Real Exchange Rate in Forward-Looking Rules: Advantages and Pitfalls *

Felipe Leal †

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Abstract

In a small open economy model, we assess the advantages and cost of a systematic policy response to the real exchange rate. In particular, in the context of the Gali and Monacelli (2005) model, we use the method of undetermined coefficients to derive closed form solutions for the evolution of all the variables under an augmented Taylor rule that, besides reacting to domestic inflation and output gap, also respond to expected real exchange rate fluctuations. We perform this exercise for three orthogonal innovations: demand, supply and country risk premium shocks. We also consider alternative information sets available to the central bank. Our main findings are as follows. First, when the central bank observes the natural rate of interest, an aggressive policy response to expected exchange rate has the potential of inducing a decline in the volatility of domestic inflation and the output gap in the face of all shocks. This equilibrium, however, is in general not unique: it induces instability due to self-fulfilling expectations. In other words, an aggressive policy response to exchange rate induces indeterminacy, pretty much in line with Uribe (2003) and Woodford (2000). The only exception is the potential decline in domestic inflation volatility. In this case, we derive the conditions under which a positive response to expected exchange rate can reduce the volatility of domestic inflation and induce determinacy. This response will lead, however, and increase in output gap variance. Second, when the central bank is not able to observe the natural rate of interest, reacting to expected exchange rate depreciation can be an efficient response in the face of demand shocks. In particular, we show that there is a unique exchange rate reaction coefficient that can mimic the optimal policy response under full information and also induces determinacy. This coefficient is directly linked to the degree of openness in the economy: as the economy becomes more open, the response to expected exchange rate fluctuations increase. This optimal exchange rate response generates a zero domestic inflation and output gap. Alternative policy responses, in which this policy coefficient is set to zero are unable to induce the optimal allocation.

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†Contact: fleal@fen.uchile.cl

1 Introduction

In a seminal contribution, Taylor (1993) was able to describe the systematic behavior of the FED with a simple rule in which the monetary authority reacted to both, inflation and the output gap. Since then, a vast theoretical and empirical literature, characterizing the behavior of inflation-targeting (IT) central banks, has emerged. In a closed economy New Keynesian model, with sticky prices, the efficient allocation can be implemented if inflation is fully stabilized (see Clarida et. al (1999); Gali and Monacelli (2016); among others). In the context of a simple New Keynesian model, stabilizing inflation removes two distortions associated to sticky prices. First, it stabilizes average markups at their frictionless level, ensuring that employment and output are at their efficient level. Second, price stickiness induces relative prices to move in a way unwarranted by changes in preferences and technologies, as a result inflation induces inefficient relative price movements. This second distortion is also removed when inflation is fully stabilized. It has been shown that, in the presence of demand shocks, a simple Taylor rule is the optimal policy response: it induces the efficient allocation.

In the context of a small open economy the policy problem is isomorphic to the one in a closed economy: the efficient allocation is reached if *domestic* inflation is fully stabilized. [1] This allocation can be implemented, again, by following a simple Taylor rule in which the policy rate reacts only to domestic variables: domestic inflation, the output gap and the natural rate of interest (which depends only on exogenous productivity shocks). In this case, fluctuations in the exchange rate and in CPI inflation are efficient as long as domestic inflation is on target (see Clarida et. al (2001); Gali and Monacelli (2005)). As a result, the efficient Taylor rule should not contain a response to movements in foreign variables: reacting to exchange rate fluctuations does not generate welfare gains.

Despite the policy prescriptions derived from the canonical open economy New Keynesian model, i.e. the Gali and Monacelli (2005) model, there is substantial evidence that central banks react to real exchange rate fluctuations. Daude et. al. (2016) find that in emerging economies with a flexible exchange rate regime, central banks frequently respond to variations in this variable, even without specifying an exchange rate target. For developing economies, Mohanty and Klau (2004) and Aizenman et al. (2011) find that the exchange rate is part of a simple Taylor rule which also contains a response to inflation and output. Similar qualitative results emerge for developed countries. In particular, Clarida et al. (1998) find that some european countries place some weight on real exchange rate fluctuations in simple instrument rules. Lubik and Schorfheide (2007) conclude

[1] This is true, as long as price stickiness is related to domestic prices. If imported prices are also sticky, then the efficient allocation could be implemented by stabilizing CPI inflation. More discussion on this issue can be found in Engel (2011): Currency Misalignments and Optimal Monetary Policy: A Reexamination, AER.

that the Bank of Canada and the Bank of England have a systematic response to movements in the exchange rate. Dennis (2003) review the Australian experience with different models, finding that the optimal response of the authority should not be focused only on the inflation variable, but also on the real exchange rate fluctuations and terms of trade. Adolfson et. al. (2008) and Caglayan et. al. (2016) using DSGE models, find that exchange rate movements affect policy decisions made by central banks in developed countries. More recently, Caporale et. al. (2018) study the conduct of monetary policy in different countries, finding that an augmented forward-looking Taylor rule, specifically one that considers a response to expected exchange rate fluctuations, in addition to inflation and output gap, has a better fit when compared to a standard Taylor rule.

Now, the previous evidence does not necessarily indicate that central banks care, per se, about exchange rate fluctuations. In particular, a policy response to exchange rate, in a simple rule, may help to stabilize some of the variables that enter the welfare loss of the monetary authority: inflation, the output gap or the interest rate itself. In recent contributions, Kam et. al (2009), for developed countries, and Gomez et. al. (2018) for developing economies, estimate the underlying structural macroeconomic policy objectives of inflation targeting central banks, in the context of a small open economy dynamic stochastic general equilibrium model. The main results of these contributions is that inflation stabilization in these countries has a high priority relative to other macroeconomic objectives. In most of the countries and specifications central banks have preference for stabilizing inflation, output and the policy instrument, with no concern about exchange rate fluctuations.

Given the evidence on the systematic policy response to exchange rate fluctuations and the absence of policy concern about this variable, in the central bank preference set, the purpose of this paper is to assess the advantages and cost of simple Taylor rules that react to expected real exchange rate fluctuations. In particular, in the context of the Gali and Monacelli (2005) model [2], we use the method of undetermined coefficients to derive closed form solutions for the evolution of all the variables under an augmented Taylor rule that, besides reacting to domestic inflation and output gap, also respond to expected real exchange rate fluctuations. Unlike the standard policy analysis in Gali and Monacelli (2005), we perform this exercise for three orthogonal innovations: demand, supply and country risk premium shocks. We also consider two alternative information sets available to the central bank. In the first one the monetary authority can observe the natural rate of interest, whereas in the second one the central bank is unable to observe this variable.

Our main findings are as follows:

[2] This is a simple and tractable model containing several important insights about the monetary transmission mechanisms in open economies. In particular, it derives a welfare based central bank loss function that depends only on domestic inflation and output gap volatility, with no policy concerns about any other variable.

First, when the central bank observes the natural rate of interest, an aggressive policy response to expected exchange rate has the potential of inducing a decline in the volatility of domestic inflation and the output gap. This equilibrium, however, is not unique. In particular, we demonstrate that an aggressive response to exchange rate induces multiple equilibria (i.e. induce indeterminacy). In other words, this type of policy reaction generates aggregate instability due to self-fulfilling expectations. Our result is related to Uribe (2003) which, in a very different context, shows that the mere existence of PPP rules can generate endogenous aggregate instability by allowing for the existence of equilibria in which agents base their expectations about economic variables on non-fundamental signals. Our result is also related to Woodford (2000) which shows that one of the pitfalls of forward-looking rules is that they have the potential of inducing multiple equilibria. However, in this same context, when the economy is affected by a supply shock, there is the possibility of decreasing the variance of domestic inflation, without inducing multiple equilibria (indeterminacy). In this case there will be an increase in the volatility of the output gap. This result is in line with the literature that establishes the existence of a policy trade-off between the stabilization of the output gap and domestic inflation in the face of cost-push shocks (Svensson 1999, 2000; Clarida et. al. 2000; Benigno and Benigno 2002, 2003b; among others).

Second, when the central bank is not able to observe the natural rate of interest, reacting to expected exchange rate depreciation can be efficient in the face of demand shocks. In particular, we show that there is a unique exchange rate reaction coefficient that can mimic the optimal policy response under full information and also induces determinacy. This coefficient is directly linked to the degree of openness in the economy: as the economy becomes more open, the response to expected exchange rate fluctuations increase.

The remainder of the thesis is organized as follows. Section 2 extends the standard Gali and Monacelli (2005) New Keynesian DSGE model for a small open economy in two dimensions. First it includes an augmented Taylor rule that react to the expected real exchange rate depreciation. Second, it includes not only the the demand shock considered in the standard version of the model, but also a supply and country risk premium innovation. Section 3 solves the model under the assumption that the central bank is able to observe the natural rate of interest, using the method of undetermined coefficients. In this case we derive closed form solutions for all the relevant variables and variances in the model. In particular, the variances of all the variables depend on the policy response coefficients and structural parameters. In Section 4 we derive the stability conditions of our extended model and determine whether the conditions that induce a decline in the relevant variances also induce a unique solution. Section 5 assesses the properties of augmented Taylor rules in a context in which the central bank is unable to observe the natural rate of interest. Section 6 presents the main conclusions.

2 Model

The theoretical framework in which our model is developed is based on the canonical version of the Gali and Monacelli (2005). We specify a Dynamic Stochastic General Equilibrium (DSGE) model for a small open economy (SOE). The economy described by the model allows finding an aggregate IS curve through the consumers Euler equation, while on supply side a New Keynesian Phillips curve (NKPC). The monetary policy is described by an augmented Taylor rule that will be specified bellow and we also explicitly derive the uncovered interest parity (UIP) condition for this economy. The world economy is composed of a continuum of SOEs (within the range of 0 to 1), modeled as an AR(1) process. Given the characteristics of these economies, each of their policy decisions does not affect the rest of the world. In addition, we assume that all of them share identical production technology, preferences, and market dynamics.

2.1 Households

In our model the demand side is determinated by a representative household who maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \tag{1}$$

where N_t is hours of labour and C_t is an index of aggregate consumption, determined by the consumption of domestic goods $C_{H,t}$, and of foreign goods $C_{F,t}$[3]. The utility function is given by

$$U(C_t, N_t) \equiv \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \tag{2}$$

where $\sigma \geq 0$ and $\varphi \geq 0$ are the intertemporal elasticity of subtitution and the disutility of labor, respectively. The composition of the consumption index is defined by

$$C_t \equiv \left[(1-\alpha)^{\frac{1}{\eta}} \left(C_{H,t}\right)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} \left(C_{F,t}\right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \tag{3}$$

where $\alpha \in [0,1]$ is an index of openess and $\eta > 0$ represents the degree of substitution between domestic and foreign goods.

[3]Both the consumption of domestic and foreign goods are modeled through CES functions given by

$$C_{H,t} \equiv \left(\int_0^1 C_{H,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}} \quad ; \quad C_{F,t} \equiv \left(\int_0^1 (C_{i,t})^{\frac{\gamma-1}{\gamma}} di \right)^{\frac{\gamma}{\gamma-1}} \quad ; \quad C_{i,t} \equiv \left(\int_0^1 C_{i,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

where $j \in [0,1]$ refers the good variety. The index $C_{i,t}$ refers to the amount of goods imported from country i and consumed by the domestic economy. The parameter $\varepsilon > 1$ indicates the elasticity of substitution between differents varities and γ the substitutability between goods produced in the rest of the world (different foreign countries).

The household maximization is subject to a following budget constraint

$$\int_0^1 P_{H,t}(j)C_{H,t}(j)dj + \int_0^1 \int_0^1 P_{i,t}(j)C_{i,t}(j)djdi + E_t\{Q_{t,t+1}D_{t+1}\} \leq D_t + W_tN_t + T_t \tag{4}$$

where P_H and $P_i(j)$ are the prices of domestic goods and of the variety j imported from country i, respectively. Households have a portfolio that has a nominal payment D_{t+1} in the period $t+1$, which includes shares of firms. $Q_{t,t+1}$ is the stochastic discount factor that accompanies the relevant nominal pay-offs. W_t and T_t are the nominal wage and lump-sum transfers, respectively.

Considering that the markets clearing in the local economy, in addition to the Euler equation, after algebra it is possible write the IS curve as:

$$x_t = x_{t+1} - \frac{1}{\sigma_\alpha}\left(i_t - \pi_{H,t+1} - r_t^n\right) \tag{5}$$

where $\sigma_\alpha = \frac{\sigma}{(1-\alpha)+\alpha w}$. The monetary policy rate is represented by i_t and $r_t^n \equiv \rho - \sigma_\alpha\Gamma(1-\rho_a)a_t + \alpha\sigma_\alpha(\Theta+\Psi)E_t\{\Delta y^*_{t+1}\}$ is the natural rate of interest [4].

The output gap x_t is the difference between the effective product and its natural level, i.e.,

$$\begin{aligned} x_t = \quad & y_t - y_t^n \\ & y_t - (\Omega + \Gamma a_t + \alpha\Psi y_t^*) \end{aligned} \tag{6}$$

where y_t^* denotes the world output defined by an AR(1) process $y_t^* = \rho_y y^*_{t-1} + \varepsilon_t^*$; $\Omega \equiv \frac{v-\mu}{\sigma_\alpha+\varphi}$, $\Lambda \equiv \frac{1+\varphi}{\sigma_\alpha+\varphi} > 0$, and $\Psi \equiv -\frac{\Theta\sigma_\alpha}{\sigma_\alpha+\varphi}$. The rate of change in the index of domestic good prices is defined by $\pi_{H,t} = p_{H,t+1} - p_{H,t}$. This forward-looking IS curve allows to leave the output gap of the current period as a function of his future values. In addition, α is the openness degree of the local economy relative to the foreign economy (when it is equal to 0 converges to a closed economy), which affects the sensitivity of the output gap to interest rate changes. The index for CPI inflation is defined as $\pi_t = p_t - p_{t-1}$, and $s_t \equiv p_{F,t} - p_{H,t}$ denotes the effective terms of trade, where $p_{F,t}$ is the price index for imported goods (in domestic currency). Then, the domestic inflation and CPI inflation are linked according to $\pi_t = \pi_{H,t} + \alpha\Delta s_t$, leaving both measures of inflation in a gap proportional to a change in terms of trade. Furthermore, we assume that the law of one price holds, implying that $p_{F,t} = e_t + p_t^*$. Ultimately, the relationship between output gap and terms of trade is defined as $x_t = y_t^* + \frac{1}{\sigma_\alpha}s_t$.

[4] The parameter Θ is defined as

$$\Theta \equiv (\sigma\gamma - 1) + (1-\alpha)(\sigma\eta - 1) = \omega - 1$$

2.2 Supply Side

By the supply side, we consider a continuum of firms indexed by $j \in [0,1]$ with identical technology, represented by a production function given by

$$Y_t(j) = A_t N_t(j)$$

where A_t is the techonology, modeled as a stochastic AR(1) process $a_t = \rho_a a_{t-1} + \varepsilon_t$ with $a_t \equiv \log A_t$.

We consider that firms are in a Calvo-price setting. Hence, a portion $1-\theta$ of firms sets new prices (under optimization process) in each period. After optimization process, the optimal price-setting strategy can be defined by the following approximated (log-linear) rule

$$\bar{p}_{H,t} = \mu + (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\{mc_{t+k} + p_{H,t}\} \tag{7}$$

where $\bar{p}_{H,t}$ is the new price, and $\mu \equiv \log\left(\frac{\varepsilon}{\varepsilon-1}\right)$ denotes the (gross) mark-up in steady state.

In the small open economy, it is possible define the relationship between the dynamics of domestic inflation and the real marginal cost as follows

$$\pi_{H,t} = \beta E_t\{\pi_{H,t+1}\} + \lambda\widehat{mc}_t \tag{8}$$

where $\lambda \equiv \frac{(1-\beta\theta)(1-\theta)}{\theta}$. The marginal cost increases with terms of trade and world output, affecting the real wage through the wealth effect on labour supply drived by domestic consumption. Besides, terms of trade have a direct effect on the product wage (see Gali and Monacelli (2005)). Additionally, the relationship between the real marginal cost and the output gap is given by

$$\widehat{mc}_t = (\sigma_\alpha + \varphi)x_t \tag{9}$$

Combining the previous equation (9) with (8), after some algebra, we can derive the log-linear approximation of the optimal price decision as a New Keynesian Phillips curve given by

$$\pi_{H,t} = \beta\pi_{H,t+1} + \kappa_\alpha x_t \tag{10}$$

where $k_\alpha = \lambda(\sigma_\alpha + \varphi)$. The slope, that is, the response of the level of domestic inflation to changes in the output gap depends, as in the case of the IS curve, on the degree of openness of the economy and the degree of substitution between domestic and foreign goods.

2.3 Shocks

As we mention at the begining, we specify three different innovations associated with the demand, supply and country risk premium shocks, specified as a_t, v_t and z_t respectively. Each of these is modeled as a stochastic AR(1) process as follow

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a \tag{11}$$

$$v_t = \rho_v v_{t-1} + \varepsilon_t^v \tag{12}$$

$$z_t = \rho_z z_{t-1} + \varepsilon_t^z \tag{13}$$

where $\rho_a, \rho_v, \rho_z \in [0, 1)$ and ε_t^a, ε_t^v, ε_t^a are $i.i.d$ $\mathcal{N}(0, \varsigma^2)$.

The formulation of the first shock, a_t, is identical to the one derived in the baseline model in Gali and Monacelli (2005), associated with a perturbance in the productivity. Specifically, having a positive productivity shock, the natural rate of interest (r_t^n) fall, as well as the product gap (x_t).

The second shock, v_t, corresponds to a perturbation in the NKPC, being it added to the equation (10). This "cost-push" shock has interesting consequences for monetary policy, this because it generates a dilemma for the authority. On the one hand, it is desirable for the central bank to avoid suboptimal fluctuations in the relative prices of the economy, that is, the stabilization of the price level. However, on the other hand, this stabilization of prices generates suboptimal fluctuations in output. Therefore, the presence of "cost-push" shock implies that the authority should allow some price flexibility to allow some level of stabilization on the output (Svensson 1999, 2000; Clarida et. al. 2000; Benigno and Benigno 2002, 2003b; among others). In the case of open economy models, it is possible to have different intensity of pass-through with changes in the exchange rate, for which the monetary authority should allow a certain degree of flexibility in the producer price level to achieve some level of stability in the nominal exchange rate. If this shock is positive, contemporary domestic inflation will increase one by one on magnitude of the shock. In other words, the equation (10) is re-defined as follows

$$\pi_{H,t} = \beta \pi_{H,t+1} + \kappa_\alpha x_t + v_t \tag{14}$$

For the third shock, z_t, we explicitly derive from the Euler equation, the dynamics of inflation and the terms of trade, in addition to the "risk sharing" (relation between the level of domestic consumption and the level of consumption of the foreign economy, together with the terms of trade) the uncovered interest parity condition in which we add the shock.[5] In addition, the real exchange

[5]See Appendix A.

rate (RER) is defined as $q_t = e_t + p_t^* - p_t$, where e_t is the nominal exchange rate. Then the real uncovered interest parity (UIP) condition that describes the evolution of q_t is described by

$$E_t(q_{t+1} - q_t) = (i_t - E_t\pi_{t+1}) - (i_t^* - E_t\pi_{t+1}^*) + z_t \tag{15}$$

where i^* is the international interest rate. This shock could be interpreted as an innovation in the country risk premium (CRP). As shown by Neumeyer and Perri (2005), this premium may be affected by domestic fundamentals (expected productivity) and, through the presence of working capital, may exacerbate the effect of this shock on real activity. This spread could also be influenced by external elements, like an increase in the foreign interest rate, as shown by Uribe and Yue (2006). In addition, this shock could also be interpreted as changes in capital flows or, indeed, capital controls imposed by the domestic economy, that have an impact on interest rate differentials. [6] In Kam et al. (2009) this innovation is interpreted as an exogenous CRP, reflecting foreign and domestic elements not explicitly modeled. In this context, it is important to note that the z_t innovation may be capturing anticipated shocks (news) as well as unanticipated ones.[7] As noted by Chen and Zhang (2015), economic news may be capturing shocks to the RER that are expected by markets participants. The relative importance of anticipated and unanticipated shocks is certainly a relevant question, but in the present context z_t is reflecting both types of shocks.

2.4 Monetary policy: a forward-looking Taylor-rule

Since the 1990s, different central banks around the world have adopted inflation targeting schemes for to manage the monetary policy. In this sense, the response of central banks to deviations from inflation with respect to their objective values has been widely discussed in the literature. Taylor (1993) explained that a good way to generate significant reductions of uncertainty in economic agents could be through a policy rule that mechanized the response of the central bank to variations in economic variables such as inflation or product gap. In addition, as we mention previously, other variable that turns out to have big importance in open economies is the exchange rate. This key relative price has strong repercussions on economic performance, levels of growth and also the inflation rate (pass-through) (Ca'Zorzi et al., 2007; Goldberg et. al., 2010; Ball 1999b; Lubik et. al., 2007; Aizenman et. al., 2008; Caglayan et. al., 2016; Caporale et. al., 2018).

Ball (1999) shows that is necessary to consider the variations of the exchange rate in the Taylor rules when open economies are analyzed. Lubik & Schorfheide (2007) develop a general structural equilibrium model for a small open economy estimated through Bayesian methods, finding that different central banks react systematically to variations in the exchange rate. In addition, for central bank policymakers, besides to reacting to variations in inflation levels and the output gap,

[6]As shown by Herrera and Valdes (2001), in emerging economies the effect of capital controls on interest rate differentials, and on the RER, is considerably smaller than what static calculations suggest.

[7]See Nam and Wang (2015).

also do so with respect to the exchange rate, which turns out to be an important variable at the time of the election of policy rate, with an even stronger effect in countries with high commodity exports (Aizenman et. al., 2008; Caglayan et. al., 2016). Caporale et. al. (2018) studies an augmented Taylor rule for emerging economies that includes the expected real exchange rate, finding that the behavior of the monetary policy rule responds constantly to movements in this variable.

In this paper, we incorporate an augmented Taylor rule, where the monetary policy rate reacts to deviations from its objective values for both inflation and the output gap, in addition to expected real exchange rate fluctuations. As in Caporale et. al. (2018), we consider for central bank the possibility of a reaction to the expected real exchange rate, which in our case is through the difference with respect to the effective value in t, i.e. is defined by $E_t\{\Delta q_{t+1}\} = E_t\{q_{t+1}\} - q_t$. So, Taylor-rule is defined as follows

$$i_t = r_t^n + \phi_\pi \pi_{H,t} + \phi_x x_t + \phi_q E_t\{\Delta q_{t+1}\} \tag{16}$$

The coefficients ϕ_π, ϕ_x and ϕ_q are the response values of the central bank associated with the inflation, the product gap and the expected real exchange rate fluctuations, respectively. All of this parameters must be fulfilled that ϕ_π, ϕ_x, $\phi_q \geq 0$.

3 Solving the model: undeterminated coefficients method

With the equations of the model showed in the previous section, we can derive analytically the variances of the variables, all of them as a function of policy response coefficients and structural parameters. For this, we use the undetermined coefficients method (Christiano, 1998). This method allow us obtain a linear approximation to the solution of a dynamic rational expectation model, which can be used to derive the model implications for second moments. Specifically, since we have a system of dynamic equations, we assume that exists a solution that depends only on an exogenous component, that is, a shock. To write the non policy block we have to obtain the variables in the following form: $\mathbb{X} = \mathbb{B}V(\varepsilon_t^S)$, where $\mathbb{X} = \{x_t, \pi_t, i_t\}$ are the variables and $\mathbb{B} = \{\Psi_x, \Psi_\pi, \Psi_i\}$ the coefficients, $\forall s \in \{a, v, z\}$ associated with each shock. The matrix form is defined as

$$\begin{bmatrix} x_t \\ \\ \pi_t \\ \\ i_t \end{bmatrix} = \begin{bmatrix} \Psi_{xs} \\ \\ \Psi_{\pi s} \\ \\ \Psi_{zs} \end{bmatrix} V(\varepsilon_t^S)$$

3.1 Solving for productivity shock

Below we solve the method for productivity shock (for the other shocks the approach is the same). First, we re-write the variables as follows: for the case of output gap, we defined it as $x_t \equiv \Psi_{xa}\, a_t$, where the coefficient Ψ_{xa} is associated with the output gap (x) and the productivity shock (a). The rest of the variables are defined in the same way. Replacing the Taylor rule (16) in the IS curve (5) we obtain

$$x_t = E_t\{x_{t+1}\} - \frac{1}{\sigma_\alpha}\left(r_t^n + E_t\{\phi_\pi \pi_{H,t} + \phi_x x_t + \phi_q \Delta q_{t+1} - \pi_{H,t+1}\} - r_t^n\right)$$

now replacing with the re-definition showed above and using the relationship between the RER and terms of trade $\Delta q_{t+1} = (1-\alpha)(s_{t+1} - s_t)$, and the fact that $\pi_{H,t} = \Psi_{\pi a}\, a_t$ and $s_t = \Psi_{sa}\, a_t$, we obtain

$$\Psi_{xa} a_t = \Psi_{xa}\rho_a a_t - \frac{1}{\sigma_\alpha}\left(\phi_\pi \Psi_{\pi a} a_t + \phi_x \Psi_{xa} a_t + \phi_q((1-\alpha)(\Psi_{sa}\rho_a a_t - \Psi_{sa} a_t)) - \Psi_{\pi a}\rho_a a_t\right)$$

$$0 = \left[\left(\frac{\sigma_\alpha(\rho_a - 1) - \phi_x}{\sigma_\alpha}\right)\Psi_{xa} - \left(\frac{\phi_\pi - \rho_a}{\sigma_\alpha}\right)\Psi_{\pi a} - \left(\frac{\phi_q(1-\alpha)(\rho_a - 1)}{\sigma_\alpha}\right)\Psi_{sa}\right] a_t$$

iterating the process for the other equilibrium equations, specifically for the NKPC, the definition of the output gap in terms of natural product and the relationship between the product and terms of trade, we have 4 equations for 4 unknows variables (coefficients). Then, it is possible re-write the system of equations in a matrix form as follows

$$\begin{bmatrix} 0 \\ \vdots \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & \frac{\sigma_\alpha(\rho_a - 1) - \phi_x}{\sigma_\alpha} & \frac{\rho_a - \phi_\pi}{\sigma_\alpha} & \frac{\phi_q(1-\alpha)(1-\rho_a)}{\sigma_\alpha} & 0 \\ 0 & \kappa_\alpha & \beta\rho_a - 1 & 0 & 0 \\ -\Gamma & -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & \frac{1}{\sigma_\alpha} & -1 \end{bmatrix} \begin{bmatrix} 1 \\ \Psi_{xa} \\ \Psi_{\pi a} \\ \Psi_{sa} \\ \Psi_{ya} \end{bmatrix} a_t$$

Therefore, when we solve this system it is possible to find the coefficients that will allow us to write the variables of our model based on structural parameters of the economy as well as the variance of each shock. For this example case for productivity shock, the coefficients that we obtain are defined by

$$\Psi_{xa} = \frac{[\sigma_\alpha(\rho_a - 1) - \phi_x]\Gamma\sigma_\alpha(1-\beta\rho_a) - (\phi_\pi - \rho_a)\kappa_\alpha\Gamma\sigma_\alpha}{\sigma_\alpha\left[[\sigma_\alpha(\rho_a - 1) - \phi_x](1-\beta\rho_a) - (\phi_\pi - \rho_a)\kappa_\alpha - \phi_q\sigma_\alpha(1-\beta\rho_a)(1-\alpha)(\rho_a - 1)\right]} - \Gamma$$

$$\Psi_{\pi a} = \frac{\kappa_\alpha \left[[\sigma_\alpha(\rho_a - 1) - \phi_x]\Gamma\sigma_\alpha(1-\beta\rho_a) - (\phi_\pi - \rho_a)\kappa_\alpha\Gamma\sigma_\alpha \right]}{\sigma_\alpha(1-\beta\rho_a)\left[[\sigma_\alpha(\rho_a - 1) - \phi_x](1-\beta\rho_a) - (\phi_\pi - \rho_a)\kappa_\alpha - \phi_q\sigma_\alpha(1-\beta\rho_a)(1-\alpha)(\rho_a - 1) \right]} - \frac{\kappa_\alpha\Gamma}{(1-\beta\rho_a)}$$

$$\Psi_{sa} = \frac{[\sigma_\alpha(\rho_a - 1) - \phi_x]\Gamma\sigma_\alpha(1-\beta\rho_a) - (\phi_\pi - \rho_a)\kappa_\alpha\Gamma\sigma_\alpha}{[\sigma_\alpha(\rho_a - 1) - \phi_x](1-\beta\rho_a) - (\phi_\pi - \rho_a)\kappa_\alpha - \phi_q\sigma_\alpha(1-\beta\rho_a)(1-\alpha)(\rho_a - 1)}$$

$$\Psi_{ya} = \frac{[\sigma_\alpha(\rho_a - 1) - \phi_x]\Gamma\sigma_\alpha(1-\beta\rho_a) - (\phi_\pi - \rho_a)\kappa_\alpha\Gamma\sigma_\alpha}{\sigma_\alpha\left[[\sigma_\alpha(\rho_a - 1) - \phi_x](1-\beta\rho_a) - (\phi_\pi - \rho_a)\kappa_\alpha - \phi_q\sigma_\alpha(1-\beta\rho_a)(1-\alpha)(\rho_a - 1) \right]}$$

3.2 Analytical variances: closed form solution

To get tractable solutions, we assume $\rho_i = 0 \quad \forall i \in \mathbb{A}$, where $\mathbb{A} = \{a, v, z\}$ associated with each shock. For the variances, we simply apply the operator $V(\cdot)$ in the variables defined as a function of the coefficients and variance of the shock. For the case of the output gap, the variance will be defined by

$$x_t \equiv \Psi_{xa} a_t$$

$$V(x_t) = \left(\frac{\Psi_{sa}}{\sigma_\alpha} - \Gamma \right)^2 \frac{V(\varepsilon_t^a)}{1-\rho_a^2}$$

where $V(\varepsilon_t^a)$ is the productivity shock variance.

How we are studying three different shocks, the results depend on which shock we are considering, so we put in the following table all the expressions associated with each shock:

Table 1: Variances. Taylor-rule with know r_t^n

Variable	Productivity Shock	NKPC Shock	UIP Shock
x_t	$\left[\frac{\Gamma\phi_q\sigma_\alpha(1-\alpha)}{\phi_q\sigma_\alpha(1-\alpha)-\phi_\pi\kappa_\alpha-\phi_x-\sigma_\alpha}\right]^2 V(\varepsilon_t^a)$	$\left[\frac{\phi_\pi}{\phi_q\sigma_\alpha(1-\alpha)-\phi_\pi\kappa_\alpha-\phi_x-\sigma_\alpha}\right]^2 V(\varepsilon_t^a)$	$\left[\frac{\phi_q\sigma_\alpha(1-\alpha)}{\sigma^2(\phi_q\sigma_\alpha(1-\alpha)-\phi_\pi\kappa_\alpha-\phi_x-\sigma_\alpha)}\right]^2 V(\varepsilon_t^z)$
$\pi_{H,t}$	$\kappa_\alpha^2\left[\frac{\Gamma\phi_q\sigma_\alpha(1-\alpha)}{\phi_q\sigma_\alpha(1-\alpha)-\phi_\pi\kappa_\alpha-\phi_x-\sigma_\alpha}\right]^2 V(\varepsilon_t^a)$	$\left[\frac{\phi_q\sigma_\alpha(1-\alpha)-\phi_x-\sigma_\alpha}{\phi_q\sigma_\alpha(1-\alpha)-\phi_\pi\kappa_\alpha-\phi_x-\sigma_\alpha}\right]^2 V(\varepsilon_t^v)$	$\kappa_\alpha^2\left[\frac{\phi_q\sigma_\alpha(1-\alpha)}{\sigma^2(\phi_q\sigma_\alpha(1-\alpha)-\phi_\pi\kappa_\alpha-\phi_x-\sigma_\alpha)}\right]^2 V(\varepsilon_t^z)$
Δq_t	$2\Psi_{sa}^2(1-\alpha)^2 V(\varepsilon_t^a)$	$2\Psi_{sv}^2(1-\alpha)^2 V(\varepsilon_t^v)$	$2\Psi_{sz}^2(1-\alpha)^2 V(\varepsilon_t^z)$

where $\Psi_{sa} = \frac{-\Gamma\sigma_\alpha(\sigma_\alpha+\phi_x+\kappa_\alpha\phi_\pi)}{\phi_q\sigma_\alpha(1-\alpha)-\phi_\pi\kappa_\alpha-\phi_x-\sigma_\alpha}$, $\Psi_{sv} = \frac{\phi_\pi\sigma_\alpha}{\phi_q\sigma_\alpha(1-\alpha)-\phi_\pi\kappa_\alpha-\phi_x-\sigma_\alpha}$, $\Psi_{sz} = -\frac{\sigma_\alpha(\sigma_\alpha+\phi_x+\phi_\pi\kappa_\alpha)}{\sigma(\phi_q\sigma_\alpha(1-\alpha)-\phi_\pi\kappa_\alpha-\phi_x-\sigma_\alpha)}$

.

Table 1 shows the variances obtained through the undeterminated coefficients method.[8] Intuitively, while greater the variance associated with each shock, the variance of the variables when also will be greater if they are affected by the same shock.

3.3 Efficiency gains from reacting to Δq_{t+1}

Using these analytical expressions of the Table 1, it is possible see what happens when it increases the coefficient of the central bank for the expected variation of real exchange rate. Again, since we are evaluating three differents shocks, we will derive the expressions for each one. Table 2 shows the derivatives for the output gap, domestic inflation and contemporaneous fluctuation of the RER. All the expressions are very similar through the different shocks and variables, all for each derivative. Specifically, calibrating the parameters of the model using the standard literature[9], for each expression shown bellow, the numerators will always be negative[10], so the "action" (sign) of the derivative will be a function of the denominator.

Table 2: Derivates. Taylor-rule with know r_t^n

Derivate	**Productivity Shock**	**NKPC Shock**	**UIP Shock**
$\frac{\partial V(x_t)}{\partial \phi_q}$	$\frac{-2\phi_q(1-\alpha)^2\Gamma^2\sigma_\alpha^4(\sigma_\alpha+\phi_x+\kappa_\alpha\phi_\pi)V(\varepsilon_t^a)}{[\phi_q\sigma_\alpha(1-\alpha)-\kappa_\alpha\phi_\pi-\phi_x-\sigma_\alpha]^3}$	$\frac{-2\phi_\pi^2\sigma_\alpha^2(1-\alpha)V(\varepsilon_t^a)}{[\phi_q\sigma_\alpha(1-\alpha)-\kappa_\alpha\phi_\pi-\phi_x-\sigma_\alpha]^3}$	$\frac{-2\phi_q(1-\alpha)^2\sigma_\alpha^2(\sigma_\alpha+\phi_x+\kappa_\alpha\phi_\pi)V(\varepsilon_t^a)}{[\phi_q\sigma_\alpha(1-\alpha)-\kappa_\alpha\phi_\pi-\phi_x-\sigma_\alpha]^3\sigma^2}$
$\frac{\partial V(\pi_{H,t})}{\partial \phi_q}$	$\frac{-2\kappa_\alpha^2\phi_q\Gamma^2\sigma_\alpha^4(1-\alpha)^2(\sigma_\alpha+\phi_x+\kappa_\alpha\phi_\pi)V(\varepsilon_t^a)}{[\phi_q\sigma_\alpha(1-\alpha)-\kappa_\alpha\phi_\pi-\phi_x-\sigma_\alpha]^3}$	$\frac{-2\phi_\pi\kappa_\alpha\sigma_\alpha(1-\alpha)(\phi_q\sigma_\alpha(1-\alpha)-\phi_x-\sigma_\alpha)V(\varepsilon_t^a)}{[\phi_q\sigma_\alpha(1-\alpha)-\kappa_\alpha\phi_\pi-\phi_x-\sigma_\alpha]^3}$	$\frac{-2\kappa_\alpha\phi_q(1-\alpha)^2\sigma_\alpha^2(\sigma_\alpha+\phi_x+\kappa_\alpha\phi_\pi)V(\varepsilon_t^a)}{[\phi_q\sigma_\alpha(1-\alpha)-\kappa_\alpha\phi_\pi-\phi_x-\sigma_\alpha]^3\sigma^2}$
$\frac{\partial V(\Delta q_t)}{\partial \phi_q}$	$\frac{-4\Gamma^2\sigma_\alpha^3(1-\alpha)^3(\sigma_\alpha+\phi_x+\kappa_\alpha\phi_\pi)^2V(\varepsilon_t^a)}{[\phi_q\sigma_\alpha(1-\alpha)-\kappa_\alpha\phi_\pi-\phi_x-\sigma_\alpha]^3}$	$\frac{-4(1-\alpha)^3\sigma_\alpha^3\phi_\pi^2V(\varepsilon_t^a)}{[\phi_q\sigma_\alpha(1-\alpha)-\kappa_\alpha\phi_\pi-\phi_x-\sigma_\alpha]^3}$	$\frac{-4(1-\alpha)^3\sigma_\alpha^3(\sigma_\alpha+\phi_x+\kappa_\alpha\phi_\pi)^2V(\varepsilon_t^a)}{[\phi_q\sigma_\alpha(1-\alpha)-\kappa_\alpha\phi_\pi-\phi_x-\sigma_\alpha]^3\sigma^2}$

Focusing on the denominator of each of the derivatives, we find two facts that are important. First, the expressions turns out to be identical for all variables, regardless of the type of shock we are considering. Second, in order to reduce the variable variances, the monetary authority must react aggressively to changes in the expected real exchange rate fluctuations, that is, the denominator must be positive. In other words, it must be fulfilled that:

$$\phi_q\sigma_\alpha(1-\alpha) - \kappa_\alpha\phi_\pi - \phi_x - \sigma_\alpha > 0$$

[8]The rest of the variables can be reviewed in Appendix D.

[9]See Appendix E.

[10]Except for domestic inflation with NKPC shock, it is a special case the we review later on the paper.

$$\phi_q > \frac{\kappa_\alpha \phi_\pi + \phi_x + \sigma_\alpha}{\sigma_\alpha(1-\alpha)} \tag{17}$$

As the previous condition allow us place in a closer frontier to the origin in a space for the variances, i.e., lower variances for any variable, onwards we will call to (16) as the *Efficient RER Taylor-rule*. Previous condition (17) will be easier to achieve with a lower openness economy degree, and also when the sensitivity of domestic inflation to the output gap is lower. Satisfy this condition require that the central bank response to expected real exchange rate fluctuations be very strong compared to the values of the inflation and/or output gap coefficients. Notice that if we assume that the central bank does not react neither to inflation nor to the output gap ($\phi_\pi = 0$ and $\phi_x = 0$), for any value of α, even so ϕ_q would have at least greater than one, that is, $\phi_q > \frac{1}{1-\alpha}$. Nevertheless, if we forcing the fulfillment of condition (17), the Blanchard-Khan conditions are not satisfied, not being able to solve the model.

This issue about the non-satisfaction of B-K conditions found raises the question of whether it is possible to find a solution that will be feasible to solve the model considering differents values of the coefficients associated with the preferences of the central bank (maintaining a standard calibration for the other parameters), in other words, analyze the determinacy and indeterminacy regions of the model.

4 Is the *Efficient RER Taylor-rule* feasible? Determinacy conditions

Using Taylor-rule (16) described before, where ϕ_π, ϕ_x and ϕ_q are non-negative coefficients, substituting it in (5), doing a bit of algebra and rearranging terms, we can represent the dynamic equilibrium as a 2x2 matrix system of difference equations:

$$\begin{bmatrix} x_t \\ \\ \pi_{H,t} \end{bmatrix} = A_F \begin{bmatrix} E_t\{x_{t+1}\} \\ \\ E_t\{\pi_{H,t+1}\} \end{bmatrix}$$

where

$$A_F \equiv \Omega \begin{bmatrix} \sigma_\alpha[1-\phi_q(1-\alpha)] & 1-\phi_\pi\beta \\ \\ \kappa_\alpha\sigma_\alpha\left[1-\phi_q(1-\alpha)\right] & \beta[\sigma_\alpha+\phi_x-\phi_q(1-\alpha)\sigma_\alpha]+\kappa_\alpha \end{bmatrix}$$

and $\Omega \equiv \frac{1}{\phi_\pi \kappa_\alpha + \phi_x + \sigma_\alpha - \phi_q(1-\alpha)\sigma_\alpha}$.

For there exist a unique (local) equilibrium, both eigenvalues of the matrix A_F must be within the unit circle. The charasteristic polynomial is given by $p(\lambda) = \lambda^2 + a_1\lambda + a_0$. So to find the values that satisfy the condition for the eigenvalues we solve $|A_F - \lambda I| = 0$, then we have

$$det \begin{bmatrix} \frac{\sigma_\alpha[1-\phi_q(1-\alpha)]}{\phi_\pi \kappa_\alpha + \phi_x + \sigma_\alpha - \phi_q(1-\alpha)\sigma_\alpha} - \lambda_i & \frac{1-\phi_\pi \beta}{\phi_\pi \kappa_\alpha + \phi_x + \sigma_\alpha - \phi_q(1-\alpha)\sigma_\alpha} \\ \frac{\kappa_\alpha \sigma_\alpha[1-\phi_q(1-\alpha)]}{\phi_\pi \kappa_\alpha + \phi_x + \sigma_\alpha - \phi_q(1-\alpha)\sigma_\alpha} & \frac{\beta[\sigma_\alpha + \phi_x - \phi_q(1-\alpha)\sigma_\alpha] + \kappa_\alpha}{\phi_\pi \kappa_\alpha + \phi_x + \sigma_\alpha - \phi_q(1-\alpha)\sigma_\alpha} - \lambda_i \end{bmatrix} = 0$$

where λ_i are the eigenvalues, with $i = 1, 2$. After some algebra we obtain:

$$\frac{\beta\sigma_\alpha(1-\phi_q(1-\alpha))}{\phi_\pi \kappa_\alpha + \phi_x + \sigma_\alpha - \phi_q(1-\alpha)\sigma_\alpha} - \frac{\sigma_\alpha[1-\phi_q(1-\alpha)]}{\phi_\pi \kappa_\alpha + \phi_x + \sigma_\alpha - \phi_q(1-\alpha)\sigma_\alpha}\lambda - \frac{\beta[\sigma_\alpha + \phi_x - \phi_q(1-\alpha)\sigma_\alpha] + \kappa_\alpha}{\phi_\pi \kappa_\alpha + \phi_x + \sigma_\alpha - \phi_q(1-\alpha)\sigma_\alpha}\lambda + \lambda^2 = 0$$

$$\frac{\beta\sigma_\alpha(1-\phi_q(1-\alpha))}{\phi_\pi \kappa_\alpha + \phi_x + \sigma_\alpha - \phi_q(1-\alpha)\sigma_\alpha} + \frac{\phi_q(1-\alpha)\sigma_\alpha(1+\beta) - \beta(\sigma_\alpha + \phi_x) - \sigma_\alpha - \kappa_\alpha}{\phi_\pi \kappa_\alpha + \phi_x + \sigma_\alpha - \phi_q(1-\alpha)\sigma_\alpha}\lambda + \lambda^2 = 0$$

Besides, we define

$$a_0 \equiv \frac{\beta\sigma_\alpha(1-\phi_q(1-\alpha))}{\phi_\pi \kappa_\alpha + \phi_x + \sigma_\alpha - \phi_q(1-\alpha)\sigma_\alpha}$$

$$a_1 \equiv \frac{\phi_q(1-\alpha)\sigma_\alpha(1+\beta) - \beta(\sigma_\alpha + \phi_x) - \sigma_\alpha - \kappa_\alpha}{\phi_\pi \kappa_\alpha + \phi_x + \sigma_\alpha - \phi_q(1-\alpha)\sigma_\alpha}$$

and to achieve that both eigenvalues are within the unit circle, i.e. determinacy, it must be satisfied two conditions

$$|a_0| < 1 \tag{18}$$

$$|a_1| < 1 + a_0 \tag{19}$$

Condition (19), implies two conditions[11]:

1. $-1 - a_0 < a_1$

$$\phi_x(1-\beta) + \kappa_\alpha(\phi_\pi - 1) > 0 \tag{20}$$

2. $a_1 < 1 + a_0$

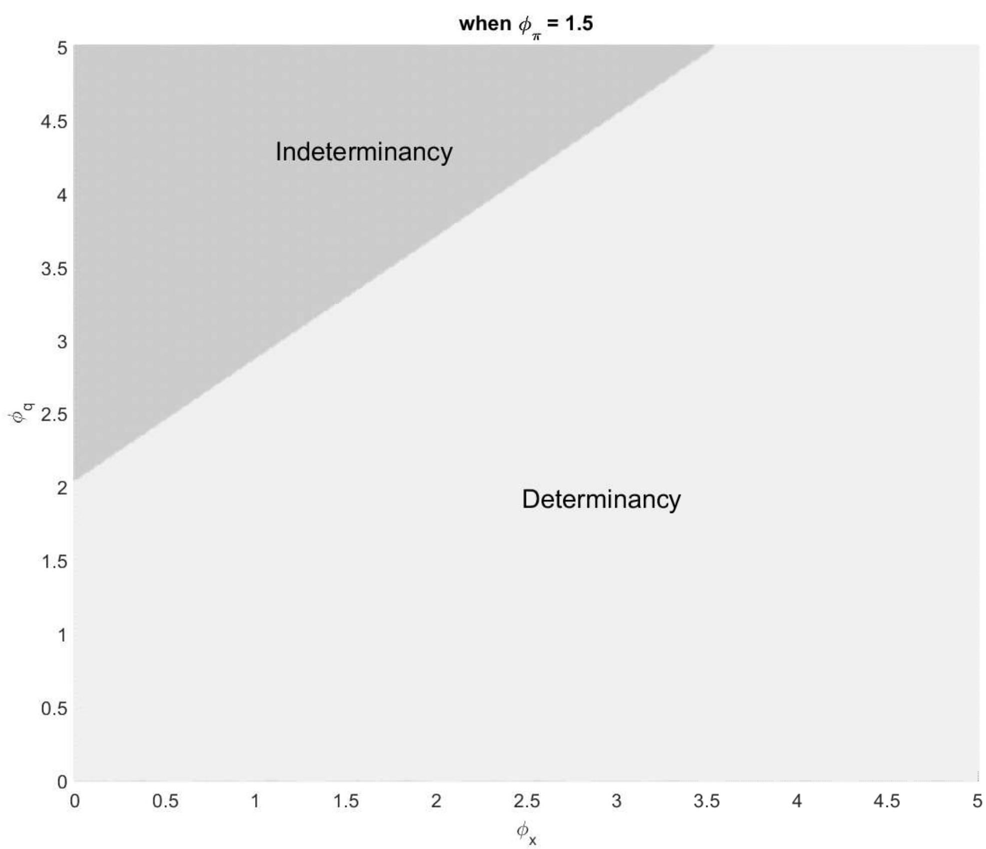
$$(1+\beta)(2\phi_q\sigma_\alpha(1-\alpha) - \phi_x) - \kappa_\alpha(\phi_\pi + 1) < 2\sigma_\alpha(1+\beta) \tag{21}$$

[11]Following Bullard and Mitra (2002), condition (18) is already satisfied in this context.

Note that condition (20) is identical to that found in Gali (2008) for the case of contemporary rules, as well as for forward-looking rules. Thus, the equilibrium will be unique if the coefficients associated with the output gap and deviations of inflation, ϕ_x and ϕ_π respectively, are large enough to ensure that the real interest rate will increase in the face of increases in inflation, i.e. Taylor principle must be fulfilled. However, satisfying the Taylor principle is a necessary condition, but not sufficient for determinacy. As shown by Bernanke and Woodford (1997), Bullard and Mitra (2002) and Levine et. al. (2007), for the forward-looking Taylor rules, it is possible induce indeterminacy even fullfiling (20), which in our case will be if the response to movements in RER it is large enough compared to inflation and output gap, i.e. if condition (21) is not satisfied.

Figure 1 illustrates graphically determinacy and indeterminacy regions in (ϕ_q, ϕ_x) space, considering that the coefficient associated with inflation deviations, ϕ_π, is fixed and equal to 1.5. We see in this case that when the central bank reacts to deviations from inflation with respect to its target value ($\phi_\pi = 1.5$), a weak reaction against the output gap, ϕ_x, could lead to indeterminacy when the coefficient of the exchange rate, ϕ_q, is bigger.

Figure 1: Determinacy and Indeterminacy Regions



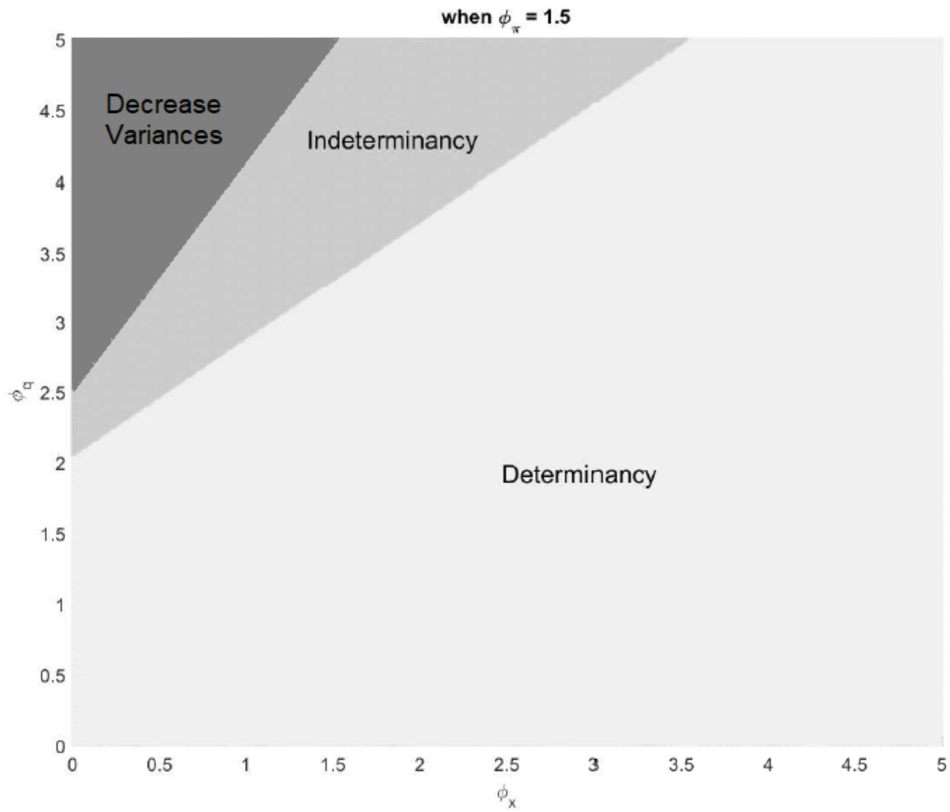
4.1 Comparing and testing conditions

Now we have two groups of conditions. On the one hand, a condition that tells us how we can decrease variances of all the variables (17), while on the other we have determinacy conditions. Since we have restrictions that point to different focus, our idea is to find a solution that allows us to satisfy all conditions at the same time.

As shown above, for the variances condition of the variables to be satisfied, it is necessary that the central bank to react strongly to real exchange rate fluctuations, ϕ_q, in comparison with its response to deviations from inflation of its target value and/or the output gap. On the other hand, we also saw that to avoid being in the indeterminacy region of the model, the value of the coefficient associated with the expected RER fluctuations can not be much higher than those that reflect the response of the central bank for inflation and output gap. Therefore, theoretically the solution that would allow all conditions to be satisfied at the same time should be characterized by have a strong response for the central bank to the expected real exchange rate fluctuations, but low enough to be in the detreminancy region of the model.

Figure 2 represents what happens when we add the condition to decrease the variances through of determinacy/indeterminacy regions. As in Figure 1, we ilustrate the differents areas in (ϕ_q, ϕ_x) space, with the same value for ϕ_π. Here we can see that the region where it is possible to decrease the variances, i.e. satisfies the condition (17), is limited exclusively to the indeterminacy region of the model.

Figure 2: Determinacy and Indeterminacy Regions with Variances Condition



Previous result can be mathematically demonstrate, excersice that we show bellow. We consider the three relevants conditions (17), (20) and (21). We can re-write the condition (17) as follows

$$\phi_q \sigma_\alpha (1-\alpha) - \kappa_\alpha \phi_\pi - \phi_x - \sigma_\alpha > 0$$

$$\Rightarrow \phi_q = \frac{\kappa_\alpha \phi_\pi + \phi_x + \sigma_\alpha}{\sigma_\alpha (1-\alpha)} + \varepsilon \tag{22}$$

where $\varepsilon \rightsquigarrow 0$. Now replacing (22) in (21)

$$(1+\beta)(2\kappa_\alpha \phi_\pi + \phi_x + 2\sigma_\alpha + 2\sigma_\alpha(1-\alpha)\varepsilon) - \kappa_\alpha(\phi_\pi + 1) < 2\sigma_\alpha(1+\beta)$$

$$2\sigma_\alpha(1+\beta)(1-\alpha)\varepsilon + 2\sigma_\alpha(1+\beta) + (1+\beta)2\kappa_\alpha\phi_\pi + (1+\beta)\phi_x - \kappa_\alpha\phi_\pi - \kappa_\alpha < 2\sigma_\alpha(1+\beta)$$

$$2\sigma_\alpha(1+\beta)(1-\alpha)\varepsilon + \kappa_\alpha\phi_\pi(1+2\beta) + (1+\beta)\phi_x - \kappa_\alpha < 0$$

$$2\sigma_\alpha(1+\beta)(1-\alpha)\varepsilon + \kappa_\alpha\phi_\pi(1+\beta) + \beta\kappa_\alpha\phi_\pi + (1+\beta)\phi_x - \kappa_\alpha < 0$$

$$(1+\beta)(\kappa_\alpha\phi_\pi + \phi_x - \kappa_\alpha) + \beta\kappa_\alpha\phi_\pi - \kappa_\alpha + (1+\beta)\kappa_\alpha < 0$$

$$\underbrace{2\sigma_\alpha(1+\beta)(1-\alpha)\varepsilon}_{>0} + \underbrace{(1+\beta)}_{>0}\underbrace{(\kappa_\alpha(\phi_\pi - 1) + \phi_x)}_{>0} + \underbrace{\beta\kappa_\alpha(\phi_\pi + 1)}_{>0} < 0 \quad \Rightarrow \quad \text{-Contradiction-}$$

It can be shown that $\kappa_\alpha(\phi_\pi - 1) + \phi_x > 0$ given the Taylor principle below

$$\phi_x(1-\beta) + \kappa_\alpha(\phi_\pi - 1) > 0$$

$$\Rightarrow \phi_x + \kappa_\alpha(\phi_\pi - 1) > \phi_x(1-\beta) + \kappa_\alpha(\phi_\pi - 1) > 0$$

Taking into account the previous results, we can establish with certainty that there is a conflict between the determinacy of the model and the decrease in the variances of the variables. This conflict is resolved in favor of the determinacy, therefore, it is not possible in this context to reduce the variances of the variables.

Table 3 shows in summary the results that we have in the determinacy area of the model, using this type of Taylor rule. We see that it is not possible to obatin gains in the majority of cases, except for domestic inflation when the economy is affected by a NKPC shock. Specifically, we see that the variance may decrease or increase, which will be determined by the conditions that we will see below in the next subsection.

Table 3: Derivates. Taylor-rule with know r_t^n

Derivate	**Productivity Shock**	**NKPC Shock**	**UIP Shock**
$\frac{\partial V(x_t)}{\partial \phi_q}$	> 0	> 0	> 0
$\frac{\partial V(\pi_{H,t})}{\partial \phi_q}$	> 0	$\gtreqless 0$	> 0
$\frac{\partial V(\Delta q_t)}{\partial \phi_q}$	> 0	> 0	> 0

4.2 Special case: domestic inflation and NKPC shock

In the face of NKPC shock, i.e., a supply shock, there is a potential reaction to expected real exchange rate that can reduce the volatility of domestic inflation and induce determinacy. As we demonstrated previously, the condition that theoretically allows us to decrease the variances is not feasible due to the irreparable conflict that exists with the determinacy conditions of the model. However, in the case of a supply shock this conclusion is not necessarily true. Below we demonstrate why in this case we can reduce the volatility of domestic inflation and, at the same time, ensure determinacy.

In this case, it is not required that the denominator of the expression for the derivative of domestic inflation be positive to reduce domestic inflation volatility (see Table 2). This is because there is a relationship between the response of the central bank associated with the real exchange rate ϕ_q and the output gap ϕ_x that allows, for certain values, to reduce the variance of domestic inflation even in the case in which the denominator of the derivative in Table 2 is negative. Assuming that we are within the determinacy region of the model, that is, the conditions (20) and (21) are satisfied, it will be possible to decrease the variance of domestic inflation if the following conditions hold (expression that is in the numerator):

$$\phi_q \sigma_\alpha (1 - \alpha) - \phi_x - \sigma_\alpha < 0$$

$$\phi_q < \frac{\phi_x + \sigma_\alpha}{\sigma_\alpha (1 - \alpha)} \tag{23}$$

Contrary to what we saw earlier, the central bank response to expected real exchange rate movements could be positive and still induce determinacy. In particular, not be very strong compared to the coefficient associated with the product gap, in addition to the elasticity of substitution and the level of openness of the economy. Finally, the decrease in variance has a lower limit that is different from 0, that is, it is not possible fully stabilization of domestic inflation. This would be possible when $\phi_q = \frac{\phi_x + \sigma_\alpha}{\sigma_\alpha (1-\alpha)}$, implying that $\phi_q = 2.5$, value that is in the indeterminacy area of the model (see Figure 1).

5 Are implementable Taylor-rules efficient?

All our previous analysis has been assuming that we know the natural rate of interest, which is why we have included it in the *Efficient RER Taylor-rule* (16). Now, we will consider a scenario where there is no complete information (for any possible reason), so we do not observe the natural rate of interest. For this reason, we specify the following Taylor-rule:

$$i_t = \phi_\pi \pi_{H,t} + \phi_x x_t + \phi_q E_t\{\Delta q_{t+1}\} \tag{24}$$

With this new rule, by construction, we will only have changes in the variances associated with the productivity shock (for the rest of the shocks and the determinacy conditions, the results are maintained as in the previous section). Consequently, the new variances associated with the productivity shock will be defined in the following table:

Table 4: Variances. Taylor-rule with unknow r_t^n

Variable	Productivity Shock
x_t	$\left(\frac{\Gamma\sigma_\alpha(\phi_q(1-\alpha)-1)}{\phi_q(1-\alpha)\sigma_\alpha-\phi_\pi\kappa_\alpha-\phi_x-\sigma_\alpha}\right)^2 V(\varepsilon_t^a)$
$\pi_{H,t}$	$\kappa_\alpha^2\left(\frac{\Gamma\sigma_\alpha(\phi_q(1-\alpha)-1)}{\phi_q(1-\alpha)\sigma_\alpha-\phi_\pi\kappa_\alpha-\phi_x-\sigma_\alpha}\right)^2 V(\varepsilon_t^a)$
Δq_t	$2(1-\alpha)^2\left(\frac{\Gamma\sigma_\alpha(\phi_x+\phi_\pi\kappa_\alpha)}{\phi_q\sigma_\alpha(1-\alpha)-\kappa_\alpha\phi_\pi-\phi_x-\sigma_\alpha}\right)^2 V(\varepsilon_t^a)$

Table 4 shows the variances of output gap, domestic inflation and real exchange rate fluctuation, obtained for the productivity shock considering the Taylor rule defined above.[12] In this case we find that the variance of the output gap and domestic inflation can be fully stabilized $V(x_t) = 0$ and $V(\pi_{H,t}) = 0$. To achieve this efficient allocation, it is possible see in the expressions of the Table 4 that it is enough to make the numerator equal to 0, that is, the following condition must be satisfied:

$$\begin{aligned} \phi_q(1-\alpha) - 1 &= 0 \\ \phi_q &= \frac{1}{1-\alpha} \end{aligned} \tag{25}$$

this condition will be within determinacy region of the model, i.e. is a unique feasible equilibrium.

Now, Table 5 consider the same variables as the previous Table 4, but calculating the derivatives of the expressions with respect to the coefficient associated with the expected real exchange rate fluctuations (similar to Table 2). In this case, we see that the relevant condition is the same as the one we saw above, but now we can analyze something more about the behavior of these variances (domestic inflation and product gap). Specifically we will have that: for certain values of ϕ_q the variances decrease $\left(\phi_q < \frac{1}{1-\alpha}\right)$, in the breakpoint when $\phi_q = \frac{1}{1-\alpha}$ the variances are completely stabilized, and then after this limit variances value begins to rise $\left(\phi_q > \frac{1}{1-\alpha}\right)$.

[12] The way to solve and obey the values was the same as in the general case seen for the case with complete information (with know r_t^n).

Table 5: Derivatives. Taylor-rule with unknow r_t^n

Derivative	**Productivity Shock**	$\phi_q = \frac{1}{1-\alpha}$	$\phi_q > \frac{1}{1-\alpha}$	$\phi_q < \frac{1}{1-\alpha}$
$\frac{\partial V(x_t)}{\partial \phi_q}$	$\frac{-2\Gamma^2\sigma_\alpha^2(1-\alpha)(\phi_q(1-\alpha)-1)(\phi_x+\phi_\pi\kappa_\alpha)V(\varepsilon_t^a)}{[\phi_q\sigma_\alpha(1-\alpha)-\kappa_\alpha\phi_\pi-\phi_x-\sigma_\alpha]^3}$	0	> 0	< 0
$\frac{\partial V(\pi_{H,t})}{\partial \phi_q}$	$\frac{-2\kappa_\alpha\Gamma^2\sigma_\alpha^2(1-\alpha)(\phi_q(1-\alpha)-1)(\phi_x+\phi_\pi\kappa_\alpha)V(\varepsilon_t^a)}{[\phi_q\sigma_\alpha(1-\alpha)-\kappa_\alpha\phi_\pi-\phi_x-\sigma_\alpha]^3}$	0	> 0	< 0
$\frac{\partial V(\Delta q_t)}{\partial \phi_q}$	$\frac{-4\Gamma^2\sigma_\alpha^3(1-\alpha)^3(\phi_x+\kappa_\alpha\phi_\pi)^2V(\varepsilon_t^a)}{[\phi_q\sigma_\alpha(1-\alpha)-\kappa_\alpha\phi_\pi-\phi_x-\sigma_\alpha]^3}$	> 0	> 0	> 0

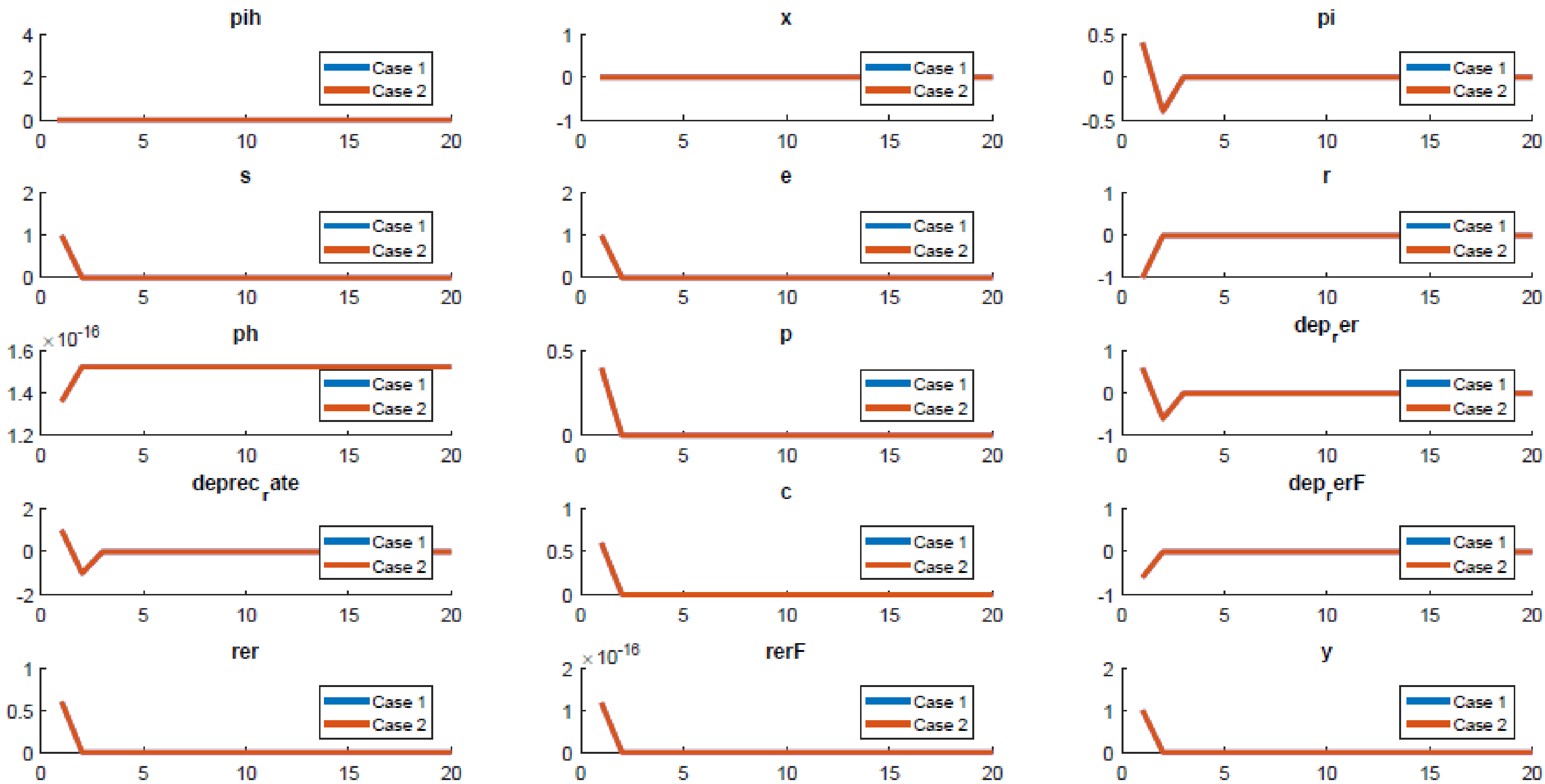
Note: For the case when $\phi_q > \frac{1}{1-\alpha}$, there will be a unique equilibrium as long as $\phi_q < 2.5$.

5.1 IRF analysis

Figure 3 shows the impulse response functions (IRFs) comparing two cases, both considering the productivity shock. Case 1 is defined considering the natural rate of interest (r_t^n) in the Taylor-rule, and assuming that ϕ_q has a value equal to 0, that is, the central bank does not react to the expected real exchange rate fluctuations. Case 2 does not consider the natural rate of interest in the Taylor-rule, and we assume that $\phi_q = \frac{1}{1-\alpha}$, a condition that we saw earlier allows obtaining the efficient allocation for domestic inflation and the output gap.

It is possible to notice that the reaction to a productivity shock is the same in front of both scenarios. In this sense, the inclusion of a response to the expected real exchange rate fluctuations in the Taylor rule can mimic the scenario with complete information (knowing the natural rate of interest).

Figure 3: IRFs



Case 1: Taylor-rule with know r_t^n and $\phi_q = 0$. Case 2: Taylor-rule with unknow r_t^n and $\phi_q = \frac{1}{1-\alpha}$. Name of the variables are described in Appendix G.

5.1.1 How expensive is it to react in this way?

Since we were previously evaluating the effects of a productivity shock, a relevant question is what would happen in the case that the economy is not affected by a shock of this nature, being a supply or UIP shock. It is possible to notice in Table 6 that in the case of a shock to the UIP the results do not change with respect to the case when we know the natural rate of interest and we do not react to the expected real exchange rate fluctuations. However, if the economy is affected by a supply shock, it is possible to have efficiency gains in the case of domestic inflation, being able to reduce its variance.

Table 6: Derivates. Taylor-rule with unknow r_t^n and $\phi_q = \frac{1}{1-\alpha}$

Derivate	**NKPC Shock**	**UIP Shock**
$\frac{\partial V(x_t)}{\partial \phi_q}$	> 0	> 0
$\frac{\partial V(\pi_{H,t})}{\partial \phi_q}$	< 0	> 0
$\frac{\partial V(\Delta q_t)}{\partial \phi_q}$	> 0	> 0

6 Conclusion

Understanding the extent to which a central bank, in open economies, can reduce the volatility of inflation and output gap is crucial for designing efficient monetary policy responses. In this paper we assess the advantages an potential costs of a systematic policy response to expected real exchange rate fluctuations.

In particular, in the context of the Gali and Monacelli (2005) model [13], we use the method of undetermined coefficients to derive closed form solutions for the evolution of all the variables under an augmented Taylor rule that, besides reacting to domestic inflation and output gap, also respond to expected real exchange rate fluctuations. Unlike the standard policy analysis in Gali and Monacelli (2005), we perform this exercise for three orthogonal innovations: demand, supply and country risk premium shocks. We also consider two alternative information sets available to the central bank. In the first one the monetary authority can observe the natural rate of interest, whereas in the second one the central bank is unable to observe this variable.

In this line, our main findings are as follows. First, when the central bank observes the natural rate of interest, an aggressive policy response to expected exchange rate has the potential of inducing a decline in the volatility of domestic inflation and the output gap. This equilibrium, however, is not unique. In particular, we demonstrate that an aggressive response to exchange rate induces multiple equilibria (i.e. induce indeterminacy). In other words, this type of policy reaction generates aggregate instability due to self-fulfilling expectations. Our result is related to Uribe

[13]This is a simple and tractable model containing several important insights about the monetary transmission mechanisms in open economies. In particular, it derives a welfare based central bank loss function that depends only on domestic inflation and output gap volatility, with no policy concerns about any other variable.

(2003) which, in a very different context, shows that the mere existence of PPP rules can generate endogenous aggregate instability by allowing for the existence of equilibria in which agents base their expectations about economic variables on non-fundamental signals. Our result is also related to Woodford (2000) which shows that one of the pitfalls of forward-looking rules is that they have the potential of inducing multiple equilibria. However, in this same context, when the economy is affected by a supply shock, there is the possibility of decreasing the variance of domestic inflation, not having the problem related to the existence of multiple equilibria (indeterminacy). This result is in line with the literature that establishes the existence of a trade-off between the stabilization of the product and inflation against cost-push shocks (Svensson 1999, 2000; Clarida et. al. 2000; Benigno and Benigno 2002, 2003b; among others).

Second, when the central bank is not able to observe the natural rate of interest, reacting to expected exchange rate depreciation can be efficient in the face of demand shocks. In particular, we show that there is a unique exchange rate reaction coefficient that can mimic the optimal policy response under full information and also induces determinacy. This coefficient is directly linked to the degree of openness in the economy: as the economy becomes more open, the response to expected exchange rate fluctuations increase.

There are several potential extension to our contribution. First, it would be useful to assess the extent to which the advantages of our augmented Taylor rule are indeed desirable features. In particular, given the welfare loss criterion in Gali and Monacelli (2005) we will analyze whether under our calibration a response to expected exchange rate depreciation should be different from zero. Also, it would be interesting to see if closed form solutions are possible to derive when responding to contemporaneous exchange rate changes.

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Appendix A. Uncovered Interest Parity Derivation

Using the euler equation of the home and foreign economy in the first difference of risk sharing equation we can obtain

$$i_{t-1} - \pi_t = i^*_{t-1} - \pi^*_t + (1-\alpha)\Delta s_t$$

Also if we add to the previous equation the relationship between domestic and CPI inflation, which is determined by terms of trade and the coefficient given by the index of openess α, and after some algebra we obtain the UIP equation

$$i_{t-1} - \pi_{H,t} = i^*_{t-1} - \pi^*_t + \Delta e_t + \pi^*_t - \pi_{H,t}$$

$$\Delta e_t = i_{t-1} - i^*_{t-1} \tag{26}$$

UIP Shock

We add z_t shock in the equilibrium equation derivated above for UIP (26) as follows

$$\Delta e_t = i_{t-1} - i^*_{t-1} + z_t$$

doing the inverse process to get the equation that we use in our general equilibrium model, we can obtain after some algebra

$$y_t = y^*_t + \frac{1-\alpha(1-\omega)}{\sigma_\alpha} s_t - \frac{1}{\sigma} z_t \tag{27}$$

where $\omega = \sigma\gamma + (1-\alpha)(\sigma\eta - 1)$.

Appendix B. Analytical Variances

Table X contains the summary with all the variances of the relevant variables of the model, all following the example shown above, being therefore defined as a function of the coefficients found and the variance of each shock.

Table 7: Variances

Variable	Productivity Shock	NKPC Shock	UIP Shock
x_t	$A^2V(\varepsilon_t^a)$	$(\Psi_{sv}/\sigma_\alpha)^2V(\varepsilon_t^a)$	$C^2V(\varepsilon_t^z)$
$\pi_{H,t}$	$\kappa_\alpha^2A^2V(\varepsilon_t^a)$	$B^2V(\varepsilon_t^v)$	$\kappa_\alpha^2C^2V(\varepsilon_t^z)$
π_t	$[\kappa_\alpha^2A^2+2\alpha\Psi_{sa}(\kappa_\alpha A+\alpha\Psi_{sa})]V(\varepsilon_t^a)$	$[B^2+2\alpha\Psi_{sv}(B+\alpha\Psi_{sv})]V(\varepsilon_t^v)$	$[\kappa_\alpha^2C^2+2\alpha\Psi_{sz}(\kappa_\alpha C+\alpha\Psi_{sz})]V(\varepsilon_t^z)$
Δe_t	$[\kappa_\alpha^2A^2+2\Psi_{sa}(\kappa_\alpha A+\Psi_{sa})]V(\varepsilon_t^a)$	$[B^2+2\Psi_{sv}(B+\Psi_{sv})]V(\varepsilon_t^v)$	$[\kappa_\alpha^2C^2+2\Psi_{sz}(\kappa_\alpha C+\Psi_{sz})]V(\varepsilon_t^z)$
Δe_{t+1}	$\Psi_{sa}^2V(\varepsilon_t^a)$	$\Psi_{sv}^2V(\varepsilon_t^v)$	$\Psi_{sz}^2V(\varepsilon_t^z)$
Δq_t	$2\Psi_{sa}^2(1-\alpha)^2V(\varepsilon_t^a)$	$2\Psi_{sv}^2(1-\alpha)^2V(\varepsilon_t^v)$	$2\Psi_{sz}^2(1-\alpha)^2V(\varepsilon_t^z)$
Δq_{t+1}	$\Psi_{sa}^2(1-\alpha)^2V(\varepsilon_t^a)$	$\Psi_{sv}^2(1-\alpha)^2V(\varepsilon_t^v)$	$\Psi_{sz}^2(1-\alpha)^2V(\varepsilon_t^z)$

where $A \equiv \frac{\Psi_{sa}-\sigma_\alpha\Gamma}{\sigma_\alpha}$, $B \equiv \frac{\kappa_\alpha\Psi_{sv}+\sigma_\alpha}{\sigma_\alpha}$, $C \equiv \frac{\Psi_{sz}\sigma-\sigma_\alpha}{\sigma_\alpha\sigma}$.

and $\Psi_{sa} = \frac{\Gamma\sigma_\alpha(-\sigma_\alpha-\phi_x)-\Gamma\kappa_\alpha\sigma_\alpha\phi_\pi}{-\sigma_\alpha-\phi_x-\phi_\pi\kappa_\alpha+\phi_e\sigma_\alpha(1-\alpha)}$, $\Psi_{sv} = \frac{\phi_\pi\sigma_\alpha}{-\sigma_\alpha-\phi_x-\phi_\pi\kappa_\alpha+\phi_e\sigma_\alpha(1-\alpha)}$, $\Psi_{sz} = \frac{\sigma_\alpha(-\sigma_\alpha-\phi_x)-\phi_\pi\kappa_\alpha\sigma_\alpha}{\sigma(-\sigma_\alpha-\phi_x)-\phi_\pi\kappa_\alpha\sigma+\phi_e\sigma_\alpha\sigma(1-\alpha)}$

.

Appendix C.

Figure 4: Determinacy and Indeterminacy Regions

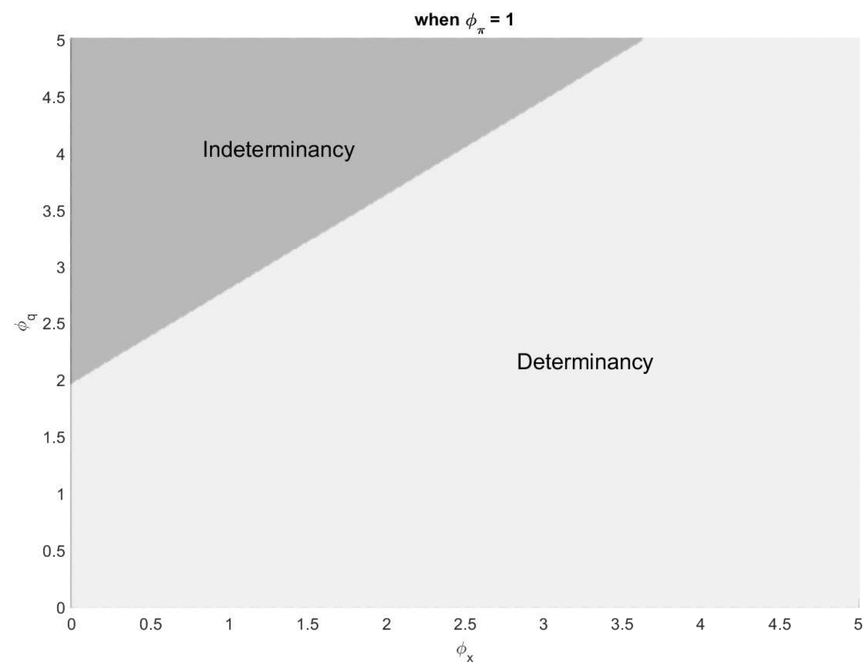


Figure 5: Determinacy and Indeterminacy Regions with Variances Condition

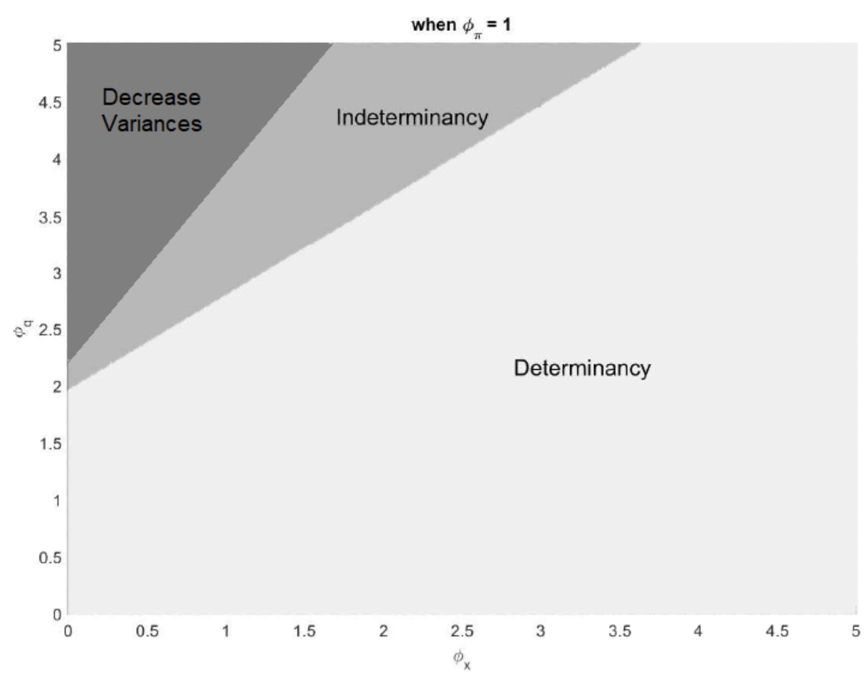


Figure 6: Determinacy and Indeterminacy Regions

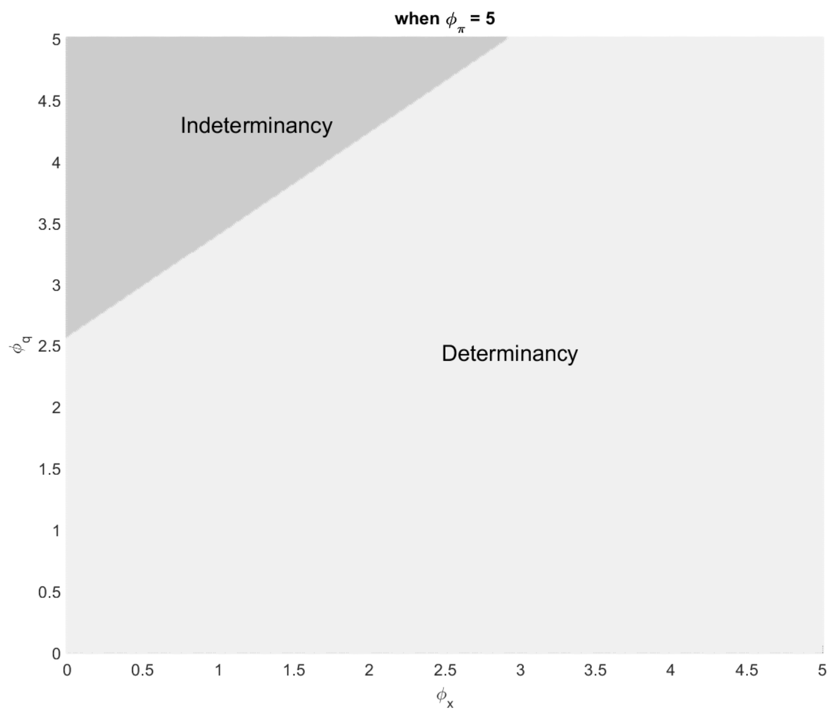


Figure 7: Determinacy and Indeterminacy Regions with Variances Condition

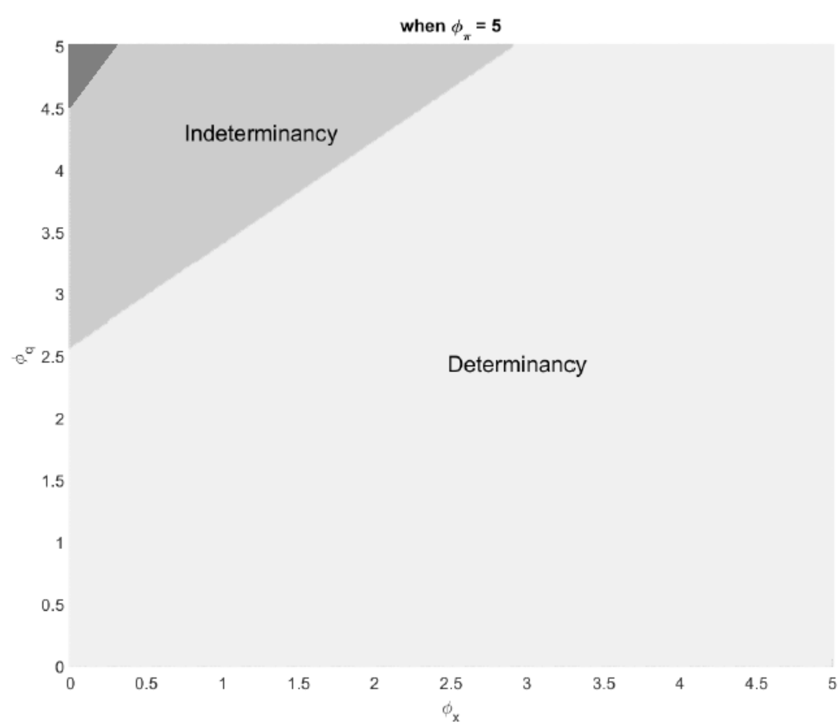


Figure 8: Determinacy and Indeterminacy Regions

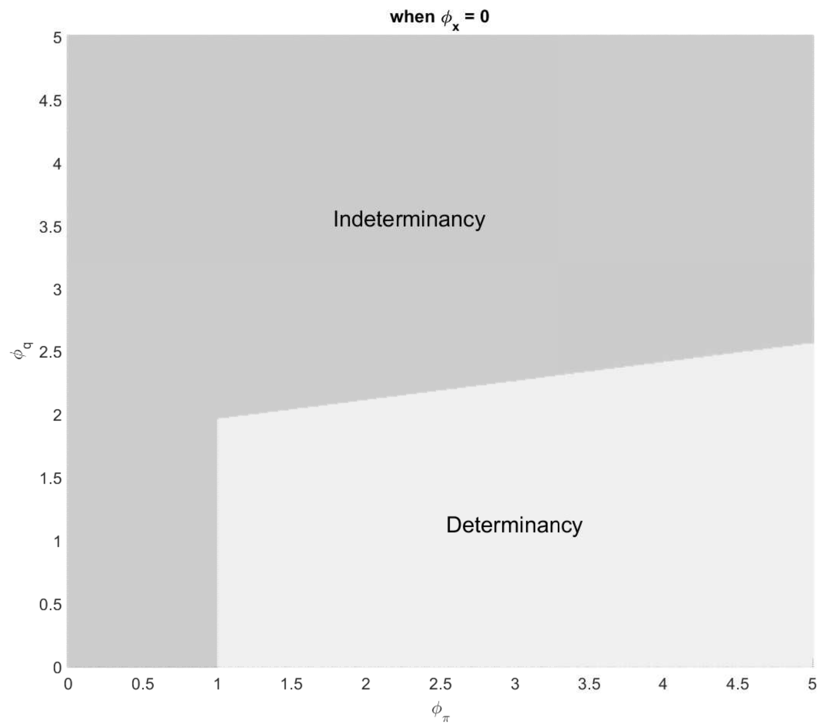


Figure 9: Determinacy and Indeterminacy Regions with Variances Condition

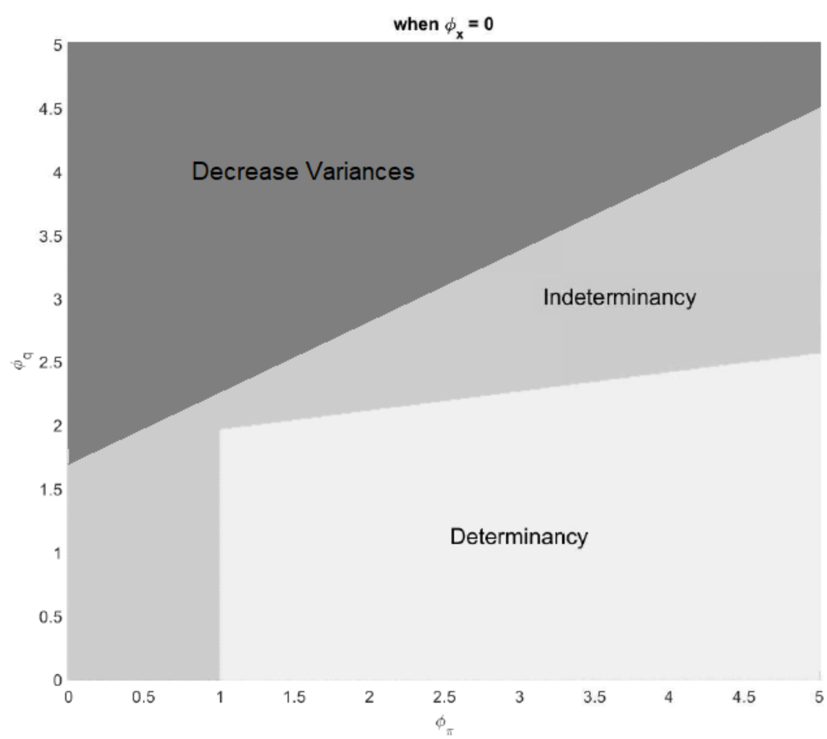


Figure 10: Determinancy and Indeterminacy Regions

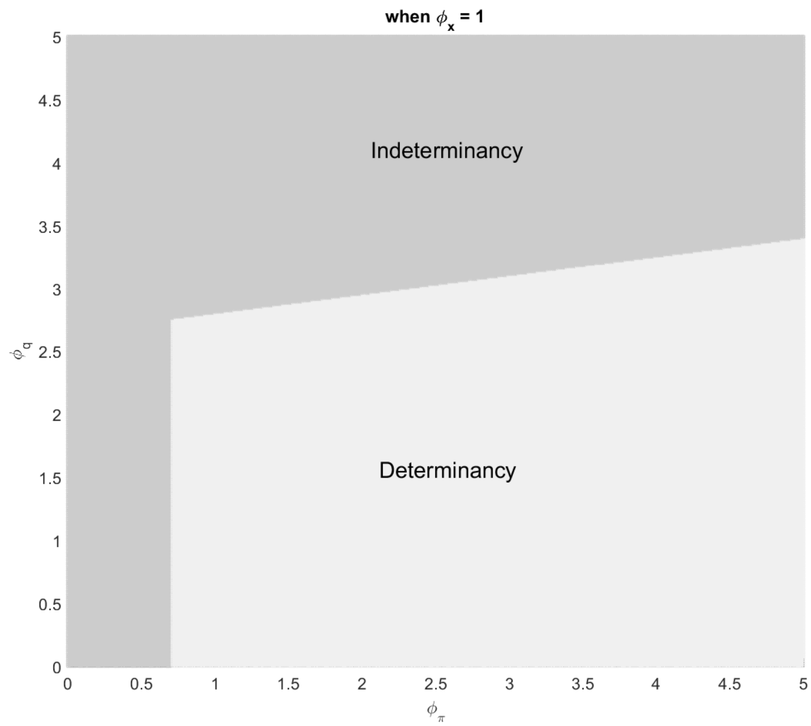
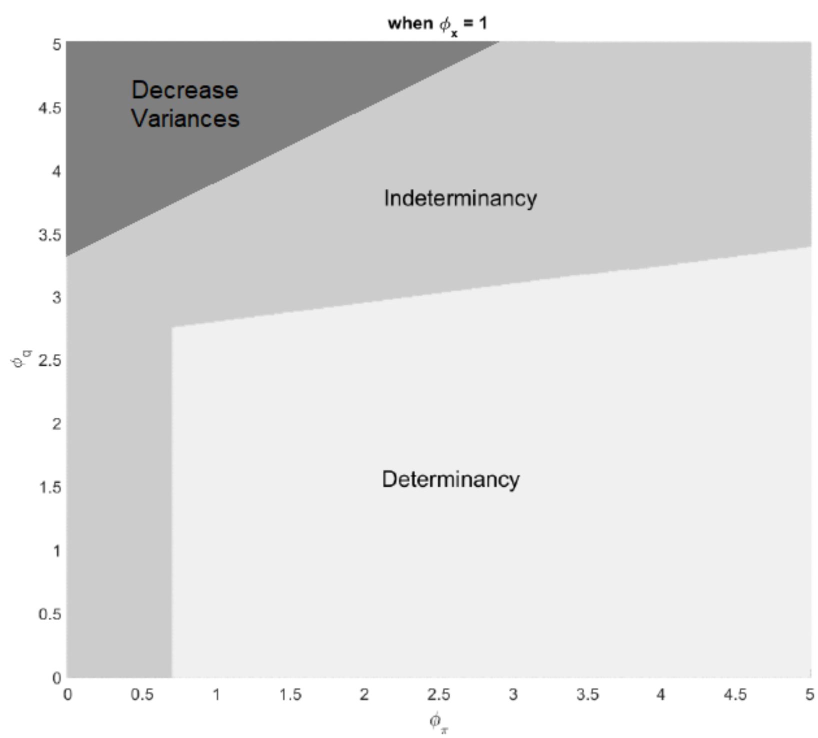


Figure 11: Determinancy and Indeterminacy Regions with Variances Condition



Appendix D. Extended Variances and Derivatives

Productivity Shock

$$V(x_t) = \left(\frac{\Psi_{sa}}{\sigma_\alpha} - \Gamma\right)^2 \frac{V(\varepsilon_t^a)}{1-\rho_a^2} = 0.00748 \tag{28}$$

$$V(\pi_{H,t}) = \left(\frac{\kappa_\alpha(\Psi_{sa} - \Gamma\sigma_\alpha)}{(1-\beta\rho_a)\sigma_\alpha}\right)^2 \frac{V(\varepsilon_t^a)}{1-\rho_a^2} = 0.00734 \tag{29}$$

$$V(\pi_t) = \left(\frac{\kappa_\alpha(\Psi_{sa} - \Gamma\sigma_\alpha)}{(1-\beta\rho_a)\sigma_\alpha}\right)^2 \frac{V(\varepsilon_t^a)}{1-\rho_a^2} + 2\alpha\Psi_{sa}\left(\frac{\kappa_\alpha(\Psi_{sa} - \Gamma\sigma_\alpha)}{(1-\beta\rho_a)\sigma_\alpha} + \alpha\Psi_{sa}\right)\frac{V(\varepsilon_t^a)}{1+\rho_a} = 0.25900 \tag{30}$$

$$V(\Delta e_t) = \left(\frac{\kappa_\alpha(\Psi_{sa} - \Gamma\sigma_\alpha)}{(1-\beta\rho_a)\sigma_\alpha}\right)^2 \frac{V(\varepsilon_t^a)}{1-\rho_a^2} + 2\Psi_{sa}\left(\frac{\kappa_\alpha(\Psi_{sa} - \Gamma\sigma_\alpha)}{(1-\beta\rho_a)\sigma_\alpha} + \Psi_{sa}\right)\frac{V(\varepsilon_t^a)}{1+\rho_a} = 1.45635 \tag{31}$$

$$V(\Delta e_{t+1}) = \rho_a^2\left(\frac{\kappa_\alpha(\Psi_{sa} - \Gamma\sigma_\alpha)}{(1-\beta\rho_a)\sigma_\alpha}\right)^2 \frac{V(\varepsilon_t^a)}{1-\rho_a^2} - \Psi_{sa}\left(\frac{2\rho_a\kappa_\alpha(\Psi_{sa} - \Gamma\sigma_\alpha)}{(1-\beta\rho_a)\sigma_\alpha} - \Psi_{sa}(1-\rho_a)\right)\frac{V(\varepsilon_t^a)}{1+\rho_a} = 0.18100 \tag{32}$$

$$V(\Delta q_t) = 2\Psi_{sa}^2(1-\alpha)^2\frac{V(\varepsilon_t^a)}{1+\rho_a} = 0.49192 \tag{33}$$

$$V(\Delta q_{t+1}) = \Psi_{sa}^2(1-\alpha)^2(\rho_a-1)^2\frac{V(\varepsilon_t^a)}{1-\rho_a^2} = 0.08363 \tag{34}$$

where $\Psi_{sa} = \frac{\Gamma\sigma_\alpha(1-\beta\rho_a)(\sigma_\alpha\rho_a-\sigma_\alpha-\phi_x)-\Gamma\kappa_\alpha\sigma_\alpha(\phi_\pi-\rho_a)}{(\sigma_\alpha\rho_a-\sigma_\alpha-\phi_x)(1-\beta\rho_a)-(\phi_\pi-\rho_a)\kappa_\alpha-\phi_e\sigma_\alpha(1-\beta\rho_a)(1-\alpha)(\rho_a-1)} = 1.06$

NKPC Shock

$$V(x_t) = \left(\frac{\Psi_{sv}}{\sigma_\alpha}\right)^2 \frac{V(\varepsilon_t^v)}{1-\rho_v^2} = 4.22152 \tag{35}$$

$$V(\pi_{H,t}) = \left(\frac{\kappa_\alpha\Psi_{sv} + \sigma_\alpha}{(1-\beta\rho_v)\sigma_\alpha}\right)^2 \frac{V(\varepsilon_t^v)}{1-\rho_v^2} = 3.25854 \tag{36}$$

$$V(\pi_t) = \left(\frac{\kappa_\alpha\Psi_{sv} + \sigma_\alpha}{(1-\beta\rho_v)\sigma_\alpha}\right)^2 \frac{V(\varepsilon_t^v)}{1-\rho_v^2} + 2\alpha\Psi_{sv}\left(\frac{\kappa_\alpha\Psi_{sv} + \sigma_\alpha}{(1-\beta\rho_v)\sigma_\alpha} + \alpha\Psi_{sv}\right)\frac{V(\varepsilon_t^v)}{1+\rho_v} = 2.70902 \tag{37}$$

$$V(\Delta e_t) = \left(\frac{\kappa_\alpha\Psi_{sv} + \sigma_\alpha}{(1-\beta\rho_v)\sigma_\alpha}\right)^2 \frac{V(\varepsilon_t^v)}{1-\rho_v^2} + 2\Psi_{sv}\left(\frac{\kappa_\alpha\Psi_{sv} + \sigma_\alpha}{(1-\beta\rho_v)\sigma_\alpha} + \Psi_{sv}\right)\frac{V(\varepsilon_t^v)}{1+\rho_v} = 3.60712 \tag{38}$$

$$V(\Delta e_{t+1}) = \rho_v^2 \left(\frac{\kappa_\alpha \Psi_{sv} + \sigma_\alpha}{(1-\beta\rho_v)\sigma_\alpha} \right)^2 \frac{V(\varepsilon_t^v)}{1-\rho_v^2} - \Psi_{sv} \left(\frac{2\rho_v(\kappa_\alpha \Psi_{sv} + \sigma_\alpha)}{(1-\beta\rho_v)\sigma_\alpha} - \Psi_{sv}(1-\rho_v) \right) \frac{V(\varepsilon_t^v)}{1+\rho_v} = 3.57199 \quad (39)$$

$$V(\Delta q_t) = 2\Psi_{sv}^2 (1-\alpha)^2 \frac{V(\varepsilon_t^v)}{1+\rho_v} = 1.03343 \quad (40)$$

$$V(\Delta q_{t+1}) = \Psi_{sv}^2 (1-\alpha)^2 (\rho_v - 1)^2 \frac{V(\varepsilon_t^v)}{1-\rho_v^2} = 0.17568 \quad (41)$$

where $\Psi_{sv} = \frac{(\phi_\pi - \rho_v)\sigma_\alpha}{(\sigma_\alpha \rho_v - \sigma_\alpha - \phi_x)(1-\beta\rho_v) - (\phi_\pi - \rho_v)\kappa_\alpha - \phi_e \sigma_\alpha (1-\beta\rho_v)(1-\alpha)(\rho_v - 1)} = -1.54$

UIP Shock

$$V(x_t) = \left(\frac{\Psi_{sz}}{\sigma_\alpha} - \frac{1}{\sigma} \right)^2 \frac{V(\varepsilon_t^z)}{1-\rho_z^2} = 0.00748 \quad (42)$$

$$V(\pi_{H,t}) = \left(\frac{\kappa_\alpha (\Psi_{sz}\sigma - \sigma_\alpha)}{(1-\beta\rho_z)\sigma_\alpha \sigma} \right)^2 \frac{V(\varepsilon_t^z)}{1-\rho_z^2} = 0.00734 \quad (43)$$

$$V(\pi_t) = \left(\frac{\kappa_\alpha (\Psi_{sz}\sigma - \sigma_\alpha)}{(1-\beta\rho_z)\sigma_\alpha \sigma} \right)^2 \frac{V(\varepsilon_t^z)}{1-\rho_z^2} + 2\alpha\Psi_{sz} \left(\frac{\kappa_\alpha (\Psi_{sz}\sigma - \sigma_\alpha)}{(1-\beta\rho_z)\sigma_\alpha \sigma} + \alpha\Psi_{sz} \right) \frac{V(\varepsilon_t^z)}{1+\rho_z} = 0.25900 \quad (44)$$

$$V(\Delta e_t) = \left(\frac{\kappa_\alpha (\Psi_{sz}\sigma - \sigma_\alpha)}{(1-\beta\rho_z)\sigma_\alpha \sigma} \right)^2 \frac{V(\varepsilon_t^z)}{1-\rho_z^2} + 2\Psi_{sz} \left(\frac{\kappa_\alpha (\Psi_{sz}\sigma - \sigma_\alpha)}{(1-\beta\rho_z)\sigma_\alpha \sigma} + \Psi_{sz} \right) \frac{V(\varepsilon_t^z)}{1+\rho_z} = 1.45635 \quad (45)$$

$$V(\Delta e_{t+1}) = \rho_z^2 \left(\frac{\kappa_\alpha (\Psi_{sz}\sigma - \sigma_\alpha)}{(1-\beta\rho_z)\sigma_\alpha \sigma} \right)^2 \frac{V(\varepsilon_t^z)}{1-\rho_z^2} - \Psi_{sz} \left(\frac{2\rho_z \kappa_\alpha (\Psi_{sz}\sigma - \sigma_\alpha)}{(1-\beta\rho_z)\sigma_\alpha \sigma} - \Psi_{sz}(1-\rho_z) \right) \frac{V(\varepsilon_t^z)}{1+\rho_z} = 0.18100 \quad (46)$$

$$V(\Delta q_t) = 2\Psi_{sz}^2 (1-\alpha)^2 \frac{V(\varepsilon_t^z)}{1+\rho_z} = 0.49192 \quad (47)$$

$$V(\Delta q_{t+1}) = \Psi_{sz}^2 (1-\alpha)^2 (\rho_z - 1)^2 \frac{V(\varepsilon_t^z)}{1-\rho_z^2} = 0.08363 \quad (48)$$

where $\Psi_{sz} = \frac{\sigma_\alpha (1-\beta\rho_z)(\rho_z \sigma_\alpha - \sigma_\alpha - \phi_x) - (\phi_\pi - \rho_z)\kappa_\alpha \sigma_\alpha}{\sigma(1-\beta\rho_z)(\rho_z \sigma_\alpha - \sigma_\alpha - \phi_x) - (\phi_\pi - \rho_z)\kappa_\alpha \sigma - \phi_e \sigma_\alpha \sigma (1-\beta\rho_z)(1-\alpha)(\rho_z - 1)} = 1.06$

Table 8: Productivity shock

Variable	Derivate
$\frac{\partial V(x_t)}{\partial \phi_q}$	$\frac{-2\phi_q(1-\alpha)^2(1+\phi_x+\kappa_\alpha\phi_\pi)V(\varepsilon_t^a)}{[\phi_q(1-\alpha)-\kappa_\alpha\phi_\pi-\phi_x-1]^3}$
$\frac{\partial V(\pi_{H,t})}{\partial \phi_q}$	$\frac{-2\kappa_\alpha^2\phi_q(1-\alpha)^2(1+\phi_x+\kappa_\alpha\phi_\pi)V(\varepsilon_t^a)}{[\phi_q(1-\alpha)-\kappa_\alpha\phi_\pi-\phi_x-1]^3}$
$\frac{\partial V(\pi_t)}{\partial \phi_q}$	$\frac{-2(1-\alpha)(1+\phi_x+\kappa_\alpha\phi_\pi)[\kappa_\alpha\phi_q(1-\alpha)(\kappa_\alpha+\alpha)+\alpha(1+\phi_x+\kappa_\alpha\phi_\pi)(\kappa_\alpha+2\alpha)]V(\varepsilon_t^a)}{[\phi_q(1-\alpha)-\kappa_\alpha\phi_\pi-\phi_x-1]^3}$
$\frac{\partial V(\Delta e_t)}{\partial \phi_q}$	$\frac{-2(1-\alpha)(1+\phi_x+\kappa_\alpha\phi_\pi)[\kappa_\alpha\phi_q(1-\alpha)(\kappa_\alpha+1)+\alpha(1+\phi_x+\kappa_\alpha\phi_\pi)(\kappa_\alpha+2)]V(\varepsilon_t^a)}{[\phi_q(1-\alpha)-\kappa_\alpha\phi_\pi-\phi_x-1]^3}$
$\frac{\partial V(\Delta e_{t+1})}{\partial \phi_q}$	$\frac{-2(1-\alpha)(1+\phi_x+\kappa_\alpha\phi_\pi)^2V(\varepsilon_t^a)}{[\phi_q(1-\alpha)-\kappa_\alpha\phi_\pi-\phi_x-1]^3}$
$\frac{\partial V(\Delta q_t)}{\partial \phi_q}$	$\frac{-4(1-\alpha)^3(1+\phi_x+\kappa_\alpha\phi_\pi)^2V(\varepsilon_t^a)}{[\phi_q(1-\alpha)-\kappa_\alpha\phi_\pi-\phi_x-1]^3}$
$\frac{\partial V(\Delta q_{t+1})}{\partial \phi_q}$	$\frac{-2(1-\alpha)^3(1+\phi_x+\kappa_\alpha\phi_\pi)^2V(\varepsilon_t^a)}{[\phi_q(1-\alpha)-\kappa_\alpha\phi_\pi-\phi_x-1]^3}$

Table 9: NKPC shock

Variable	Derivate
$\frac{\partial V(x_t)}{\partial \phi_q}$	$\frac{-2\phi_\pi^2(1-\alpha)V(\varepsilon_t^a)}{[\phi_q(1-\alpha)-\kappa_\alpha\phi_\pi-\phi_x-1]^3}$
$\frac{\partial V(\pi_{H,t})}{\partial \phi_q}$	$\frac{-2\phi_\pi(1-\alpha)(\phi_q(1-\alpha)-\phi_x-1)V(\varepsilon_t^a)}{[\phi_q(1-\alpha)-\kappa_\alpha\phi_\pi-\phi_x-1]^3}$
$\frac{\partial V(\pi_t)}{\partial \phi_q}$	$\frac{-2\phi_\pi(1-\alpha)[(1+\alpha)(\phi_q(1-\alpha)-\phi_x-1)+\alpha\phi_\pi(\kappa_\alpha+2\alpha)]V(\varepsilon_t^a)}{[\phi_q(1-\alpha)-\kappa_\alpha\phi_\pi-\phi_x-1]^3}$
$\frac{\partial V(\Delta e_t)}{\partial \phi_q}$	$\frac{-2\phi_\pi(1-\alpha)[2(\phi_q(1-\alpha)-\phi_x-1)+\alpha\phi_\pi(\kappa_\alpha+2\alpha)]V(\varepsilon_t^a)}{[\phi_q(1-\alpha)-\kappa_\alpha\phi_\pi-\phi_x-1]^3}$
$\frac{\partial V(\Delta e_{t+1})}{\partial \phi_q}$	$\frac{-2(1-\alpha)\phi_\pi^2V(\varepsilon_t^a)}{[\phi_q(1-\alpha)-\kappa_\alpha\phi_\pi-\phi_x-1]^3}$
$\frac{\partial V(\Delta q_t)}{\partial \phi_q}$	$\frac{-4(1-\alpha)^3\phi_\pi^2V(\varepsilon_t^a)}{[\phi_q(1-\alpha)-\kappa_\alpha\phi_\pi-\phi_x-1]^3}$
$\frac{\partial V(\Delta q_{t+1})}{\partial \phi_q}$	$\frac{-2(1-\alpha)^3\phi_\pi^2V(\varepsilon_t^a)}{[\phi_q(1-\alpha)-\kappa_\alpha\phi_\pi-\phi_x-1]^3}$

Table 10: Productivity shock

Variable	Derivate
$\frac{\partial V(x_t)}{\partial \phi_q}$	$\frac{-2\phi_q(1-\alpha)^2(1+\phi_x+\kappa_\alpha\phi_\pi)V(\varepsilon_t^a)}{[\phi_q(1-\alpha)-\kappa_\alpha\phi_\pi-\phi_x-1]^3}$
$\frac{\partial V(\pi_{H,t})}{\partial \phi_q}$	$\frac{-2\kappa_\alpha^2\phi_q(1-\alpha)^2(1+\phi_x+\kappa_\alpha\phi_\pi)V(\varepsilon_t^a)}{[\phi_q(1-\alpha)-\kappa_\alpha\phi_\pi-\phi_x-1]^3}$
$\frac{\partial V(\pi_t)}{\partial \phi_q}$	$\frac{-2(1-\alpha)(1+\phi_x+\kappa_\alpha\phi_\pi)[\kappa_\alpha\phi_q(1-\alpha)(\kappa_\alpha+\alpha)+\alpha(1+\phi_x+\kappa_\alpha\phi_\pi)(\kappa_\alpha+2\alpha)]V(\varepsilon_t^a)}{[\phi_q(1-\alpha)-\kappa_\alpha\phi_\pi-\phi_x-1]^3}$
$\frac{\partial V(\Delta e_t)}{\partial \phi_q}$	$\frac{-2(1-\alpha)(1+\phi_x+\kappa_\alpha\phi_\pi)[\kappa_\alpha\phi_q(1-\alpha)(\kappa_\alpha+1)+\alpha(1+\phi_x+\kappa_\alpha\phi_\pi)(\kappa_\alpha+2)]V(\varepsilon_t^a)}{[\phi_q(1-\alpha)-\kappa_\alpha\phi_\pi-\phi_x-1]^3}$
$\frac{\partial V(\Delta e_{t+1})}{\partial \phi_q}$	$\frac{-2(1-\alpha)(1+\phi_x+\kappa_\alpha\phi_\pi)^2V(\varepsilon_t^a)}{[\phi_q(1-\alpha)-\kappa_\alpha\phi_\pi-\phi_x-1]^3}$
$\frac{\partial V(\Delta q_t)}{\partial \phi_q}$	$\frac{-4(1-\alpha)^3(1+\phi_x+\kappa_\alpha\phi_\pi)^2V(\varepsilon_t^a)}{[\phi_q(1-\alpha)-\kappa_\alpha\phi_\pi-\phi_x-1]^3}$
$\frac{\partial V(\Delta q_{t+1})}{\partial \phi_q}$	$\frac{-2(1-\alpha)^3(1+\phi_x+\kappa_\alpha\phi_\pi)^2V(\varepsilon_t^a)}{[\phi_q(1-\alpha)-\kappa_\alpha\phi_\pi-\phi_x-1]^3}$

Appendix E. Configuration model

Table 11: Deep parameters values

Parameter	Description	Value
σ	Curvature of the utility of consumption	1.0
η	Degree of substitution between domestic and foreign goods	1.0
γ	Degree of substitutability between goods produced in the rest of the world	1.0
ϵ	Degree of substitutability between differents varities	6.0
φ	Disutility of labor	3.0
θ	Index of price stickiness	0.8
β	Discount factor	0.9
α	Openess degree	0.4
ϕ_π	CB preference by inflation	1.5
ϕ_x	CB preference by output gap	0.5
ϕ_q	CB preference by expected RER variation	0.5
ρ_a, ρ_v, ρ_z	Shock persitence for a_t, v_t and z_t	0

Appendix F. In what dimension forward-looking rules are successful?

Although the individual variances of variables such as inflation, product gap and exchange rate variations can't be diminished given the feasible determination space of the model, there is a dimension in which forward-looking rules such as the one used in this paper they are successful. In a frontier of efficiency furthest from the origin, the relative variances can effectively decrease, in some cases always, while in others needs very little restrictive conditions.

Table 12: Relative Variances

Variable	Productivity Shock	NKPC Shock	UIP Shock
$\frac{V(x_t)}{V(\pi_{H,t})}$	$\left[\frac{\Psi_{sa}-\Gamma}{\kappa_\alpha(\Psi_{sa}-\Gamma\sigma_\alpha)}\right]^2$	$\left[\frac{\Psi_{sv}}{\kappa_\alpha\Psi_{sv}+\sigma_\alpha}\right]^2$	$\left[\frac{\Psi_{sz}\sigma-\sigma_\alpha}{\kappa_\alpha(\Psi_{sz}\sigma-\sigma_\alpha)}\right]^2$
$\frac{V(\Delta q_t)}{V(\pi_{H,t})}$	$2\left[\frac{\Psi_{sa}\sigma_\alpha(1-\alpha)}{\kappa_\alpha(\Psi_{sa}-\Gamma\sigma_\alpha)}\right]^2$	$2\left[\frac{\Psi_{sv}\sigma_\alpha(1-\alpha)}{\kappa_\alpha\Psi_{sv}+\sigma_\alpha}\right]^2$	$2\left[\frac{\Psi_{sz}\sigma_\alpha\sigma}{\kappa_\alpha(\Psi_{sz}\sigma-\sigma_\alpha)}\right]^2$

Table 13: Derivative Relative Variances

Variable	Productivity Shock	NKPC Shock	UIP Shock
$\frac{\partial\left(\frac{V(x_t)}{V(\pi_{H,t})}\right)}{\partial\phi_q}$	0	$\frac{2\sigma_\alpha^2\phi_\pi D^2}{(\phi_q\sigma_\alpha^2(1-\alpha)-\phi_x\sigma_\alpha-\sigma_\alpha^2)^3}\frac{\partial\Psi_{sv}}{\partial\phi_q}$	0
$\frac{\partial\left(\frac{V(\Delta q_t)}{V(\pi_{H,t})}\right)}{\partial\phi_q}$	$-\frac{4\sigma_\alpha^3\Gamma\kappa_\alpha^2(1-\alpha)^2A^2B}{C^3}\frac{\partial\Psi_{sa}}{\partial\phi_q}$	$\frac{4\sigma_\alpha^4(1-\alpha)^2\phi_\pi D^2}{(\phi_q\sigma_\alpha^2(1-\alpha)-\phi_x\sigma_\alpha-\sigma_\alpha^2)^3}\frac{\partial\Psi_{sv}}{\partial\phi_q}$	$-\frac{4\sigma_\alpha^3\sigma^2\kappa_\alpha(1-\alpha)^2EF^2}{G^3}\frac{\partial\Psi_{sz}}{\partial\phi_q}$

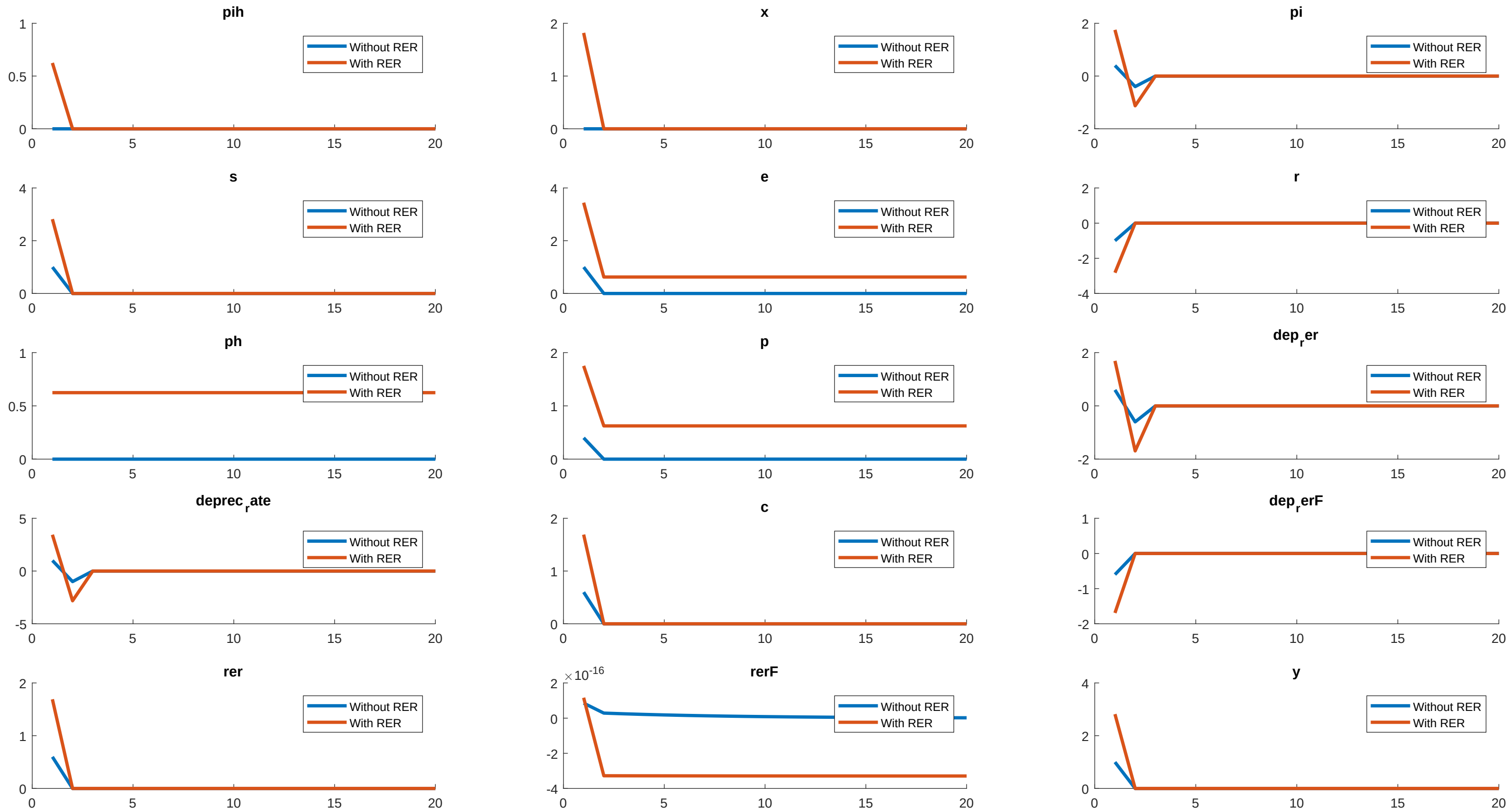
$$A \equiv (\phi_q\sigma_\alpha(1-\alpha) - \phi_\pi\kappa_\alpha - \phi_x - \sigma_\alpha)$$
$$B \equiv (\Gamma\sigma_\alpha^2 + \phi_x + \phi_\pi\kappa_\alpha\Gamma\sigma_\alpha)$$

$C \equiv (\phi_q \sigma_\alpha^2 \Gamma(1-\alpha) + \phi_x(1-\Gamma\sigma_\alpha))$
$D \equiv (\phi_q \sigma_\alpha^2 (1-\alpha) - \phi_\pi \kappa_\alpha - \phi_x - \sigma_\alpha)$
$E \equiv (\sigma_\alpha^2 + \phi_x \sigma_\alpha + \phi_\pi \kappa_\alpha \sigma_\alpha)$
$F \equiv (\phi_q \sigma \sigma_\alpha (1-\alpha) - \phi_\pi \kappa_\alpha \sigma - \phi_x \sigma - \sigma_\alpha \sigma)$
$G \equiv (\phi_q \sigma \sigma_\alpha^2 (1-\alpha))$

Appendix G. IRFs analysis

- Rules
  - Without RER: $r_t = r_t^n + \phi_\pi \pi_{H,t} + \phi_x x_t$
  - With RER: $r_t = r_t^n + \phi_\pi \pi_{H,t} + \phi_x x_t + \phi_q \Delta q_{t+1}$
- Variables
  - pih: $\pi_{H,t}$ domestic inflation
  - x: $x_{H,t}$ output gap
  - pi: π_t CPI inflation
  - s: s_t terms of trade
  - e: e_t nominal exchange rate
  - r: r_t nominal interest rate
  - ph: $p_{H,t}$ domestic price level
  - p: s_t CPI price level
  - dep_rer: Δq_t RER depreciation
  - deprec_rate: Δe_t ER depreciation
  - c: c_t consumption
  - dep_rerF: $E_t\{\Delta q_{t+1}\}$ Expected RER depreciation
  - rer: q_t RER
  - rerF: $E_t\{q_{t+1}\}$ expected RER
  - y: y_t GDP

Figure 12: Productivity Shock

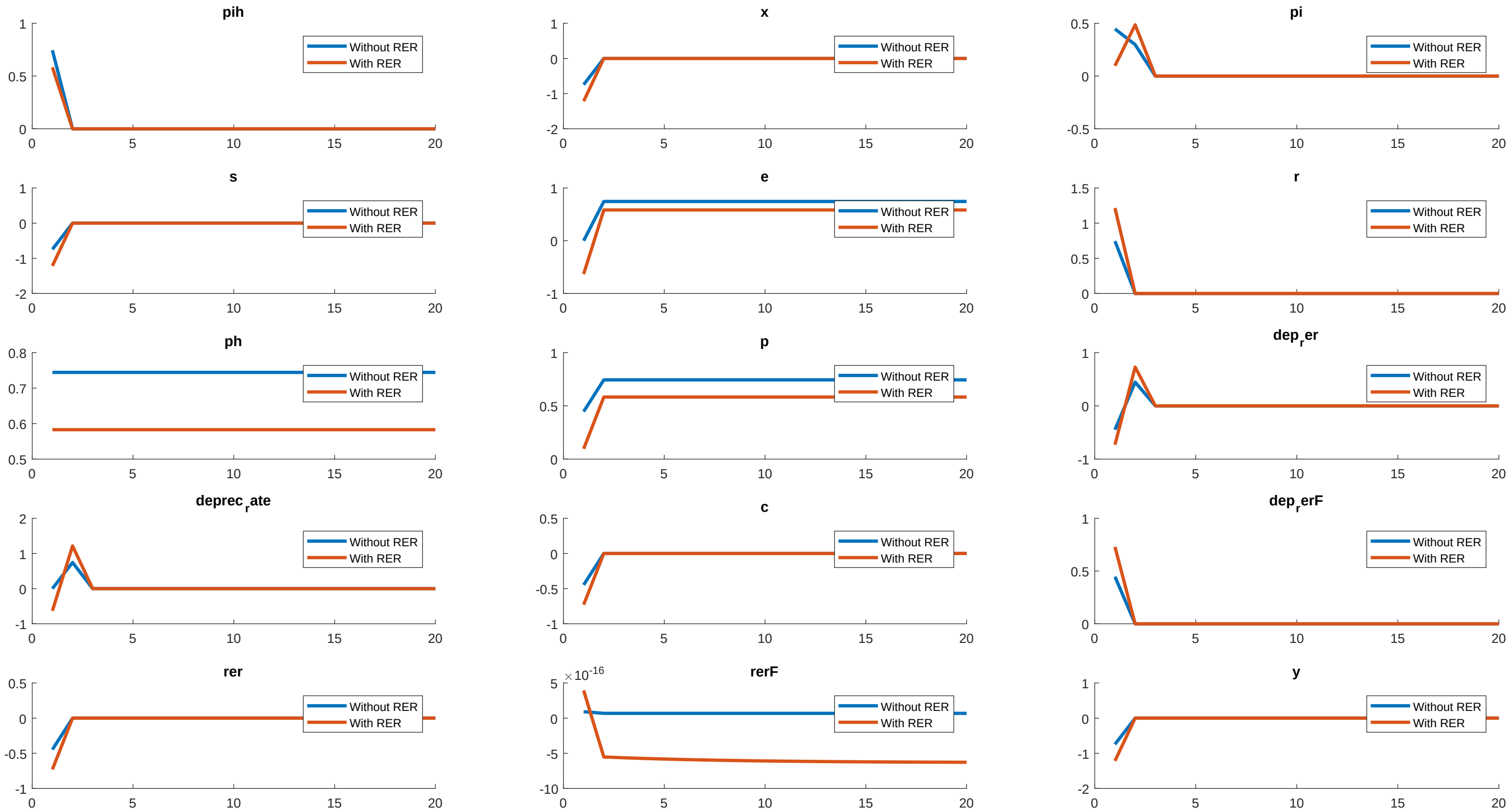


Note: In rule with RER, $\phi_q = 1.3rer$

Table 14: Variances - Productivity Shock

Variable	without RER	with RER	dif
Domestic Inflation	0,000	0,390	-0,390
Output gap	0,000	3,306	-3,306
CPI Inflation	0,320	4,339	-4,019
RER Depreciation	0,720	5,718	-4,998
RER	0,360	2,859	-2,499
GDP	1,000	7,942	-6,942

Figure 13: NKPC Shock

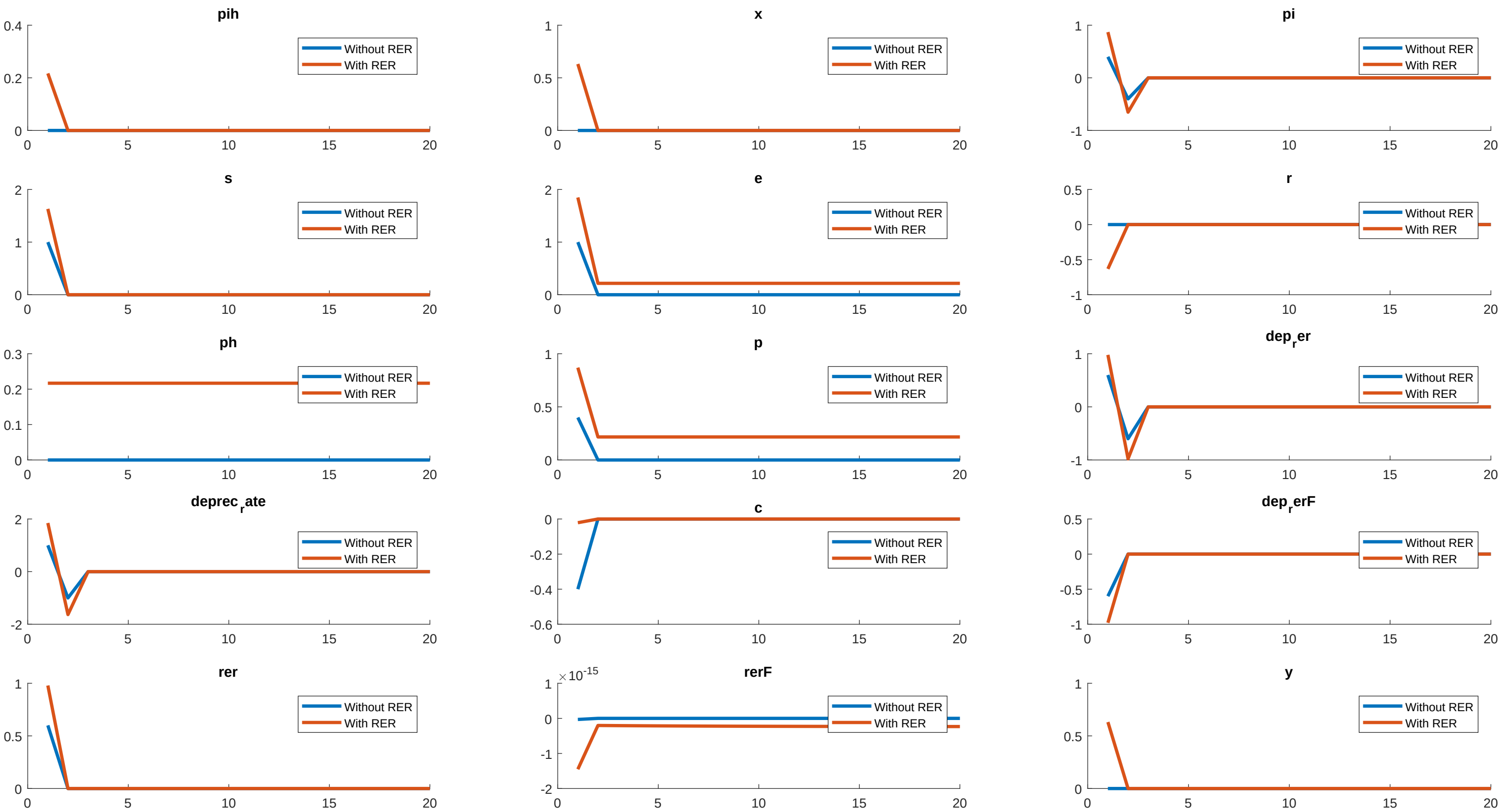


Note: In rule with RER, $\phi_q = 1.3$

Table 15: Variances - NKPC Shock

Variable	**without RER**	**with RER**	**dif**
Domestic Inflation	0,554	0,340	0,214
Output gap	0,554	1,475	-0,921
CPI Inflation	0,288	0,246	0,043
RER Depreciation	0,399	1,062	-0,663
RER	0,200	0,531	-0,332
GDP	0,554	1,475	-0,921

Figure 14: UIP Shock



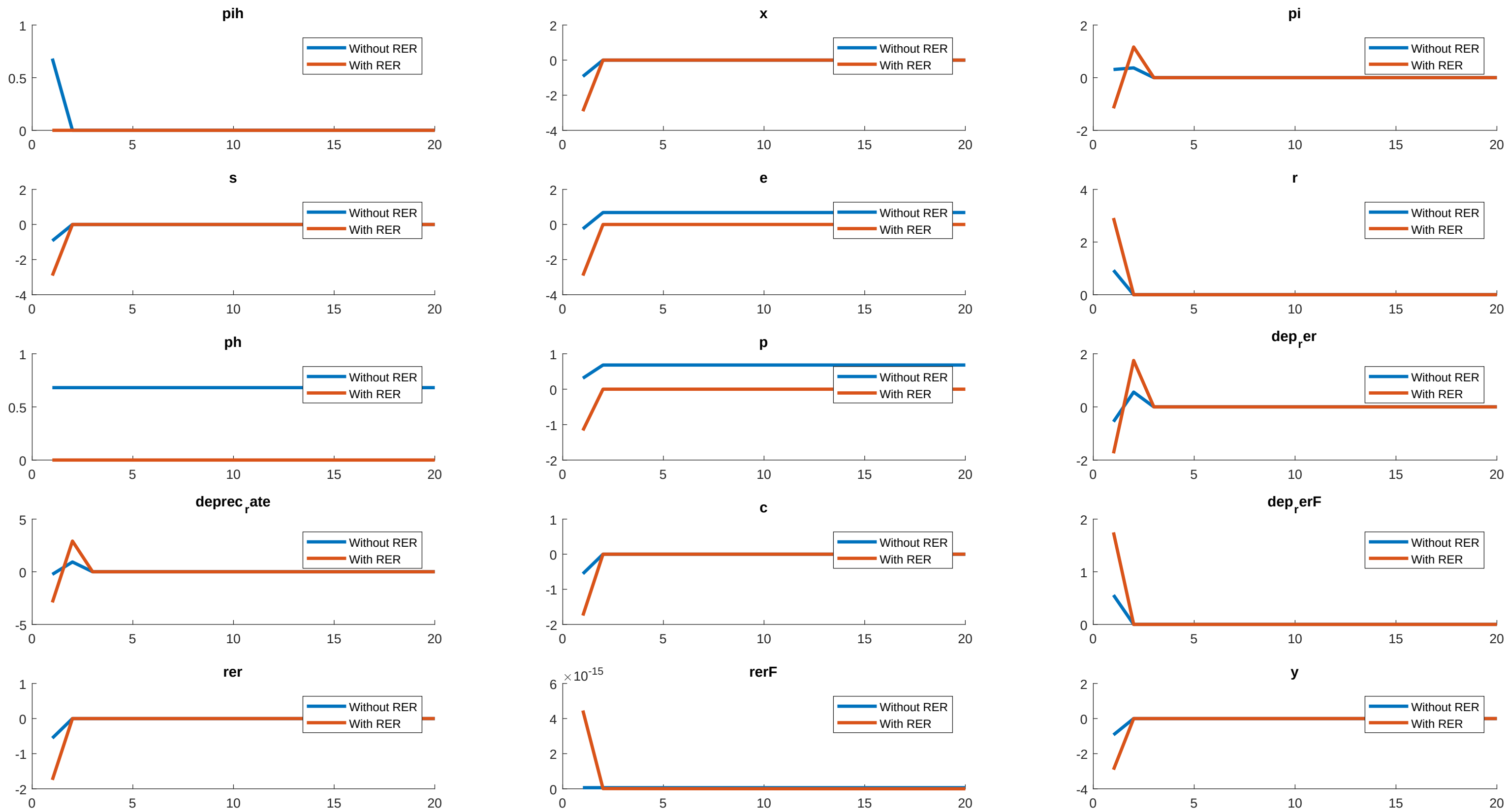
Note: In rule with RER, $\phi_q = 1.3$

Table 16: Variances - UIP Shock

Variable	**without RER**	**with RER**	**dif**
Domestic Inflation	0,000	0,047	-0,047
Output gap	0,000	0,399	-0,399
CPI Inflation	0,320	1,182	-0,862
RER Depreciation	0,720	1,917	-1,197
RER	0,360	0,958	-0,598
GDP	0,000	0,399	-0,399

NKPC Shock. 3 cases

Figure 15: NKPC Shock

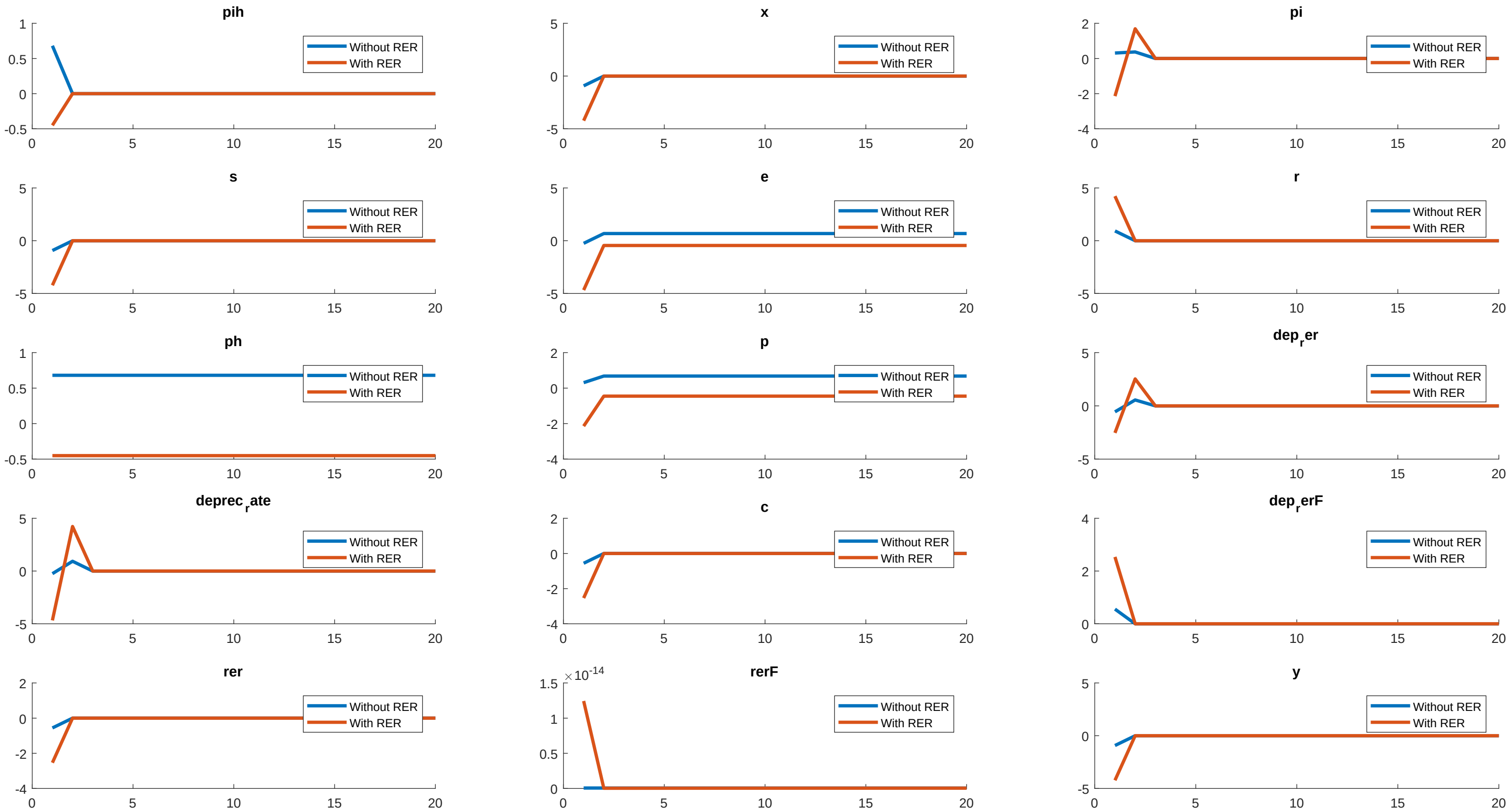


Note: In rule with RER, $\phi_q = 1.8333$ and $\phi_x = 0.1$

Table 17: Variances - NKPC Shock

Variable	**without RER**	**with RER**	**dif**
Domestic Inflation	0,464	0,000	0,464
Output gap	0,863	8,483	-7,621
CPI Inflation	0,234	2,715	-2,481
RER depreciation	0,621	6,108	-5,487
RER	0,311	3,054	-2,743
GDP	0,863	8,483	-7,621

Figure 16: Variances - NKPC Shock

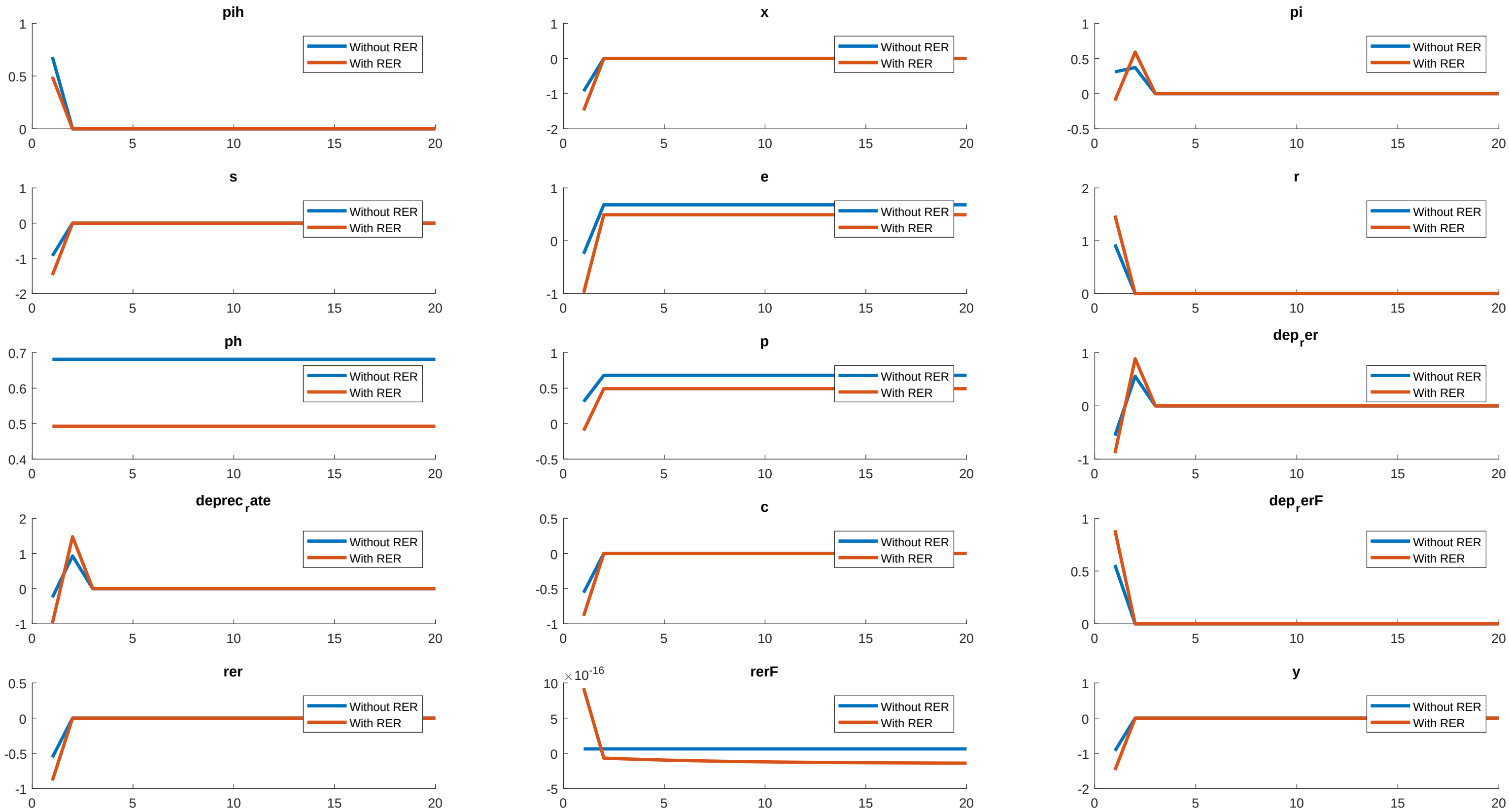


Note: In rule with RER, $\phi_q = 1.8333$ and $\phi_x = 0.1$

Table 18: Add caption

Variable	without RER	with RER	dif
Domestic Inflation	0,464	0,203	0,261
Output gap	0,863	17,854	-16,991
CPI Inflation	0,234	7,440	-7,206
RER depreciation	0,621	12,855	-12,234
RER	0,311	6,427	-6,117
GDP	0,863	17,854	-16,991

Figure 17: Variances - NKPC Shock

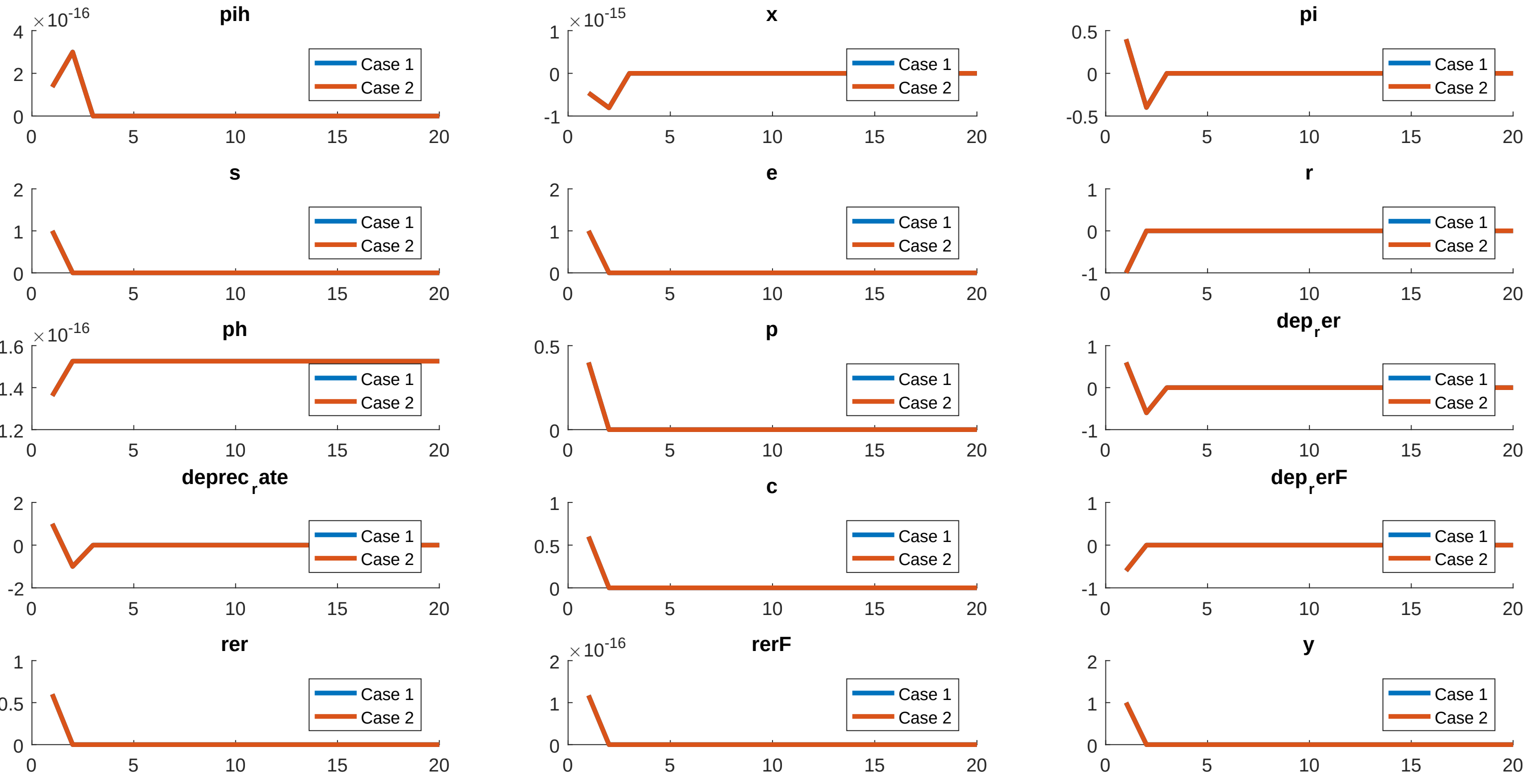


Note: In rule with RER, $\phi_q = 1.8333$ and $\phi_x = 0.1$

Table 19: Add caption

Variable	**without RER**	**with RER**	**dif**
Domestic Inflation	0,464	0,243	0,221
Output gap	0,863	2,184	-1,321
CPI Inflation	0,234	0,359	-0,125
RER depreciation	0,621	1,573	-0,951
RER	0,311	0,786	-0,476
GDP	0,863	2,184	-1,321

Figure 18: IRFs.



Case 1: Taylor-rule with know r_t^n and $\phi_q = 0$. Case 2: Taylor-rule with unknow r_t^n and $\phi_q = \frac{1}{1-\alpha}$