Transportation Research Part A 138 (2020) 187–212

Contents lists available at ScienceDirect

Transportation Research Part A

journal homepage: www.elsevier.com/locate/tra

Comparing social costs of public transport networks structured around an Open and Closed BRT corridor in medium sized cities

Francisco Proboste[a,d], Juan Carlos Muñoz[a,d,*], Antonio Gschwender[b,c]

[a] *Pontificia Universidad Católica de Chile, Vicuña Mackenna 4860, Macul, Santiago 7820436, Chile*
[b] *Universidad de Chile, Blanco Encalada 2002, Santiago 8370449, Chile*
[c] *Instituto Sistemas Complejos de Ingeniería (ISCI), Chile*
[d] *Bus Rapid Transit Centre for Excellence, Chile*

A R T I C L E I N F O

Keywords:
Network design
Bus rapid transit
Medium sized city
Continuum approximation
Public transport

A B S T R A C T

Bus Rapid Transit (BRT) has proven to be an effective and affordable transportation option for large-sized cities. In these cities, BRT is usually considered an effective complement or substitute for rail-based systems, playing a key role in complex multimodal networks with several massive transport corridors. More recently, medium-sized cities of less than 200,000 inhabitants have also considering implementing BRT as a means of mass transit. These cities usually need only a few of these massive transport corridors (often just one), and they must decide how to structure their services. This report discusses which of the two types of BRT-based networks is best for the social interest in the case of medium-sized cities: (1) Closed BRT, in which buses operating inside and outside the corridor are separated and have different designs, or (2) Open BRT, in which the same buses operate inside and outside the corridor, entering and exiting at different points along a route. To answer this question two models with different levels of detail in terms of a city's characteristics were developed to represent both agency and user costs. In the first model a classic idealized city approach is addressed, while in the second model the problem is solved for the specific geographic characteristics and constraints of a real city. The results based on both models show that when it is optimally configured, Closed BRT networks offer mid-sized cities higher frequencies and lower waiting times. However, these benefits do not offset the cost associated with higher number of transfers that Closed BRT networks require, as compared to Open BRT networks. Transfers not only affect users due to the transferring experience, but also end up making the entire system slower. Overall, Open BRT shows significantly less Total Costs than Closed BRT in most of the scenarios that were analyzed.

1. Introduction

Bus Rapid Transit (BRT) was developed in large metropolitan areas, but many medium-sized cities also consider it as an effective and affordable transportation option (Cervero, 2013). Trip length, demand patterns, road network infrastructure and intermodality are quite different in large and medium-sized cities (Kline et al., 2012). Medium-sized cities are also less complex and allow for simpler analyses than for larger cities. This research contributes to the discussion of how massive bus-based public transport could be designed for medium-sized cities in which as few as one segregated corridor is needed.

In this paper, we assume that the city in question will have a single bus-segregated corridor, structured as a BRT, around which all

* Corresponding author.
E-mail address: jcm@ing.puc.cl (J.C. Muñoz).

https://doi.org/10.1016/j.tra.2020.06.005
Received 21 July 2019; Received in revised form 23 May 2020; Accepted 3 June 2020
Available online 20 June 2020

other regular bus services will be organized. We examine the degree of segregation of the operations inside and outside the corridor, comparing a Closed and an Open BRT. These two BRT network schemes are the focus of this research. We assume that in both cases the system is designed in terms of routes, bus sizes, frequencies and distances between stops by a central planner. Additionally, we assume that in both cases the fare is the same for all trips and that a a fare is not requested to transfer (integrated flat fare managed by a transit authority).

Many case studies and reviews of BRT systems can be found in the literature of the last decades. For example, Levinson et al. (2002) discuss BRT in the context of transport planning processes in the United States, providing guidelines and recommendations. Hensher and Golob (2008) review 44 BRT systems in operation throughout the world and argue that high capacity integrated BRT systems are in many contexts an attractive alternative. The review by Wirasinghe et al. (2013) highlights that each BRT system is unique and capital costs cannot be generalized from one place to another. Recently, Kathuria et al. (2016) provide a detailed review of the explosive implementation of BRT systems in India in the last years.

1.1. Closed BRT

In a Closed BRT the operations inside and outside the corridor are separated. This allows for the allocation of different types of buses to the two types of service. Buses operating inside the corridor as the trunk service will be larger than those operating in the feeder services. The horizontal line in Fig. 1 (labeled as line T) represents the trunk service along the corridor, while the five vertical ones (labeled 1 to 5) represent the feeder services operating outside of it. In a Closed BRT, since feeder buses are not allowed to enter the corridor, transfer stations allowing users to move from one service to another are necessary; in this sense, a closed BRT is similar to a Metro line (in which users must also transfer, since feeder buses cannot use the Metro infrastructure). This BRT scheme is also referred to in the literature as a Trunk and Feeder configuration. It is worth noting that this configuration is similar to a rail/feeder-bus system, as has been studied for an urban corridor by Chien and Schonfled (1998)[1] and in a rectangular grid with a many to one demand pattern by Wirasinghe (1980).

1.2. Open BRT

In an Open BRT the buses operating outside the corridor also provide their service along it, entering and leaving the corridor at different points towards the central business district (CBD). Therefore, the need for users to transfer to reach their destinations is lower than in a Closed BRT. In a first stage of this study, we assumed that all lines of the Open BRT network end at the CBD, yielding one line per periphery micro-zone. This leads to more lines than in the Closed BRT case where there is one feeder line for every two periphery micro-zones. However, we later considered pairing a westbound and an eastbound line at the CBD into a single line, in order to reduce the number of transfers. This is a much more attractive option if the CBD is centrally located along the corridor since the number of connections between lines coming to the CBD from opposite sides of the city is maximized. On the contrary, if the CBD is located in an extreme of the corridor, connecting pairs of lines would not provide a convenient service for passengers traveling between different periphery areas in the city, as they would rather transfer in a corridor station before reaching the CBD. Also, by operating independent lines in each side of the city the frequency of each one could be optimized as a separate variable. Thus, in such a case the Open BRT will operate with lines connecting each urban area with the CBD as its terminal station and therefore not visiting the opposite side of the corridor without a transfer. Hence, when the CBD is not in the middle of the corridor, the Open BRT will have more lines than the Closed BRT for an identical spatial coverage.

Fig. 2a shows a diagram representing an Open BRT, in which all the lines (labeled 1 to 5) from the system use the corridor infrastructure and where the CBD is located in the center of the corridor, similar to the model of Newell (1979). Fig. 2b shows the case of an Open BRT where the CBD is located in one extreme. In this case, most lines use the corridor, but the line that crosses the CBD connecting two periphery micro-zones does not. In the application in Section 3.2, we will use this case with the CBD in one extreme. Notice that in an Open BRT the network lacks a trunk line operating strictly in the corridor. In the literature Open BRT networks are also referred to as either Direct or Flexible Services.

1.3. Research objectives and methodology

The goal of this research is to compare the performance of Open and a Closed BRT networks in the context of medium-sized cities, to understand the tradeoffs involved in this comparison, and to recognize under which circumstances each one is the most suitable. While many of the conclusions we reach could potentially be applicable in large cities, we make assumptions that are characteristic of medium-sized cities, such as low road congestion, little bus interaction at stops, and low variability of travel times. We also assume that the public transport network can be structured around a single BRT corridor. Thus, the results obtained from our model should be applied to cities meeting these requirements.

The problem stated in this research fits into the context of the Transit Route Network Design Problem (TRNDP). This is a classic research area in Transit Planning, involving the strategic decisions of this planning process (Guihaire and Hao, 2008; Kepaptsoglou

[1] As will be shown in detail in section 2, our model improves the one by Chien and Schonfled (1998), as well as others reviewed here, in several directions. Cycle time and users' in-vehicle time depend on the number of passengers boarding and alighting, costs depend on the size of the vehicle, vehicle size is optimized, and a transfer penalty is included.

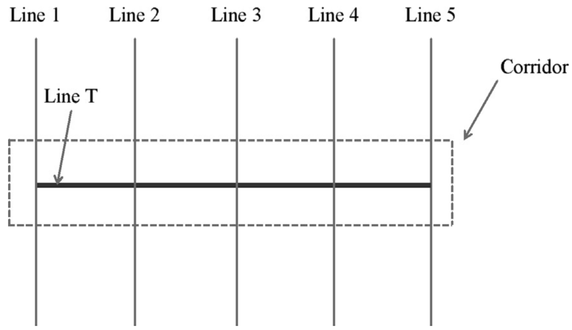


Fig. 1. Closed BRT network.

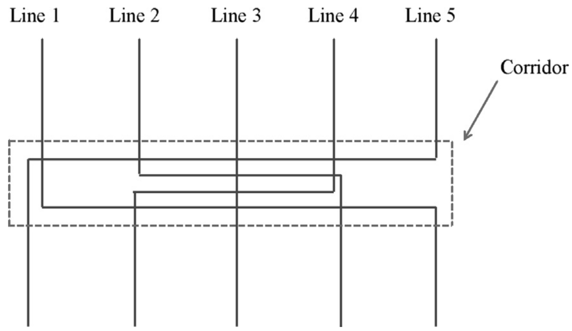


Fig. 2a. Open BRT network with CBD in the center.

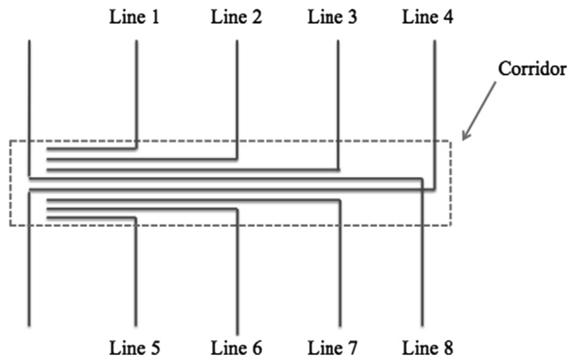


Fig. 2b. Open BRT network with CBD in an extreme.

and Karlaftis, 2009). This problem has been addressed quite extensively in the last decade. For a thorough review, we refer to Ibarra-Rojas et al. (2015).

Several authors have categorized TRNDP in different research lines (Kepaptsoglou and Karlaftis, 2009). There are two main approaches to modeling networks. One is based on analytical models that use continuous approximations to model the main network characteristics. These models lead to simple analytical expressions that can often be optimized, highlighting the tradeoffs between elements of the system. Ceder (2001) argues that this approach is helpful both for policy analyses and for the strategic design of real-size transit networks. Under this approach, Gschwender et al. (2016) compared direct services with a feeder-trunk scheme – although without taking advantage of BRT characteristics – on a Y-shaped network, finding that the best solutions depend on the level of demand, the length of the trips and the value of transfer costs, among other parameters. Fielbaum et al. (2016) compared different bus lines structures, both with and without transfers, on a parametric city model where monocentric or dispersed cases can be represented parametrically. This shape of the demand affects the optimal lines structure, as well as the total demand level. Kocur and Hendrickson (1982) and Chang and Schonfeld (1991) analyzed the optimal line spacing and their optimal frequency, on a rectangular zone where trips with a single destination are originated. Kocur and Hendrickson solved the problem numerically with an elastic demand, but Chang and Schonfeld simplified it to obtain closed analytical solutions. Using a parametric demand and other simplifying assumptions, they found that both optimal frequency and optimal number of lines are proportional to the cube root of the demand. Kuah and Perl (1988) also used the continuous approximation approach for the optimal line spacing and frequency of bus routes, adding the stop spacing as a decision variable. Wirasinghe et al. (1977) obtained explicit solutions for the optimal railway frequency and distance between stations, and the bus feeder boundary in a model of a circular city with bus-railway and direct bus options. Several other studies have looked into the best public transport lines structures on a regular rectangular grid (Newell, 1979; Daganzo, 2010; Badia et al., 2016), on a regular radial-concentric grid (Byrne, 1975; Tirachini et al., 2010; Saidi et al., 2016) or both (Chen et al., 2015). The second approach uses detailed discrete models leading to NP-Hard optimization problems. Since these can be optimally solved only for very small networks, heuristic and meta-heuristic procedures are used for the resolution. This is the case in Dubois et al. (1979), Ceder and Wilson (1986) and Cenek (2010), who propose different heuristic algorithms. The second approach requires extensive use of computer resources. In this paper, we use the continuous approximation approach to build an analytical model that allows the identification of relationships among variables. Nevertheless, due to the complexity of the expressions the model must be solved numerically.

In Section 2 of this paper, we assume a stylized generic medium-sized city model and borrow ideas from the Continuum Approximation Approach to optimize the transit network that will operate in this city (Daganzo and Ouyang, 2019; Newell, 1979). In Section 3 the model is used to fit the context of the city of Valdivia in Chile in two approaches: (i) as a generic rectangular city, and (ii) using a detailed model based on the real origin destination demand matrix and Geographic Information System data of the city, and considering the road infrastructure which limited the maximum number of bus lines that could be offered. In both cases, the two BRT networks (Open and Closed) are evaluated in terms of user and agency costs, aiming at revealing the key parameters that determine which one is the best option for a medium-sized city. Finally, in Section 4 we present the conclusions of the paper.

2. Continuum approximation model for a generic rectangular city

In this section we introduce the continuum approximation model to compare the convenience of implementing an open or a closed BRT in an idealized city. The model considers several assumptions that should be highlighted. We not only assume a rectangular city with a road network configuration that is later described in this Section, but also several assumptions that are a consequence of a deterministic consideration of the operation of the system and its demand. This deterministic approach is not different from most of the papers listed in our literature review. The main assumptions are:

- The origin destination matrix between zones is given and inelastic to the level of service. During the whole period of analysis, the intervals between the times that consecutive passengers start their trip from a specific origin to a specific destination are constant.
- The optimal frequency is continuous and has no upper bound.
- Bus lines operate without a fixed schedule (no timetables), therefore passengers arrive randomly through a Poisson process to the first stop of their trip. Buses pass evenly according to the programmed headway of the line (the inverse of its frequency), but some degree of bunching is considered. Therefore, waiting time is equal to k/f, with k being a constant and f the programmed frequency of the line. The constant k would be 0.5 if there were no bunching but is larger otherwise. Waiting times occur every time a passenger boards a new bus (including transfers). As frequencies will be relatively large, no extra buffer waiting times are considered.
- The model optimizes vehicle capacity, assuming that it can take a continuous size. The operation cost of each bus depends on this size. The bus size of each line is obtained from its maximum load which every bus in the service reaches. Passengers always board the first arriving bus.
- The number of lines in the model must take a discrete value
- Stop spacing and line spacing are both optimized. However, strictly different results from these two variables would yield different transfer experiences at the BRT corridor. In this model we neglect this detail by assuming a constant walking penalty at each transfer, implicitly included in the transfer penalty.
- Stops spacing is identical everywhere in the periphery. The BRT corridor stations are also evenly spaced.
- Travel times between stations are assumed constant so there is no vehicle congestion considered.
- The dwell time of each bus at each station is affected by the number of boarding and alighting passengers. Thus, increasing the frequency of a service reduces its dwell times, and therefore increases its operational speed.
- Infrastructure costs are neglected

2.1. The city and network models

We developed a generic medium-sized rectangular city model using the typical urban spatial layout of Chilean medium-sized cities as a reference. This model divides the city into five macro-zones as shown in Fig. 3: a CBD, with the city's main avenue crossing it from west to east; two zones along the main avenue (CW and CE), west and east of the CBD; and two periphery zones at the north and south of the main avenue (PN and PS). The dimensions of the city, as well as the position of the different macro-zones are determined by the parameters α, α_1, β, β_1, R_1, R_2, as can be seen in Fig. 3.

The two types of BRT networks will be evaluated using this generic city model. As shown in Figs. 1 and 2, both networks use a BRT corridor to cross the city from east to west along the main avenue. In addition, there is bus coverage going in the North-South direction through secondary streets. In the Closed BRT network, East-West service is provided by a single trunk line that runs exclusively on the corridor, and feeder buses, which do not enter the corridor, provide the North-South services. In the Open BRT network, buses head from the periphery towards the center, enter the corridor and provide East-West service in a certain section, and then leave the corridor to connect another zone in the opposite periphery. We will design the Open BRT lines such that they meet in the CBD, providing a single transfer experience for any trip in the city, no matter where the CBD is located along the BRT corridor. As

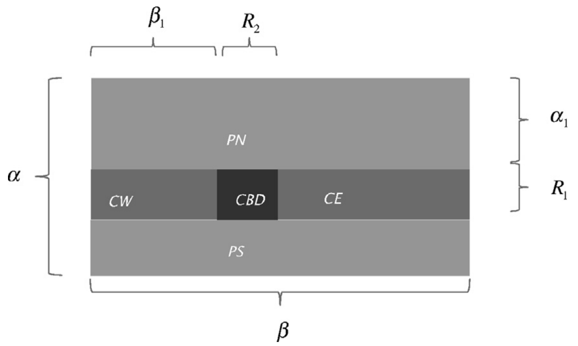


Fig. 3. Generic medium-sized city model.

shown in Fig. 2a, our design of open BRT services pairs zones from the extreme east with the extreme west, allowing a direct trip for some origin–destination pairs through the corridor.

As is mentioned above, this model uses concepts from Continuum Approximation methodology, in which it is assumed that local geographical attributes (e.g., demand) change smoothly over the whole modeled area. This assumption allows the use of analytical formulas that will reduce the complexity of the model. In this case this assumption holds within each macrozone. We are assuming that a demand discontinuity exists at the border between macrozones. The Continuum Approximation expressions derived in this paper are only valid within each macrozone.

2.2. Zones

Although we already have five macro-zones, for both BRT schemes it is necessary to subdivide the peripheral macro-zones (PN and PS) into micro-zones representing the area of influence of each line. The influence area of a periphery line consists of all the geographical points for which it is the closest transit service. The peripheral micro-zones are represented by the rectangles marked with dashed vertical lines in Fig. 4 (illustrating the open BRT case). The number of peripheral micro-zones is given by the number of lines that run in each periphery. This number can be different for the Open and Closed BRT.

The zoning inside the corridor is slightly different and is not determined through accessibility. We assume that any origin or destination in this area will be reached walking directly to or from the corridor. For them, the level of service varies depending on how many lines operate in the specific place where they access the corridor. One more or one less line at a given station in the corridor affects connectivity opportunities and waiting times for those users that can take the first line that is convenient to them. Thus, the corridor micro-zones are defined by the areas in which the corridor is served by the same set of lines, as shown in Fig. 5. This yields that the boundaries of the corridor micro-zones are determined by the entrance points of the peripheral lines. These entrance points coincide for the lines of both northern and southern peripheries, although they are drawn in slightly different positions in the schematic representations in Figs. 4 and 5 to facilitate following the path of each line. Note that the frontiers of these micro-zones do not coincide with the frontiers of the micro-zones of the periphery. For the sake of simplicity, we assume that the corridor micro-zones are all rectangular. However, it could be argued that for some trips originating (or ending) in a corridor

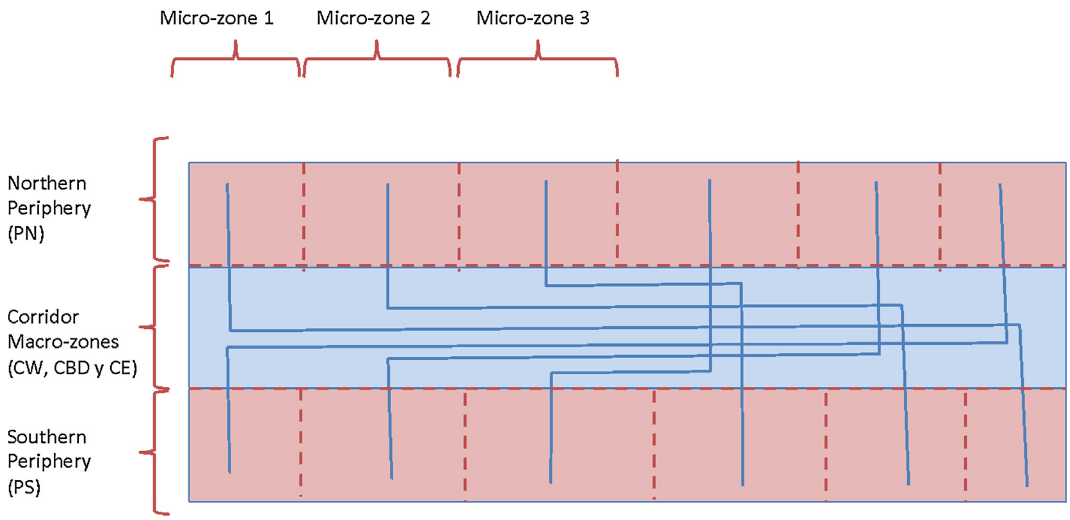


Fig. 4. Peripheral micro-zones.

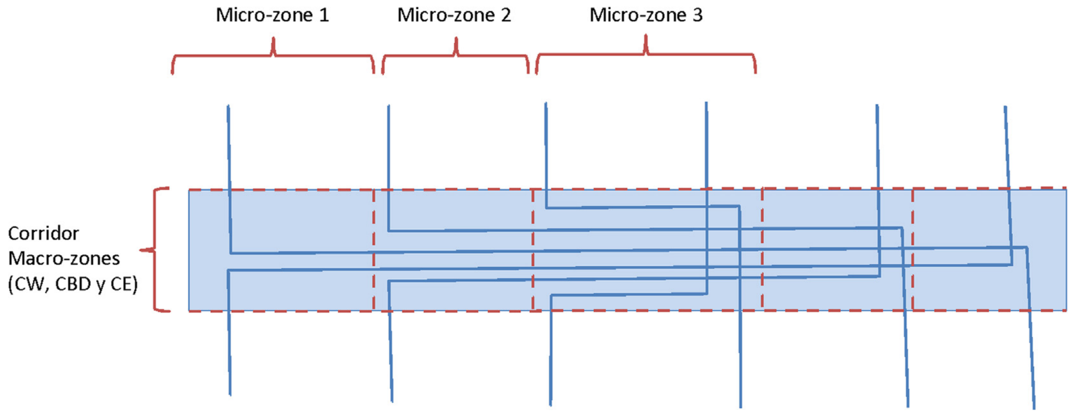


Fig. 5. Corridor micro-zones.

macrozone it could be more convenient to walk towards a stop in the segment where the line runs orthogonal to the corridor. This is more likely to happen for the Open BRT case, since in the Closed BRT it would involve an extra transfer in the trip. Thus, this simplification should mostly harm the modeled performance of the Open BRT. Similarly, we assume that passengers starting (or ending) their trip in a microzone, walk to (or from) the closest stop. However, in reality some passengers may prefer to walk directly to (from) the corridor to avoid a transfer. Similarly, passengers traveling from one microzone to another one may prefer a line that, not being the minimum walking choice, provides a direct trip. These behaviors are not captured by the model.

Once the city zoning and network of lines are defined, we explain in the next section how demand is incorporated into the model, the different metrics used to evaluate the performance of the networks, and the process implemented to compare the optimal design of each network type.

Notice that the number of zones grows with the number of lines in the network. Since the origin–destination matrix will be built using these zones, the demand structure will vary with the number of lines. This prevents us from simultaneously optimizing the network features (frequency, capacity of the vehicles, and distance between stops) and the number of lines. Instead we will optimize these features of the network for a given number of lines, and therefore for a given origin–destination matrix. The combination of a certain BRT network type (open or closed) and a certain number of lines will define a scenario to be analyzed.

The set of scenarios to be analyzed should consider that the number of lines in the system cannot exceed what is physically possible to insert in the city. Indeed, based on the above network configurations, the street layout of a city determines a maximum number of lines to operate in an Open BRT system and a maximum number of lines to feed a Closed BRT corridor.

2.3. Demand, evaluation metrics and optimization model

In this model, we assume that the demand for trips being generated and attracted in each unit area is known and does not depend on service quality (captive demand). This density is constant inside each macro-zone and therefore does not vary from scenario to scenario. In each scenario (i.e. Open or Closed BRT and a given number of lines), an origin–destination matrix between all micro-zones is built; e.g., using a double constrained gravitation model. Thus, for any given number of micro-zones, the origin–destination matrix used for analyzing an Open and a Closed BRT are identical.

Once the origin–destination matrix is obtained, demand is assigned to the lines of the network. Given the particular structure of the network, the specific routes taken by each of the passengers can be imputed before the characteristics of each line is determined (i.e. frequency, capacity of the vehicle, distance between consecutive stops, etc.) by a simple procedure. Thus, demand assignment consists of a simple model that considers common lines (i.e. that for trip legs originating inside the BRT corridor the user may take the first line showing up at the station from a set of convenient lines) and consists of the following steps:

- In PN and PS, users enter and exit the network through the line and stop that is closest to their origins and destinations, respectively. In the corridor macro-zones, users always enter and exit the network through corridor stops.
- At the first stop of their trip, users disregard any routes that require more than the minimum possible number of transfers. It should be noted that given the structure of both networks (shown in Figs. 1 and 2), the minimum travel time trip (waiting plus in-vehicle) always has the minimum number of transfers. Also notice that the network structures avoid passengers from needing to backtrack to reach their destination.
- From the set of all possible routes with the minimum number of transfers, a user will choose the route for which the sum of wait and travel times are minimized.

The output of the assignment procedure is a strategy followed by all users traveling from an origin in micro-zone i to a destination in micro-zone j. This strategy will consist of the set of transfer stops that they will visit, and the set of common lines that they will consider convenient in each trip leg. This strategy for travelers between i and j is independent of the specific origin and destination inside the respective micro-zones. Notice that this output is also independent of the number of trips in each origin–destination pair, and of the frequencies of each line.

Once the demand of a scenario is assigned, we proceed to determine the optimal frequency, vehicle capacity and fleet size of each line, and the stop spacing inside each micro-zone. Since a deterministic demand is assumed, vehicles will be full in the link where the maximum load is observed. Thus, since in reality demand will spread unevenly across buses, a conservative bus capacity should be considered to incorporate this factor. These are the main variables of the model built to address each scenario. However, as we will show later the optimal capacity, fleet size and stop spacing can be expressed as a function of the frequency of the lines in the network. Thus, the network design problem for a given scenario can be formulated in terms of a single variable family: the lines frequencies. Problem (1) shows the general formulation of our optimization model, which minimizes a social cost function composed by agency and users costs for a given scenario.

$$\begin{aligned} &z(n) = \textstyle\sum_{l\in L}(C^l_{op-km} + C^l_{op-fleet}) + \sum_{i,j\in M} Q_{ij}(\gamma_a T^{ij}_a + \gamma_w T^{ij}_w + \gamma_w T^{ij}_{tw} + \gamma_v T^{ij}_v + \tau_{ij}\Delta)\\ &K_l = \rho_l/f_l,\ \forall\ l\in L\\ &f_l \geq 0,\ \forall\ l \in L \end{aligned} \tag{1}$$

The agency costs, also called operation costs, are added across all lines l in L and are divided into two categories. The first category, C^l_{op-km}, contains costs related to distance travelled, made up of fuel and maintenance. The second group, $C^l_{op-fleet}$, comprises costs related to fleet size, including capital costs, drivers' wages, and insurance. Infrastructure costs are not included in this analysis; however, Hook and Wright (2007) argue that a Closed BRT might imply higher infrastructure costs due to the need of larger transfer stations.

The user costs are added across all origin–destination pairs between micro-zones or macrozones i and j (i, j in M). The costs experienced by riders traveling between each of these OD pairs, Q_{ij}, includes the sum of the costs associated with access/egress times (T^{ij}_a), initial (T^{ij}_w) and transfer (T^{ij}_{tw}) wait times, and in-vehicle travel time (T^{ij}_v) experienced by each of these trips. These travel time components must be weighted according to their associated values (γ_a,γ_w, and γ_v *respectively*). A transfer has three impacts on the traveler: additional waiting time, additional walking time and the disruption of the trip. The additional waiting time is explicitly considered in the objective function, whereas the other two are merged into a single parameter. Therefore, users costs also include a penalty which reflects the disruption and walking necessary for transferring, Δ, multiplied by the number of transfers in each trip, τ_{ij}.

The first set of constraints in (1) guarantee that the vehicle capacity of each line l, K_l, will be large enough to carry the maximum load of the line, ρ_l. In this paper we assume K_l to be a continuous variable. Since in our objective function the social cost function grows with vehicle size (for a given set of frequencies), the vehicle capacity becomes an active constraint at the maximum load segment of each line. Therefore, we express these constraints as an equation instead of an inequality.

Notice that while agency costs grow with f, user cost decreases with it. The effect of f on user cost is perceived not only in waiting times, but also in in-vehicle travel time, since higher frequencies reduce the number of passengers boarding and alighting in each bus. Also, when the optimal strategy for travel between micro-zones i and j involves common lines, each respective bus line's frequency affects how users are distributed. Finally, although agency costs are proportional to frequencies, the proportionality factor is affected by cycle times and vehicle size, both of which decrease with frequency. Our model and the optimization methodology recognize these effects, as will be shown in detail later in this section.

2.3.1. Agency costs

Agency costs are all those costs that transit agencies must necessarily incur to provide transportation service. Since our model optimizes not just the frequencies, but also the vehicle size used in each line, the operational cost both per kilometer and per bus are modeled as linear functions growing with K_l. This is shown in Eq. (2).

$$\textstyle\sum_{l\in L}(C^l_{op-km} + C^l_{op-fleet}) = \sum_{l\in L} 2D_l(a_d + b_d K_l)f_l + (a_t + b_t K_l)R_l f_l \tag{2}$$

where:

D_l	:	One-direction route distance of line l
a_d	:	Fixed operation cost per bus-km.
b_d	:	Variable operation cost per bus-km per vehicle size unit.
a_t	:	Fixed operation cost per bus-hr.
b_t	:	Variable operation cost per bus-hr per vehicle size unit.
R_l	:	Cycle time of line l.

In this expression, $2D_l f_l$ represents the total number of kilometers traveled by the fleet in each hour, whereas $R_l f_l$ represents the fleet size needed to provide service at such a frequency. In this paper, we treat the fleet size as a continuous variable. Notice that as in the case of K_l, R_l depends on $f_l$as well. R_l is divided into two components: motion time and dwell time. Motion time is the hypothetical time taken by a bus to complete a full cycle, without stopping at any bus stops, while dwell time is the extra time needed to stop at all stops and allow passengers to board and alight. It is this last component of cycle time that depends on the set of frequencies, since frequency affects the number of passengers boarding and alighting each bus, as well as the optimal spacing between stations. These two components of cycle time are presented in Eq. (3).

$$R_l = \left[\frac{D_l^p}{V_p} + \frac{D_l^c}{V_c}\right] + [(t_{ba}^{pl} + t_{ba}^{cl}) + (t_s^p n_l^p + t_s^c n_l^c)] \tag{3}$$

Expression (3) takes route structure into account. Each route consists of segments along the periphery (of total distance D_l^p) and along the corridor (of total distance D_l^c). The speed of buses in motion can be different outside (V_p) and inside (V_c) the corridor.

Dwell time at stops consist of two components. First, the time needed for all passengers to board and alight in the periphery (t_{ba}^{pl}) and the corridor (t_{ba}^{cl}). We assume that the ticket paying method is different in the corridor and on the periphery, so these first components of the dwell times are calculated differently (Tirachini, 2014). We also assume that all stops in the corridor have off-board fare collection, allowing users to board the buses through all doors simultaneously, once passenger alighting has finished. Thus, t_{ba}^{cl}can be expressed as:

$$t_{ba}^{cl} = t_b^c B_l^c + t_a^c A_l^c \tag{4}$$

where t_b^c and t_a^c correspond to boarding and alighting times per passenger in the corridor stations. In this model we assume that these unitary times are independent of the size of the bus (i.e. we ignore the effect of the number and size of the doors in the bus). The total number of passengers boarding and alighting each bus along the corridor are called B_l^c and A_l^c respectively. Notice that they are not obtained directly from the demand equation because in the case of common line sections, the number of passengers attracted by a line depends on its relative frequency in a given section.

On the periphery, we assume that users pay when they enter the bus through the front door. Since we assume that passengers alight through the back door, boarding and alighting processes happen simultaneously. Thus, the time needed to board and alight in a stop can be expressed as:

$$t_{ba}^{pl} = \sum_{k\in C_l^p} Max\{t_b^p B_{kl}^p; t_a^p A_{kl}^p\} \tag{5}$$

The second component of dwell time consists of the extra time needed to decelerate, accelerate, and open and close the doors (t_s^p and t_s^c, respectively, which are different since the cruising speeds are different) at all stops along the line. This component depends directly on the number of stops (n_l^p and n_l^c, respectively), and therefore on the spacing between stops in each segment. To determine the optimal stop spacing in a segment we need to balance the following costs:

- Access Time. More space between stops forces users to walk further.
- In-Vehicle Travel Time. More space between stops allows buses to stop less and therefore to travel faster.
- Agency Costs due to stops. Increasing the distance between stops allows buses to stop less and therefore reduce cycle times. Thus, for a given frequency the needed fleet drops, reducing the associated operational costs.

We address the trade-offs between these costs assuming that the spacing between consecutive stops should correspond to a homogeneous standard at the city periphery and a homogeneous standard for all the length of the corridor. As shown in the Appendix A, the minimization of these three components yields the following expression for an approximation of the optimal spacing between stops, $s_z{}^*$, for the periphery (z = p) and the corridor (z = c).

$$s_z{}^* = \sqrt{\frac{\bar{f}_z t_s^z(\gamma_v \bar{Q}_z + a_t + b_t \bar{K}_z)}{\frac{\gamma_a \bar{\lambda}_z}{4V_a}}} \tag{6}$$

where:

$\bar{f}_z$	:	The average frequency of buses in z.
$\bar{Q}_z$	:	The average passenger load per bus observed in z.
$\bar{\lambda}_z$	:	The average number of passengers per unit of distance and unit of time walking from z to a stop.
$\bar{K}_z$	:	The average vehicle size operating in z.
V_a	:	Access/egress speed.

Notice that this spacing between stops (in the periphery and the corridor) depends on the following decision variables of the model: average frequencies, vehicle capacities, and loads per bus, which are all decision variables in our equation. However, vehicle capacities and load per bus depend directly on the frequency being offered since the demand profile and the maximum load allowed for each route is assumed given. Thus, the single variable affecting stop spacing in this expression is the bus frequency and hence we only optimize the frequency and find the corresponding spacing as a sub-problem for simplicity reasons. Since the demand profile and the maximum load per bus are inversely proportional to the frequency, the positive marginal impact of the frequency in the stop spacing is expected to be rather small. This will be confirmed in the results of the graphs presented in Appendix A related to the case study of the paper. We should expect that Appendix B summarizes all the variables that directly depend on the set of frequencies for a given scenario (number of lines given).

2.3.2. User cost

Travelers for each origin–destination pair all travel the same known route. In this section we formulate user cost for a generic (i,j) pair. The first of the five components of user cost in (1) is travelers' access/egress time. This cost is expressed as the average distance that users of a micro-zone have to cover to access/egress to/from their closest stop, divided by their access/egress speed V_a. Since we assume that people walk as in a Manhattan metric, walking distance is calculated as the sum of the perpendicular distance to the bus line and the distance from this point to the closest stop. Since the model is solved for a given number of lines, the distance walked perpendicular to the line is also known. For instance, in the case of a trip originated in the periphery, the access time should be equal to one fourth of time needed to cover the spacing between lines plus one fourth of the time needed to go along the spacing between periphery stops (Daganzo and Ouyang, 2019). We compute the access (or egress) distance for trips originating (or ending) in micro-zone m as L_m. Thus, the access and egress times experienced by a trip going from micro-zone i to j can be expressed as shown in Eq. (7).

$$T_a^{ij} = \frac{L_i}{V_a} + \frac{L_j}{V_a} \tag{7}$$

The second term of user cost in (1) is the initial waiting time. This is the time users have to wait to board the first bus of their trip. In this paper we assume that all the lines operate without a schedule, i.e. the user only knows the frequency of the line, but not the time buses will show up at the stop. This is the common practice in high frequency corridors around the world (i.e. average headways lower than 5 min). The demand assignment procedure yielded the sequence of lines that every user utilizes in each trip stage. As this assignment considers common lines, the initial waiting time is calculated considering the sum of frequencies from the common lines set, L_1^{ij}, as shown in (8).

$$T_w^{ij} = \frac{k}{\sum_{l \in L_1^{ij}} f_l} \tag{8}$$

In our model we assume a constant k, which would equal 0.5 if buses arrive at evenly spaced intervals and passengers arrive randomly (independently of each other and of bus arrival times). Nevertheless, we assumed k = 0.6 for the Closed BRT operation within the corridor, and k = 0.8 in the periphery in the Closed BRT and both in the corridor and the periphery in the Open BRT, We make this assumption since headway variability is easier to handle in a Closed corrido than in an Open corridor, given that there is no line overlapping and lines are shorter. Moreover, in the case of the Open BRT the variability produced outside the corridor negatively affects variability in the corridor.

The third element of user cost in (1) is wait times for transfers. This aspect is modeled in the same way as initial waiting time, but using the line frequencies from the respective trip stage. Note that in the open BRT network, trips are either one or two stages, whereas in the closed BRT network, trips can reach up to three stages.

In-vehicle travel time, the fourth element of user cost in (1), has the same components as cycle times defined in (3). The in-vehicle travel time of a trip stage on a given line, l, will be given by the corresponding fragments of R_l in which the stage takes place. Notice that since the number of boardings and alightings of each bus varies at different stops, operational speeds will also vary in consecutive links along the segment (even though the motion speed and stop spacing are kept constant). Also, since different lines serving the same trip stage will face different boarding and alighting patterns, their operational speeds will also vary. Therefore the expected in-motion travel time for the trip stage will correspond to the weighted average of the travel times of the lines considered convenient by users. The last user cost component corresponds to the transfer penalty (Δ) that represents all other costs that users perceive when they transfer (including walking and trip interruptions). These costs are influenced by the context in which a transfer occurs (such as neighborhood safety, weather, etc.); see Raveau et al. (2014) and Currie (2005).

2.4. Solution method

As a result of the mathematical derivations of all elements involved in Problem (1), the resulting model is built exclusively on the following independent decision variables: 1) the number of lines and 2) the frequency of each line. Once these variables are defined, all the remaining design variables of the system are obtained, i.e. optimal distance between stops, optimal bus size and optimal fleet size.

As was mentioned above, each scenario is determined by a combination of a certain BRT network type (Open or Closed) and a given number of lines. Since the number of lines needed for medium-sized cities is not very large, in this paper we exhaustively analyze all the possible number of lines within a reasonable range. We verify that the objective function of the design model increases when the number of lines is lower than the lower bound or higher than the upper bound. It is important to note that the optimization model for a given set of lines is based only on continuous variables. To solve each of them, we identify an optimal set of frequencies using the Constrained Optimization By Linear Approximation (COBYLA) method, developed by Powell (1994) and available in the open source library SciPy. Since this process is based on a variant of the gradient method, we verify the optimality of the solution by starting our search from different initial solutions. In every scenario we solved, the optimal solution obtained was the same for every initial solution. The rapid convergence of frequencies optimization for the optimal networks presented in Section 3.2 can be observed in Appendix E. This process was efficient enough for us to address the main contribution of this paper, which is to compare as robustly as possible the performance of open and closed BRT networks.

3. Application of the approach to a medium-sized city

In this section we apply the model developed in Section 2 to the city of Valdivia in the south of Chile, in order to compare the performance of Closed and Open BRT systems. We consider two different representations of the city. First, we adapt the characteristics of the city to fit the rectangular model. Second, using this model's results for Open and Closed BRT systems, we design a bus network for each one, this time taking the real road network and its characteristics (e.g. hierarchy, speed) and city's demand function into account.

3.1. The city of Valdivia

Valdivia is a medium-sized city located in the mid-south area of Chile, at a distance of 841 km from Santiago de Chile, the country's capital. It has 154,445 inhabitants according to the 2012 census and an approximate average (PPP) annual income of US $12,341. This city is the capital of the *Los Ríos* (Rivers) province. As can be imagined, several rivers and wetlands surround the city, creating a beautiful and uneven landscape. According to its last origin–destination survey from 2013 (Trasa Ingenieria, 2014), buses are used in almost 24% of the city trips. Buses are the only form of public transportation in the city besides conventional taxis and shared taxis with pre-established routes (called *colectivos*).

The current bus network is composed by 9 lines connecting most neighbourhoods and also the suburban town *Niebla*, 17 km away from Valdivia. Each line of the system is operated by a single private company. The operation is regulated very loosely by the local authorities. The network's lines were not planned or designed by the local transit authority; rather, the routes and frequencies were proposed by the companies themselves, and the authority simply granted them the right to operate. While the operation of each of these lines can be expected to be a profitable venture, they do not necessarily form a cost-efficient network. However, the National Public Transportation Law was recently modified, allowing transportation authorities to get much more involved in the bus network design of medium-sized cities like Valdivia. A BRT-based network is an important option for this new network.

Fig. 6 presents the road network of the city and the BRT corridor examined in the present research. The best corridor for a BRT would be Avenue Picarte, which connects the southeastern part of the city to the downtown area. This avenue is the widest in the city and contains the largest public transport supply.

We use information from the most recent Origin/Destination Survey in Valdivia, which took place in 2013 (Trasa Ingenieria, 2014). In the detailed model, we directly use the origin–destination matrix coming from this survey. Nevertheless, in the generic rectangular city model presented in the next section, we only use the total demand and distribute it in the area of the rectangle.

Fig. 6. BRT corridor layout on the road network of Valdivia.

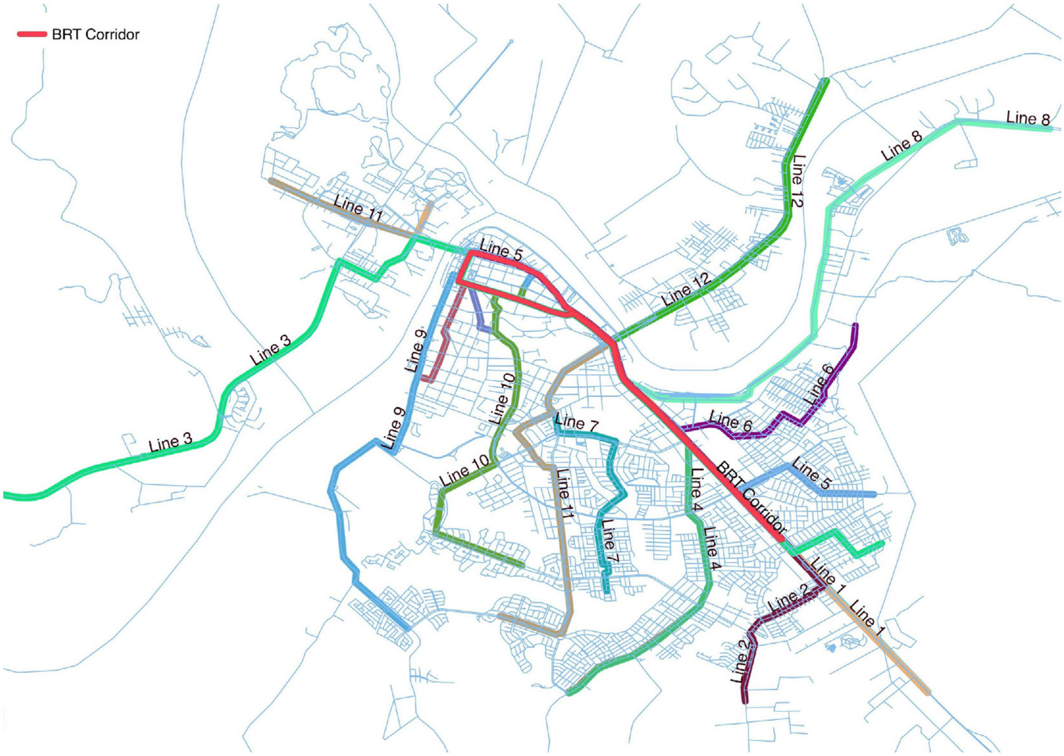


Fig. 7. Open BRT network in Valdivia; all lines enter the BRT corridor.

3.2. Results and discussion for the generic rectangular city

In order to fit the characteristics of Valdivia, our chosen city, to the rectangular model shown in Section 2, one must first locate the CBD. Valdivia's CBD is located in one extreme of Avenue *Picarte*, so we decided to locate the model's CBD in one extreme of the BRT corridor. Then we identified the maximum number of lines that the urban road grid could fit for an Open and a Closed BRT. Only the main avenues and important secondary streets of the city were included in the set of streets to be used. In the case of a Closed BRT the lines should be orthogonal to the BRT corridor, but in the case of the Open BRT it may make sense to take advantage of avenues that reach the BRT corridor heading towards the CBD. Figs. 7 and 8, present the networks considering the maximum number of lines that the primary road network allowed (10 in the Closed BRT and 12 in the Open BRT). In the Open BRT network of Fig. 7 each line starts in a periphery area, enters the BRT corridor and ends in the CBD. In the network of Fig. 8 the BRT corridor acts as a closed BRT, while each feeder line crosses the corridor connecting neighborhoods in both sides of the corridor. As is clear from the Figures, we made a slight adjustment in the route design for the extreme southeastern area of the city. In the case of the Closed BRT, lines are expected to provide a fast connection with the BRT and a direct connection between opposite sides of the corridor. Therefore, we forced the lines to run perpendicular to the corridor. However, for the Open BRT we chose the most important roadways in this area who run diagonally towards Picarte Avenue providing a direct trip towards the CBD. Notice that Figs. 7 and 8 only provide the maximum number of lines for each system to be used for the rectangular city model; i.e. the network is still modeled as in Fig. 4.

We compute the city's average demand and its approximate dimensions as inputs for the model. We use a double-constrained gravitation model to build an origin–destination matrix between micro-zones. This model uses the aggregated trip generation and attraction of each micro-zone, a cost function that depends on the in-network distance of an origin–destination pair, and a fixed parameter representing the demand sensitivity of that distance. With those elements, a doubly constrained gravitational model was calibrated using an iterative process similar to the Furness calibration method (Ortúzar and Willumsen, 2011). This double constrained gravitational model generates an OD matrix that fits the total demand originated and attracted in each zone. The model considers two additional inputs: (1) the distance that a user must travel to go from an origin i to a destination j, and (2) travelers' sensitivity to the distance of their trips to choose their destinations. All the parameters required by the gravitational model are shown in Appendix C.

As will be argued later in the paper, transfer penalties play a key role in the difference between the Open and Closed BRT's performance. In Currie (2005), it is observed that the transfer penalty Δ is highly variable between different world public transport systems, making it very difficult to choose one value for this parameter. We decided to use penalties obtained in Raveau et al. (2014) for the Metro of Santiago de Chile, where an average transfer penalty has a cost comparable to 5.86 min of in-vehicle travel time. The parameters obtained from Raveau et al. (2014) are also presented in Appendix C.

As discussed in Section 2, we optimize the number of lines for the system by solving problem (1) assuming that the number of

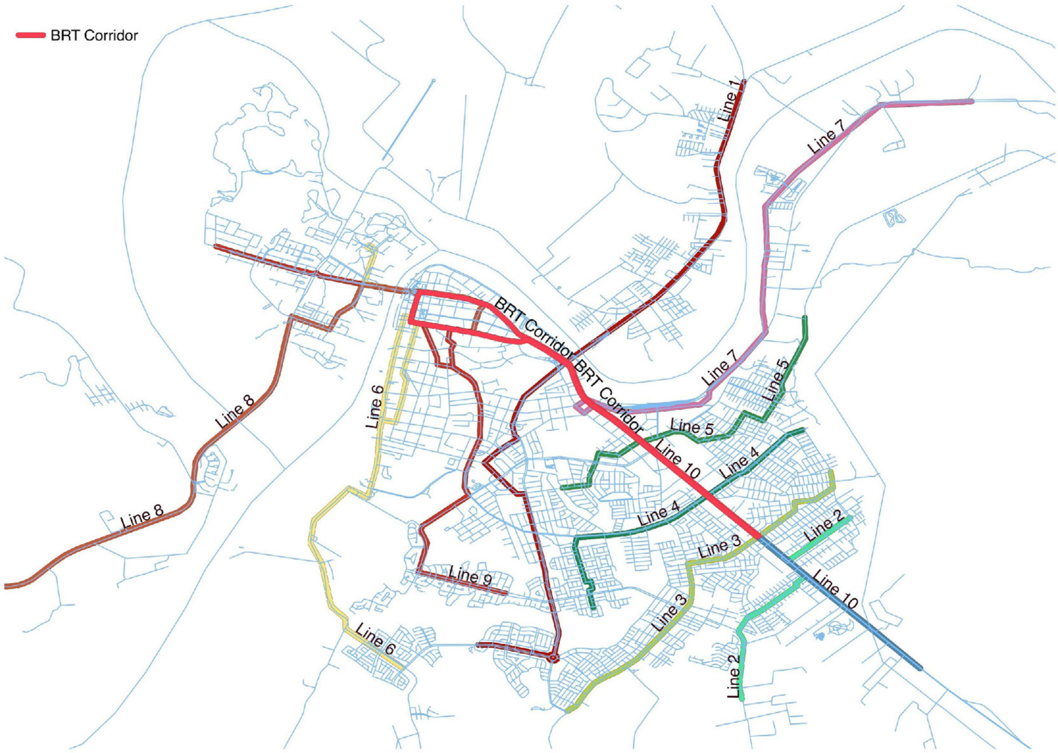


Fig. 8. Closed BRT network in Valdivia; Line 10 is the corridor line.

lines in the city is given, and varying it in a wide enough range. The maximum number of lines in each network is limited by the road grid configuration which was identified as 10 for the Closed BRT and 12 for the Open BRT. Still, to explore the shape of the cost curves beyond these limits, and understand the impact of these bounds on the optimal social cost we solve the problem for line configurations exceeding the maximum number of lines that the road network of this idealized city can support. Thus, we solve problem (1) for the idealized rectangular city for a range of 5 to 42 lines for the Open BRT, and of 4 to 25 lines for the Closed BRT.

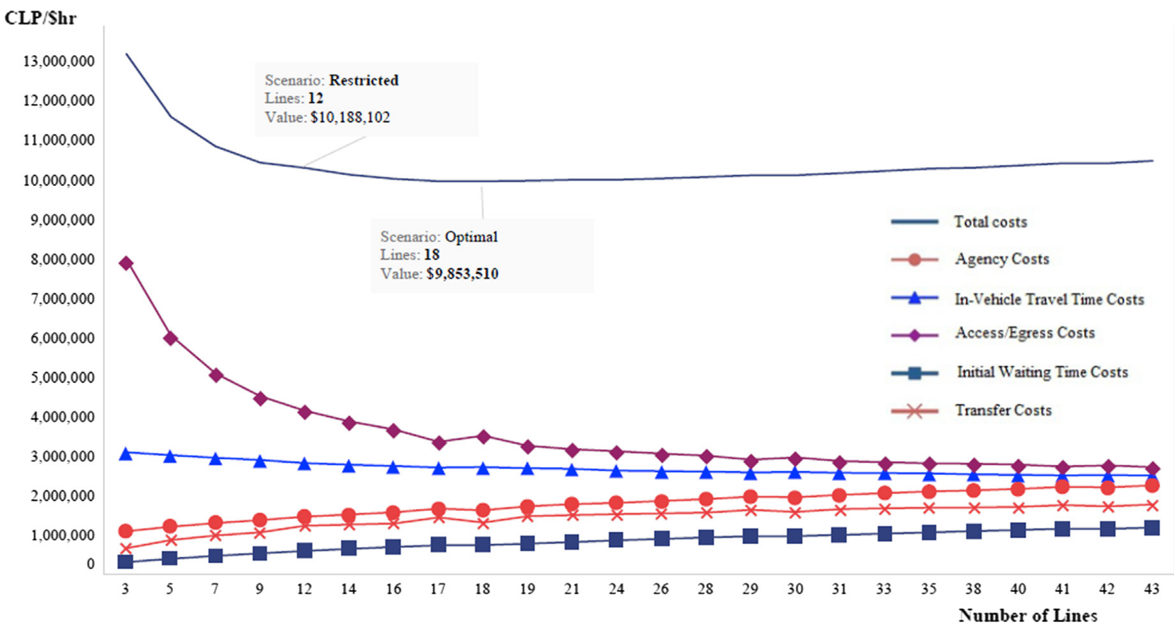


Fig. 9. Open BRT Costs for different number of lines.

Fig. 10. Closed BRT Costs for different number of lines.

The cost components for each number of lines are shown in Figs. 9 and 10, yielding an unbounded optimal number of 18 and 14 lines respectively. The reason for this significant difference in the number of lines in each network is due to the location of the CBD in an extreme of the corridor as was explained in Section 1.2. As can be observed from the Figures, the optimal cost is quite insensitive to the number of lines in the vicinity of the optimal solutions. Also, we observe that the optimal number of lines that can be fit in the city for each network when the urban road grid is considered corresponds to the upper bound imposed in the model; i.e. 12 for the Open BRT and 10 for the Closed BRT associated to networks of Figs. 7 and 8. This yields 6 micro-zones in each side of the corridor for the Open BRT, and 9 micro-zones in each side of the Closed BRT. The analysis presented below corresponds to these networks.

Table 1 shows the cost results for the optimal number of lines in each type of network, while Appendix D.1 and D.2 present operational indicators per line for each one. The total cost of the Open BRT is 6.3% lower than that of the Closed BRT. As expected, the main benefit comes from reducing transfers by more than a half (see Table 2) with an associated drop in waiting time while transferring and associated penalties. Interestingly, waiting time for the first leg of the trip is higher for the Open BRT by 16.6% due to lower frequencies. But the total waiting when all trip legs are added is only 6.2% higher. The higher operational speed of the Open BRT shown in Table 2 is also a consequence of requiring fewer transfers. Faster buses impact the performance twofold: it reduces the in-vehicle time as shown in Table 1 and allows the provision of a given frequency with fewer buses yielding a 5.3% lower agency costs for the Open BRT. This agency cost drop happens despite that the feeder-trunk configuration of the Closed BRT adapts its frequencies and vehicle sizes being offered in the periphery and the corridor independently, while the Open BRT line must keep its vehicle size and its frequency in both the periphery and the corridor.

Regarding the access cost of a user, it is composed of the distance to approach the periphery line and the distance to reach a stop within the line. In our case, the distance between lines is given by the city road structure forcing the first element to be approximately twice as long than the second one. The optimal stop spacing displayed in (6) grows slightly with the aggregated frequency. Thus, the space between stops in the periphery is slightly larger in the Closed BRT system, because the average frequency in the periphery is higher. But its effect cannot overcome the longer distance to reach the line faced in the Open BRT case. Conversely, inside the corridor, the Open BRT system has a higher average frequency and more space between stops. Citywide, the access cost of the Open

Table 1
Continuum approximation model costs results.

Cost [CLP$/h]	Closed BRT (10 lines)	Open BRT (12 lines)	Difference [%]	Difference [CLP$/h]
Total Costs	10,874,829	10,188,102	−6.3%	−686,727
Agency Costs	1,536,280	1,454,761	−5.3%	−81,519
In-Vehicle Travel Time Costs	2,823,433	2,798,448	−0.9%	−24,985
Access/Egress Costs	3,453,514	4,107,549	18.9%	654,035
Initial Waiting Time Costs	512,546	597,807	16.6%	85,260
Transferring Waiting Time Costs	263,937	226,824	−14.1%	−1,319,518
Transfer Penalty Costs	2,285,119	1,002,714	−56.1%	−1,282,405
Total Costs without Transfer Penalty	8,589,710	9,185,388	6.9%	595,678

Table 2
Continuum approximation model KPI results.

Indicator	Unit	Closed BRT (10 lines)	Open BRT (12 lines)	Difference[%]
Average Frequency	Cycles/hr	36.8	27.6	−25%
Bus-Kilometers	Buses*km/h	3,190	3,160	−1%
Load Factor	%	44.8%	44.9%	0.1 pp
Total Fleet	Buses	180	160	−11%
Average Vehicle Capacity	Seats/Bus	21,1	22,7	9%
Available Seat Kilometres	Seats*km/h	66,990	72,680	8%
Corridor Stop Spacing	m/Stop	392	417	6%
Periphery Stop Spacing	m/Stop	175	159	−9%
Average Operational Speed	km/h	18	20	10%
In-Motion Time	h	960	973	1%
Fixed Dwell Time	h	428	433	1%
Variable Dwell Time	h	496	462	−7%
Average Trip Length	km	3.0	3.1	2%
Transferring Demand	Pax/h	7,451	4,366	−41%
Number of Transfers	Transfers/h	9,949	4,366	−56%

BRT is 18.9% higher. If the network configuration of the Open BRT allowed two extra lines, then this difference would drop to just 11.2% (this is not displayed in the Table). Of course, in such a case other costs would raise keeping total costs almost unchanged as shown in Fig. 9.

It is worth noting that the optimal stop spacing in equation (6) decreases if ceteris paribus (i.e. constant frequency) vehicles become smaller. Therefore, the relatively short spacing between stops (specially in the periphery) is explained by the small size of buses (15 to 20 passengers per bus) given by the optimization process. Previous models minimizing users' and agency costs are consistent with this. For example, Jansson (1980) obtained buses in the 15 to 60 range, Jara-Díaz and Gschwender (2003b) obtained the optimal bus size to be constrained in 24 or 61 passengers depending on some parameters, Jara-Díaz et al. (2017b) obtained optimal buses of 10 to 30 passengers jointly optimizing peak and off-peak periods, Badia et al. (2016) obtained buses in the 40 to 60 passengers range for one of the lines structures considered. Jara-Díaz and Gschwender (2009) showed how the consideration of users' cost in the optimization critically affects the optimal bus size. Minimizing users and agency cost, they obtain an optimal bus size between 20 and 40 passengers per bus. But, for the same set of parameters, when only agency cost is minimized, the optimal bus size increases to 130 passengers per bus. The relation between optimal spacings, average frequency, vehicle load and vehicle capacity along the frequency optimization process of the optimal networks (10 and 12 lines) was recorded and can be observed in Appendix A4 and A5.

Regarding waiting times, the initial waiting time cost is higher in the Open BRT because frequencies in the periphery (where most of the trips start or end) are lower. However, in this network fewer users need to transfer (as shown in Table 2), implying a lower transferring waiting time costs, despite the fact that these transfers suffer a higher average waiting time due to a higher value of parameter k and lower frequencies, than for the Closed BRT network.

As can be observed from Table 1, the Closed BRT presents a 6.9% lower cost if transfer penalties are ignored. However, if penalties are valued the balance is reversed and is the Open BRT the one presenting 6.3% lower costs.

3.3. *Sensitivity analysis for the rectangular city*

In this subsection we provide a sensitivity analysis for the previous results in six key parameters of the model:

i. Total demand (changing demand density)
ii. Average trip length
iii. Value of time
iv. Transfer penalty
v. Headway variability parameter, *k*
vi. City dimensions (for a given transport demand)

For each new scenario, the model is solved in the same way as in Section 3.2, optimizing the number of lines and their frequencies, and considering the maximum number of lines for the Open and Closed BRTs as 12 and 10 respectively.

3.3.1. *Total demand*

Three new demand scenarios where considered in which the demand density was multiplied by a factor of 0.5, 2 and 4. Table 3 presents the main results for each case.

The results show that except for the 0.5 demand factor, the networks being compared correspond to the largest ones allowed by the urban road grid. In these cases, the benefits of an Open BRT versus a Closed BRT grow with the demand. Only when the demand drops by half, the cost difference trend is reversed, but this is only due to the fact that the optimal Closed BRT responds to the

Table 3
Results of the sensitivity analysis on the demand density.

Demand Density Factor	Total Demand [pax/h]	Closed BRT			Open BRT			Total Cost Difference
		Total Cost [CLP $/h]	Unitary Cost [CLP $/pax]	Lines	Total Cost [CLP $/h]	Unitary Cost [CLP $/pax]	Lines	
0.5	5,385	5,759,090	1,069	10	5,369,112	997	9	−6.8%
Base	10,770	10,874,829	1,010	10	10,188,102	939	12	−6.3%
2	21,540	20,714,111	962	10	18,456,992	857	12	−10.9%
4	43,080	40,303,922	936	10	35,229,090	818	12	−12.6%

constrained network, while the Open BRT network does not. If either the Closed BRT constraint were lifted or if the Open BRT constraint were imposed to 12 lines, then the cost difference between both systems would be lower than 6.3%. This trend in which an Open BRT becomes more convenient when demand grows is consistent with previous literature where users' and operators' costs are minimized for different public transport network structures. Jara-Díaz and Gschwender (2003a) found that lines structures without transfers dominate for large values of the demand. Fielbaum et al. (2016) found that lines structures with transfers may be the best for very low demand volumes or when there are extreme cases of monocentric or polycentric cities, but not for (more realistic) medium cases, where structures that avoid transfers dominate. Badia et al. (2016) also found that the structure without transfers increase its area of applicability when demand grows, but the best structure also depends on the level of concentration of the demand, the city size and the value of the transfer penalty.

As observed, the total demand also has an effect in the optimum number of lines. If the demand is low enough, the optimal number of lines may be below the maximum number of available avenues that the city can support to operate the feeding buses, Thus, the constraint would become inactive as was the case for the Open BRT case. This effect is also observed in all the sensitivity analyses presented in the next subsections.

3.3.2. Average trip length

In this section we deliver a sensitivity analysis of the results with respect to the average distance traveled by the users, while keeping the city dimensions unchanged. To do this we generated five new scenarios in which the parameter in the gravitational model representing the sensitivity of the demand to the distance (θ) was multiplied by a factor of 0, 0.5, 1.5, 2 and 3. This yields average trip length in the range of 2.6 and 3.1 km. Table 4 presents the main results for each case.

The results show that there is no significant difference in the Total Cost of both networks, however shorter trips slightly benefit the Closed BRT network. This is a consequence of a lower proportion of trips needing to transfer. The Table also highlights the percentage of passengers needing to transfer at least once. As expected the Open BRT requires significantly fewer passengers to transfer, and the longer the distance the more passengers in both systems need to transfer to reach their destinations.

3.3.3. Value of time

As a proxy to represent a poorer and a wealthier city two new scenarios were considered, in which the value of time was multiplied by a factor of 0.5 and 2.

Table 5 presents the main results for each case.

Our results show that the Open BRT performs better than the Closed BRT when the value of time grows. This should not come as a surprise since the Closed BRT has a higher waiting and in-vehicle time than the Open BRT, and a slightly lower access time.

3.3.4. Transfer penalty

Five new scenarios where considered in which the transfer penalty was multiplied by a factor of 0, 0.5, 1.5, 2 and 3.

Table 6 presents the main results for each case.

The results show that the decision is significantly sensitive to this parameter, and in the case of no transfer penalty the Closed BRT becomes more convenient than the Open BRT. As expected, larger transfer penalties increase the cost difference between both

Table 4
Results of the sensitivity analysis on the average trip length.

Avg. Trip Length θ	Closed BRT				Open BRT				Total Cost Diff.
	Avg. Trip Length [km]	Total Cost [CLP $/hr]	Demand that transfers [%]	Lines	Avg. Trip Length [km]	Total Cost [CLP $/hr]	Demand that transfers [%]	Lines	
0x	3.173	11,119,162	72%	10	3.240	10,377,903	42%	12	−6.7%
0.5x	3.085	11,000,711	70%	10	3.148	10,286,824	41%	12	−6.5%
Base	2.994	10,874,829	69%	10	3.052	10,188,102	41%	12	−6.3%
1.5x	2.901	10,742,160	68%	10	2.952	10,082,146	40%	12	−6.1%
2x	2.808	10,603,595	66%	10	2.850	9,969,915	39%	12	−6.0%
3x	2.622	10,312,774	63%	10	2.643	9,729,127	36%	12	−5.7%

Table 5
Results of the sensitivity analysis on the value of time.

Value of Time Factor	Unitary Travel Time Cost [CLP$/pax-hr]	Closed BRT			Open BRT			Total Cost Difference
		Total Cost [CLP $/hr]	Total User Costs [CLP$/hr]	Lines	Total Cost [CLP $/hr]	Total User Costs [CLP$/hr]]	Lines	
0.5	749	6,085,684	4,911,312	10	5,715,948	4,602,585	12	−6.1%
Base	1,498	10,874,829	9,338,549	10	10,188,102	8,733,341	12	−6.3%
2	2,996	20,046,707	17,994,314	10	18,740,554	16,800,119	12	−6.5%

Table 6
Results of the sensitivity analysis on the transfer penalty.

Transfer Penalty Factor	Closed BRT			Open BRT			Total Cost Difference
	Transfer Costs [CLP $/hr]	Total Cost [CLP$/hr]	Lines	Transfer Costs [CLP $/hr]	Total Cost [CLP$/hr]	Lines	
0	263,937	8,589,710	10	226,824	9,185,388	12	+6.9%
0.5	1,406,496	9,732,270	10	728,181	9,686,745	12	−0.5%
Base	2,549,056	10,874,829	10	1,229,538	10,188,102	12	−6.3%
1.5	3,691,615	12,017,389	10	1,730,895	10,689,459	12	−11.1%
2	4,834,175	13,159,948	10	2,232,252	11,190,816	12	−15.0%
3	7,119,294	15,445,067	10	3,234,966	12,193,530	12	−21.1%

systems benefitting the Open BRT. This is consistent with Jara-Díaz et al. (2017a) who found the value of the transfer penalty to be a key factor in the optimal design for the public transport networks they analyze, claiming that feeder-trunk structures only dominate for some extreme cases of the demand pattern and very low value of the transfer penalty.

3.3.5. Headway variability parameter, k

The models have neglected the impact of the chosen BRT design into headways regularity which directly affect waiting times, reliability and comfort. We expect that handling headway variability should be easier for a Closed BRT corridor than an Open one since the lines are shorter, and line overlap is avoided. In addition, in the case of the Open BRT the operation inside the corridor is affected by headway variability happening in local streets upstream from entering it. To consider this effect, we performed a sensitivity analysis on the headway variability parameter affecting the waiting time (k) for every bus operation affected by mixed traffic. In the base scenario we assumed $k = 0.6$ for the Closed BRT operation within the corridor, while a $k = 0.8$ was used in all other situations, i.e. periphery in the Closed BRT and both corridor and periphery in the Open BRT.

Five new scenarios were considered, where we tested the optimal network configuration for each system considering higher levels of headway variability affecting the Open BRT services and the feeder services of the Closed BRT (the Closed BRT corridor service was kept unchanged at $k = 0.6$). For doing this, the value of $k = 0.8$ affecting these services was increased by 20%, 40%, 60%, 80% and 100%, as shown in Table 7.

The results show that headway variability is quite relevant, affecting the comparative performance of both systems. Recall that in the Closed BRT system passengers experience more transfers and therefore must wait more times per trip. Still, higher headway variability due to the operation in local streets affect the Open BRT more. However, even when the value of *k* is doubled, the Open BRT still presents a lower total cost than the Closed BRT. The significant impact in the total cost of both systems highlights the importance of implementing an active headway control strategy.

Table 7
Results of the sensitivity analysis on the headway variability parameter.

Waiting Time Constant k	Closed BRT			Open BRT			Total Cost Difference
	Initial Waiting Time [CLP$/hr]	Total Cost [CLP $/hr]	Lines	Initial Waiting Time [CLP$/hr]	Total Cost [CLP $/hr]	Lines	
Base	512,546	10,874,829	10	597,807	10,188,102	12	−6.3%
1.2x	575,037	10,963,906	10	670,392	10,347,440	12	−5.6%
1.4x	632,781	11,048,578	10	736,858	10,496,962	12	−5.0%
1.6x	686,622	11,129,439	10	798,443	10,638,285	12	−4.4%
1.8x	737,191	11,206,960	10	856,031	10,772,627	12	−3.9%
2.0x	784,972	11,281,525	10	910,274	10,900,928	12	−3.4%

Table 8
Results of the sensitivity analysis on the city dimensions.

Geometry scale factor	Closed BRT			Open BRT			Total Cost Difference
	In-Vehicle Travel Time [CLP$/hr]	Total Cost [CLP $/hr]	Lines	In-Vehicle Travel Time [CLP$/hr]	Total Cost [CLP $/hr]	Lines	
0.7x	2,066,692	8,798,794	7	2,090,465	8,234,393	7	−6.4%
Base	2,823,433	10,874,829	10	2,798,447	10,188,102	12	−6.3%
2.0x	4,914,450	16,022,481	18	4,846,972	14,879,025	17	−7.1%
3.0x	6,794,500	20,188,038	22	6,660,000	19,065,127	21	−5.6%

3.3.6. City dimensions

Finally, we did a sensitivity analysis on the spatial dimensions of the city, while keeping the total number of trips constant. For doing this, we multiplied the width and the length of the rectangular urban geometry (α and β) by a constant scale factor (taking values of 0.7, 2.0 and 3.0), while keeping constant the width of the corridor's area of influence and the dimensions of the CBD (R_1 and R_2). Thus, the average trip density in the city changed inversely proportional to the square of the scale factor. As explained in Section 3.2, a maximum of 10 and 12 lines were considered for the Open and Closed BRT respectively due to spatial constraints of the city of Valdivia. In this sensitivity analysis we kept this road network constraint, by requiring each solution to keep parallel lines apart by at least the same distance as was found in the city.

As shown in Table 8, the results show that the optimal number of lines for the Base case and the 0.7 case were governed by the minimum distance across parallel lines. However, for the 2.0 and 3.0 case the optimal number of lines was lower than what proportionality would show, indicating that the chosen distance between parallel lines was higher than the one observed for the city of Valdivia. The results also show that as the city grows in size, the In-Vehicle Travel Time Costs takes a higher share of the Total Cost as the city grows in surface. In all cases analyzed, the relative difference between both BRT networks did not change much, while the Open BRT remained as the one yielding the lowest Total Cost.

3.4. Results and discussion for a detailed model of Valdivia, Chile

In this section we move a step further, comparing the optimal design for an Open and a Closed BRT network for a real city instead of an idealized rectangular one, considering all the constraints from geographical obstacles and the road networks. As will be seen, the result of this experiment is not as straightforward as was the conclusion drawn from the idealized model. Implementing the model results for Open and Closed BRT networks in the context of a real city forces us to adapt to that city's specific characteristics. For example, the city in question does not have a rectangular shape, demand is not homogeneously distributed in rectangular zones, there are geographical obstacles like hills and rivers, and the road network is not a continuous Manhattan metric as modeled in Fig. 4. Given these differences between both models, the numerical results are expected to differ to some extent. Since for this city the optimal number of lines in each system was given by the maximum number of lines that the city can support, the networks chosen for each case are still given by Figs. 7 and 8.

To compare the performance of both networks, we used the origin–destination matrix from the most recent Origin/Destination Survey, which took place in 2013 (Trasa Ingenieria, 2014). In this matrix, trips were aggregated into 65 zones that include the city of Valdivia and the suburban area of Niebla located at the west. In our study, we only used the trips from the morning peak hour of 07:00 AM to 08:00 am, which is the busiest time of each workday. We adapted the procedure explained in Section 2.3 to this zone configuration and assigned the trips from the origin–destination matrix to the network. The parameters used in this model are the same ones shown in Appendix C. For each zone, we assigned an average walking distance to reach its closest line. Then a second component of access and egress time (along the direction of the bus line) is needed to reach the stop. This distance depends on the optimal stop spacing obtained from the model as we did in the rectangular model. Finally, frequencies, vehicle capacity, fleet size, and stop spacing for Open and Closed networks were simultaneously optimized, also following the model previously stipulated.

The aggregated costs and some key performance indicators of both optimized networks are shown in Tables 9 and 10, while Appendix D.3 and D.4 present operational indicators per line for each network. As in the results from the continuum approximation

Table 9
Detailed Model results.

Cost [CLP$/h]	Closed BRT (10 lines)	Open BRT (12 lines)	Difference [%]	Difference [CLP$/h]
Total Costs	16,168,801	14,479,498	−10.4%	−1,689,303
Agency Costs	2,565,759	2,326,121	−9.3%	−239,638
In-Vehicle Travel Time Costs	5,775,971	5,768,710	−0.1%	−7,261
Access/Egress Costs	4,026,720	4,021,885	−0.1%	−4,835
Initial Waiting Time Costs	514,434	731,285	42.2%	216,851
Transferring Waiting Time Costs	460,927	346,854	−24.7%	−114,073
Transferring Penalty Costs	2,824,990	1,284,643	−54.5%	−1,540,348
Total Costs without Transfer Costs	12,882,884	12,848,001	−0.3%	−34,883

Table 10
Detailed model KPI results.

Indicator	Unit	Closed BRT (10 lines)	Open BRT (12 lines)	Difference [%]
Average Frequency	Cycles/hr	41.3	26.5	−36%
Bus-Kilometers	Buses*km/h	4,934	4,762	−3%
Load Factor	%	46.0%	53.7%	7.7 pp
Total Fleet	Buses	311	262	−16%
Average Vehicle Capacity	Seats/Bus	24.2	25.3	5%
Available Seat Kilometres	Seats*km/h	119,418	120,686	1%
Highway Stop Spacing (to Niebla)	m/Stop	491	645	31%
Corridor Stop Spacing	m/Stop	326	347	6%
Periphery Stop Spacing	m/Stop	161	186	15%
Average Operational Speed	km/h	17.0	19.2	13%
In-Motion Time	h	2,130	2,309	8%
Fixed Dwell Time	h	1,116	1,080	−3%
Variable Dwell Time	h	609	462	−24%
Average Trip Length	km	5.4	6.3	16%
Transferring Demand	Pax/h	8,657	5,593	−35%
Number of Transfers	Transfers/h	12,300	5,593	−55%

model, the Open BRT showed significantly lower total costs than the Closed BRT. This is again explained mainly by transfer costs. As before, a significant strength of the Open BRT was its higher operational speed: ending up being 10% higher than the speed obtained for the Closed BRT system, mainly due to fewer boarding and alighting (fewer transfers).

As can be observed from comparing Tables 1–2 with Tables 9–10 the results from applying the model to the real city yields higher costs in several items than when the rectangular idealized city is optimized. This is a consequence of having longer distances for each line (some very large lines appear in the detailed model) and therefore for the average trip. The average trip length, which was 3.0 and 3.1 km for the rectangular city model, increases in the detailed model to 5.4 km for the Closed BRT and 6.3 km for the Open BRT respectively. In reality, bus services must overcome several geographical obstacles as rivers, wetlands and creeks. In addition, this causes that the demand is far from being homogenously distributed across the city, and its origin destination patterns are different from the one considered for the rectangular model. In fact, in the real city around 1,200 extra passengers per hour require transferring independent of the BRT type. However, in the case of the Closed BRT 2,350 extra transfers are needed when compared with the rectangular city.

The longer distances faced in the real city directly affect the in-vehicle travel times and agency costs, as a higher level of supply is needed to fulfill these longer trips (the fleet also grow accordingly). These higher operational and in-vehicle costs induce lower frequencies, which increase the total waiting time in the real city scenario and demand larger vehicles. The access/egress cost in the real city model are almost equal since the spatial coverage of both networks is quite similar.

As a result, the balance between the Open and Closed BRT systems still tilts towards the Open BRT. Furthermore, now this system is preferred to the Closed BRT even if the transfer penalties are ignored. This is consistent with the sensitivity analysis presented in Section 3.3.2 in which an Open BRT becomes more attractive when the average trip distance grows.

These results show that an optimal transit network design on a real city will very likely differ significantly from our idealized rectangular model. The reasons for this difference lie in the uneven distribution of the demand and the different length of feeder routes and the presence of suburban lines. The contribution of the rectangular model is to allow us to understand the trade-offs present in the design and the impact of key variables in other design decisions, as has thoroughly discussed all along this section. It also allowed us to realize that in general the open BRT presents lower costs than the closed BRT since this was true in almost any scenario we analyzed.

4. Conclusions

In this paper we formulated a mathematical model that compared the performance of Closed and Open BRT systems structured around a single corridor. In both cases, bus frequencies, vehicle capacity, the number of lines, and other key system attributes are obtained from minimizing social costs. The idealized rectangular model was then used to determine which of the two public transport systems should perform better for a medium-sized city with the characteristics of Valdivia, Chile. Finally, the model was used to compare the performance of both systems once the specific characteristics (such as trip demand in each area of the city and street layout) were considered.

We argue that many mid-sized cities (those with less than 500,000 inhabitants) have grown around a single main street. This street is likely not only to pass through the city's central business district, but also to be the most important transport corridor. Thus, to increase the capacity of the corridor and improve its performance, this street becomes the natural place to install a Bus Rapid Transit corridor. In the Chilean cities we have analyzed, this argument seems to hold, and the main transport corridor of the city becomes easily apparent. Of course, installing a BRT is always challenging due to the fight for urban space among different transport modes and local activities (Paget-Seekins, 2016).

Even though the idealized rectangular model and the detailed city model shared key parameters as total demand, maximum number of lines and city size, the results are similar in structure but quite different in some cost magnitudes. This shows the attractiveness of simple idealized models to understand solution patterns and relationships among variables, as described in the next paragraph, but also shows the importance of relying on more realistic models for a definitive decision and a precise network design.

The results of the two experiments are quite consistent in their structure. Both showed that Open BRT networks present lower social costs than Closed BRT networks in every case except when the transfer penalty Δ drops to 40% of its base value or lower. The sensitivity analysis in five key parameters of the rectangular model showed that this conclusion was strengthened for higher demand and higher transfer penalties which is consistent with previous works (Jara-Díaz and Gschwender, 2003a, 2003b; Fielbaum et al., 2016; Badia et al., 2016). The analysis also showed that the performance of the Open BRT gets slightly stronger when trip length and value of time grows. Finally, buses operating in mixed traffic tend to present high headway variance, severely affecting waiting times. The higher this variance, the more attractive the Closed BRT becomes, because the operation inside the corridor in the case of the Open BRT gets affected by this irregularity.

Another interesting conclusion is that the supposed economies of scale in terms of agency costs in a Closed BRT network are not so clear. Clearly, using large trunk buses in the dense environment of the corridor, and smaller feeder buses outside of it, can better fit supply to demand. However, the large number of transfers that such configuration requires, creates several operational inefficiencies due to considerably higher dwell times. The large amount of transfers in the Closed BRT system made in-vehicle travel time higher than the Open BRT system. Longer dwell times in the case of a Closed BRT also affected the commercial speed of the buses in that system. All in all, transfers do not only affect transferring users considerably, but also made the system slower in general, generating inefficiencies in operation and level of service. The mismatch of bus size and demand observed in Open BRT is largely paid off through the benefits of effective and direct trips.

An unexpected result of the design optimization process is that in our case the number of parallel lines feeding the BRT corridor (Open or Closed) was almost always actively constrained by the city road network. We expect that for cities with low public transport demand density (few trips or extended cities), this finding may not hold, as shown by our sensitivity analysis. This could also happen in cities with low value of time or when transfer penalties are considered too high.

The discussion regarding whether open or closed BRTs should be preferred is quite an open question, far from being solved. This paper contributes in this discussion by presenting a model that allow such comparison considering the most important impacts to passengers and operators arising from the operational design, and by quantifying each of them under an optimal design for each case.

A real-world decision should involve some other policy aspects that have not been addressed in our model, as infrastructure, bus characteristics or the simplicity of the closed BRT for the understanding of users. Regarding infrastructure one might expect that stations in a Closed BRT would be larger, since buses would be larger and more passengers at a time would have to wait at these stations. This specialized infrastructure should allow for faster and smoother boarding and alighting. The effect on the traffic signals of having buses entering and leaving the corridor has also been ignored in the case of an Open BRT. These operations could become problematic and could affect the capacity of the corridor if frequency is high. Also high frequencies may trigger queuing at stops, affecting in-vehicle travel times. This is especially true in the case of the Open BRT in which the aggregated frequency over the corridor can become quite high (recall that in this case buses are smaller than those used in the trunk service of the Closed BRT system). For medium-sized cities this should be easily handled by proper high-capacity bus infrastructure (i.e. several bus docks per stop and an overpassing lane). However, if the area available for bus stops in the corridor could not exceed a given amount, then this should be added to the model as a constraint which would result in buses with higher capacity. Regarding buses, doors may be needed at both sides of the bus if stops are located at the center of the corridor. All these infrastructure and bus configuration decisions are quite case specific and therefore the possible configurations (number of lanes, overpassing infrastructure, location of the stops at the median or curb side of the corridor, etc.) could be designed on a separate process from our analysis, and the cost difference compared with the impact found in our operational analysis.

The comparison between both BRT schemes has been made for the peak period. During off-peak periods where the frequency may be very low the system may operate under a schedule. In such a case the waiting times should be reformulated since informed passengers would wait little and transfers might be coordinated. Also, it could be argued that considering the full dynamics of the daily passenger demand could affect the comparison between an Open and a Closed BRT. We should also recognize that a Closed BRT is simpler to navigate for the user. This is particularly relevant in the case of a network of corridors. However, in the case of a single corridor city like this one, the Open BRT network that is being proposed could be simple enough for an uneducated traveler to understand.

We should also mention that a generalization of our results to “any city” should be avoided since we focus our analysis for middle size cities with a single BRT corridor. An attractive future research might extend our model to the case of a large city equipped with a network of BRT corridors, instead of a single one. Finally, this paper has compared strict open and closed BRT corridors. However, we could explore the merits of a mixed system in which an open operation coexists with a service operating strictly inside the corridor.

This paper was written based on the Master Thesis of Francisco Proboste. The research topic and methodology was proposed by his supervisor, Juan Carlos Munoz. Francisco Proboste gathered all the information needed for this work and did all the calculations and analysis presented in the paper. Juan Carlos Munoz oriented the work identifying research opportunities and analyzing the results, correcting the model and identifying some policy insights that it delivered. Antonio Gschwender was a member of the evaluation committee of the Thesis. Antonio Gschwender was key in determining the structure of the paper, providing an important literature background, and analyzing the results in order to highlight the main contributions of this research. Francisco Proboste, Juan Carlos Munoz and Antonio Gschwender did the writing of the paper as a team. All authors have read and agreed to the published version of the manuscript

Acknowledgements

This research was supported by CEDEUS Chile, ANID/FONDAP 15110020 Chile, the Bus Rapid Transit Centre of Excellence funded by Volvo Research and Educational Foundations (VREF), FONDECYT 1150657, the Master of Science Program of the Engineering School of Pontificia Universidad Católica de Chile and CONICYT PIA/BASAL AFB180003.

Appendix A. Optimal spacing

The Stop Spacing optimization process was inspired by ideas obtained in Daganzo and Ouyang (2019) and Wirasinghe and Ghoneim (1981). Both cases used a Continuum Approximation Approach, where optimal spacing of a certain portion of a line is obtained analytically.

This process is divided in three steps:

- The definition of the temporal and spatial horizon in which to optimize the stop spacing.
- The definition of an Objective Function that includes all relevant costs related to the stop spacing.
- Optimizing the problem in order to obtain a closed analytical expression of the optimal spacing.

This is how these three steps are applied into our model:

A.1. Temporal and spatial horizon definition

In order to use standardized spacing in all places of the city, we was decided to use only two different spacing values: one for periphery stops, and another for corridor stops. Hence the spacing optimization model was applied to two independent spatial contexts: (z = p) periphery and (z = c) corridor. Additionally, the temporal horizon was to be the same used in the general optimization model of this model; in other words, the rush hour between 8:00AM and 9:00AM.

The units of the objective function being minimized is monetary cost per unit of time and per unit of distance along the route.

A.2. Costs involved

Three relevant costs considered were included in this optimization model:

1. Access/Egress Time

$$Access\ \ Cost = \left(\frac{s_z}{4V_A}\gamma_A\right)(\lambda_z s_z)\left(\frac{1}{s_z}\right)$$

where:

$\left(\frac{s_z}{4V_A}\gamma_A\right)$: *Average unitary cost of access to a stop.*

$(\lambda_z s_z)$: *Incoming demand rate to a certain stop.*

$\left(\frac{1}{s_z}\right)$: *Average stop quantity in a unit of space.*

2. Travel Time due to stops

$$Travel\ \ Time^{Stops} = (t_s^z\gamma_v)\,\underline{Q_z}\left(\frac{1}{\underline{s_z}}f_z\right)$$

where:

$(t_s^z\gamma_v)$: *Unitary cost of a stop for an user inside a bus that is stopping*

$\underline{Q_z}$: *Average load of a bus traveling in z*

$\left(\frac{1}{\underline{s_z}}f_z\right)$: *Number of stops that all buses have to perform*

3. Agency Costs due to stops

$$Agency\ \ Costs^{Stops} = (a_t + b_t K_z) t_s^z \frac{1}{s_{z}} f_z$$

$(a_t + b_t K_z) t_s^z$: *Unitary agency cost for a bus that is stopping*

$\left(\frac{1}{s_{z}} f_z\right)$: *Number of stops that all buses have to perform*

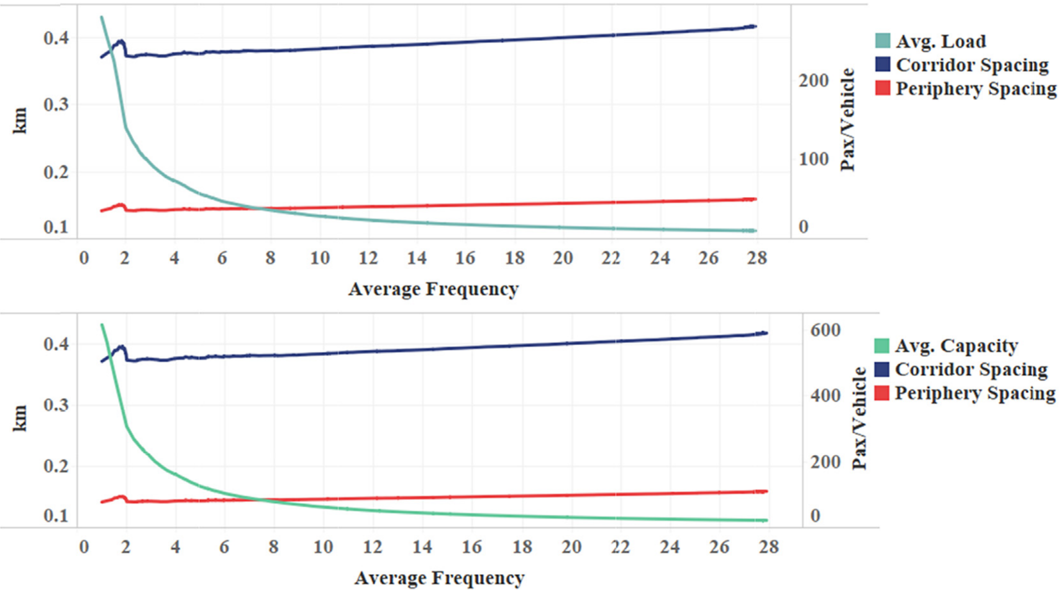
A.3. Optimization

Once the related costs are identified, they are added in a single objective function to be optimized in terms of the stop spacing s. The first order condition leads to the following expression for the optimal stop spacing s_z^* for periphery (z = p) and the corridor (z = c):

$$s_z^* = \sqrt{\frac{f_z t_s^z (\gamma_v Q_z + a_t + b_t K_z)}{\frac{\gamma_a \lambda_z}{4V_a}}}$$

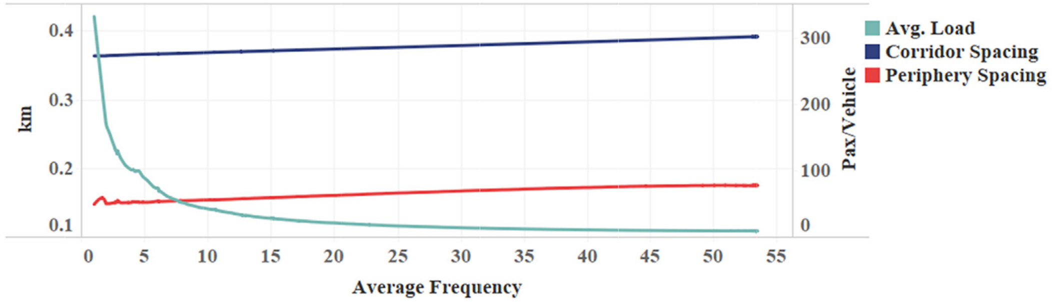
A.4. Average capacity, load and spacing along Open BRT frequencies optimization

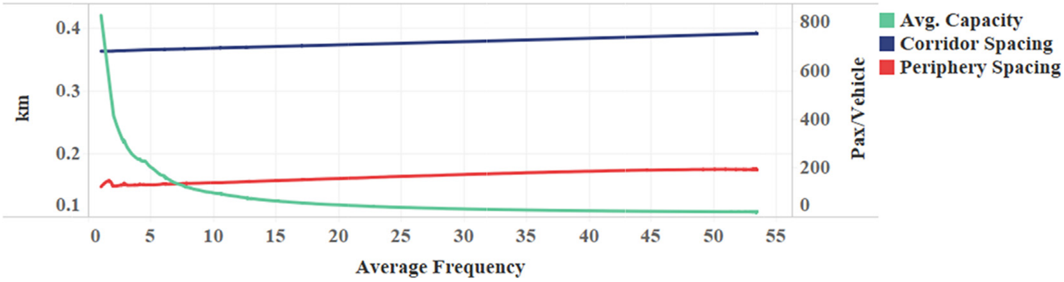
The top Figure in this Annex presents the average bus load (left axis) and the optimal periphery and corridor stop spacings (right axis) for different average frequencies for the Open BRT. The bottom Figure presents the average capacity together with the optimal spacings for different average frequencies.



A.5. Average capacity, load and spacing along Closed BRT frequencies optimization

The top Figure in this Annex presents the average bus load (left axis) and the optimal periphery and corridor stop spacings (right axis) for different average frequencies for the Closed BRT. The bottom Figure presents the average capacity together with the optimal spacings for different average frequencies.





Appendix B. List of variables directly depending on the frequencies (assuming a given number of lines)

Variable	Symbol
Agency costs related to distance travelled	C^{l}_{op-km}
Agency costs related to fleet size	$C^{l}_{op-fleet}$
Spacing between stops	s_z
Access/egress time	T^{ij}_{a}
Initial waiting time	T^{ij}_{w}
Transfer waiting time	T^{ij}_{tw}
In-vehicle travel time	T^{ij}_{v}
Speed	V
Vehicle capacity	K_l
Maximum load of the line	ρ_l
Cycle time of line l	R_l
Number of stops	n_l
Number of passengers boarding	B
Number of passengers alighting	A

Appendix C. Parameters

Parameter	Symbol and Units	Value	Source
Travel Time Value	γ_v [CLP$/h]	1,498	División de Evaluación Social de Inversiones, 2015
Waiting Time Value	γ_e [CLP$/h]	2,351	Raveau et al., 2014
Walking Time Value	γ_a [CLP$/h]	3,273	Raveau et al., 2014
Transfer Penalty	Δ [travel time minutes/transfer]	5.86	Raveau et al., 2014
Access Speed	V_a [km/h]	4.50	Our own estimation
Periphery Fixed Dwell Time	t^p_s [s/stop]	11.76	Our own estimation
Corridor Fixed Dwell Time	t^c_s [s/stop]	15.24	Our own estimation
Periphery Boarding Time	t^p_b [s/pax]	4.04	Tirachini, 2014
Periphery Alighting Time	t^p_a [s/pax]	2.36	Tirachini, 2014
Corridor Boarding Time	t^c_b [s/pax]	1.75	Tirachini, 2014
Corridor Alighting Time	t^c_a [s/pax]	1.75	Tirachini, 2014
Fixed Agency Cost of operation per bus kilometer	a_d [CLP$/bus*km]	182	MTT, 2013
Variable Agency Cost of operation per bus kilometer	b_d [CLP$/bus*size*km]	2.28	MTT, 2013
Fixed Agency Cost of operation per hour	a_t [CLP$/bus*hr]	4,189	MTT, 2013
Variable Agency Cost of operation per hour	b_t [CLP$/bus*size*hr]	13.0	MTT, 2013
Length of the north–south edge of Valdivia	α [km]	3.9	Our own estimation
Length of the north–south edge of the northern periphery	α_1 [km]	1.1	Our own estimation

Length of the west-east edge of Valdivia	β[km]	5.5	Our own estimation
Length of the west-east distance from western corridor	β_1[km]	4.0	Our own estimation
Length of the north–south edge of the CBD in Valdivia	R_1[km]	0.6	Our own estimation
Length of the west-east edge of the CBD in Valdivia	R_2[km]	1.5	Our own estimation

Appendix D. Indicators by line

D.1. Continuum approximation model-closed BRT lines detail

Line	Frequency [cycle/hour]	Operational Speed [km/hr]	Vehicle Capacity [Seats/Bus]	Fleet [Buses]	Load Factor [%]
1	29.9	16.0	17.1	14.6	37.1%
2	29.9	16.0	17.2	14.7	37.2%
3	29.8	16.0	17.3	14.6	37.2%
4	29.7	15.9	17.3	14.6	37.2%
5	29.8	15.9	17.3	14.6	37.2%
6	29.8	15.9	17.3	14.6	37.2%
7	29.8	15.9	17.3	14.6	37.2%
8	30.1	15.9	17.1	14.8	37.2%
9	30.0	15.9	17.1	14.7	37.2%
10	98.9	22.6	28.6	48.2	59.6%

D.2. Continuum approximation model-open BRT lines detail

Line	Frequency [cycle/hour]	Operational Speed [km/hr]	Vehicle Capacity [Seats/Bus]	Fleet [Buses]	Load Factor [%]
1	22.6	22.1	21.2	13.5	62.2%
2	30.2	20.6	27.6	22.6	55.8%
3	21.1	22.0	18.5	10.9	51.6%
4	27.7	20.3	27.2	18.5	45.9%
5	21.6	21.5	18.1	9.6	41.9%
6	28.3	19.7	26.8	16.8	39.6%
7	23.5	20.8	16.7	8.7	37.0%
8	30.0	18.9	25.3	15.7	36.5%
9	26.4	19.6	14.8	7.9	35.3%
10	32.5	17.8	23.3	14.7	35.6%
11	31.1	17.8	12.6	7.1	39.5%
12	35.7	16.4	21.1	13.6	39.1%

D.3. Detailed Model-Closed BRT lines detail

Line	Frequency [cycle/hour]	Operational Speed [km/hr]	Vehicle Capacity [Seats/Bus]	Fleet [Buses]	Load Factor [%]
1	27.5	15.1	18.4	32.2	44.7%
2	56.9	14.3	10.3	22.0	30.4%
3	43.8	13.5	19.3	26.8	31.6%
4	41.5	13.5	14.2	21.4	47.5%
5	39.0	14.5	11.4	20.3	47.6%
6	40.5	12.0	18.0	47.0	49.2%
7	14.5	18.0	13.8	10.1	38.4%
8	14.7	27.7	29.0	20.3	57.9%
9	31.6	11.9	29.9	37.3	23.9%
10	102.5	19.6	35.1	73.3	54.6%

D.4. Detailed model-open BRT lines detail

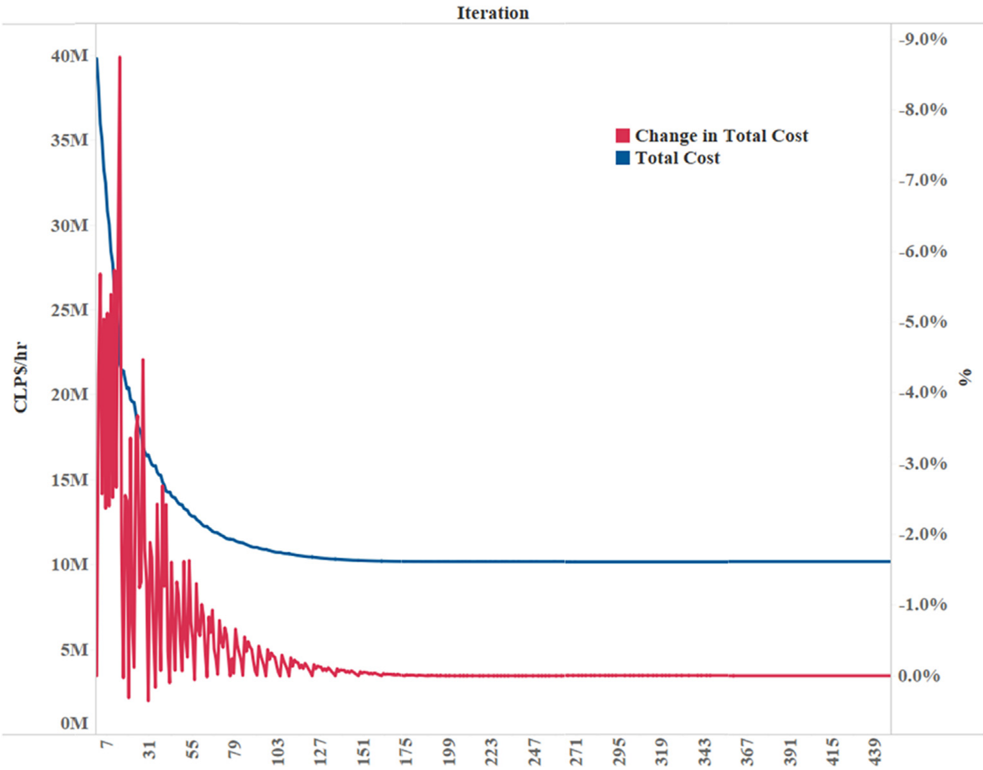
Line	Frequency [cycle/hour]	Operational Speed [km/hr]	Vehicle Capacity [Seats/Bus]	Fleet [Buses]	Load Factor [%]
1	20.7	18.3	28.7	17.8	49.9%
2	39.6	21.6	17.8	25.8	58.4%
3	17.9	27.1	36.9	31.8	67.4%
4	33.9	17.8	26.2	26.0	50.5%
5	19.3	20.8	13.4	10.1	63.5%
6	24.1	19.6	18.5	14.0	47.5%
7	35.0	16.4	23.3	24.1	61.4%

8	12.3	20.9	16.2	9.9	48.1%
9	28.5	13.4	24.8	25.4	54.8%
10	36.5	12.5	25.9	29.5	25.3%
11	20.1	15.3	25.3	22.6	46.5%
12	29.8	16.4	25.1	24.9	49.8%

Appendix E. Frequencies optimization convergence

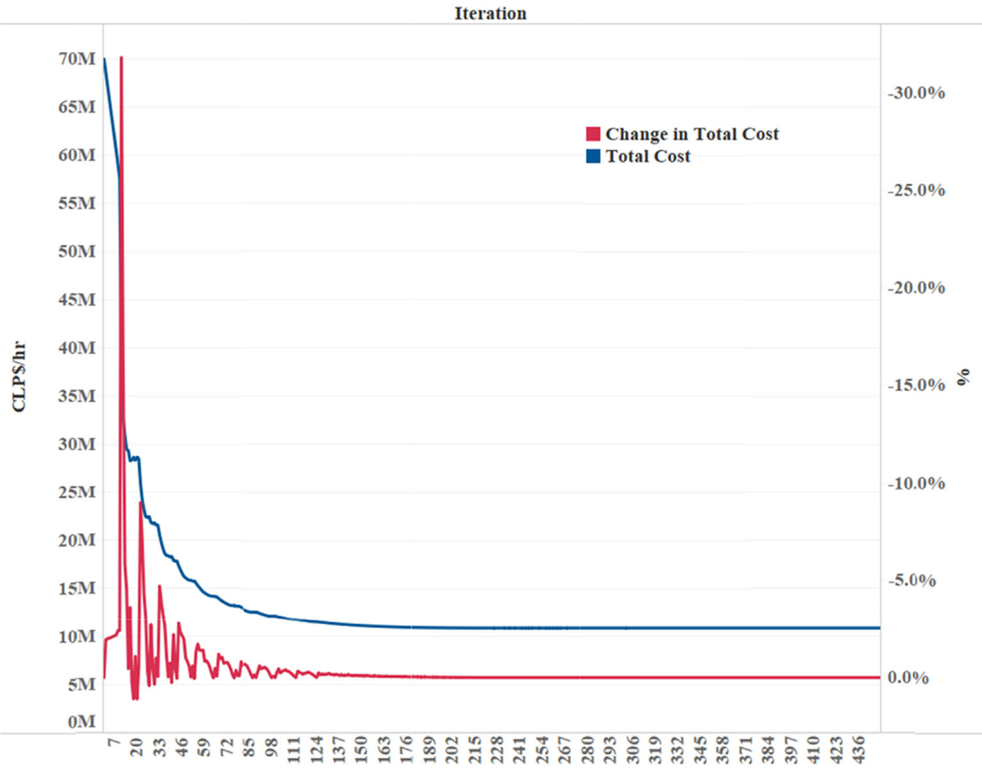
E.1. Total cost along the Open BRT frequencies optimization process

This Figure presents the convergence of the Open BRT frequency optimization process, by plotting the total cost in each iteration and the percentage change between consecutive iterations.



E.2. Total cost along the Closed BRT frequencies optimization process

This Figure presents the convergence of the Closed BRT frequency optimization process, by plotting the total cost in each iteration and the percentage change between consecutive iterations.



References

Badia, H., Estrada, M., Robusté, F., 2016. Bus network structure and mobility pattern: A monocentric analytical approach on a grid street layout. Transp. Res. Part B 93, 37–56.

Byrne, B.F., 1975. Public transportation line positions and headways for minimum user and system cost in a radial case. Transp. Res. 9, 97–102.

Ceder, A., 2001. Operational objective functions in designing public transport routes. J. Adv. Transp. 35, 125–144.

Ceder, A., Wilson, N.H., 1986. Bus network design. Transp. Res. Part B 20, 331–344.

Cenek, J., 2010. Line routing algorithm. J. Inform. Control Manage. Syst. 8, 3–10.

Cervero, R., 2013. Bus Rapid Transit (BRT): An Efficient and Competitive Mode of Public Transport. In: IURD Working Paper 2013-01, pp. 1–36. Retrieved from http://escholarship.org/uc/item/4sn2f5wc.pdf.

Chang, S., Schonfeld, P., 1991. Multiple period optimization of bus transit systems. Transp. Res. Part B 25, 453–478.

Chen, H., Gu, W., Cassidy, M.J., Daganzo, C.F., 2015. Optimal transit service atop ring-radial and grid street networks: A continuum approximation design method and comparisons. Transp. Res. Part B 81, 755–774.

Chien, S., Schonfled, P., 1998. Joint optimization of a rail transit line and its feeder bus system. J. Adv. Transp. 32, 253–284.

Currie, G., 2005. The demand performance of bus rapid transit. J. Public Transp. 8 (1), 41–55.

Daganzo, C.F., 2010. Structure of competitive transit networks. Transp. Res. Part B 44, 434–446.

Daganzo, C.F., Ouyang, Y., 2019. Public Transportation Systems: Principles of System Design, Operations Planning and Real-Time Control. World Scientific Publishing Company, Singapore.

División de Evaluación Social de Inversiones, 2015. Precios Sociales Vigentes 2015. Retrieved from http://www.ministeriodesarrollosocial.gob.cl/.

Dubois, D., Bel, G., Llibre, M., 1979. A set of methods in transportation network synthesis and analysis. J. Operat. Res. Soc. 30, 797–808.

Gschwender, A., Jara-Díaz, S., Bravo, C., 2016. Feeder-trunk or direct lines? Economies of density, transfer costs and transit structure in an urban context. Transp. Res. Part A 88, 209–222.

Fielbaum, A., Jara-Díaz, S., Gschwender, A., 2016. Optimal public transport networks for a general urban structure. Transp. Res. Part B 94, 298–313.

Guihaire, V., Hao, J.-K., 2008. Transit network design and scheduling: A global review. Transportation Research Part A: Policy and Practice, 42, 1251–1273. Hensher, D. & Golob, T., 2008. Bus rapid transit systems: a comparative assessment. Transportation 35, 501–518.

Hook, W., Wright, L., 2007. Bus rapid transit planning guide. Retrieved from https://www.itdp.org/the-brt-planning-guide/.

Ibarra-Rojas, O., Delgado, F., Giesen, R., Munoz, J.C., 2015. Planning, Operation, and Control of Bus Transport Systems: A Literature Review. Transportation Research Part B, 77, 38–75.Jansson, J. O., 1980. A simple bus line model for optimization of service frequency and bus size. J. Transp. Econ. Policy 14, 53–80.

Jansson, J.O., 1980. A Simple Bus Line Model for Optimization of Service Frequency and Bus Size. J. Transp. Econ. Policy 14, 53–80.

Jara-Díaz, S., Gschwender, A., 2003a. From the single line model to the spatial structure of transit services: Corridor or Direct? J. Transp. Econ. Policy 37, 261–277.

Jara-Díaz, S., Gschwender, A., 2003b. Towards a general microeconomic model for the operation of public transport. Transp. Rev. 23, 453–469.

Jara-Díaz, S., Gschwender, A., 2009. The Effect of Financial Constraints on the Optimal Design of Public Transport Services. Transportation 36, 65–75.

Jara-Díaz, S., Gschwender, A., Bravo, C., 2017a. Total cost minimizing transit route structures considering trips towards CBD and periphery. Transportation. https://doi.org/10.1007/s11116-017-9777-z.

Jara-Díaz, S., Fielbaum, A., Gschwender, A., 2017b. Optimal fleet size, frequencies and vehicle capacities considering peak and off-peak periods in public transport. Transp. Res. Part A 106, 65–74.

Kathuria, A., Parida, M., Sekhar, Ch.R., Sharma, A., 2016. A review of bus rapid transit in India. Cog. Eng. 3.

Kepaptsoglou, K., Karlaftis, M., 2009. Transit route network design problem: review. J. Transp. Eng. 135, 491–505.

Kline, S., Forbes, S., Hughes, J., 2012. Midsize Cities on the Move. Retrieved from http://www.reconnectingamerica.org/assets/Uploads/20121206midsizefinal.pdf.

Kocur, G., Hendrickson, C., 1982. Design of local bus service with demand equilibrium. Transp. Sci. 16, 149–170.
Kuah, G., Perl, J., 1988. Optimization of feeder bus routes and bus-stop spacing. J. Transp. Eng. 114, 341–354.
Levinson, H., Zimmerman, S., Clinger, J., Rutherford, G.S., 2002. Bus rapid transit: an overview. J. Public Transp. 5, 1–30.
MTT, 2013. Estudio para la actualización de la estructura de costos y evaluación del equilibrio económico de los contratos de operadores de vías del sistema de transporte público de Santiago mediante buses. Ministerio de Transportes y Telecomunicaciones de Chile, Directorio de Transporte Público Metropolitano.
Newell, G.F., 1979. Some issues relating to the optimal design of bus routes. Transp. Sci. 13, 20–35.
Ortúzar, J. de D., Willumsen, L.G., 2011. Modelling Transport, fourth ed. John Wiley and Sons. Chichester.
Paget-Seekins, L., 2016 Conflict over public space. In Muñoz, J.C., Paget-Seekins, L. (Eds.), Bus Rapid Transit and the Restructuring of Public Transit. Policy Press, Bristol, UK, pp. 1–14.
Powell, M.J.D., 1994. A direct search optimization method that models the objective and constraint functions by linear interpolation. In: Gomez, S., Hennart, J-P. (Eds.), Advances in Optimization and Numerical Analysis. Kluwer Academic (Dordrecht), pp. 51–67.
Raveau, S., Guo, Z., Muñoz, J., Wilson, N., 2014. A behavioural comparison of route choice on metro networks: Time, transfers, crowding, topology and socio-demographics. Transp. Res. Part A: Policy Pract. 66, 185–195.
Saidi, S., Wirasinghe, S.C., Kattan, L., 2016. Long-term planning for ring-radial urban rail transit networks. Transp. Res. Part B 86, 128–146.
Trasa Ingenieria, 2014. Actualizacion Plan de Transporte Valdivia y Desarrollo de Anteproyecto, Etapa 1. Retrieved from http://www.sectra.gob.cl/biblioteca/detalle1.asp?mfn=3290.
Tirachini, A., 2014. The economics and engineering of bus stops: Spacing, design and congestion. Transp. Res. Part A: Policy Pract. 59, 37–57.
Tirachini, A., Hensher, D.A., Jara-Díaz, S.R., 2010. Comparing operator and users costs of light rail, heavy rail and bus rapid transit over a radial public transport network. Res. Transp. Econ. 29, 231–242.
Wirasinghe, S., 1980. Nearly optimal parameters for a rail/feeder-bus system on a rectangular grid. Transp. Res. Part A 14, 33–40.
Wirasinghe, S., Ghoneim, N., 1981. Spacing of Bus-Stops for Many to Many Travel Demand. Transportation Science. Retrieved from http://pubsonline.informs.org/doi/abs/10.1287/trsc.15.3.210.
Wirasinghe, S., Hurdle, V., Newell, G., 1977. Optimal parameters for a coordinated rail and bus transit system. Transp. Sci. 11, 359–374.
Wirasinghe, S., Kattan, L., Rahman, M., Hubbell, J., Thilakaratne, R., Anowar, S., 2013. Bus rapid transit: a review. Int. J. Urban Sci. 17, 1–31.