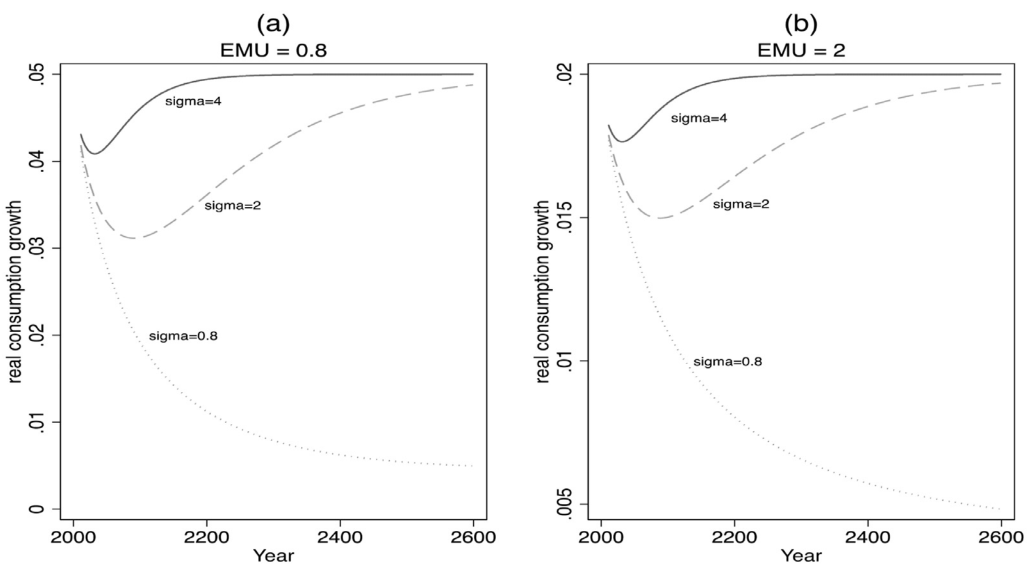

Fig. 3. Real consumption growth rates for different values of σ and *EMU*.

Panel (b) of Fig. 3 illustrates the case when $EMU = 2$. If $\sigma < 1$, then the long-run growth rate remains positive, but falls below toward the technological growth rate ($\zeta = 0.005$). However, as predicted by Proposition 1, if $\sigma = 2$ then the rate of economic growth converges to the potential growth rate M/a, and the temporary growth sacrifice imposed by environmental sustainability is smaller than the previous case. In this case, the maximum reduction of the rate of economic growth is about 0.5 percentage points. In the short-run, the growth sacrifice caused by the pollution tax is 0.2 percentage points, from a 2% annual growth when no environmental tax is implemented to about 1.8% when the tax imposed.[26]

[26] According to Panel (a) in Fig. 3, where we assumed that $EMU = 0.8$, we find that the economy took about 56 years and 273 years to recover back to the pre-policy level of real income growth rate for $\sigma = 4$ and $\sigma = 2$ respectively. In case of $\sigma = 0.8$, the consumption level never recovered back to that of pre-policy level of income growth rate but remained positive. Similarly, from Panel (b), where we assumed that $EMU = 2$, we find that it took about 52 years and 224 years to recover back to the pre-policy level of real income growth rate.

8.4. Stock effects

We now consider the possibility of irreversible disaster assuming Cobb-Douglas utility and production functions, and $EMU > 1$. Although there is no clear consensus on the structure of the carbon cycle, recent scientific studies find that the lifetime of carbon in the air spans a few centuries. The Intergovernmental Panel on Climate Change (IPCC) (2007) suggests about half of the CO2 increase will be removed from the atmosphere within 30 years, implying a 1.6% regeneration rate of clean air per annum. Then, Eq. (19) implies that $x(t*) = 0.016\underline{E}$.

Given the Cobb-Douglas specification, the cost share of clean input in production,S_k, and the consumers' budget share of the dirty final good, s, are constant. Assuming service output and labor input are less pollution intensive than manufacturing output and energy-intensive inputs, we use estimates for the share of clean input and clean final goods in world GDP for illustration purposes, and set $S_k = 0.5$ and $s = 0.54$ (Guscina, 2006;

World Bank, 2012). Using the same values used for the previous simulation for the other parameters ($a = 2; \zeta = 0.005; M = 0.04; \eta = 1$), we obtain from Eq. (16) that $x(t) = x_0 \exp(-\vartheta t)$, where $\vartheta = 0.0085$, implying that the optimal pollution decreasing rate is equal to 0.85% per annum.

Since there is no direct measure to gauge absolutely clean air stock, we construct the so-called relative clean air stock (RCAS) index. Let $Carbon_t$ and $Carbon^D$ represent the current global carbon stock in year t and the disaster-rendering magnitude of the global carbon stock, respectively, both measured in *ppm*. Define *RCAS* index as follows;

$$E(t) = RCAS(t) = Carbon^D / Carbon_t.$$

For illustrative purposes, we assume that the disaster-rendering level of the carbon stock is 650 ppm.[27] In addition, we set the initial value (year 2013) and pre-industrial value of global carbon stock level in the atmosphere at 395 ppm and 280 ppm, respectively (NOAA, 2013). Then, the pre-industrial *RCAS* level which, we consider environmentally pristine, is $\bar{E} = 650/280 \approx 2.32$, while the current level and disaster-rendering level of clean air stock are $E_{2013} = 650/395 \approx 1.64$ and $\underline{E} = 650/650 = 1$, respectively.[28]

To solve numerically for the corresponding critical level of emission, x^c_{2013}, we first note, using Eq. (19), that

$$x^c_{2013} \exp(-\vartheta t*) = \psi \underline{E} = \psi \quad (21)$$

Also, from Eqs. (4') and (20), we have,

$$E(t*) = \exp(\psi t*)(E_{2013} - \int_0^{t*} x_{2013} \exp(-\vartheta t) dt) = \underline{E} = 1.$$

Using the expression for the pollution emissions in the Cobb–Douglas case, $x(t) = x_0 \exp(-\vartheta t)$ and integrating, it follows that the previous expression can be written as:

$$\exp(\psi t*) \left(E_{2013} + \frac{x^c_{2013}}{\vartheta + \psi} (\exp(-(\vartheta + \psi) t*) - 1) \right) = 1. \quad (22)$$

Solving Eq. (21) and Eq. (22) using numerical methods gives the level of carbon in the year 2013 which is located in the envelope C in Fig. 1, obtaining $x^c_{2013} = 0.038$ given that $E_{2013} \approx 1.64$. We then generate the time profiles of pollution emissions and the stock of clean air under alternative scenarios.

We consider five alternative scenarios.

Scenario 1 (Optimistic case): The government reduces emissions by 15% in the first year below the critical level, x^c_{2013}, and the rate of pollution emissions growth is to be regulated optimally according to Eq. (16).

Scenario 2 (Sufficient case): The government takes measures to reduce emissions exactly to the critical level, x^c_{2013}, and the rate of pollution emissions growth is to be regulated optimally according to Eq. (16).

Scenario 3 (Insufficient, late disaster case): The government is unable to reduce pollution emissions to the initial critical level, x^c_{2013}, and instead allows emission 15% above such level in the first year, while still restricting the rate of pollution emissions growth optimally according to Eq. (16).

Scenario 4 (Business as usual, early disaster case): Pollution emissions are 15% above the critical level, x^c_{2013}, and they grow by 3.1% per annum, which corresponds to the historical growth rate of carbon emissions over the 2000–10 period (Peters et al., 2012).

Scenario 5 (Sufficient but stringent target case): Environmental disaster-rendering carbon concentration level is set to be equal to 550 ppm and the government takes measures to reduce initial emissions exactly to the critical level, x^c_{2013}.[29] The rate of pollution emissions growth is to be regulated optimally according to Eq. (16).

Table 1 shows the simulation results for the time profiles of $x(t)$ and $E(t)$ under the above scenarios. Under Scenario 1, sustainable development takes place, and the turnaround point of the clean air stock occurs in 2046, reaching an environmentally pristine condition by 2122. Under Scenario 2, sustainable development is also feasible as the clean air stock never falls below the threshold level, and starts growing in 2117. Under Scenario 3, an environmental disaster is unavoidable; by 2054, the stock of the clean air falls below the threshold level. An environmental disaster occurs despite the assumption that the government is able to regulate emissions growth according to the optimal rate of change. Under Scenario 4, an environmental disaster occurs by the year 2028. Lastly, under Scenario 5, sustainable development is still feasible as long as government is willing to drastically cut initial carbon emissions to the level that is much lower than that of Scenario 2 where we assume 650 ppm as a disaster rendering level of carbon concentration. Once the initial emissions level reaches the critical emissions level that leads to sustainable growth, the economy achieves an environmentally pristine condition by 2194.

9. Conclusion

This paper examines the sustainable development of an economy where the net effect of the two primary sources of growth, capital accumulation and technological change, is pollution increasing. Sustainable development can be achieved under a variety of plausible technological conditions if an optimal Pigouvian pollution tax is implemented as the only policy instrument. If the often-used assumption of *EMU* greater than 1 holds, then sustainable development is almost automatically satisfied if either the elasticity of substitution in production or in consumption is positive. An optimal pollution tax profile rules optimal pollution changes over time as characterized by Eq. (16), and is sufficiently high to set the initial pollution level below a critical level to avoid disastrous stock effects of pollution.

Moreover, even if the initial pollution tax is suboptimal, sustainable development still takes place as long as the initial tax level is sufficient to set the initial pollution flow less than or equal to its critical level, and the tax rate of change over time is at the rate necessary to induce optimal pollution changes over time. Such a critical level is well defined once the initial level of renewable resource stock, such as clean air, is identified.

Sustainable development mainly becomes an issue when *EMU* is less than 1. Sustainability may also occur in this case if consumer substitution between clean and dirty goods are high, even if the production elasticity of substitution between clean and dirty inputs is low. In contrast to the assumption of high producer flexibility made by the standard growth models, the assumption of consumer flexibility required in this case appears to be more adequately supported by empirical studies. This paper has demonstrated that neither strong production substitution nor technological optimism is necessary for environmentally sustainable growth.

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[27] Although the disaster-rendering magnitude of the stock of CO_2 differs among various experts, commonly accepted carbon concentration levels lie somewhere between 550 ppm and 750 ppm, implying a 3 degrees Celsius and 4 degrees Celsius increase, respectively (e.g., Glasby, 2006; Pearson et al., 2009).

[28] A pre-industrial level of carbon stock is often considered an environmentally clean air condition (e.g., Acemoglu et al., 2012).

[29] IPCC (https://www.ipcc.ch/sr15/) suggests that to avoid the disaster earth temperature should stay below an increase of 1.5 degrees Celsius which is equivalent to 500–550 ppm carbon concentration level. If we set 550 ppm as a disaster rendering level then $\bar{E} = 550/280 \approx 1.96$. Solving Eqs. (21) and (22) using numerical methods gives the point for the year 2013 located in the envelope C, which corresponds to $x^c_{2013} = 0.0322$ and $E_{2013} = 550/395 \approx 1.40$.

Table 1
Time path of pollution emissions and clean air stock under different scenarios. .

	Scenario 1 Optimistic case ($\hat{x} = -0.0085$)		Scenario 2 Sufficient case ($\hat{x} = -0.0085$)		Scenario 3 Insufficient, late disaster case ($\hat{x} = -0.0085$)		Scenario 4 Business as usual, early disaster case ($\hat{x} = 0.031$)		Scenario 5 Sufficient but stringent target case ($\hat{x} = -0.0085$)	
Year(t)	$x(t)$	$E(t)$	$x(t)$	$E(t)$	$x(t)$	$E(t)$	$x(t)$	$E(t)$	$x(t)$	$E(t)$
2013	**0.0327**	1.640	**0.0385**	1.640	**0.0442**	1.809	**0.0442**	1.809	0.0322	1.407
2027	0.0307	1.609	0.0341	1.478	0.0389	1.412	0.0993	1.064	0.0316	1.388
2046	**0.0244**	**1.542**	0.0290	1.299	0.0331	1.112	**Environmental disaster**		0.0285	1.283
2053	0.0230	1.547	0.0274	1.244	0.0311	1.006			0.0283	1.275
2095	0.0162	1.837	0.0191	1.029	**Environmental disaster**				**0.0160**	**1.000**
2117	0.0136	2.219	**0.0160**	**1.000**					0.0120	0.0180
2122	0.0130	2.335	0.0153	1.001						
2140	**Pristine condition**		0.0131	1.037					0.0109	1.156
2194			0.0083	1.523					**0.0069**	**1.962**
2229			**0.0061**	**2.324**					**Pristine condition**	
			Pristine condition							

Notes: (1) $x(t)$ and $E(t)$ denote the yearly index of pollution emissions and relative clean air stock, respectively, (2) for each scenario, Eq. (4') is used to generate $E(t)$ over time starting from the initial year of 2013. *Source:* Authors' calculations.

Although the informational requirement for implementing government intervention to ensure sustainable development is not formidable, mitigating the political and institutional obstacles to the implementation of optimal pollution taxes as part of the initial necessary measures to reduce emissions is not an easy task. This paper shows the scope of discretionary government intervention by characterizing a family of suboptimal sustainable growth paths.

Acknowledgement

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Appendix

Proofs of Remarks, Propositions and Assertions in the text

Proof of Remark 2. The marginal product of pollution is defined as

$$\frac{\partial A_d F(k_d, bx)}{\partial x} = bA_d F_2(k_d, bx)$$

It changes with technological progress as follows.

$$\frac{\partial (bF_2)}{\partial b} = F_2\left(1 + bx\frac{F_{22}}{F_2}\right), \tag{A.1}$$

Since $F(k_d, bx)$ is homogenous of degree 1, by Euler's Theorem it follows that $k_d F_{21}(k_d, bx) + bxF_{22}(k_d, bx) = 0$.Using this in (A.1) we obtain,

$$\frac{\partial (bF_2)}{\partial b} = F_2\left(1 - \frac{k_d F_{21}}{F_{22}}\frac{F_{22}}{F_2}\right) = F_2\left(1 - \frac{S_k}{\omega}\right), \text{ where}$$

$$\omega = \frac{F_1 F_2}{F_{21} F}, \text{ and } S_k = \frac{k_d F_1}{F}.$$

Derivation of Eq ((9))**.** :

Use Roy's identity to derive the demand for the dirty good from the indirect utility function as follows.

$$c_d = \frac{c}{e(1,p)} e_2(1,p). \tag{A.2}$$

Logarithmic time differentiation yields,

$$\hat{c}_d = \hat{c} + \hat{e}_2(1,p) - \hat{e}(1,p). \tag{A.3}$$

Totally differentiating both sides of first order condition Eq. (5) with respect to time and using Eq. (6), we have,

$$\hat{c} = \left(\frac{a-1}{a}\right)\hat{e} + \frac{M}{a}. \tag{A.4}$$

The second term of the right-hand side of Eq. (A.3) can be written as,

$$\hat{e}_2 = \frac{d\log e_2}{dp}\frac{dp}{dt}. \tag{A.5}$$

Using the CES utility function we obtain,

$$\frac{d\log e_2}{dp} = \left(\frac{\sigma}{1-\sigma}\right)\frac{\gamma_d(1-\sigma)p^{-\sigma}}{\gamma_c + \gamma_d p^{1-\sigma}} - \frac{\sigma}{p} = \frac{\sigma}{p}(s(p) - 1). \tag{A.6}$$

On the other hand, using Shephard's lemma on the expenditure function $e(1,p)$ we have,

$$\hat{e}(1,p) = \frac{pe_2}{e}\hat{p} = s(p)\hat{p}. \tag{A.7}$$

Using Eq. (A.6) into Eq. (A.5) and then using (A.4), (A.5) and (A.7) in (A.3) we find,

$$\hat{c}_d = \left(\frac{1-a}{a}\right)\left[M - s(p)\hat{p}\right] + \sigma(s(p)-1)\hat{p} - s(p)\hat{p} \tag{A.8}$$

$$= \frac{1}{a}M - \left[\frac{s(p)}{a} + (1-s(p))\sigma\right]\hat{p}.$$

Derivation of Eq ((13))**.** :

Logarithmic total differentiation of both sides of the first order condition Eq. (8),

$$\eta\hat{x} - \hat{\lambda} = \hat{p} + g_D + \hat{b} + \left(F_2\left(\hat{k}_d, bx\right)\right). \tag{A.9}$$

Also, since the function F is CES, we have,

$$\left(F_2\left(\hat{k}_d, bx\right)\right) = \frac{\alpha}{\omega}\frac{\left(\frac{\hat{k}_d}{bx}\right)}{\left[(1-\alpha)\left(\frac{k_d}{bx}\right)^{-\frac{\omega-1}{\omega}} + \alpha\right]} = \frac{S_k}{\omega}\left(\frac{\hat{k}_d}{bx}\right). \tag{A.10}$$

Rearranging (A.9) and using (A.10) and $\hat{b} \equiv \zeta$, we arrive at

$$\hat{p} + \frac{S_k}{\omega}\left(\frac{\hat{k}_d}{bx}\right) - \eta\hat{x} = M - \zeta - g_D. \tag{A.11}$$

Derivation of Eqs ((14)–(16))**.** The system of Eqs. (11)–(13) in matrix form can be written as,

$$\begin{bmatrix} z & S_k & 1 \\ 1 & -\frac{1}{\omega}(1-S_k) & 0 \\ 1 & \frac{1}{\omega}S_k & -\eta \end{bmatrix}\begin{bmatrix} \hat{p} \\ \left(\frac{\hat{k}_d}{bx}\right) \\ \hat{x} \end{bmatrix} = \begin{bmatrix} \frac{M}{a} - g_d - \zeta \\ -g_d \\ M - g_d - \zeta \end{bmatrix}.$$

Using Cramer's rule and noting that the determinant,

$$|W| = \begin{vmatrix} z & S_k & 1 \\ 1 & -\frac{1}{\omega}(1-S_k) & 0 \\ 1 & \frac{1}{\omega}S_k & -\eta \end{vmatrix} = \frac{1}{\omega}[(1-S_k)(1+z\eta)+S_k] + \eta S_k > 0,$$

we arrive at the solutions that are given in Eqs. (14)–(16).

Proof of Proposition 1.

(i) The growth rate of real consumption is $(\frac{\hat{c}}{e}) = \hat{c} - \hat{e}$. Using Eqs. (A.4) and (A.7), it follows that

$$\left(\frac{\hat{c}}{e}\right) = \frac{1}{a}\left[M - s(p)\hat{p}\right]. \tag{A.12}$$

(ii) Eq. (A.12) implies that real consumption grows over time as long as $\hat{p} < \frac{M}{s(p)}$. From Eq. (14), we can decompose $\hat{p}$ as follows; $\hat{p} \equiv \hat{p}_0 + \hat{p}_b + \hat{p}_g$.

where $\hat{p}_0 = \frac{\frac{M}{\omega}(1-S_k)[\frac{\eta}{a}+1]}{|W|}$, $\hat{p}_b = \frac{-\zeta[(1-S_k)(\eta+1)]}{\omega|W|}$ and

$$\hat{p}_g = \frac{-g_d\left[(1-S_k)(\eta+1) + \omega S_k\left(\eta + \frac{1}{\omega}\right)\right]}{\omega|W|}.$$

Then since $\hat{p}_b < 0$ and $\hat{p}_g < 0$, we find that a sufficient condition for $\hat{p} < \frac{M}{s(p)}$ to hold is,

$$\hat{p}_0 = \frac{(1/\omega)M(1-S_k)[(\eta/a)+1]}{(1/\omega)[(1-S_k)(1+z\eta)+S_k] + \eta S_k} < \frac{M}{s(p)}. \tag{A.13}$$

Rearranging (A.13) we have,

$$(1-S_k)\left(\frac{\eta}{a}+1\right)s(p) < [(1-S_k)(1+z\eta)] + S_k + \eta S_k \omega. \tag{A.14}$$

Since $(S_k + \eta S_k \omega) > 0$ and $z \equiv \frac{s(p)}{a} + (1-s(p))\sigma$, (A.14) is satisfied if the following inequality holds,

$$\frac{\eta s(p)}{a} + s(p) < 1 + \frac{\eta s(p)}{a} + (1-s(p))\sigma\eta, \tag{A.15}$$

or, equivalently if $0 < (1-s(p))(1+\sigma\eta)$, which is always true for $0 < s(p) < 1$. Thus, we have $\hat{p} < (M/s(p))$ at any finite point of time and for all finite σ and ω. That is, real consumption growth is positive along the equilibrium dynamic path.

(i) If $\omega > 1$, then $\lim_{t\to\infty} S_k = 1$ and $\lim_{t\to\infty} \hat{p} = -g_d$ for any $\sigma > 0$. If $\sigma < 1, \lim_{t\to\infty} s(p) = 0$. Suppose that $\omega < 1$ and $\sigma > 1$. Then we have $\lim_{t\to\infty} S_k = 0$ and the relative price of dirty goods monotonically increases over time under Assumption 2. It then follows that $\lim_{t\to\infty} s(p) = 0$. In either case we find that $s(p)\hat{p}$ approaches to zero. Thus, from (A.11) it follows that the growth rate of real consumption converges from below to M/a if either $\omega > 1$ or $\sigma > 1$, but not both. When $\omega > 1$, and $\sigma > 1$, then $\lim_{t\to\infty} \hat{p} = -g_d$ and $\lim_{t\to\infty} s(p) = 1$. It follows that $s(p)\hat{p}$ converges to $-g_d$ and the consumption growth rate converges to $(M + g_d)/a$.

(ii) If $\omega < 1$ and $\sigma < 1$, then $\lim_{t\to\infty} S_k = 0$ and $\lim_{t\to\infty} s(p) = 1$.

This implies that $\lim_{t\to\infty} \hat{p} = \frac{(1+\eta/a)M-(1+\eta)(\zeta+g_d)}{1+z\eta} > 0$. But since $\lim_{t\to\infty} s(p) = 1$, we have that $\lim_{t\to\infty} z = 1/a$. It follows that $\lim_{t\to\infty} \hat{p} = M - \frac{(1+\eta)(\zeta+g_d)}{1+(\eta/a)}$. Thus, using this expression in (A.11) and considering the fact that $\lim_{t\to\infty} s(p) = 1$ we have,

$$\lim_{t\to\infty}\left(\frac{\hat{c}}{e}\right) = \left(\frac{1+\eta}{a+\eta}\right)(\zeta + g_d).$$

Finally, we show that $s(p)\hat{p}$ is increasing over time, meaning that $(\frac{\hat{c}}{e})$ converges towards the limit from above. Substituting the definitions of $|W|$ and z into Eq. (14) we can write,

$$s\hat{p} = \frac{(1+\eta)\left[\frac{1+\eta/a}{1+\eta}M - (\zeta+g_d)\right] + \frac{S_k}{1-S_k}(1+\eta\omega)g_d}{1 + \frac{s\eta}{a} + \frac{(1-s)}{s}\sigma\eta + \frac{S_k}{1-S_k}(1+\eta\omega)}.$$

Clearly, this expression is increasing in s and decreasing in S_k. If $\sigma < 1$ it follows that s is increasing over time as p increases. Also, since k_d/bx increases over time, the assumption that $\omega < 1$ implies that S_k is falling. Thus, along the equilibrium growth path $s\hat{p}$ is increasing when g_d is sufficiently small. Hence, we have that $(\frac{\hat{c}}{e}) = \frac{1}{a}[M - s(p)\hat{p}]$ must be falling over time. That is, the rate of growth of real consumption converges to a positive rate $\frac{1+\eta}{a+\eta}\zeta$ from above. In other words, if $\sigma < 1$ and $\omega < 1$, then the rate of economic growth is declining over time. To show that $M/a > \frac{1+\eta}{a+\eta}(\zeta + g_d)$ note that this inequality can be written as $M + \eta M/a > (\zeta + g_d) + \eta(\zeta + g_d)$, which is true under Assumption 2. □

Proof of Proposition 2

Proposition 1. *already shows that the growth rate of real consumption always remains positive for any positive ω and σ. Here we show that positive growth is accompanied by a decreasing level of pollution over the long run, that $\lim_{t\to\infty} \hat{x} < 0$ as long as $a > 1$. We first note from Eq. (15) that k_d/bx always increases over time which implies that $\lim_{t\to\infty} S_k = 1$ for $\omega > 1$, and $\lim_{t\to\infty} S_k = 0$ for $\omega < 1$. Then from Eq. (14) and Assumption 2, we find that $\lim_{t\to\infty} \hat{p} > 0$ for $\omega < 1$, and $\lim_{t\to\infty} \hat{p} < 0$ for $\omega > 1$.*

Case 1: $\omega > 1$ and $\sigma > 1$

We have $\lim_{t\to\infty} s = 1$; $\lim_{t\to\infty} z = 1/a$; $\lim_{t\to\infty} S_k = 1$.

Plugging these values into Eq. (16),

$$\lim_{t\to\infty} \hat{x} = \frac{1}{(1+\omega\eta)}\left\{\left(\frac{M}{a} - \zeta\right) - \omega(M-\zeta) + g_d\left(\frac{1}{a} - 1\right)\right\}.$$

Assumption 2 implies that $\lim_{t\to\infty} \hat{x} < 0$ if $a > 1$. This is also valid if technological change is absent,ζ=g_d=0.

Case 2: $\omega > 1$ and $\sigma < 1$

We have $\lim_{t\to\infty} s = 0$; $\lim_{t\to\infty} z = \sigma$; $\lim_{t\to\infty} S_k = 1$.

Plugging these values into Eq. (16),

$$\lim_{t\to\infty} \hat{x} = \frac{1}{(1+\omega\eta)}\left\{\left(\frac{M}{a} - \zeta\right) - \omega(M-\zeta) + g_d(\sigma - 1)\right\}.$$

Assumption 2 implies that $\lim_{t\to\infty} \hat{x} < 0$.

Case 3: $\omega < 1$ and $\sigma > 1$

We have $\lim_{t\to\infty} s = 0$; $\lim_{t\to\infty} z = \sigma$; $\lim_{t\to\infty} S_k = 0$.

Plugging these values into Eq. (16),

$$\lim_{t\to\infty} \hat{x} = \frac{1}{(1+\sigma\eta)}\left\{\left(\frac{M}{a} - \zeta - g_d\right) - \sigma(M - \zeta - g_d)\right\}.$$

Since $a > 1$, we have $(\frac{M}{a} - \zeta - g_d)/(M - \zeta - g_d) < 1 < \sigma$, and $\lim_{t\to\infty} \hat{x} < 0$.

Case 4: $\omega < 1$ and $\sigma < 1$

We have $\lim_{t\to\infty} s = 1$; $\lim_{t\to\infty} z = 1/a$; $\lim_{t\to\infty} S_k = 0$.

Plugging these values into Eq. (16), $\lim_{t\to\infty} \hat{x} = \frac{1}{(1+\eta/a)}\{(g_d+\zeta)(\frac{1}{a} - 1)\} < 0$.

Case 5: $\omega \neq 1$ and $\sigma = 1$

We have $0 < s = \beta < 1$ and $z = \frac{\beta}{a} + (1-\beta) < 1$ for $a > 1$. We consider two cases.

If $\omega > 1$, then $\lim_{t\to\infty} S_k = 1$ and $\lim_{t\to\infty} \hat{x} = \frac{1}{1+\omega\eta}\{(\frac{M}{a} - \zeta) - \omega(M-\zeta) + g_d(z-1)\}$. Since $z < 1$, Assumption 2 implies that $\lim_{t\to\infty} \hat{x} < 0$. If $\omega < 1$, then $\lim_{t\to\infty} S_k = 0$ and $\lim_{t\to\infty} \hat{x} = \frac{1}{1+\eta z}(M(\frac{1}{a} - z) - \xi(1-z) + g_d(z-1))$.

Since $\frac{1}{a} < z < 1$, we have $\lim_{t\to\infty} \hat{x} < 0$ for $a > 1$.

Case 6: $\omega = 1$ and $\sigma \neq 1$

Since $0 < S_k = \alpha < 1$, we have;

$$\lim_{t\to\infty} \hat{p} = \left(\lim_{t\to\infty} \frac{1}{|W|\omega}\right)\left[(1-\alpha)\left((\frac{M}{a} - \zeta)\eta + (M-\zeta)\right) - g_d(1+\eta)\right].$$

It follows that $\lim_{t\to\infty} \hat{p} > (<) 0$ if and only if $g_d < (>) \frac{(1-\alpha)((\frac{M}{a}-\zeta)\eta + (M-\zeta))}{1+\eta}$. We consider four alternative cases.

(i) $\sigma < 1$ and $g_d < \frac{(1-\alpha)((\frac{M}{a}-\zeta)\eta + (M-\zeta))}{1+\eta}$.

Since $\sigma < 1$, we have $\lim_{t\to\infty} s(p) = 1$ and $\lim_{t\to\infty} z = 1/a$.

It follows that $\lim_{t\to\infty} \hat{x} = \frac{(\frac{1}{a}-1)(M\alpha + g_d + \zeta(1-\alpha))}{((1-\alpha)(1+\frac{\eta}{a}) + \alpha(1+\eta))} < 0$ for $a > 1$ regardless of magnitude of $g_d > 0$.

(i) ($\sigma > 1$ and $g_d < \frac{(1-\alpha)((\frac{M}{a}-\zeta)\eta + (M-\zeta))}{1+\eta}$. We have $\lim_{t\to\infty} s(p) = 0$ and $\lim_{t\to\infty} z = \sigma$. It follows that

$$\lim_{t\to\infty} \hat{x} = \frac{M(\frac{1}{a}-1) - (1-\alpha)(\sigma-1)(M-\zeta) + g_d(\sigma-1)}{(1-\alpha)(1+\sigma\eta) + (1+\eta)\alpha}.$$

The first term of the numerator is negative, while the sum of second and third term becomes negative since

$$-(1-\alpha)(\sigma-1)(M-\zeta) + g_d(\sigma-1) < -(1-\alpha)(\sigma-1)(M-\zeta) + \frac{(1-\alpha)\left((\frac{M}{a}-\zeta)\eta + (M-\zeta)\right)(\sigma-1)}{1+\eta} < 0$$

(i) $\sigma < 1$ and $g_d > \frac{(1-\alpha)((\frac{M}{a}-\zeta)\eta + (M-\zeta))}{1+\eta}$ .We have $\lim_{t\to\infty} s(p) = 0$ and $\lim_{t\to\infty} z = \sigma$.

It follows that $\lim_{t\to\infty} \hat{x} = \frac{(\frac{1}{a}-\sigma)M + M\alpha(\sigma-1) + g_d(\sigma-1) + \zeta(1-\alpha)(\sigma-1)}{(1-\alpha)(1+\sigma\eta) + (1+\eta)\alpha} < 0$ for $a > 1$.

(i) $\sigma > 1$ and $g_d > \frac{(1-\alpha)((\frac{M}{a}-\zeta)\eta + (M-\zeta))}{1+\eta}$. We have $\lim_{t\to\infty} s(p) = 1$ and $\lim_{t\to\infty} z = 1/a$. It follows that $\lim_{t\to\infty} \hat{x} = \frac{(\frac{1}{a}-1)(M\alpha + g_d + \zeta(1-\alpha))}{((1-\alpha)(1+\frac{\eta}{a}) + \alpha(1+\eta))} < 0$ for $a > 1$.

Case 7: $\omega = 1$ and $\sigma = 1$

We always have $0 < S_k = \alpha < 1$, $0 < s(p) = \beta < 1$, and $z = \frac{\beta}{a} + (1-\beta) < 1$.

Then $\lim_{t\to\infty} \hat{x} < 0$ if and only if $M(\frac{1}{a} - (1-\alpha)z - \alpha) - \zeta(1 - (1-\alpha)z - \alpha) < 0$.

Rearranging, we have,

$$M\left(\frac{1}{a} - (1-\alpha)z - \alpha\right) - \zeta(1 - (1-\alpha)z - \alpha) = \left[M\left(\frac{1}{a} - z\right) - \zeta(1-z)\right] + (M-\zeta)\alpha(z-1).$$

The first term is negative since $\frac{(M/a-\zeta)}{(M-\zeta)} < \frac{1}{a} < \frac{\beta}{a} + (1-\beta) = z$, and the second term is also negative since $z < 1$. □

Proof of Proposition 3.

(i) First we assume $\omega > 1$. For any $\sigma > 0$, Eq. (18) applies with $g_d = 0$,

$\lim_{t\to\infty} \hat{x} = \frac{M(\frac{1}{a\omega}-1) - \zeta(\frac{1}{\omega}-1)}{\frac{1}{\omega}+\eta} < 0$ if and only if $\omega > d(M, a; \zeta, 0) = \frac{\frac{M}{a}-\zeta}{M-\zeta}$. Since the minimum value of $d(M,\ a;\ \zeta,\ 0)$ is $\frac{1}{a} > 1$ for $0 < a < 1$, we have $d(M,\ a;\ \zeta,\ 0) > 1$.

(i) Consider now the case where $\omega < 1$. If $\sigma > 1$, Eq. (18) applies with $g_d = 0$,

$\lim_{t\to\infty} \hat{x} = \frac{M(\frac{1}{a}-\sigma) - \zeta(1-\sigma)}{(1+\sigma\eta)} < 0$ if and only if $\sigma > d(M, a; \zeta, 0) = \frac{\frac{M}{a}-\zeta}{M-\zeta} > 1$ for $0 < a < 1$. □

Capital-augmenting technological change

If we allow capital-augmenting technological change, $\dot{n}/n = \theta > 0$, in addition to pollution-augmenting and neutral technological change in the dirty sector, the equilibrium growth rates of $\hat{p}$, $(\frac{n\hat{k}_d}{bx})$ and $\hat{x}$ become as follow:

$$\hat{p} = \frac{1}{|W|\omega}\left(\begin{matrix}\left[M(1-S_k)\left(\frac{\eta}{a}+1\right)\right] - g_d\left[(1-S_k)(\eta+1) + \omega S_k\left(\eta + \frac{1}{\omega}\right)\right] \\ -\zeta[(1-S_k)(\eta+1)] - \theta S_k(\eta + \frac{1}{\omega})\end{matrix}\right), \quad \text{(A.17)}$$

$$\left(\frac{n\hat{k}_d}{bx}\right) = \frac{1}{|W|}\left[M\left(\frac{\eta}{a}+1\right) + g_d\eta(z-1) - \zeta(\eta+1) + \theta(z\eta+1)\right] > 0 \quad \text{(A.18)}$$

$$\hat{x} = \frac{1}{|W|\omega}\left\{M\left(\frac{1}{a} - z(1-S_k) - \omega S_k\right) + g_d(z-1) + \theta S_k(z-\omega) + \zeta(z(1-S_k) + \omega S_k - 1)\right\}, \quad \text{(A.19)}$$

where $|W| \equiv \frac{1}{\omega}[(1-S_k)(1+z\eta) + S_k] + \eta S_k > 0$.

Case 1: $\omega > 1$ and $\sigma > 1$

By Eq. (A.18) for $\omega > 1$, we have $\lim_{t\to\infty} S_k = 1$. Plugging this into Eq. (A.17), we have; $\lim_{t\to\infty} \hat{p} = -\frac{1}{1+\eta\omega}(\eta + \frac{1}{\omega})(g_d + \theta) < 0$. It follows that for $\sigma > 1, \lim_{t\to\infty} s = 1$, and $\lim_{t\to\infty} z = 1/a$.Then Eq. (A.19) implies; $\lim_{t\to\infty} \hat{x} = \frac{1}{|W|\omega}\{(\frac{M}{a} - \zeta) - \omega(M-\zeta) + g_d(\frac{1}{a}-1) + \theta(\frac{1}{a}-\omega)\}$. Since $(\frac{M}{a} - \zeta) - \omega(M-\zeta) < 0$ for $a > 1$ and $\omega > 1$, it follows that $\lim_{t\to\infty} \hat{x} < 0$.

Case 2: $\omega > 1$ and $\sigma < 1$

By Eq. (A.18) for $\omega > 1$, we have $\lim_{t\to\infty} S_k = 1$. Plugging this into Eq. (A.17), we have; $\lim_{t\to\infty} \hat{p} = -\frac{1}{1+\eta\omega}(\eta + \frac{1}{\omega})(g_d + \theta) < 0$. It follows that for $\sigma < 1$, $\lim_{t\to\infty} s = 0$ and $\lim_{t\to\infty} z = \sigma$.Then Eq. (A.19) becomes; $\lim_{t\to\infty} \hat{x} = \frac{1}{|W|\omega}\{(\frac{M}{a} - \zeta) - \omega(M-\zeta) + g_d(\sigma-1) + \theta(\sigma-\omega)\}$. We find that $\lim_{t\to\infty} \hat{x} < 0$ if $(\frac{M}{a} - \zeta) < \omega(M-\zeta)$, which is always true for $a > 1$.

Case 3: $\omega < 1$ and $\sigma > 1$

By Eq. (A.18) for $\omega > 1$, we have $\lim_{t\to\infty} S_k = 0$. Plugging this into Eq. (A.17), we have; $\lim_{t\to\infty} \hat{p} = \frac{1}{(\lim_{t\to\infty} |W|\omega)}(M(\frac{\eta}{a}+1) - (g_d+\zeta)(\eta+1)) > 0$. Therefore for $\sigma > 1$, we have that $\lim_{t\to\infty} s = 0$ and $\lim_{t\to\infty} z = \sigma$ so that $\lim_{t\to\infty} \hat{p} = \frac{1}{1+\sigma\eta}(M(\frac{\eta}{a}+1) - (g_d+\zeta)(\eta+1)) > 0$. Then by Eq. (A.19), $\lim_{t\to\infty} \hat{x} = \frac{1}{1+\sigma\eta}\{M(\frac{1}{a} - \sigma) + (g_d+\zeta)(\sigma-1)\} < 0$ if and only if $\sigma > \frac{\frac{M}{a}-\zeta-g_d}{M-\zeta-g_d} = h_3(\zeta, g_d)$.

For $a > 1$, this requirement is automatically satisfied since $h_3(\zeta, g_d) < 1$.

Case 4: $\omega < 1$ and $\sigma < 1$

From Eq. (A.18) for $\omega > 1$, we have $\lim_{t\to\infty} S_k = 0$. It follows that $\lim_{t\to\infty} \hat{p} > 0$. Since $\sigma < 1$, we have that $\lim_{t\to\infty} s = 1$ and $\lim_{t\to\infty} z = 1/a$, and therefore $\lim_{t\to\infty} \hat{p} = M - \frac{1+\eta}{1+(\eta/a)}(\zeta + g_d) > 0$ and $\lim_{t\to\infty} (\hat{c}/e) = (\frac{1+\eta}{a+\eta})(\zeta + g_d)$. By Eq. (A.19), $\lim_{t\to\infty} \hat{x} = \frac{(\frac{1}{a}-1)(g_d+\zeta)}{(1+(\eta/a))} < 0$ for $a > 1$.

Case 5: $\omega = 1,\ \sigma \neq 1$

Since $0 < S_k = \alpha < 1$ we have,

$$\lim_{t\to\infty} \hat{p} = \left(\lim_{t\to\infty} \frac{1}{|W|\omega}\right)\left[(1-\alpha)\left((\frac{M}{a}-\zeta)\eta + (M-\zeta)\right) - g_d(1+\eta) - \theta\alpha\left(\eta + \frac{1}{\omega}\right)\right].$$

It follows that $\lim_{t\to\infty} \hat{p} > (<) 0$ if and only if

$$g_d < (>) \frac{(1-\alpha)\left((\frac{M}{a}-\zeta)\eta + (M-\zeta)\right) - \theta\alpha(1+\eta)}{1+\eta} = \bar{g}.$$

We consider four different sub-cases.

5-1) $\sigma < 1$ and $g_d < \bar{g}$: We have $\lim_{t\to\infty} s(p) = 1$ and $\lim_{t\to\infty} z = 1/a$. It follows that

$$\lim_{t\to\infty} \hat{x} = \frac{\left(\frac{1}{a}-1\right)(M\alpha + g_d + \zeta(1-\alpha) + \theta\alpha)}{\left((1-\alpha)\left(1+\frac{\eta}{a}\right) + \alpha(1+\eta)\right)} < 0 \text{ for } a > 1$$

regardless of magnitude of $g_d > 0$.

5-1) $\sigma > 1$ and $g_d < \bar{g}$: We have $\lim_{t\to\infty} s(p) = 0$ and $\lim_{t\to\infty} z = \sigma$. It follows that

$$\lim_{t\to\infty} \hat{x} = \frac{\left[M\left(\frac{1}{a}-1\right)\right] - \left[(1-\alpha)(\sigma-1)(M-\zeta)\right] + \left[(\sigma-1)(g_d+\theta\alpha)\right]}{(1-\alpha)(1+\sigma\eta) + (1+\eta)\alpha}.$$

The first term of the numerator is negative, while the sum of second and third term becomes negative since

$$\begin{aligned}&-(1-\alpha)(\sigma-1)(M-\zeta) + (\sigma-1)(g_d+\theta\alpha)\\ &\quad < -(1-\alpha)(\sigma-1)(M-\zeta)\\ &\quad + \left[\frac{(1-\alpha)\left((\frac{M}{a}-\zeta)\eta + (M-\zeta)\right) - \theta\alpha(\eta+1) + \theta\alpha(1+\eta)}{1+\eta}\right](\sigma-1) < 0.\end{aligned}$$

Therefore, $\lim_{t\to\infty} \hat{x} < 0$.

5-1) $\sigma < 1$ and $g_d > \bar{g}$: We have $\lim_{t\to\infty} s(p) = 0$ and $\lim_{t\to\infty} z = \sigma$. It follows that

$$\lim_{t\to\infty} \hat{x} = \frac{\left(\frac{1}{a}-\sigma\right)M + M\alpha(\sigma-1) + g_d(\sigma-1) + \zeta(1-\alpha)(\sigma-1) + \theta\alpha(\sigma-1)}{(1-\alpha)(1+\sigma\eta) + (1+\eta)\alpha} < 0 \text{ for } a > 1.$$

5-2) $\sigma > 1$ and $g_d > \bar{g}$: We have $\lim_{t\to\infty} s(p) = 1$ and $\lim_{t\to\infty} z = 1/a$. It follows that

$$\lim_{t\to\infty} \hat{x} = \frac{\left(\frac{1}{a}-1\right)(M\alpha + g_d + \zeta(1-\alpha) + \theta\alpha)}{\left((1-\alpha)\left(1+\frac{\eta}{a}\right) + \alpha(1+\eta)\right)} < 0 \text{ for } a > 1.$$

Case 6: $\omega = 1$ and $\sigma = 1$

We always have $0 < S_k = \alpha < 1$, $0 < s(p) = \beta < 1$, and $z = \frac{\beta}{a} + (1-\beta) < 1$. Eq. (A.18) implies that $\lim_{t\to\infty} \hat{x} < 0$ if and only if $M(\frac{1}{a} - (1-\alpha)z - \alpha) - \zeta(1-(1-\alpha)z - \alpha) + \theta\alpha(z-1) < 0$. Rearranging terms in the left-hand side, we have,

$$\begin{aligned}&M\left(\frac{1}{a} - (1-\alpha)z - \alpha\right) - \zeta(1-(1-\alpha)z-\alpha) + \theta\alpha(z-1)\\ &= \left[M\left(\frac{1}{a} - z\right) - \zeta(1-z)\right] + (M-\zeta+\theta)\alpha(z-1)\end{aligned}$$

The first term is negative since $\frac{(M/a-\zeta)}{(M-\zeta)} < \frac{1}{a} < \frac{\beta}{a} + (1-\beta) = z$, and the second term is also negative since $z < 1$. □

References

Acemoglu, D., Aghion, P., Bursztyn, L., Hemous, D., 2012. The environment and directed technical change. Am. Econ. Rev. 102 (1), 131–166.

Aghion, P., Howitt, P.W., Howitt, P., Brant-Collett, M., García-Peñalosa, C., 1998. Endogenous Growth Theory. MIT press.

Ahmad, N., Wyckoff, A., 2003. Carbon dioxide emissions embodied in international trade of goods. OECD Science, Technology and Industry Working Papers, 2003/15. OECD Publishing doi:10.1787/421482436815.

Arkesteijn, K., Oerlemans, L., 2005. The early adoption of green power by Dutch households: an empirical exploration of factors influencing the early adoption of green electricity for domestic purposes. Energy Policy 33 (2), 183–196.

Arrow, K.J., Dasgupta, P., Goulder, L.H., Mumford, K.J., Oleson, K., 2012. Sustainability and the measurement of wealth. Environ. Dev. Econ. 17 (3), 317–353.

Attanasio, O.P., Browning, M., 1995. Consumption over the life cycle and over the business cycle. Am. Econ. Rev. 85 (5), 1118–1137.

Banerjee, A., Solomon, B.D., 2003. Eco-labeling for energy efficiency and sustainability: a meta-evaluation of US programs. Energy policy 31 (2), 109–123.

Bansal, R., Yaron, A., 2004. Risks for the long run: a potential resolution of asset pricing puzzles. J. Finance 59 (4), 1481–1509.

Barbier, E.B., 2016. Sustainability and development. Ann. Rev. Resour. Econ. 8, 261–280.

Barbier, E., Markandya, A., 1990. The conditions for achieving environmentally sustainable development. Eur. Econ.Rev. 34 (2-3), 659–669.

Barbier, E.B., 1999. Endogenous growth and natural resource scarcity. Environ. Resour. Econ. 14 (1), 51–74.

Barro, R.J., 2001. Human capital and growth. Am. Econ. Rev. 91 (2), 12–17.

Barro, R.J., Lee, J.W., 2013. A new data set of educational attainment in the world, 1950–2010. J. Dev. Econ. 104, 184–198.

Bond, T.C., Doherty, S.J., Fahey, D.W., Forster, P.M., Berntsen, T., DeAngelo, B.J., ... Kinne, S., 2013. Bounding the role of black carbon in the climate system: a scientific assessment. J. Geophys. Res. 118 (11), 5380–5552.

Bovenberg, A.L., Smulders, S., 1995. Environmental quality and pollution-augmenting technological change in a two-sector endogenous growth model. J. Public Econ. 57 (3), 369–391.

Bovenberg, A.L., Smulders, S.A., 1996. Transitional impacts of environmental policy in an endogenous growth model. Int. Econ. Rev. 37 (4), 861–893.

Bretschger, L., Smulders, S., 2012. Sustainability and substitution of exhaustible natural resources: how structural change affects long-term R&D-investments. J. Econ. Dyn. Control 36 (4), 536–549.

Brock, W.A., Taylor, M.S., 2010. The green Solow model. J. Econ. Growth 15 (2), 127–153.

Bye, B., 2000. Environmental tax reform and producer foresight: an intertemporal computable general equilibrium analysis. J. Policy Model. 22 (6), 719–752.

Colacito, R., Croce, M.M., 2013. International asset pricing with recursive preferences. J. Finance 68 (4), 2651–2686.

Cohen, D., Soto, M., 2007. Growth and human capital: good data, good results. J. Econ. Growth 12 (1), 51–76.

Cole, M.A., Elliott, R.J., 2003. Determining the trade–environment composition effect: the role of capital, labor and environmental regulations. J. Environ. Econ. Manag. 46 (3), 363–383.

Copeland, B.R., Taylor, M.S., 2004. Trade, growth, and the environment. J. Econ. Lit. 42 (1), 7–71.

Colacito, R., Croce, M.M., 2011. Risks for the long run and the real exchange rate. J. Polit. Econ. 119 (1), 153–181.

Corrado, C., Hulten, C., Sichel, D., 2009. Intangible capital and US economic growth. Rev. Income Wealth 55 (3), 661–685.

Cropper, M.L., 1976. Regulating activities with catastrophic environmental effects. J. Environ. Econ. Manag. 3 (1), 1–15.

Cropper, M.L., Oates, W.E., 1992. Environmental economics: a survey. J. Econ. Lit. 30 (2), 675–740.

Energy Information Administration, 2012. Fuel Competition in Power Generation and Elasticities of Substitution. U.S. Department of Energy, Washington D.C.

Galarraga, I., González-Eguino, M., Markandya, A., 2011. Willingness to pay and price elasticities of demand for energy-efficient appliances: combining the hedonic approach and demand systems. Energy Econ. 33, S66–S74.

Galor, O., Moav, O., 2004. From physical to human capital accumulation: inequality and the process of development. Rev. Econ. Stud. 71 (4), 1001–1026.

Gillingham, K., Jenn, A., Azevedo, I.M., 2015. Heterogeneity in the response to gasoline prices: evidence from Pennsylvania and implications for the rebound effect. Energy Econ. 52, S41–S52.

Glasby, G.P., 2006. Drastic reductions in utilizable fossil fuel reserves: an environmental imperative. Environ. Dev. Sustain. 8 (1), 197–215.

Glaser, L.K., Thompson, G.D., 2000. Demand for organic and conventional beverage milk. Annual Meeting, June 29-July 1, 2000. Western Agricultural Economics Association.

Grossman, G.M., Krueger, A.B., 1995. Economic growth and the environment. Quart. J. Econ. 110 (2), 353–377.

Greening, L.A., Greene, D.L., Difiglio, C., 2000. Energy efficiency and consumption—the rebound effect—a survey. Energy Policy 28 (6), 389–401.

Guscina, A., 2006. Effects of Globalization on Labor's Share in National Income. International Monetary Fund.

Hassler, J., Krusell, P., Olovsson, C., 2012. *Energy-Saving Technical Change* (No. w18456). National Bureau of Economic Research.

Hartman, R., Kwon, O.S., 2005. Sustainable growth and the environmental Kuznets curve. J. Econ. Dyn. Control 29 (10), 1701–1736.

Herring, H., 2006. Energy efficiency—a critical view. Energy 31 (1), 10–20.

IPCC (Intergovernmental Panel on Climate Change), 2007. Climate Change (2007): The Physical Science Basis. Contribution of Working Group I to the Fourth Assessment Report of the Intergovernmental Panel on Climate Change. In: Solomon, S., Qin, D., Manning, M., Chen, Z., Marquis, M., Averyt, K.B., Tignor, M., Miller, H.L. (Eds.), Climate Change (2007): The Physical Science Basis. Contribution of Working Group I to the Fourth Assessment Report of the Intergovernmental Panel on Climate Change. Cambridge University Press, Cambridge, United Kingdom and New York, NY, USA.

Jones, L., Manuelli, R., Siu, H., 2000. *Growth and Business Cycles* (No. w7633). National Bureau of Economic Research.
Kemfert, C., Welsch, H., 2000. Energy-capital-labor substitution and the economic effects of CO2 abatement: evidence for Germany. J. Policy Model. 22 (6), 641–660.
Kotchen, M.J., Moore, M.R., 2007. Private provision of environmental public goods: household participation in green-electricity programs. J. Environ. Econ. Manag. 53 (1), 1–16.
Kydland, F.E., Prescott, E.C., 1982. Time to build and aggregate fluctuations. Econometrica 1345–1370.
Layard, R., Mayraz, G., Nickell, S., 2008. The marginal utility of income. J. Public Econ. 92 (8-9), 1846–1857.
Lin, B.H., Smith, T.A., Huang, C.L., 2008. Organic premiums of US fresh produce. Renew. Agric. Food Syst. 23 (3), 208–216.
Liu, Y., Jayanthakumaran, K., Neri, F., 2013. Who is responsible for the CO_2 emissions that China produces? Energy Policy 62, 1412–1419.
López, R.E., 1994. The environment as a factor of production: the effects of economic growth and trade liberalization. J. Environ. Econ. Manag. 27 (2), 163–184.
López, R.E., Yoon, S.W., 2014. Pollution–income dynamics. Econ. Lett. 124 (3), 504–507.
López, R.E., Yoon, S.W., 2016. Sustainable growth with irreversible stock effects of renewable resources. Econ. Lett. 147, 153–156.
Nordhaus, W.D., 2007. A review of the Stern review on the economics of climate change. J. Econ. Lit. 45 (3), 686–702.
Nordhaus, W.D., 2012. Economic policy in the face of severe tail events. J. Public Econ. Theory 14 (2), 197–219.
National Oceanic and Atmospheric Administration (NOAA). 2013 (database). [Accessed: 5 April 2013].
Ogaki, M., Reinhart, C.M., 1998. Measuring intertemporal substitution: the role of durable goods. J. Polit. Econ. 106 (5), 1078–1098.
Ortu, F., Tamoni, A., Tebaldi, C., 2013. Long-run risk and the persistence of consumption shocks. Rev. Financ. Stud. 26 (11), 2876–2915.
Pearson, P.N., Foster, G.L., Wade, B.S., 2009. Atmospheric carbon dioxide through the Eocene–Oligocene climate transition. Nature 461 (7267), 1110.
Peretto, P.F., 2009. Energy taxes and endogenous technological change. J. Environ. Econ. Manag. 57 (3), 269–283.
Peters, G.P., 2008. From production-based to consumption-based national emission inventories. Ecol. Econ. 65 (1), 13–23.
Peters, G.P., Marland, G., Le Quéré, C., Boden, T., Canadell, J.G., Raupach, M.R., 2012. Rapid growth in CO 2 emissions after the 2008–2009 global financial crisis. Nat. Clim. Change 2 (1), 2.
Sorrell, S., Dimitropoulos, J., 2008. The rebound effect: microeconomic definitions, limitations and extensions. Ecol. Econ. 65 (3), 636–649.
Stokey, N.L., 1998. Are there limits to growth? Int. Econ. Rev. 1–31.
Thøgersen, J., Ölander, F., 2002. Human values and the emergence of a sustainable consumption pattern: a panel study. J. Econ. Psychol. 23 (5), 605–630.
Thompson, G.D., Glaser, L.K., 2001. National demand for organic and conventional baby food. Annual Meeting, July 8–11. Western Agricultural Economics Association 36139.
Van der Werf, E., 2008. Production functions for climate policy modeling: an empirical analysis. Energy Econ. 30 (6), 2964–2979.
Vissing-Jørgensen, A., 2002. Limited asset market participation and the elasticity of intertemporal substitution. J. Polit. Econ. 110 (4), 825–853.
Weber, C.L., Peters, G.P., Guan, D., Hubacek, K., 2008. The contribution of Chinese exports to climate change. Energy Policy 36 (9), 3572–3577.
Weitzman, M.L., 2007. A review of the Stern Review on the economics of climate change. J. Econ. Lit. 45 (3), 703–724.
Weitzman, M.L., 2009. On modeling and interpreting the economics of catastrophic climate change. Rev. Econ. Stat. 91 (1), 1–19.
World Bank (2012). Data retrieved August 2, 2012, from World Development Indicators Online Database.