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A comparison of characteristic parameters of mining related and tectonic seismic aftershock sequences

Rodrigo Estay [a,*], Javier Vallejos [b], Claudia Pavez [c], Marco Brönner [c,d]

[a] Department of Metallurgy and Materials Engineering, Federico Santa María Technical University, Chile. Av. Vicuña Mackenna 3939, San Joaquín, 8940000, Santiago, Chile

[b] Department of Mining Engineering, University of Chile, Chile. Av. Beaucheff 859, 8320000, Santiago, Chile

[c] Norwegian Geological Survey, Trondheim, Norway. Leiv Erikssons vei 39, 7040, Trondheim, Norway

[d] Norwegian University of Science and Technology, Trondheim, Norway. Høgskoleringen 1, 7491, Trondheim, Norway

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ABSTRACT

The temporal decay pattern of seismicity following rock bursts and large magnitude events is studied for aftershock sequences in several mines in Ontario, Canada. The decay phenomenon is described by the so – called *Modified Omori's law*. The dependency of the parameters *K, c and p* with the magnitude of large mine seismic events is analyzed. A total of eleven mining related seismic sequences, with mainshock magnitudes ranging from 0.5 to 4.0 M_n (Nuttli magnitude scale) and from four different mine environments are examined. Results, including the calculation of the Gutenberg - Richter's and Reasenberg – Jones parameters *b* and *a'*, are compared with previous research in tectonic seismicity using available data from New Zealand, Italy and U.S.A. Further parameters were derived and applied in a correlation study of mining related and tectonic events. The study revealed a good correlation between the productivity parameter *a'* and *b – values*. Particularly, the *b – values* show significant different number ranges for mining related and tectonic data, which was applied to derive a direct numerical relationship between mining related and tectonic parameters and the aftershock number estimation. Finally, numerical values for the *b – values* in mining related seismic activity were obtained.

1. Introduction

The survey and analysis of seismic laws constitute a highly studied topic considering the importance of understanding the behavior of the aftershock sequences.[1–6] Most of them correspond to modified and scaled aftershock rate decay models, such as the Gutenberg Richter (GR) law[7] and the Modified Omori's law.[8,9] The later one, which introduces a ray decay with time, can be combined with the regular GR law to obtain a rate decay with a time elapsed after the mainshock, resulting in the Reasenberg Jones model.[10] The related parameters, *b, K, c, p* and *a'*, respectively, represent physical properties of the aftershock sequences and the seismogenic zones. Additionally, the correlation among them can be useful in order to understand the rate decay time and behavior of the aftershock sequence after a mainshock. So far, to our knowledge all existing studies are only considering different tectonic settings,[1,11–13] mainly applicable for risk analysis in mining companies. As a new approach to this area of research, we investigate correlations between mining related micro seismicity and passive seismic events, considering the seismic parameters of different aftershock sequences as input dataset.

In this work, we study the correlation between the Gutenberg Richter, Modified Omori's Law and Reasenberg – Jones parameters for both, tectonic and mining related seismicity. The micro seismicity data were derived from eleven aftershocks sequences from different mining sites in Ontario, Canada.[14] We calculated the parameters for the physical properties and compared them with the properties derived from a series of available tectonic datasets from New Zealand, California, Italy and Japan.

As was detailed before, many studies attempt to find empirical relations between tectonic parameters in different seismogenic zones. However, none of the current research studies are showing the relation between tectonic and mining related seismic parameters, even when the tectonic results are always used as a comparative reference in mining analysis.[15–17] Therefore, the main goal of this study is to understand the physical meaning of these parameters and study potential correlations between available data for a variety of seismogenic zones and the

* Corresponding author.
E-mail address: rodrigo.estayh@usm.cl (R. Estay).

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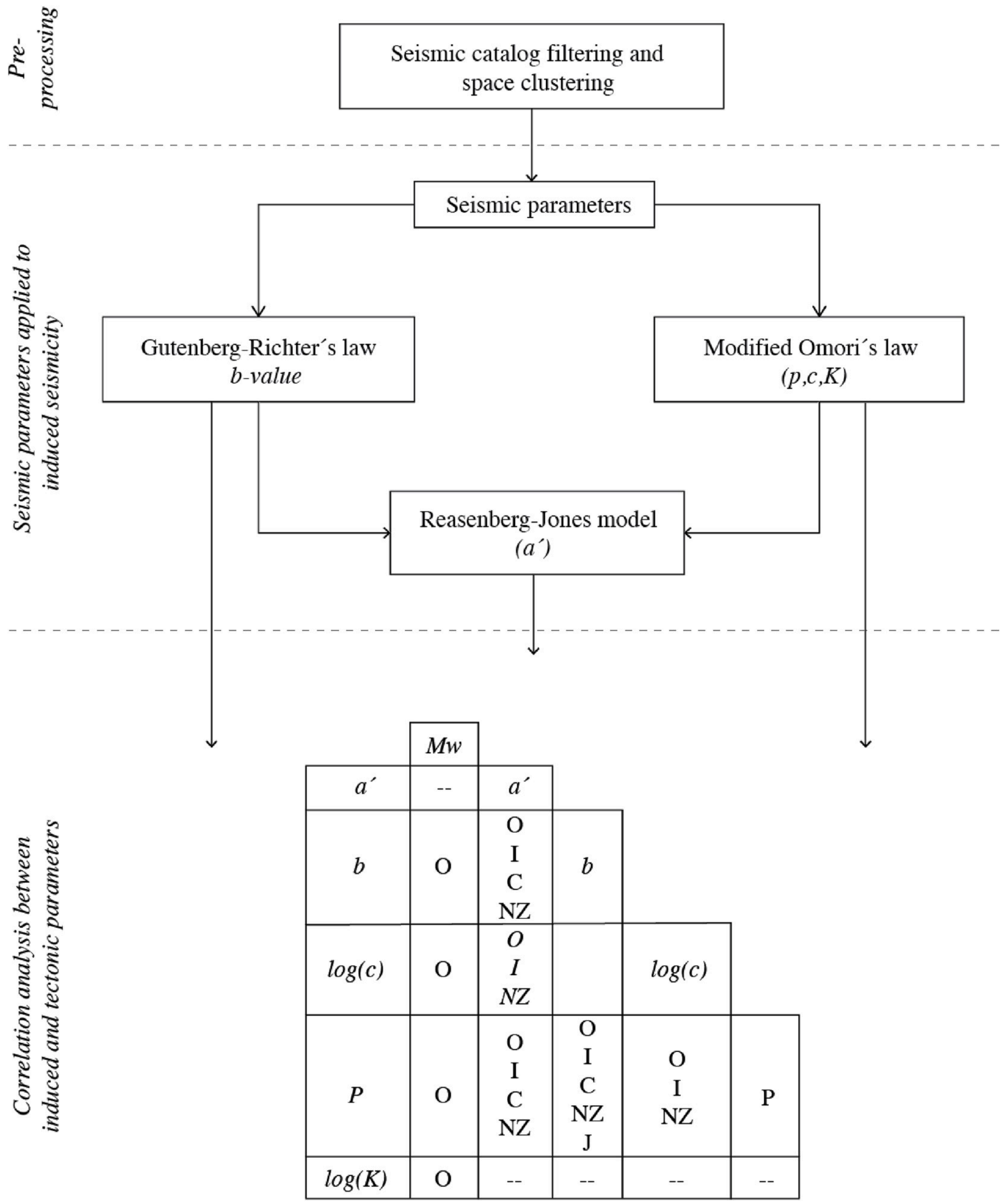

Fig. 1. Methodology and correlations between seismic parameters used in the analysis. Each letter means: Ontario (O), Italy (I), California (C), New Zealand (NZ) and Japan (J).

parameters obtained for mining related seismic sequences. Fig. 1 presents the workflow we applied for this study to investigate possible relations between the different datasets applied.

1.1. *A brief geological frame*

The micro seismicity data used throughout this study were obtained from four mining operations in Ontario, Canada. These sites are: Copper Cliff North, Craig, Creighton and Kidd Creek, all of them located in Sudbury and Timmins, Ontario. The mines have a permanent monitoring system related to the Engineering Seismology Group (ESG) company, which support the quality of the traces.[14] The micro – seismic monitoring system is composed by uniaxial and triaxial accelerometers, which are covering the sites. Each mine, with Kidd Creek exception, is located around the Sudbury Igneous Complex (SIC) which correspond to a 2.5–3.0 km thick with a 60 × 27 km elliptical igneous rock body.[18] As a general description, the zone is characterized by quartz-diorite dykes, granite and granodiorites rocks of the Creighton Pluton, and meta – volcanic to metasedimentary rocks. With respect to Kidd Creek, the mine is located near the top of a locally thickened rhyolite, which is underlain to the east by ultramafic units and overlain to the west by mafic flows and associated intrusions. More geological information about the sites can be found in Ref. 14.

2. Background

2.1. *Magnitude conversion*

In general, to describe magnitudes of mining related seismic events, two scales are mainly used. The first one corresponds to the Local magnitude scale (M_L),[19] which proved enough for primary seismicity in mines ($M_L = 1.5$ to 4.0). Secondly, the Moment magnitude scaleM_w[20] is

Table 1
List of analyzed aftershock sequences (Seq) following large magnitude events collected from Ontario mines, Canada. M_n, $M_{w,m}$ and M_L are the mainshock magnitudes of the sequence in Nuttli, Moment and Local magnitude respectively and t_N is the total duration of the sequence. The value of M_L for Kidd Creek does not comply the conditions $M_L \geq 1.0$; CCN correspond to Copper Cliff North Mine.

Seq	Site	Date (mm/dd/yyyy)	M_n	$M_{w,m}$	M_L	t_N (hours)	N° of events in sequences
1	CCN	06/10/2005	2.1	1.6	1.3	10.3	53
2	CCN	11/30/2004	2.4	1.9	1.8	29.9	192
3	CCN	09/24/2008	2.4	1.9	1.8	69.4	17
4	CCN	09/11/2008	3.8	3.3	3.9	165.6	213
5	Craig	06/22/2007	2.2	1.7	1.5	37.5	77
6	Creighton	02/07/2008	2.4	1.9	1.8	45.9	41
7	Creighton	03/14/2009	2.6	2.1	2.1	197.7	627
8	Creighton	12/06/2008	2.9	2.4	2.5	25.3	54
9	Creighton	06/15/2007	3.0	2.5	2.7	27.6	402
10	Creighton	10/07/2007	3.1	2.6	2.9	53.6	408
11	Kidd Creek	03/02/2006	1.6	1.0	0.5	23.5	21

Table 2
Magnitude of completeness, $M_{w,c}$, for each sequence (Seq). R_{GoF} correspond to the percentage of adjustment of each sequence applying Goodness of Fit method.

Seq	Site	$M_{w,c}$	R_{GoF} (%)
1	Copper Cliff North	−1.4	87.7
2	Copper Cliff North	−1.6	97.7
3	Copper Cliff North	−1.7	91.8
4	Copper Cliff North	−1.6	94.7
5	Craig	−1.8	93.6
6	Creighton	−1.5	92.2
7	Creighton	−1.7	98.2
8	Creighton	−1.5	93.0
9	Creighton	−1.4	96.3
10	Creighton	−1.4	94.2
11	Kidd Creek	−2.0	90.2

widely accepted and stable to describe both mining related micro seismicity and passive events.

Furthermore, to avoid wrong interpretations of the mining related seismic events from the Canadian mines, the Nuttli magnitude M_n was used.[21] The Nuttli magnitude scale is adopted to express the magnitude of large seismic events for mines in the Canadian Shield[22] and generally yields magnitudes about 0.3 to 0.6 units larger than the M_L scale.[23]

The correct estimation of the seismic parameters in micro seismicity and the comparison with the tectonic ones require a magnitude conversion between the Nuttli and the Moment magnitude. Therefore, we applied an empirical relationship after[24] which is valid for earthquakes with Nuttli magnitude between $1.0 \leq M_n \leq 6.0$:

$$M_w = 1.03M_n - 0.61 \tag{1}$$

Moreover, Hudyma & Potvin (2004)[25] summarize several investigations in which M_W and M_L are estimated and related to each other, using micro – seismic parameters with $M_L \geq 1.0$. One of the proposed empirical equations is[26]:

$$M_w = 0.67M_L + 0.67 \tag{2}$$

The use of this relation ensures that further estimation of the sequence parameters be valid, due to it was exclusively calculated for Canadian micro-seismicity.

Therefore, replacing equation (2) into equation (1), a relation to estimate local magnitude from Nuttli magnitude can be obtained:

$$M_L = 1.54M_n - 1.91 \tag{3}$$

Notice that equation (3) is only valid for sequences which mainshock magnitudes are $M_L \geq 1.0$.

2.2. Seismic laws

The seismic laws are statistical models that have been proposed to describe the empirical behavior of magnitude, occurrence times and spatial locations of aftershocks either for tectonic or mining related seismicity. The Gutenberg – Richter's and the Modified Omori's laws are two of the widest accepted and most studied laws in seismology, where the equation defining parameters are described both statistically and physically.

In the next subsections we will describe the estimation of those parameters and their uncertainties using the method of maximum

Table 4
Mining related micro seismicity parameters for Ontario's aftershock sequences.

Seq	Site	$M_{w,m}$	a'	b	p	K	c
1	CCN	1.6	−5.29	2.16 ± 0.14	0.73 ± 0.16	9.54 ± 2.22	0.05 ± 0.09
2	CCN	1.9	−2.51	1.20 ± 0.07	0.70 ± 0.04	21.72 ± 1.79	0.00 ± 0.00
3	CCN	1.9	−3.77	1.31 ± 0.55	1.04 ± 0.28	2.60 ± 1.19	0.01 ± 0.01
4	CCN	3.3	−5.31	1.51 ± 0.06	1.09 ± 0.07	43.76 ± 10.33	0.44 ± 0.20
5	Craig	1.7	−2.97	1.30 ± 0.13	0.74 ± 0.06	8.41 ± 1.19	0.00 ± 0.01
6	Creighton	1.9	−5.26	1.85 ± 0.29	0.74 ± 0.10	4.39 ± 1.00	0.01 ± 0.02
7	Creighton	2.1	−2.59	1.27 ± 0.01	0.79 ± 0.02	51.82 ± 4.28	0.05 ± 0.02
8	Creighton	2.4	−3.55	1.20 ± 0.08	0.82 ± 0.11	7.50 ± 1.39	0.02 ± 0.03
9	Creighton	2.5	−2.71	1.19 ± 0.04	0.79 ± 0.06	62.51 ± 8.36	0.15 ± 0.08
10	Creighton	2.6	−4.10	1.50 ± 0.04	0.80 ± 0.05	54.03 ± 7.43	0.17 ± 0.09
11	Kidd Creek	1.0	−1.50	0.83 ± 0.16	0.93 ± 0.13	2.53 ± 0.63	0.00 ± 0.01

Table 3
Correlation values between aftershock sequence parameters (b, c, K, p) and the mainshock magnitude ($M_{w,m}$). As described above, the value of $M_{w,c} = -1.30$ (in bold) is selected as a global magnitude of completeness.

$M_{w,c}$	N_{seq}	$M_{w,m}$,log(c)	$M_{w,m}$,p	$M_{w,m}$,log(K)	$M_{w,m}$,b	log(c),p	log(c),log(K)	log(c),b	p,log(K)	p,b	log(K),b
−1.4	9	0.85	0.28	0.20	0.01	0.34	0.11	0.00	0.05	0.02	0.08
−1.3	**11**	**0.60**	**0.12**	**0.48**	**0.01**	**0.09**	**0.45**	**0.11**	**0.02**	**0.06**	**0.00**
−1.2	11	0.47	0.04	0.38	0.00	0.06	0.28	0.06	0.01	0.29	0.03
−1.1	10	0.00	0.00	0.25	0.00	0.02	0.00	0.60	0.01	0.01	0.09

Table 5
Overview of some researches that estimates the Gutenberg-Richter's, Omori's and Reasenberg – Jones's parameters for tectonic aftershock sequences from different regions. The first row is including the results for mining related aftershock sequences. The values of K and c are in [events/day] and [days] respectively, except Ontario sequences which K and c – values are in events/hour and hour respectively. The last row is the worldwide tectonic aftershock sequences average and standard deviation.

a'		b		p		K		c		Number of sequences	Years	Geographic region	References
mean	std. dev	mean	std. dev	mean	std. dev	mean	std. dev	mean	std. dev				
−3.60	**1.29**	**1.39**	**0.36**	**0.83**	**0.13**	**24.44**	**23.62**	**0.08**	**0.13**	**11**	**2004–2009**	**Ontario, Canada**	
−1.79	0.74	1.01	0.15	1.04	0.31	–	–	0.08	0.11	17	1987 – 1995	New Zealand	52
−1.76	0.07	0.9	0.02	1.07	0.03	–	–	–	–	62	1933 – 1987	California	10
–	–	1.06	0.26	1.11	0.25	–	–	–	–	37	1933 – 1981	California$^{(3)}$	37
–	–	0.96	0.1	1.06	0.07	8.51	3.34	0.01	0.01	4	2000 – 2007	Japan$^{(4)}$	41
–	–	–	–	1.27	0.35	351.38	482.54	0.30	0.47	3	1948 – 1964	Japan$^{(5)}$	44
–	–	0.97	0.20	1.11	0.24	–	–	–	–	32	1972 – 1993	Japan$^{(6)}$	11
–	–	1.14	0.01	–	–	–	–	–	–	–	1978 – 1994	Northern California$^{(7)}$	38
–	–	1.24	0.01	–	–	–	–	–	–	–	1978 – 1994	Northern California w/o aftershock$^{(7)}$	38
–	–	1.18	0.01	–	–	–	–	–	–	–	1983 – 1994	Southern California$^{(7)}$	38
–	–	1.27	0.01	–	–	–	–	–	–	–	1983 – 1994	Southern California w/o aftershocks$^{(7)}$	38
−1.81	0.62	0.9	0.18	1.09	0.25	13.07	16.69	0.05	0	64	–	California$^{(8)}$	1
−1.79	0.74	1.01	0.15	0.99	0.34	45.03	29.26	0.08	0.11	17	–	New Zealand$^{(8)}$	1
−1.83	0.67	0.99	0.17	0.99	0.34	33.21	54.52	0.61	1.53	30	–	Italy$^{(8)}$	1
–	–	–	–	0.79	0.26	28.13	25.45	0.09	0.42	37	1933 – 2004	California$^{(9)}$	51
–	–	–	–	0.86	0.25	24.82	20.68	0.15	0.22	9	1976 – 2003	Italy$^{(9)}$	51
−1.80	***0.03***	***1.05***	***0.12***	***1.03***	***0.13***	***72.02***	***123.8***	***0.17***	***0.20***				

likelihood.[27] We further discuss some of the physical implications of the parameters of the seismic laws in the behavior of the aftershock sequences.

2.2.1. Gutenberg – Richter's law

In order to a better understanding of the Gutenberg – Richter's law and its parameters, a detailed demonstration will be done.

The Gutenberg – Richter's law begins with the definition of the probability density[27]:

$$f(M,\beta) = \beta e^{-\beta(M-M_{min})}, \tag{4}$$

where M is the magnitude, considered as a continuous random variable, M_{min} is the minimum magnitude allowed and β is defined by construction as $1/(\overline{M} - M_{min})$ with $\overline{M}$ as the average magnitude.

Considering equation (4), the number of seismic events N between magnitudes M and $M + dM$ per time unit, t, is defined as:

$$N = \frac{f(M) - f(M+dM)}{t} = \alpha e^{-\beta(M-M_{min})}\left(1 - e^{-\beta dM}\right) \tag{5}$$

where $\alpha = \beta/t$ is the average number of events per time unit.

If equation (5) is approximated in first order Taylor's series the number of events can be re-written as:

$$N \approx \alpha\beta e^{-\beta(M-M_{min})} dM. \tag{6}$$

When applying logarithm on both sides of equation (6), considering $a = log(\alpha\beta e^{\beta Mmin} dM)$ and $b = \beta\ log(e)$, the Gutenberg – Richter's law is obtained:

$$logN = a - bM \tag{7}$$

Given the definition of the parameters a and b, β will be estimated using the log likelihood function defined as:

$$ln\left\{\prod_i f(M_i,\beta)\right\} = Nln\beta - \beta\sum_i (M_i - M_{min}). \tag{8}$$

When equation (8) is maximized, resulting in the maximum likelihood estimation of β, the b value will be given by:

$$\hat{b} = \frac{\log(e)}{\overline{M} - M_c} \tag{9}$$

where Mc is the magnitude of completeness.

According to Marzocchi & Sandri (2003),[28] equation (9) is biased because of two reasons: $\overline{M}$ of a continuous random variable with a power law distribution is different from the average of the same "binned" random variable. To have a negligible bias for the first case, the authors recommended a width of the bin $\Delta M_{bin} = 0.1$. The second problem is that in general $Mc \neq M_{min}$, which can be corrected applying the definition presented in equation (10)

$$\hat{b} = \frac{\log(e)}{\overline{M} - M_c + \Delta M_{bin}/2} \tag{10}$$

There are several ways to get the final estimation of the b-value. However, the details about the different methodologies are beyond of this research and can be found extensively in literature.[27–29]

Considering $\Delta M_{bin} = 0.1$ the bias existing between the estimation of the b – value using the continuous and the grouped magnitudes, is corrected.[29] Later, Marzochi & Sandri (2003) proved that the bias between the corrected (eq. (9)) and non – corrected b – value (eq. (10)) is 0.13.[28]

Particularly the *b - value* depends on the tectonic and stress regimes.[12,30–33] Scholz (1968) showed on a laboratory scale and for different types of rocks the relation between the *b – value* and the fracture ratio under uniaxial compression stress, σ/σ_c.[30] As a general result,

Table 6
Coefficients, intercepts and correlation between seismic parameters of the Ontario's mining related and tectonic aftershock sequences. The first parameter corresponds to the dependent variable and the second one to the independent variable. The results of Italy, California and New Zealand, including the p' – value from the Student-t statistics, were obtained from.[1] For the Ontario mines, the p'-value was calculated with a 95% confident interval.

Parameters	Dataset	Coefficient	Intercept	ρ	Student-t p' – values
b, $M_{w,m}$	Ontario	0.08	1.22	0.14	>0.05
p, $M_{w,m}$		0.07	0.68	0.32	>0.05
$log(c)$, $M_{w,m}$		1.0	−3.75	0.77	>0.05
a', b	Ontario	−3.15	0.78	−0.88	<0.05
	Italy	−2.37	0.52	−0.59	<0.05
	California	−2.50	0.45	−0.72	<0.05
	New Zealand	−3.51	1.76	−0.71	<0.05
a', p	Ontario	−1.26	−2.55	−0.13	>0.05
	Italy	0.61	−2.43	0.31	>0.05
	California	0.20	−2.03	0.08	>0.05
	New Zealand	1.12	−2.96	0.47	>0.05
a', c	Ontario	−4.26	−3.25	−0.44	>0.05
	Italy	0.16	−1.93	0.37	<0.05
	New Zealand	2.76	−2.01	0.41	>0.05
a', $log(c)$	Ontario	−0.80	−4.92	−0.49	>0.05
	Italy	0.50	−1.36	0.57	<0.05
	New Zealand	0.47	−1.11	0.35	>0.05
b, p	Ontario	−0.67	1.95	−0.25	>0.05
	Italy	−0.10	1.09	−0.19	>0.05
	California	−0.15	1.07	−0.21	>0.05
	New Zealand	−0.06	1.07	−0.12	>0.05
p, c	Ontario	0.55	0.79	0.55	>0.05
	Italy	0.10	0.93	0.43	<0.05
	New Zealand	1.57	0.92	0.55	<0.05
p, $log(c)$	Ontario	0.05	0.92	0.30	>0.05
	Italy	0.16	1.14	0.36	<0.05
	New Zealand	0.37	1.59	0.67	<0.05

higher levels of stress are related to lower *b – values*, exactly as it is expected when using the *b – value* as a seismic risk parameter. Schorlemmer et al. (2005) described how the Gutenberg – Richter's *b – value* varies depending on the faulting type/rake angle.[12] Considering normal (nr), strike slip (ss) and inverse mechanisms (th), it is true that $b_{nr} > b_{ss} > b_{th}$. Since this relation is associated with the average stress needed to generate a fault, it is generating an inverse relationship between the *b-value* and its stress level, so $\sigma_{nr} < \sigma_{ss} < \sigma_{\text{th}}$.

Some common errors are described in the literature in the calculation of the *b – value*. One of these errors are the adjustment of the Gutenberg – Richter's law with linear Least Squares (LSQ) instead of using the Maximum Likelihood Estimation (MLE) method.[27] It was previously demonstrated that closer results to the real b-value are obtained using MLE rather than LSQ estimation.[34]

Another error is using a very small data set, a greater variability in the *b – value* is obtained. Felzer (2006) recommends using more than 2000 seismic events to have an estimation error of 0.05 with a confidence interval of 98%.[34]

Finally, the use of earthquakes with magnitude smaller than the magnitude of completeness (*Mc*) of the catalogue.

The error estimation of the *b-value* is given by the equation (11)[35]:

$$\sigma(\widehat{b}) = 2.3b^2\sigma(\overline{M}) \tag{11}$$

where the variance of the magnitude is showed in equation (12):

$$\sigma^2(\overline{M}) = \sum_{i=1}^{n} \frac{(M_i - \overline{M})^2}{n(n-1)}. \tag{12}$$

2.2.2. Modified Omori's law (MOL) and Reasenberg – Jones model parameters

Immediately following a major seismic event, triggered by a rock burst or a blast in seismically active mines, there is an increase in the seismic frequency, known as aftershocks. This frequency gradually decays to the normal levels of seismicity, creating a seismic sequence.

Omori (1894) showed that the aftershock frequency per day at Nobi's earthquake in 1891 ($M = 8.0$) decreased according to the equation[8]:

$$n(t) = \frac{K}{(c+t)} \tag{13}$$

where $n(t)$ is the rate of aftershocks per unit time interval at time t and K and c are constants.

Utsu (1961) found that equation (13) adjusts better when considering an exponent value different from one, and proposed the so call Modified Omori's Law (MOL)[9]:

$$n(t) = \frac{K}{(c+t)^p}. \tag{14}$$

where p controls the decay rate and varies for each seismic sequence.

Several investigations on tectonic seismicity are related to the factors that influence the variations of p. Mogi (1962), studying 31 seismic sequences in Japan, showed that in volcanic settings the *p – value* is high (higher rate of decay) whilst for subduction zones it is low (lower decay rate).[36] Consequently, a faster decay is obtained in areas where the temperature of the crust is higher, and the stresses can relax more quickly. These results were corroborated by Kisslinger & Jones (1991) with seismic sequences obtained in California.[37] In their results, the highest *p – values* were obtained in the western zone of the volcanic front, while the lowest values were obtained in the subducting plate. In the same way, Tahir (2011) found that the *p – value* is related to the focal mechanism associated to seismicity using earthquake sequences from western Asia, with mainshock magnitude $M_s \geq 7.0$.[13] He found that *p – values* are higher for thrust events than for strike slip events. In the same way, using the world wide global Centroid Moment Tensor (CMT) and the United States Geological Survey (USGS) seismic catalogues, the author found an even higher *p – value* for normal than for thrust or strike slip mechanisms.

The *K – value* corresponds to the seismic activity of the sequence or the number of aftershocks after the main event.[37] Also, Utsu et al. (1995) showed that the *K – value* strongly varies with changes of *Mc*, which indicates that the number of seismic events in the sequences is a relevant factor.[39]

On the other hand, the physical meaning of the *c – value* can be attributed to the complex seismicity causing rupture process[40] and is related to the displacement in time related to the aftershocks in the first part of the sequence.[41] That segment of the sequence has a lower signal amplitude and is difficult to detect by the monitoring system.[42,43] In addition, *c – value* behaves mathematically as the constant prohibiting equation (13) from diverging at $t = 0$.[6]

For this research we applied the MLE methodology to estimate the MOL's parameters. Given the occurrence times t_i (i=1, …, N) of the individual N events in a time interval [S, T] the log – likelihood function of equation (14) can be expressed by:

$$lnL(K,p,c,S,T) = NlnK - p\sum_{i=1}^{N}\ln(t_i + c) - KA(p,c,S,T) \tag{15}$$

where the value of A(p,c,S,T) depends on the value of p, which is show in equation (16):

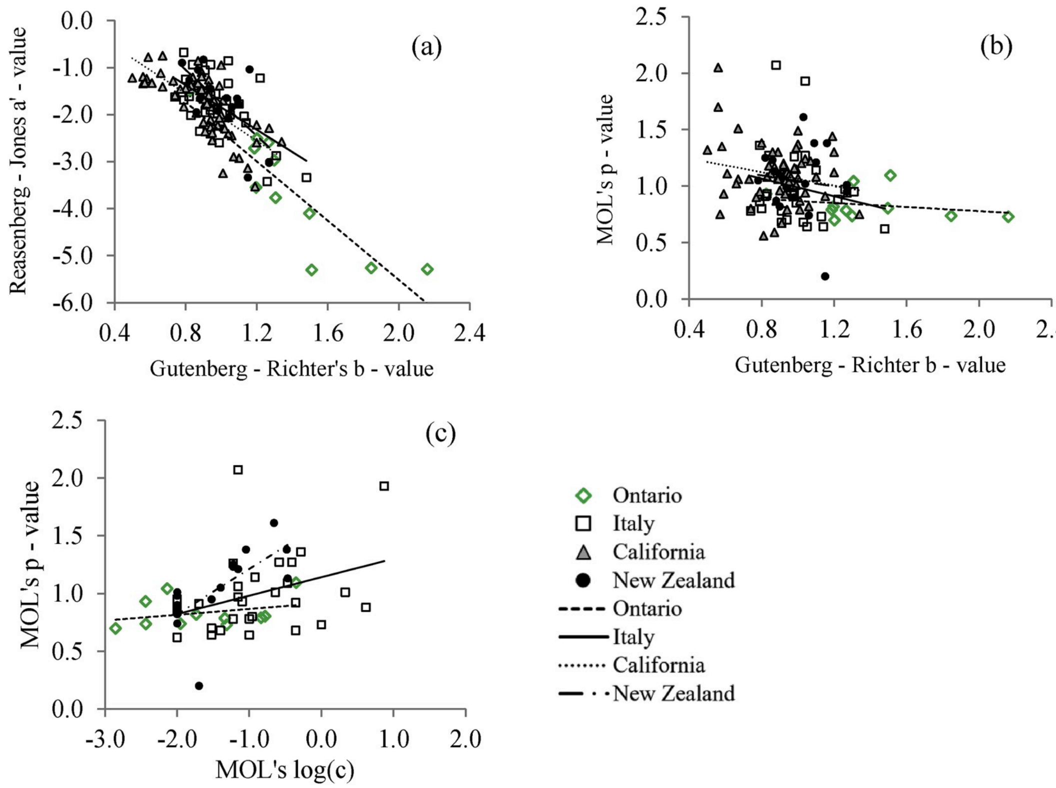

Fig. 2. Comparison between the (a) a' and b parameters, (b) b – value of Gutenberg – Richter's law and the p – value of the Modified Omori's law and (c) p and log(c) parameters of the Modified Omori's law for mining related and tectonic[1] aftershock sequences.

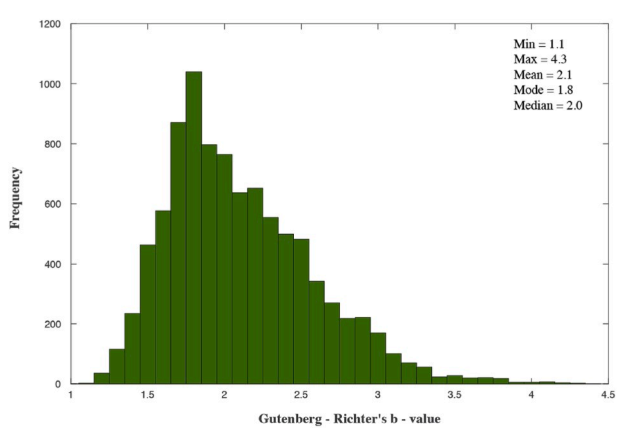

Fig. 3. Histogram of events vs Gutenberg-Richter b value for a mining site in Chile.

$$A(p,c,S,T) = \begin{cases} \ln(T+c) - \ln(S+c) & p = 1 \\ \left[(T+c)^{1-p} - (S+c)^{1-p}\right] \Big/ (1-p) & p \neq 1 \end{cases}. \tag{16}$$

The maximum likelihood estimates of K, p and c, are those values that maximize equation (15). The MOL's parameters have been estimated into the complete duration of the sequence, i.e. $[S, T] = [t_0, t_N]$, where t_0 and t_N are the occurrence times of the main and last events in the clustered sequence, respectively.

The standard errors of the maximum likelihood estimation of the parameters K, p and c, are the square roots of the diagonal elements of the inverse of the observed Fisher information matrix[44] (equation (17))

$$J(K,c,p,S,T) = \int_S^T \begin{bmatrix} K^{-1}(t+c)^{-p} & -p(t+c)^{-p-1} & -(t+c)^{-p}\ln(t+c) \\ * & Kp^2(t+c)^{-p-2} & Kp(t+c)^{-p-1}\ln(t+c) \\ * & * & K(t+c)^{-p}\{\ln(t+c)\}^2 \end{bmatrix} dt. \tag{17}$$

Combining the GR law and MOL, the aftershock occurrence can be described as a non-stationary Poisson process.[1] Reasenberg – Jones model expresses the rate λ of aftershocks with magnitude M or larger, at the time t following a main shock of magnitude $M_{w,m}$ as follows in equation (18)[10]:

$$\lambda(t,M) = \frac{10^{a' + b(M_{w,m} - M)}}{(t+c)^p} \tag{18}$$

where p and c are the modified Omori's law parameters (equation (14)) and the b - *value* is the Gutenberg-Richter's coefficient (equation (7)). Finally, a' value can be written as:

$$a' = \log(K) - b(M_{w,m} - M) \tag{19}$$

where K is also a MOL's parameter.

The authors refer to the *a' – value* (equation (19)) as the "productivity" of the sequence, which becomes a higher value for larger earthquakes.[10] In general, it can be described as the aftershock density following a mainshock.

The Reasenberg Jones model has been applied in different parts of the world, affirming that it is valid for a large variety of datasets and regions.[1]

2.2.3. Correlations between seismic parameters

The physical meaning of the parameters involved in the seismic laws and their relations constitutes a problem that has been widely discussed[1] (and references there in). Particularly, it is possible to study the link between the Gutenberg – Richter's law parameters (a, b), Modified

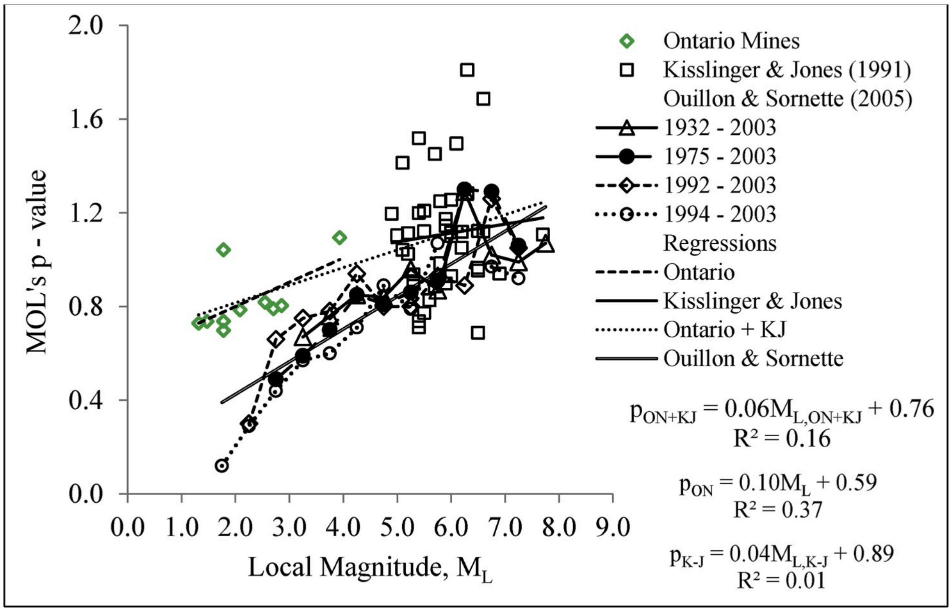

Fig. 4. *p-values as a function of the main shocks in local magnitude (M_L) for the mining related aftershock sequences (Ontario Mines), Ouillon & Sornette's California aftershock sequences from 1932 to 2003 and Kisslinger & Jones's California aftershock sequences from 1932 to 1988 (modified from[59]).*

Omori's Law parameters (c, p, K) and the Reasenberg – Jones productivity factor (a') in order to understand their correlations and the aftershock decay process. Several authors have calculated for events all over the world a', b – *value* and Omori's seismic tectonic parameters.[45–47] Particularly in Japan,[11,33,41,48–50] California,[1,10,32,37,51] Italy[1,51] and New Zealand.[1,52] A spread and most representative range of tectonic settings were considered.

For the b and p – values from worldwide sequences no significant correlation was found.[1] However, when using a regional subdivision of Japanese seismic sequences and comparing interplate against intraplate seismicity, a positive correlation between these two parameters is found.[11] Furthermore, a good correlation was found for c (or $log(c)$) and p, which is related to an interaction between these parameters.[1] As was also discussed by Gasperini & Lolli (2006), the MOL's parameters *p & c* are strictly correlated and the inclusion in the rate formulation has some effects in smoothing the decay of the modeled rate in the period shortly after the mainshock.[1] In fact, higher values of c correspond to a slower decay right after the mainshocks, counteracted by a higher p – *value* and vice versa.

Between a' and b a significant negative correlation has been observed by Gasperini & Lolli (2006).[1] They concluded that the productivity is higher with larger magnitudes events, in which the b – *value* tends to decrease.

3. Data source

Eleven aftershock sequences have been collected from large magnitude events in mines all over Ontario, Canada. Nuttli magnitude (M_n), moment magnitude ($M_{w,m}$), local magnitude (M_L), date of the mainshock, total duration of the sequences (t_N) and the total number of events (N) in each sequence for each site are presented in Table 1. For this study a wide variety of mining methods, geological and seismic settings is considered to evaluate the range of aftershock statistics that can be found in mining operations in Ontario. A brief description of mining methods, geology and micro seismic monitoring systems can be found in Refs. 14,17,39,53,54.

The total duration of the sequence (t_N) was estimated by the ratios method,[55] and following the considerations of.[38] This method evaluates the ratio

$$r(N_b, N_a) = T_{N_a} / T_{N_b} \tag{20}$$

where T_{Na} and T_{Nb} in equation (20) are the times of occurrence of the N_a^{th} and N_b^{th} events following and preceding the principal event, respectively. Subsequent events are identified as aftershocks if the above ratio is smaller than a critical value generated by a random process with a certain probability. The values used to evaluate the ratios methods were $N_a = 1$, $N_b = 5$ with a probability of 1%, giving a critical value of $r_c(5,1) = 0.002$.[38] The start of the sequence is defined if the ratio $r(5,1)$ is less than the critical value for a group of at least three consecutive events.

Each aftershock sequence was filtered by limiting the source location error (Δr) and by the magnitude of completeness ($M_{w,c}$). These two filters remove effectively poorly located seismic events and provide uniformity to the data, respectively. For the analysis, only seismic events with an associated hypocentre location error less or equal than 50 m were considered. The $M_{w,c}$ was selected with the Goodness of Fit method[56,57] (Table 2). Even though the omission of poorly reported events can bias the estimation of the statistical parameters, this method constitutes a common procedure[14,17] that guarantee the cluster reliability.

To select a unique value of $M_{w,c}$ for all the sequences, the following methodology is applied[14]: first, a tentative $M_{w,c}$* value is selected. Only aftershock sequences that satisfy $M_{w,c} \le M_{w,c}$* are considered for the analysis. The sequence is filtered (error location and magnitude of completeness) and clustered. After this, only sequences with more than 10 events are considered for the analysis. Then the Gutenberg – Richter's and the Modified Omori's Law parameters were estimated. The correlations between the seismic parameters and the magnitude of the mainshock were derived. Finally, the selected $M_{w,c}$* is the value which maximizes the number of sequences in the analysis, the correlation between the seismic parameter (Table 3) and is equal or greater than the maximum $M_{w,c}$ value of all the sequences (in this case −1.40, Table 2). Based on this procedure a value of $M_{w,c}* = -1.30$ was determined.

4. Results and discussion

We carried out statistical correlations between the gained parameters from mining related and tectonic sequences. Table 4 is showing the results of the seismic mining related micro seismicity parameters for the

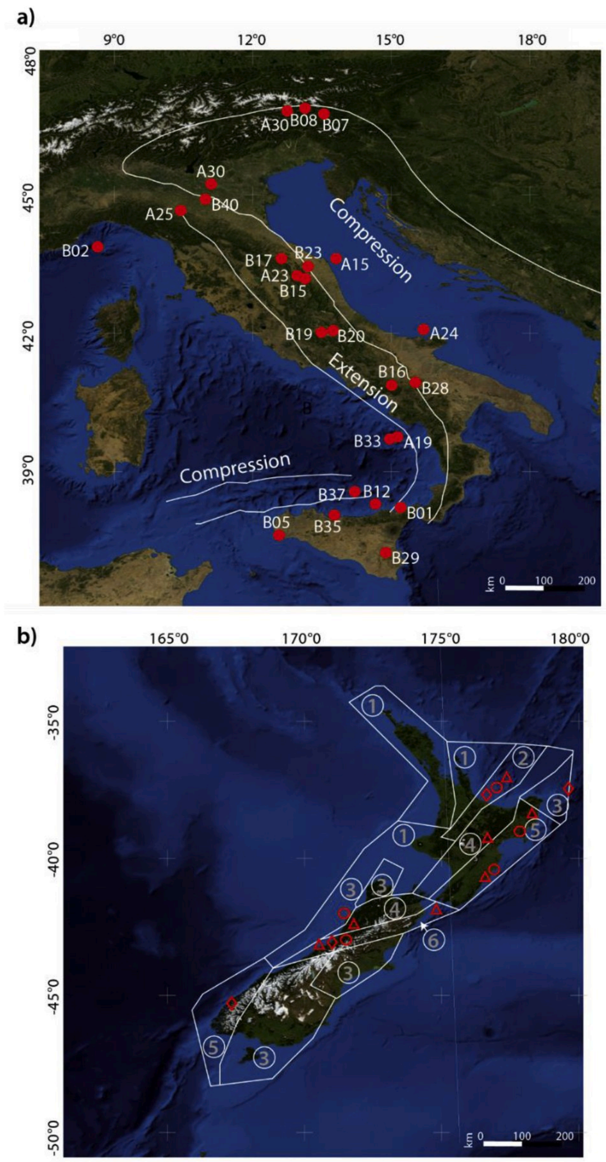

Fig. 5. Faulting zones defined by: (a) Modified from .[60] Each name in the red dots corresponds to the originally sequence's name used by[1]; (b) Modified from .[61] Each number refers to a faulting style: 1. Normal, 2. Normal (Termal Volcanic Zone), 3. Reverse, 4. Strike-slip, 5. Reverse & Strike-slip, 6. Strike-slip & Reverse. Symbols and labels are the same used by Eberhart-Phillips (1998)[52]: $M_w \geq 5.5$ (triangles), $M_w \geq 6.0$ (circles), $M_w \geq 6.5$ (rhombus). (For interpretation of the references to color in this figure legend, the reader is referred to the Web version of this article.)

Table 7
p – values for Italy and New Zealand for each type of focal mechanism. In case of New Zealand, the mechanism strike slip + normal/thrust or normal/thrust + strike slip were considered as strike slip and normal/thrust respectively.

Focal mechanism	Italy	New Zealand
Normal	1.05	1.27
Thrust	0.94	1.04
Strike slip	–	0.93

eleven seismic sequences recorded in Ontario. The errors for b – *values* and MOL's parameters are calculated using equations (8) and (17) respectively. Two seismic sequences are showing high errors: sequences 3 (CCN) and 6 (Creighton). They were removed and the calculations were rebuilt (see appendix A). However, no significant variations could be observed therefore they were preserved in the analysis. In Table 5 the values for tectonic seismic parameters are given combined with the average results obtained for mining related aftershock sequences, stated in the first row of Table 5.

The average of the b – *value* in mining related seismicity ($b = 1.39$) is higher than each b – *value* calculated for tectonic seismicity and certainly higher than its average ($b = 1.05$) (see Table 5). According to the results obtained by Schorlemmer et al. (2005), the mining related micro seismicity b – *value* obtained for the Ontario sequences is out of range with respect to any tectonic regime, even taking into consideration several seismic catalogues.[12] Meanwhile, the average of the a', p and c parameters of the mining related micro seismicity are lower than the average of the tectonic seismic parameters. Particularly, the value of a' for the tectonic sequences is approximately only a half of what was observed for the mining related aftershock sequences, related with a low productivity probably linked with the stress mining control during the process of fracture of the rock. Same for the K – *value,* which mean is considerable high for tectonic events (see Table 5).

Due to the mathematical coupling between p and *c-value*, both parameters will be analyzed at the same time in reference to the results obtained by.[17] In there, $p = 1$ (a similar value to that obtained in this research for the tectonic and mining related micro seismicity averages) is considered to analyze the coupling between p and c parameters with time. Regarding that $c = 0.08$ and $c = 0.17$ for mining related and tectonic events, respectively (Table 5), the decay rate is higher for mining related micro seismic events during the first hour, where it becomes very similar at the end of the sequence. Two hypotheses arise with respect to this point.

First, this value, beyond to be a mathematical artifact, seems to be controlling the equation for decay processes (Eq. (14)). Both parameters acting together could be consequence of the rock's preconditioning during the mining process before a rock burst. The last, could reduce the total aftershock prevalence time mainly in the first time after the main event. Finally, considering the results obtained by Hamaguchi & Hasegawa (1970), the smaller c – *values* in mining related micro seismicity also could reflect how the complexity of the rupture process affects the signal detection during the tectonic events.[42]

Second, the difference in the c *value* for mining related and tectonic seismicity could also be related with the ability of the network to detect small earthquakes. Accordingly, a lower *c-value* might only reflect the higher sensitivity with respect to sparser regional networks. Regarding this point, the inter-sensor spacing measurement could be considered as an additional method, as an option to GoF (see Table 2), to re-analyze the minimum magnitude value. The last, in reference to which earthquake size can be consistently detected by a sensor array deployed in a defined area.[58] As an example of this point, according to the linear regression obtained by,[58] the completeness of seismic record for the mines Kidd Creek (area: 300 x 9200 m^2;sequence 11) and Craig (area: 1000 x 1300 m^2; sequence 5) correspond to $M_{w,ci} = 0.3$ and $M_{w,ci} = -1.3$, respectively. These magnitudes, if bigger than those obtained in Table 2, are still small values, supporting the effect of the array sensitivity in the *c-value.*

In order to better understand the connection and possible relations between the seismic parameters, Table 6 is presenting a summary of the correlations for the aftershock sequences for mining related and tectonic seismicity, which are graphically displayed in Fig. 2 and Fig. 3. For the seismic parameters of Ontario mining related aftershock sequences, p and b *values* could be correlated with $M_{w,m.}$ whilst between p and b *values* no statistically significant correlation was found neither for mining related nor tectonic aftershock sequences. a' shows a statistically significant negative correlation with the b – *value*, and parameter c is correlated with a' and p, although these correlations are not statistically significant. However, for tectonic aftershock sequences the b – *value* has a profound negative statistically correlation with a' in all cases.

Fig. 2 is presenting a graphical comparison between the results obtained for mining related (always in green color) and tectonic aftershock sequences from California, Italy, New Zealand and Japan.

In Fig. 2a and b it is possible to see a similar tendency regarding tectonic and mining related aftershock sequences. The behaviour of the Ontario's parameters are coherent with the tectonic ones, where the values of the mining related micro seismic parameters seems to be a continuation of the tectonic parameters in the lower *a'* and *p* values and higher *b – values*. Moreover, there is a clear overlap in the mining related and tectonic seismicity curves for b - values between 0.83 and 1.34 (Fig. 2a). The few sequences that are in the overlap zone (sequences number 2, 5, 7, 9 and 11) don't have any relationship between them, regarding for example spatial location, magnitude, sequence recording time or number of registered and used events. However, all of them register b values close to 1, which is typically related to tectonic seismic sequences.[12] These results, which are even considering several tectonic settings, have a high potential in order to establish a numerical range of *a'*- and *b - values*, which could be exclusively used for mining analysis purposes. Particularly, the obtained *b - values* for mining related micro seismicity would be in the range of 1.4 and 2.3.

In order to corroborate this value range, a parallel approach was used to compare the *b-values* considering a different mining site. The *b-value* was calculated using a four years seismicity catalogue obtained from a hard rock mining site in Chile (Fig. 3).

As can be observed in Fig. 3, mean, mode and the median are inside the proposed range. The minimum value, 1.1, is still in the overlap zone (Fig. 2). To a better understanding of this behavior, it is recommended to improve the analysis using seismic sequences of this Chilean mining site.

The lower correlation between *p* and *b* parameters, although not statistically significant, could be explained considering the different values of the *b* parameter in the mining related aftershock sequence. Neglecting this data shift, *p – values* for tectonic and mining related micro seismicity are in general in the same range ($\sim 0.5 - 1.5$), therefore the mining related micro seismicity rate decay value could be related with those founded in tectonic regimes.[1,37]

In Fig. 2c, a comparison between *p* and *log(c)* is done. The mining related micro seismicity parameters from Ontario show a positive correlation, which also occurs for the data of Italy and New Zealand. This correlation was explained by Gasperini & Lolli (2006) because of the interplay between those parameters within the maximum likelihood estimation procedure.[1]

Fig. 4 compares the results obtained by,[59] and the mining related aftershock sequences. This figure indicates that mining related aftershock sequences presents higher *p – values* than those presented by[59] for seismic events with local magnitude $M_L < 4.0$, which corresponds to a faster decay of the Omori's law. A possible reason of this difference is that given the years of the seismic data (1932 – 2003) it is difficult to reach a magnitude of completeness lower than 3.0. This effect could underestimate the *p – value* or incorporate higher errors in its estimation. Moreover, the *p – value* has a relation with the type of focal mechanism (normal, strike slip and thrust). Considering the magnitude of completeness not a problem and following the results of Tahir (2011),[13] the lower values of *p* coincide with the strike slip mechanism. Thus, the higher values of *p* for the mining related aftershock sequences may represent a normal mechanism. This last point can be verified approximately using the seismic aftershock sequences of Gasperini & Lolli (2006).[1]

Finally, we relate the p - values obtained by Lolli & Gasperini (2006)[52] with the seismogenic zones proposed by Chiarabba et al (2005)[60] and Stirling et al (2002)[61] for Italy (4a) and New Zealand (4b), respectively (Fig. 5). From this comparison it can be obtained that the high *p - values* correspond to normal mechanisms, intermediate values to thrust mechanisms and low values to strike slip mechanisms (Table 7). Given the above and considering that the *p – value* has a similar behavior for both mining related and tectonics seismicity (as was discussed in this paper), we would expect that mining related micro seismicity *p – values* are also related to the character of focal mechanisms. However, this requires verification by calculating the mechanisms of the mining related micro seismicity main event and correlate it with the *p – value,* which could not be done for the presented dataset due to lack of the seismic traces.

5. Conclusions

A significant difference in the range of the seismic parameter values can be observed by establishing correlations between them. Specifically, correlation of *a'* and *b – values* seems well adapted to, mining related or tectonic, even though there still is an obvious zone where both types overlap. Why an a'-b correlation is such beneficial is most likely due stress differences during the mining related and tectonic seismicity rupture processes. As a seismological implication, we established a new tool to distinguish between mining related and tectonic seismicity and determined defined values of these seismic parameters that are characteristic for mining related micro seismic activity. According to this result, *b-values* are in the range between 1.4 and 2.3.

The inverse relation between the Reasenberg – Jones factor *a'* and the *b – value* could be extensively used for mining safety purposes. Provided the knowledge of the local tectonic *b – values* for a mining zone, an estimation about the mining related micro seismic productivity and the rate decay can be done.

Finally, empirical relations could be obtained by comparing mining related and tectonic seismic parameters in the same tectonic setting. With this step, the validity of the seismic correlations in the interpretation process can be avoided. Other implication considering this case is the possibility to include the geological frame and mining parameters - fluid saturation and content, mining production and stress/strain fields - into the analysis, regarding that probably the seismic parameters are closely related with the type of rock in which the mines are emplaced and also the mining process.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix B. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.ijrmms.2020.104242.

Appendix A

Considering the high errors that could be observed in sequences three (CCN) and six (Creighton) (Table 4), the seismic parameters were re-calculated in order to estimate their influence in the results. For this purpose, Tables 5 and 6 were partially replicated using two different approaches, which are show in Table A.1 and Table A.2 respectively.

Table A.1
Mean and standard deviation for the seismic parameters, without considering sequences three and six.

Parameters	Without sequence 3		Without sequence 3 and 6	
	Mean	Standard Deviation	Mean	Standard Deviation
a'	−3.58	1.36	−3.39	1.3
b	1.4	0.38	1.35	0.36
p	0.81	0.12	0.82	0.12
K	26.62	23.7	29.09	23.74
c	0.09	0.14	0.1	0.14

Table A.2
Coefficients, intercepts and correlation between seismic parameters of the Ontario's mining induced aftershock sequences, without considering sequence three and six. The first parameter corresponds to the dependent variable and the second one to the independent variable. The results are presented following the structure in Table 6.

Parameters	Without sequence 3				Without sequence 3 and 6			
	Coefficient	Intercept	ρ	Student-t p' – values	Coefficient	Intercept	ρ	Student-t p' – values
b, $M_{w,m}$	0.08	1.24	0.13	> 0.05	0.1	1.13	0.19	> 0.05
p, $M_{w,m}$	0.08	0.64	0.45	> 0.05	0.08	0.65	0.44	> 0.05
$log(c)$, $M_{w,m}$	0.99	−3.7	0.77	< 0.05	0.98	−3.7	0.76	< 0.05
a', b	−3.18	0.87	−0.89	< 0.05	−3.06	0.74	−0.86	< 0.05
a', p	−1.43	−2.42	−0.12	> 0.05	−2.67	−1.20	−0.25	> 0.05
a', c	−4.49	−3.18	−0.46	> 0.05	−5.55	−2.85	−0.62	> 0.05
a', log(c)	−0.85	−4.94	−0.51	> 0.05	−0.98	−4.92	−0.65	> 0.05
b, p	−0.77	2.03	−0.24	> 0.05	−0.50	1.76	−0.17	> 0.05
p, c	0.67	0.75	0.78	< 0.05	0.65	0.76	0.77	< 0.05
p, log(c)	0.07	0.93	0.49	> 0.05	0.07	0.93	0.48	> 0.05

Regarding the results in Table 5, there is no significant difference when analyzing the behavior of the seismic parameters. For this reason, the eleven sequences were considered in the final results. Even more, the obtained ranges mainly for the b-values, seems to be very stable and not being affected by the sequences with the higher errors. With respect to the relation between the parameters (Table 6), the statistical coefficients are exposed in Table A2. Same conclusion could be inferred.

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