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A multi-objective approach for solving a replacement policy problem for equipment subject to imperfect repairs

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ABSTRACT

This paper proposes a multi-objective approach to model a replacement policy problem applicable to equipment with a predetermined period of use (a planning horizon), which may undergo critical and non-critical failures. Corrective replacements and imperfect repairs are taken to restore the system to operation respectively when critical and non-critical failures occur. Generalized Renewal Process (GRP) is used to model imperfect repairs. The proposed model supports decisions on preventive replacement intervals and the number of spare parts purchased at the beginning of the planning horizon. A Multi-Objective Genetic Algorithm (MOGA) coupled with discrete event simulation (DES) is proposed to provide a set of solutions (Pareto-optimum set) committed to the different objectives of a maintenance manager in the face of a replacement policy problem, that is, maintenance cost, rate of occurrence of failures, unavailability, and investment on spare parts. The proposed MOGA is validated by an application example against the results obtained via the exhaustive approach. Moreover, examples are presented to evaluate the behavior of objective functions on Pareto set (trade-off analysis) and the impact of the repair effectiveness on the decision making.

1. Introduction

Decisions on replacement policy are common places in the field of preventive maintenance planning. Although the main purpose of decision models on replacement policy is to obtain optimal preventive replacement intervals, equipment may fail before the time defined for that. In this context, two points are central: (i) the criteria used for determining the optimal preventive replacement intervals, and (ii) the assumption on the effectiveness of repairs that are executed whenever failures occur before the preventive replacement. Indeed, the supposition on how the systems are restored to operation after the occurrence of failure significantly influences the applicability of the model in real cases.

Many works such as have investigated optimal preventive replacement policies under the perspective of minimizing the maintenance cost [1–5], while others aimed at optimizing the system reliability [6–9] or availability [5,9–11]. However, the

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objectives of determining a replacement policy with the highest reliability or availability and the lowest related maintenance costs are often conflicting. In fact, even though shorter replacement intervals may improve the system reliability, they may also increase the associated costs and unavailability in the case of an inefficient maintenance crew. Therefore, finding the solution that optimizes an individual objective may not result in adequate replacement policies.

In this way, some authors like Yang et al [5], Jiang & Ji [12], Duan et al [13], Ke & Yao [14] and Nosoohi & Hejazi [15] proposed multi-objective models for finding the optimal age to perform preventive maintenance, and Sharma et al [16] carried out a full review of the literature on maintenance optimization models, and then suggested considering simultaneously maintenance cost and reliability measures to make decisions on maintenance planning. In these situations, a solution that concurrently optimizes all objectives is very difficult to be obtained or does not even exist. Thus, in a multi-objective approach instead of having a unique solution as in single objective cases, one may obtain the Pareto set, which is composed of non-dominated solutions [17].

Regarding the second point (assumption on the effectiveness of the repair actions), some alternatives may arise. For instance, the perfect repair supposition results in an “as good as new” (AGAN) condition, while the minimal repair yields an “as bad as old” (ABAO) situation. Perfect repair assumption is suitable when equipment is entirely replaced by a new one, whereas when the repair only recovers the function with simple actions, as restarting or resetting, it can be considered as minimal. AGAN and ABAO are usually treated by Renewal Processes (RP) and Non-Homogeneous Poisson Processes (NHPP) [18].

Imperfect repairs, in turn, return a failed item to a state between AGAN and ABAO. Repairing or even replacing minor parts or components, or performing general repairs (repack, weld, etc.), adjustment (align, reset, calibrate, etc.) and refit (polish, clean, grind, paint, coat, lube, oil change, etc.) may correspond to imperfect repairs, which may be handled with Generalized Renewal Process (GRP) [19–21], which is a type of a virtual age approach. In GRP, the imperfect repair is modeled through a parameter that evaluates the effectiveness of the repair action by measuring equipment rejuvenation after performing maintenance; for further details on imperfect repairs’ methods (including GRP and others), see Pham & Wang [22].

Generally, repair actions return equipment to an imperfect/general state that is “better than old” but “worse than new”. Then, models that do not consider this possibility are not suitable enough to be applied in many real cases. Shahraki et al [6], Badía et al [23], Love et al [24], Him et al [25] and Aghezzaf et al [26], for example, have investigated maintenance policies under the assumption the system is subject to imperfect repairs if it fails before the preventive maintenance.

However, the above-mentioned works either use a multi-objective approach to determine replacement policies, supposing the system is subject to perfect and/or minimal repairs [12–15], or use an imperfect repair hypothesis, but considering a mono-objective approach to determine intervals for preventive maintenance [6,23–26].

Given that, this paper develops a multi-objective approach to model a problem that tackle simultaneous determination of the preventive replacement intervals and the number of spare parts by assuming equipment may be subject to two possible kinds of failures: (i) critical failures, which are repaired through corrective replacement (perfect repair), and (ii) non-critical failures, which are recovered through imperfect repairs, which are here handled with GRP. We assume this structure because some failures may greatly damage equipment, and then full replacement is required, while others only damage components, demanding equipment to be imperfectly repaired. Four maintenance objectives are considered to compare different solutions: maintenance costs, rate of occurrence of failures, unavailability and spare parts investment.

The four objective functions developed in this paper become rather intricate, and then an analytical treatment may be infeasible. Then, we adopt discrete event simulation (DES; [27]) to imitate system behavior through the random generation of discrete events to determine the values of the objective functions for a solution. Sharma et al [16] affirm there is an emerging trend towards using simulation for maintenance optimization. Moreover, the combinatorial nature of the problem may increase the complexity of the problem and render prohibitive the use of exhaustive procedures to evaluate all potential solutions by employing DES due to increased time and computational effort required. Therefore, probabilistic optimization heuristics such as Genetic Algorithms (GA; [28]) are suitable alternatives, and then will be here adopted. Indeed, Konak et al [29], Abdullah & Ashutosh [30], Lins & Droguett [31] and Azevedo et al [32] have already combined GA with DES in the maintenance optimization context.

In the multi-objective approach, GA permits a separated treatment of the different objectives, thus not requiring transformations of the multiple objectives into a unique function. Instead, a set of non-dominated solutions in competing for the four objectives is given: the Pareto-optimal set [33]. The decision making is performed by analyzing the preferences of the decision-maker when comparing the trade-offs among the solutions, for example defining whether, for a solution, the gain on reliability justifies its costs.

The main difference between the single objective GA and the multi-objective GA (MOGA) is the selection phase, where, in the latter, the concept of dominance is directly incorporated. For further details on single and/or multi-objective GA, see Deb [33], Coello et al [34], Eiben & Smith [28] and Konak et al [29]. Then, in this paper, a solution that couples MOGA and DES is proposed to obtain the Pareto set for the problem. Specific genetic operators will be developed for the proposed problem to avoid that MOGA evaluates unfeasible solutions.

The remainder of this paper is organized as follows. A brief overview of the GRP concepts is given in Section 2. Section 3 provides a description and a formulation of the multi-objective replacement policy problem. Section 4 explains the use of DES for calculating the values of the objective functions. The proposed model for generating Pareto-optimal solutions via MOGA and DES is presented in Section 5. Section 6 presents examples to validate the model and analyze the trade-offs among solutions in the Pareto set, including an evaluation of the effectiveness repair impact. Finally, some concluding remarks are provided in Section 7.

2. Generalized Renewal Process

GRP is a virtual age-based counting process proposed by Kijima & Sumita [19] and can be used to model systems subject to imperfect repairs. To that end, GRP uses a rejuvenation parameter q that "changes" the actual age of the equipment after a repair occurrence. This means that, although the equipment has been operational for certain calendar time, it seems like it operates as it was younger (or older), which depends on the repair' effectiveness. Let X_n be the time between the $(n\text{–}1)^{\text{th}}$ and the n^{th} failures, Y_n be the real age ($Y_n = \sum_{k=1}^{n} X_k$), and V_n be the virtual age after the n^{th} failure. In this way, V_n is calculated as follows:

$$V_n = V_{n-1} + qX_n = q\sum_{k=1}^{n} X_k = qY_n \tag{1}$$

where $V_0 = 0$. Note Eq. (1) considers the effect of the repair only reduces the additional age X_n (Kijima Type I). In general, $q \in [0, 1]$, even though other values are also possible.

The Cumulative Distribution Function (CDF) of X_n is conditioned to the $(n-1)$th system virtual age, as follows:

$$F(x_n|\upsilon_{n-1}) = P(X_n \le x_n|V_{n-1} \ge \upsilon_{n-1}) = P(Y_n \le \upsilon_{n-1} + x_n|V_{n-1} \ge \upsilon_{n-1}) = \frac{F(x_n + \upsilon_{n-1}) - F(\upsilon_{n-1})}{1 - F(\upsilon_{n-1})} \tag{2}$$

where $F(.)$ is the CDF of the Time To First Failure (TTFF). Yañez et al [35] presented a full solution, based on Monte Carlo simulation, for the Maximum Likelihood Estimators (MLE) of the GRP parameters by assuming TTFF follows a Weibull distribution. Oliveira et al [36] proposed the goodness of fit test for Weibull-based GRP.

3. Problem definition and formulation of the multi-objective problem

3.1. Description of the problem and its assumptions

Let us assume that equipment failures may be grouped into two categories: (i) failures that require equipment is correctively replaced by a new one, which corresponds to a "critical failure"; (ii) "non-critical failures" that demand the equipment is restored into operation through an imperfect corrective maintenance action that is not its full replacement.

The replacement policy aims at determining a time interval t_p, when a preventive replacement should be performed, if the equipment does not fail critically within this interval. Otherwise (if a critical failure occurs before t_p), a corrective replacement takes place. In this case, the equipment reaches the end of its stochastic lifetime at any time before it is preventively replaced. Non-critical failures may also occur before replacement, and then an imperfect repair is performed. The occurrence of non-critical failures does not comprise the end of equipment lifetime since it returns to operation by repair action.

Preventive or corrective replacements and the imperfect repairs are time-consuming, and work operations are halted during these interventions. Cycle time is defined as the summation of equipment lifetime and time to replace it with the new one. Fig. 1 illustrates these alternating processes (failure-repair) for both cycle types (preventive and corrective). We consider the spare part may be provided either at the beginning of (H_0) or during (at the time of need) the planning horizon. We also assume the cost of purchasing spare parts is lower when they are provided at H_0 instead of at the time when replacement is required. Yet, savings in downtime and maintenance costs are obtained if a spare part is already available when the replacement is necessary.

Given that, let s be the number of spare parts purchased at H_0. Then, the first replacements will be performed with spare parts available and, if more than s replacements are needed, they will have an additional cost. Thus, s influences both the maintenance cost and the system downtime. The higher the quantity s, the more replacements will be performed with spare parts available. However, the lower the quantity s, the more budget is available to be used in other investment plans. Thus, the proposed model aims at determining simultaneously t_p and s [15,36].

3.2. Multi-Objective problem formulation

In this section, the objective and constraint functions are formulated for the problem previously described. We considered as objectives to be optimized the maintenance costs, rate of occurrence of failures, system unavailability and investments on spare parts. Fig. 2 defines the parameters and random variables adopted in the multi-objective model.

3.2.1. The probability of having or not having spare parts available for replacement

We consider the probability of having spare parts at the time of replacement depends on s purchased at H_0. Then, the probability of having spare parts available P_s is given by

$$P_s = P(N_c \le s) = \sum_{n_c=0}^{s} P(N_c = n_c),$$

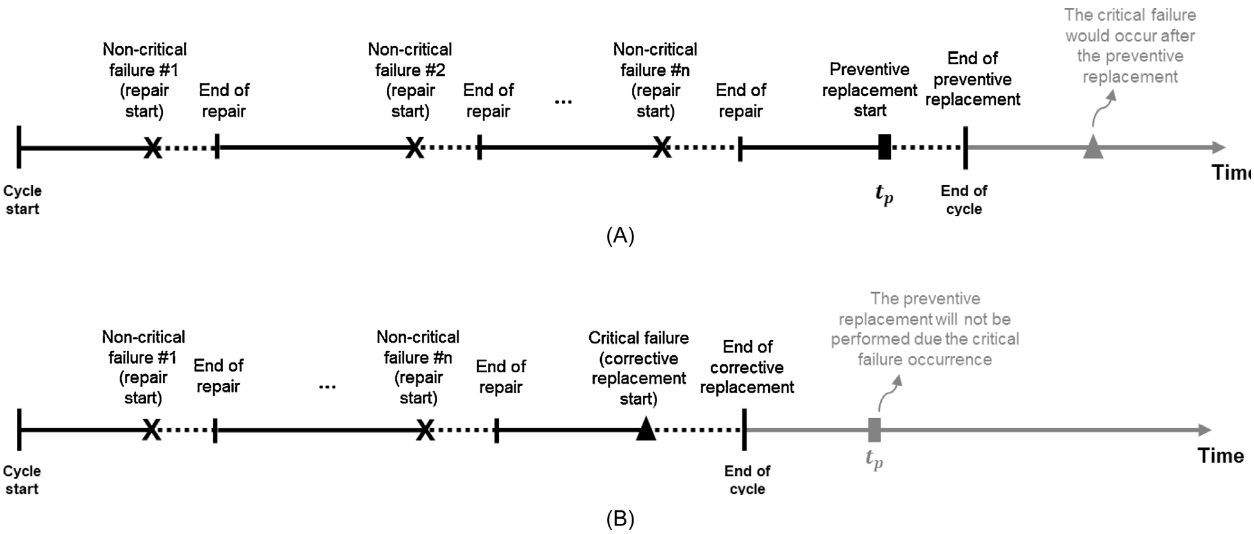


Fig. 1. Illustration of preventive (A) and corrective (B) cycles in the perspective of calendar time.

Decision Variables:			
t_p	Time interval for preventive replacement		
s	Number of spare parts purchased at H_0		
Multi-objective problem parameters:			
H	Planning Horizon	B	Available budget for purchasing spare parts
$C_r(n)$	Mean cost of performing n imperfect repairs	R_r	Mean time to perform imperfect repair
C_{p1},C_{p2}	Mean cost of preventive replacement with and without spare part available respectively	R_{p1},R_{p2}	Mean time to perform preventive replacements with and without spare parts available respectively
C_{f1},C_{f2}	Mean cost of corrective replacement with and without spare part available respectively	R_{f1},R_{f2}	Mean time to perform corrective replacements with and without spare parts available respectively
C_S	Unit cost of spare part at H_0	Θ	Maximum acceptable system unavailability
Random variables:			
T_f	System total lifetime at the critical failure	M_t	Number of non-critical failures between 0 and t
T_c	System cycle time	N_c	Total number of cycles (replacements) over H

Fig. 2. Notation for the multi-objective model.

where N_c is the number of replacements performed over H. Then, we define C_p (mean cost of preventive replacement), C_f (mean cost of corrective replacement), R_p (mean time to perform preventive replacement) and R_f (mean time to perform corrective replacement) as follows:

$$C_p = C_{p1}P_s + C_{p2}(1 - P_s) \tag{3}$$

$$C_f = C_{f1}P_s + C_{f2}(1 - P_s) \tag{4}$$

$$R_p = R_{p1}P_s + R_{p2}(1 - P_s) \tag{5}$$

$$R_f = R_{f1}P_s + R_{f2}(1 - P_s) \tag{6}$$

where index 1(2) indicates the value of the parameters as there are (no) spare parts available. This means the cost and time parameters of replacements are influenced by s.

Note this model also considers a just-in-time way for planning preventive replacements. In this situation, downtime and maintenance costs would be the same for both cases with and without spare parts available. Then, we would assume $C_{p1} = C_{p2}$ and $R_{p1} = R_{p2}$. Next, we describe the objectives.

3.2.2. Objective 1: Maintenance Cost

The maintenance cost rate is given by a ratio between the expected cost per cycle and the expected cycle length [12]. We consider a replacement cycle may end in two different situations:

(i) preventive replacement, where a critical failure does not occur before t_p (i.e., $t_f \geq t_p$), and thus the replacement is performed at t_p. Let $E[M_{t_p}]$ be the mean number of non-critical failures by t_p, which are handled with imperfect repairs. Then, the associated expected maintenance cost is given by $C_r(E[M_{t_p}]) + C_p$, where $C_r(n)$ and C_p are the mean costs of performing n imperfect repairs and a preventive replacement respectively;

(ii) corrective replacement: in this situation, a critical failure occurs before t_p (i.e., $t_f < t_p$). Then, the expected number of non-critical failures is given by the conditional expectation $E[M_{t_f}|t_f < t_p]$, and the expected maintenance cost is $C_r(E[M_{t_f}|t_f < t_p]) + C_f$, where C_f is the mean cost of a corrective replacement.

Similarly, we can define the expected times for the preventive and corrective replacement cycles. For the former, the equipment lifetime always ends at t_p, and R_p denotes the meantime to perform the preventive replacement, and thus the expected cycle length is $t_p + R_p$. For the latter, the end of the equipment lifetime is a random variable represented by T_f, and thus $E[T_f|T_f < t_p]$ is the mean lifetime and R_f is the meantime to perform a corrective replacement. Therefore, the expected length of a corrective cycle is $E[T_f|T_f < t_p] + R_f$. Eq. (7) shows the expected maintenance cost rate, $F1$, which is minimized in the multi-objective model.

$$F1 = \frac{\left[C_r\left(E\left[M_{t_p}\right]\right) + C_p\right] \times P\left(T_f \geq t_p\right) + \left[C_r\left(E\left[M_{t_f}|t_f < t_p\right]\right) + C_f\right] \times P\left(T_f < t_p\right)}{(t_P + R_P)P\left(T_f \geq t_P\right) + \left(E\left[T_f|T_f < t_p\right] + R_f\right)P(T_f < t_P)} \tag{7}$$

3.2.3. Objective 2: rate of occurrence of failures

The failure occurrences have undesirable and unpredictable consequences on maintenance, safety issues, and production operations [15]. Corrective activities in most cases constitute a major portion of unplanned maintenance expenditures such as costs with production loss and delays, besides the possibility of accident occurrences. Then, the second objective is to minimize the occurrence of corrective activities, and its consequence, as far as possible.

Nosoohi & Hejazi [15] and Azevedo et al [32] used a function that prompts the expected number of failures in a cycle to represent this objective. Thus, in [15] and [32], this objective is obtained by reducing the time interval to preventive replacements t_p. Low values of t_p diminish the probability of failure occurrences in a cycle, while the expected number of cycles increases. Given that, an objective function that relates the number of failures in a cycle and the number of cycles would be suitable.

In this paper, a mean rate of failure occurrences is used, which is presented in Eq. (8) that is a ratio between the expected number of failures per cycle (where the first and second parts correspond to the expected number of failures in preventive and corrective replacement cycles respectively) and the expected cycle time (defined in the denominator of Eq. (7)). Note that, in corrective replacement cycles, the total number of failures is the number of non-critical failures plus one, which corresponds to the critical failure.

$$F2 = \frac{E\left[M_{t_p}\right]P\left(T_f \geq t_p\right) + \left(E\left[M_{t_f}|t_f < t_p\right] + 1\right)P\left(T_f < t_p\right)}{(t_P + R_P)P\left(T_f \geq t_P\right) + \left(E\left[T_f|T_f < t_p\right] + R_f\right)P(T_f < t_P)} \tag{8}$$

3.2.4. Objective 3: system unavailability

We also consider system unavailability as an objective, which is shown in Eq. (9). This function only considers the system unavailability due to the maintenance actions; thus, the downtime in a cycle is due to times to repair non-critical failures (imperfect repairs) and the time to replace. Then, Eq. (9) is a ratio between the expected downtime in a cycle (the first and second parts correspond to expected downtime in preventive and corrective replacement cycles respectively) and the expected cycle time.

$$F3 = \frac{\left(E\left[M_{t_p}\right]R_r + R_P\right)P\left(T_f \geq t_p\right) + \left(E\left[M_{t_f}|t_f < t_p\right]R_r + R_f\right)P\left(T_f < t_p\right)}{(t_P + R_P)P\left(T_f \geq t_P\right) + \left(E\left[T_f|T_f < t_p\right] + R_f\right)P(T_f < t_P)} \tag{9}$$

where R_r is the meantime to perform the imperfect repair.

3.2.5. Objective 4: Investment in spare parts

The fourth objective is concerned about the number of spare parts to be purchased at H_0. Although there is a budget available for that, it is always interesting to minimize the total investment amount. If the whole budget available for the purchase of spare parts is not spent, a portion of it may be used in other investments that may bring beneficial returns for the organization. Moreover, inventory-holding costs can be minimized if fewer spare parts are acquired at H_0. Thus, we can argue that inventory-holding costs are indirectly considered in our model.

Then, Eq. (10) aims at minimizing the amount spent on purchasing spare parts as a linear relationship between the investment (C_s) in buying a spare part unit at H_0 and s. In other words, this objective seeks to minimize s.

$$F4 = sC_s \tag{10}$$

3.2.6. The multi-objective model

We consider three constraints in our model. First, the expected unavailability must not be higher than a predefined threshold Θ. Next, the value spent on buying spare parts must be lower than the budget B. Finally, the third constraint determines the minimum number of spare parts to be purchased at H_0. Given a defined time interval to perform the preventive replacement (t_p), the minimal number of replacements can be calculated as $H/(t_p + R_P)$.

In this context, s is integer and t_p represents a time interval. However, as t_p is used for maintenance planning, only integer values are considered. Thus, our model considers integer-valued decision variables. Therefore, combining the objective functions defined above with the constraints, the formulation of the multi-objective problem is as follows:

$$\begin{aligned} &\min_{t_p,s} F1 \\ &\min_{t_p,s} F2 \\ &\min_{t_p,s} F3 \\ &\min_{t_p,s} F4 \\ &\text{s.t. :} \\ &F3 \leq \Theta \end{aligned} \tag{11}$$

$$F4 \leq B \tag{12}$$

$$\begin{aligned} &s \geq \left\lfloor \frac{H}{t_p+R_P} \right\rfloor \\ &t_p \in \mathbb{Z}_+^* \\ &s \in \mathbb{Z}_+ \end{aligned} \tag{13}$$

This model comprises a nontrivial multi-objective optimization problem and no single solution exists that simultaneously optimizes each objective. For instance, a more preventive replacement policy (short replacement intervals) will decrease the probability of failure occurrence in a cycle ($F2$), however, it should increase the maintenance cost ($F1$) and the system unavailability ($F3$), since replacement actions are more costly and time-consuming than the repairs. Also, a greater amount of spare parts acquire in the begging of the planning horizon, despite corresponding to a high investment ($F4$), should reduce the maintenance cost ($F1$) and the system unavailability ($F3$), because it decreases the number of replacements without spare part in stock which demands more cost and downtime.

Note that functions $F1$, $F2$ and $F3$ depend on s through the probability of having spare parts available for replacement, according to Eqs. (3)–(6). In this paper, we use GRP to model the times for non-critical failures. Moreover, other stochastic processes are used to model the time to critical failure and maintenance times. Thus, some measures in the multi-objective model result from the combination of these processes (e.g.: $E[M_{t_p}]$, $E[M_{t_f}|t_f < t_p]$ and $P(T_f < t_p)$), and thus are not analytically obtainable. To calculate those measures, we use a DES-based algorithm, which will be discussed in Section 4.

4. Discrete event simulation for evaluating functions of the multi-objective model

We considered an alternate counting process [37] composed of a model related to the occurrences of failure and another to the maintenance actions, both characterizing the system behavior over time, i.e., if it is either operational or under maintenance, as shown in Fig. 1. The process related to the failure occurrences is formed by integrating two approaches: one associated with the critical failures and another one to the non-critical failures.

A GRP counting process will be adopted to model non-critical failures, which are treated by imperfect repairs, while critical failures are handled with perfect repairs, and then modeled through a Renewal Process. Thus, the failure-repair process is a composition of these two stochastic models, for which analytical handling is intricate. Therefore, we here develop a Discrete Event Simulation (DES) algorithm, which is described in this section. Table 1 shows the variables used in the DES algorithm.

The quantities $P(T_f < t_p)$, $P(T_f \geq t_p)$, $E[M_{t_p}]$, $E[M_{t_f}|t_f < t_p]$, $E[T_f|T_f < t_p]$ and P_s will be estimated via DES. To that end, we consider that the maintenance times may not be neglected, and thus the *total* equipment age is different from the *operational* equipment age. Given that, the first step in the DES methodology is the generation of the times to failures and the respective times to repair. These times are sampled to assess the failure-repair process in each cycle and are compared with the time interval for preventive replacement (t_p) for the analysis of the events of interest.

4.1. The assessment of the failure-repair process

We consider a Weibull distribution for the operational times to critical failure y_f. Then, Eq. (14) presents the CDF of y_f, where α_1 and β_1 are the scale and shape parameters. We also consider a Weibull distribution for the non-critical Times to First Failures (TTFF). Indeed, the times between non-critical failures x_n are treated by imperfect repairs, and then GRP is here used. Given that, Eq. (2) turns into Eq. (15), which is the Weibull-GRP-CDF with parameters α_2, β_2 and q (imperfect

Table 1
Variables used in the DES algorithm.

Variable	Description
OEA_f	Operational equipment age at the critical failure time
$OTB_n,(n = 1, 2, \ldots)$	Operational time between $(n-1)$th and nth non-critical failures
$OEA_n,(n = 1, 2, \ldots)$	Operational equipment age at the nth non-critical failure time ($OEA_n = \sum_{k=1}^{n} OTB_k$)
$TTIR$	Time to perform the imperfect repair
$TTCR_1$,$TTCR_2$	Time to perform corrective replacement with and without spare part available respectively
$TTPR_1$,$TTPR_2$	Time to perform preventive replacement with and without spare part available respectively
TSA_f	Total equipment age at the critical failure time
$TEA_n,(n = 1, 2, \ldots)$	Total equipment age at the nth non-critical failure time ($TEA_n = OEA_n + \sum_{k=1}^{n-1} TTIR_k$)
ELT	Equipment lifetime (total equipment age at the replacement time)
ECT	Equipment cycle time (equipment lifetime + time to replace equipment)
$NNCF$	Number of non-critical failures in a cycle
$NPR^{(i)}$,$NCR^{(i)}$	Number of preventive and corrective replacements at ith iteration of the DES algorithm
$TNR^{(i)}$	Total number of replacements at ith iteration ($TNR^{(i)} = NPR^{(i)} + NCR^{(i)}$)

repair effectiveness). Yañez et al [35] present a procedure to obtain maximum likelihood estimates for α_1, β_1, α_2, β_2 and q.

$$F\left(y_f\right) = 1 - e^{-\left(\frac{y_f}{\alpha_1}\right)^{\beta_1}} \tag{14}$$

$$F(x_n|v_{n-1}) = 1 - exp\left[\left(\frac{v_{n-1}}{\alpha_2}\right)^{\beta_2} - \left(\frac{v_{n-1}+x_n}{\alpha_2}\right)^{\beta_2}\right], n = 1, 2, \ldots \tag{15}$$

Then, assuming that u is a random variable uniformly distributed between 0 and 1, the operational times for the critical failures and the n^{th} non-critical failures can be simulated by solving the CDFs in Eqs. (14) and (15) for y_f and x_n [38]:

$$OEA_f = \alpha_1[-\ln(1-u)]^{1/\beta_1}. \tag{16}$$

$$OEA_n = \alpha_2\left[\left(\frac{q}{\alpha_2}\sum_{k=1}^{n-1}x_k\right)^{\beta_2} - \ln(1-u)\right]^{1/\beta_2} - q\sum_{k=1}^{n-1}x_k,\ \ n = 1, 2, \ldots. \tag{17}$$

We assume the maintenance times are modeled by Exponential distributions. Let Z be a random variable denoting the time to perform one of the five maintenance types (imperfect repair, corrective replacement with (or without) spare part available, preventive replacement with (or without) spare part available). Then, its CDF is given in Eq. (18):

$$F(z) = 1 - e^{-\left(\frac{z}{\mu_Z}\right)} \tag{18}$$

where μ_Z is the meantime of Z (i.e., $\mu_Z\, \varepsilon\, \{R_r, R_{p1}, R_{p2}, R_{f1}, R_{f2}\}$). A valid estimative for μ_Z is the average of the times observed in historic data (i.e. $\hat{\mu}_Z = \bar{z}$). Then, the maintenance times can be simulated solving the CDF in Eq. (18) for z:

$$z = \mu_Z[-\ln(1-u)] = \bar{z}[-\ln(1-u)]. \tag{19}$$

In the DES algorithm, z represents the variables $TTIR$, $TTPR_1$,$TTPR_2$, $TTCR_1$, or $TTCR_2$.

4.2. Calculating the functions of the multi-objective model

The variables OEA_f, OEA_n, $TTIR$, $TTPR_1$,$TTPR_2$, $TTCR_1$, and $TTCR_2$ are simulated by using the times to occurrence of failures and the times to maintenance actions, and they are used in the generation of random discrete events (the occurrences of failures and maintenances) during the simulation time. The goal is to generate a "typical" scenario for a planning horizon to allow the evaluation of some features that are of interest for calculating the functions of the multi-objective model, which is done for one possible solution (t_p,s) and it is performed in the following way:

I. *Cycle simulation*: the steps for simulating a cycle (summarized in Fig. 3) are:
  I.1. The operational time to critical failure (OEA_f) is generated as in Eq. (16);
  II.2. The operational times between non-critical failures (OTB_n), as well as their imperfect repair times ($TTIR$) are generated from Eqs. (17) and (19) until either (i) the operational age ($OEA_n = \sum_{k=1}^{n} OTB_k$) of the equipment exceeds the time to critical failure ($OEA_n > OEA_f$) or (ii) the total equipment age is higher the interval defined to preventive replacement ($OEA_n + \Sigma TTIR > t_p$), whichever occurs first;
  III.3. The number of non-critical failures ($NNCF$) and the equipment lifetime (ELT) are computed, and the number of each cycle type ($NPR^{(i)}$ and $NCR^{(i)}$) is updated: if the previous step ends on condition "(i)", $ELT = t_p$ and $NPR^{(i)} = NPR^{(i)} + 1$. Otherwise, $ELT = OEA_f + \Sigma TTIR$ and $NCR^{(i)} = NCR^{(i)} + 1$;

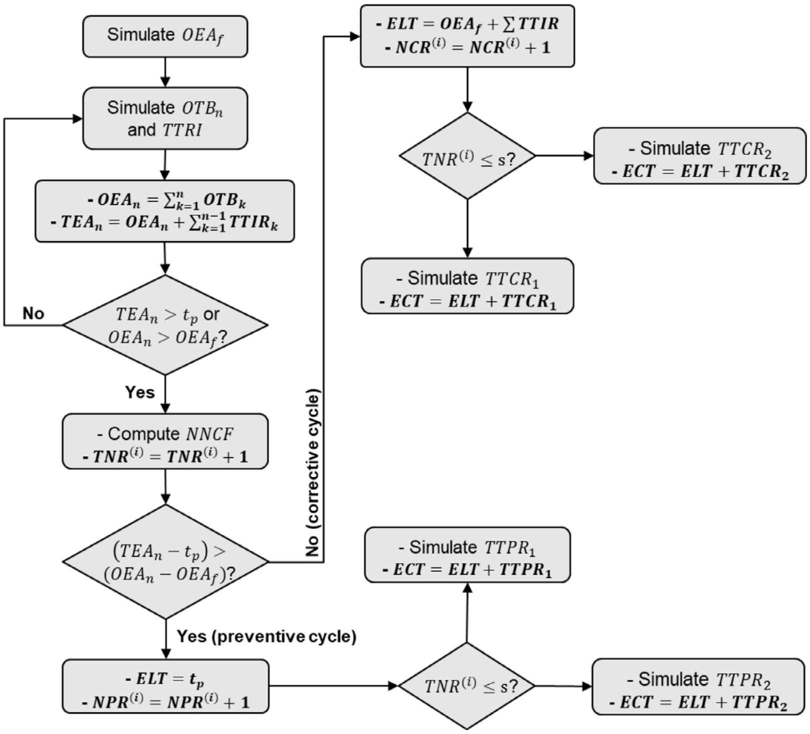


Fig. 3. Cycle simulation steps.

IV.4. The time to perform replacement is generated from Eq. (19). If condition "(i)" occurs, then the time to perform preventive replacement ($TTPR_1$, if $TNR^{(i)} \leq s$, or $TTPR_2$, otherwise) is generated. If condition "(ii)" happens, the time to perform corrective replacement ($TTCR_1$, if $TNR^{(i)} \leq s$, or $TTCR_2$, otherwise) is generated;

V.5. The cycle time is calculated as the sum of equipment lifetime and time to replace it, i.e., $ECT = \{ELT + TTPR_1, ELT + TTPR_2, ELT + TTCR_1, ELT + TTCR_2\}$.

II. *Planning Horizon simulation*: The cycle simulation (step I) is repeatedly performed until the sum of the times of simulated cycles exceeds the planning horizon time ($\Sigma ECT > H$). In this way, a planning horizon is simulated. When this step is executed, one iteration of the DES algorithm is completed. The number of simulated replacements/cycles in that iteration is, then, given by $TNR^{(i)} = NPR^{(i)} + NPR^{(i)}$.

III. *Estimation of the quantities $P(T_f < t_p)$, $P(T_f \geq t_p)$, $E[M_{t_p}]$, $E[M_{t_f}|t_f < t_p]$, $E[T_f|T_f < t_p]$ and P_s*: One iteration of the DES algorithm provides a possible scenario for the planning horizon. Some measures are evaluated for this scenario as the number of preventive and corrective cycles ($NPR^{(i)}$ and $NCR^{(i)}$) and the number of non-critical failures per cycle ($NNCF$). Simulating many planning horizons, several scenarios are evaluated and, if this number is large enough, the expected frequency of the measures of interest can be estimated. After NI iterations, the quantities of interest are estimated as:

I.1. The probability of critical failure occurs before t_p is equal to the average of the proportion of corrective replacements in each iteration (Eq. (20));

II.2. The probability of critical failure does not occur before t_p is equal to the average of the proportion of preventive replacements in each iteration (Eq. (21));

III.3. The expected number of non-critical failures in preventive cycles is equal to the average of the mean number of non-critical failures, in preventive cycles, in each iteration (Eq. (22));

IV.4. The expected number of non-critical failures in corrective cycles is equal to the average of the mean number of non-critical failures, in corrective cycles, in each iteration (Eq. (23));

V.5. The expected equipment lifetime in corrective cycles (the expected time of the critical failure) is equal to the average of the meantime to critical failures, in corrective cycles, in each iteration (Eq. (24));

VI.6. The probability of having spare parts available at the replacement time is equal to the average of the proportion of the number of replacements is higher than s (Eq. (25)).

$$\hat{P}(T_f < t_p) = \sum_{i=1}^{NI} \left(\frac{NCR^{(i)}}{TNR^{(i)}} \right) / NI \tag{20}$$

Table 2
Multi-objective GA parameters.

MOGA parameter	Description
L	Size of population Π
p_{cr}	Crossover probability
p_{mt}	Mutation probability
N_{gen}	Number of generations

$$\hat{P}\left(T_f \geq t_p\right) = 1 - \hat{P}(t_c < t_p) = \sum_{i=1}^{NI}\left(\frac{NPR^{(i)}}{TNR^{(i)}}\right)/NI \tag{21}$$

$$\hat{E}\left[M_{t_p}\right] = \sum_{i=1}^{NI}\left(\frac{\sum_{k=1}^{TNR^{(i)}}\begin{cases} NNCF_k, & \text{if cycle } ''k'' \text{ is preventive} \\ 0, & \text{if cycle } ''k'' \text{ is corrective}\end{cases}}{NPR^{(i)}}\right)/NI \tag{22}$$

$$\hat{E}\left[M_{t_f}|t_f < t_p\right] = \sum_{i=1}^{NI}\left(\frac{\sum_{k=1}^{TNR^{(i)}}\begin{cases} NNCF_k, & \text{if cycle } ''k'' \text{ is corrective} \\ 0, & \text{if cycle } ''k'' \text{ is preventive}\end{cases}}{NCR^{(i)}}\right)/NI \tag{23}$$

$$\hat{E}\left[T_f|T_f < t_p\right] = \sum_{i=1}^{NI}\left(\frac{\sum_{k=1}^{TNR^{(i)}}\begin{cases} TEA_k, & \text{if cycle } ''k'' \text{ is corrective} \\ 0, & \text{if cycle } ''k'' \text{ is preventive}\end{cases}}{NCR^{(i)}}\right)/NI \tag{24}$$

$$\hat{P}_s = \sum_{i=1}^{NI}\left(\frac{\begin{cases} \left(TNR^{(i)} - s\right), & \text{if } \left(TNR^{(i)} - s\right) > 0 \\ 0, & \text{otherwise}\end{cases}}{TNR^{(i)}}\right)/NI \tag{25}$$

Then, by estimating the quantities $P(T_f < t_p)$, $P(T_f \geq t_p)$, $E[M_{t_p}]$, $E[M_{t_f}|t_f < t_p]$, $E[T_f|T_f < t_p]$ and P_s (from Eqs. (20)–(25)), the values of the objective functions are calculated Eqs. (7)–((10)).

5. A method to get Pareto-optimal solutions for the multi-objective replacement policy problem

As the DES algorithm is adopted for estimating the functions of the multi-objective model, the use of exhaustive methods to obtain the Pareto-optimal set is prohibitive due to computational time and cost. In this way, we propose a Multi-Objective Genetic Algorithm (MOGA), which is described in this section.

An individual generated in MOGA corresponds to a pair (t_p,s). To obtain a Pareto-optimal set, the dominance relationship is evaluated based on each individual's fitness, which is a four-dimension vector $\underline{F} = [F1, F2, F3, F4]$. As the DES algorithm is used to calculate the objective functions, the coupling MOGA-DES takes place every time MOGA needs to obtain the fitness of an individual. Fig. 4 details the MOGA + DES algorithm model we here propose.

5.1. Multi-objective Genetic Algorithm

This paper uses an integer-coded MOGA, and then the phenotype of an individual is a bi-dimensional vector with each entry corresponding to the decision variables t_p and s. Moreover, our MOGA neither uses elaborated fitness metrics nor transforms multiple objectives into a unique function. Hence, each individual has an associated fitness vector with size equal to the number of the considered objectives.

Let L be the fixed size of population Π, $\Pi[j]$ be the jth individual (that represents a solution) of Π, and Π_{aux} be the auxiliary population that stores non-dominated individuals and is updated at each iteration. Table 2 defines the parameters used in the proposed MOGA, shown in Fig. 5 as pseudocode. Section 5.1.2 describes in detail the genetic operators proposed.

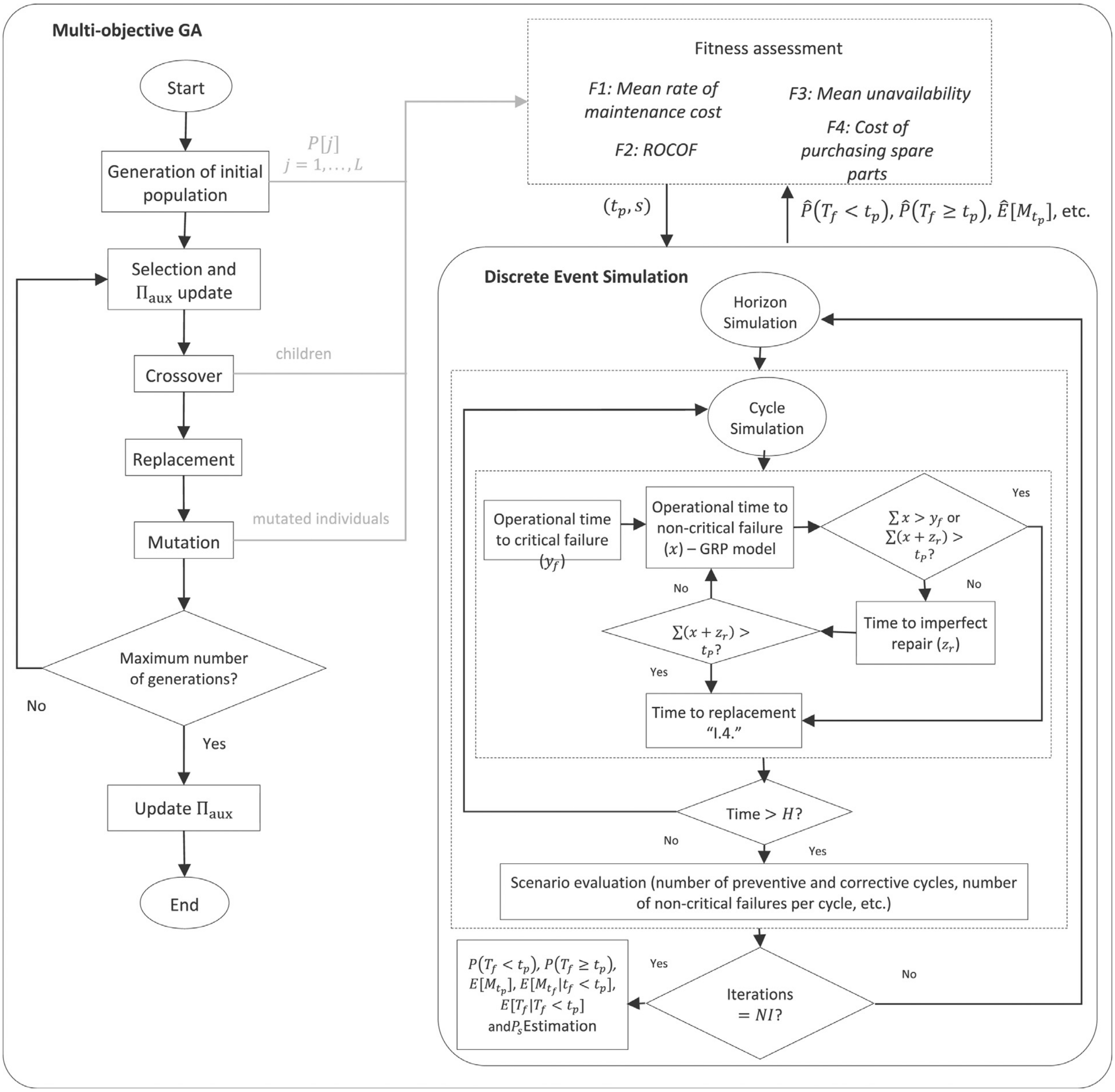


Fig. 4. Proposed MOGA + DES algorithm.

5.1.1. Individual Representation

Integer representation of individuals is used. As an illustration, suppose a planning horizon of 1,000 days, t_p represents the day at equipment replacement occurs (if the critical failure occurs after t_p); thus, $t_p \in [1;\ 1,000]$. Furthermore, given the maximum budget constraint (Eq. (13)), it is possible to determine an acceptable range for s as follow: $s \in [0;\ B/C_S]$. Fig. 6 depicts an example of the phenotype of an individual as well as a scenario of the planning horizon. The times above the timeline are the instants when the system is stopped (due to a failure occurrence or a preventive replacement), and the times below the timeline corresponds to the instants when the system returns to operation (when the maintenance ends).

Four full cycles (replacements) are observed in Fig. 6. The first and the third ones correspond to preventive cycles, i.e., equipment reaches the total age $t_p = 300$ before the occurrence of a critical failure, while the second and the fourth cycles are corrective because a critical failure occurs before the total age t_p. Yet, as $s = 3$, the fourth replacement is performed without spare part available.

```
Procedure MOGA
  for j = 1, ..., L do #generate initial population
    ⇒ generate Π[j]
  for n_gen = 1, ..., N_gen do #stopping criterion
    for j = 1, ..., L do #selection and Π_aux update
      ⇒ verify the dominance relation of Π[j] and the other individuals in Π
      if Π[j] is nondominated in Π then
        ⇒ verify the dominance relation of Π[j] and the individuals stored in Π_aux
        if Π[j] is nondominated in relation to each solution of Π_aux then
          Π_aux ⟵ Π_aux · ⟨Π[j]⟩
          if Π[j] dominates solutions in Π_aux then
            ⇒ eliminate dominated solutions from Paux
        else Π[j] does not enter in Π_aux
      else Π[j] is discarded
    ⇒ Π receives individuals randomly chosen from Π_aux to keep the size L
    for j = 1, ..., L do #crossover
      ⇒ verify if Π[j] will participate in crossover according to p_cr
    ⇒ perform crossover and replace parents by children
    for j = 1, ..., L do #mutation
      ⇒ verify if Π[j] will be mutated according to p_mt
    ⇒ perform mutation
  ⇒ update Π_aux
End Procedure
```

Fig. 5. Proposed MOGA pseudocode.

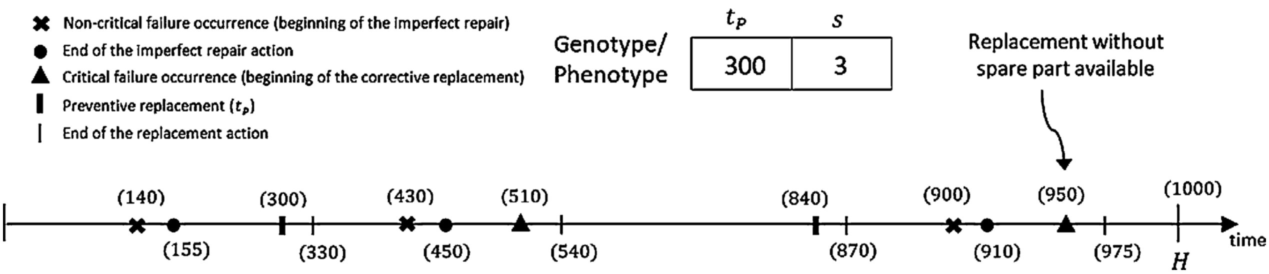


Fig. 6. Example of an integer-coded individual and a possible scenario.

5.1.2. Steps of MOGA

Generation of the initial population. The initial population is generated by randomly sampling integer values for t_p and s. Each pair (t_p,s) corresponds to the phenotype of an individual $\Pi[j]$ ($j = 1, \ldots, L$). Fig. 7 shows the pseudocode for the initial population generation. The procedure was designed so that only feasible individuals are generated, i.e., t_p is randomly generated in a way that Eqs. (8) and (9) are satisfied. If (t_p,s) meets the constraint in Eq. (7), then it is introduced into population Π. This step is performed only once, and the population Π will be transformed by the application of the genetic operators of selection, crossover, and mutation.

Selection and Π_{aux} *update.* In this step, the relation of dominance among the individuals is assessed according to their fitness values. This step works as follows:

(i) The dominance relation is assessed between pairs of individuals within the population Π. The dominated ones are removed from Π, and then the remaining are eligible to enter Π_{aux};
(ii) Π_{aux} is updated according to the following rules: (a) if a candidate is dominated by an individual in Π_{aux}, it is discarded; (b) otherwise, it is added to Π_{aux}, and if it also dominates individuals in Π_{aux}, all dominated solutions are deleted from Π_{aux}.

In the first iteration (generation 0), there are no individuals in Π_{aux}; then, Π_{aux} update (stage (ii)) is done by adding all candidates (selected in stage (i)) in Π_{aux}. After (i), the size of Π is reduced. To maintain the population with L individuals, solutions are randomly selected from Π_{aux} (after step (ii)) and inserted into Π.

Crossover. After Selection and Π_{aux} update, Crossover is performed to evaluate other solutions in good regions (since only non-dominated individuals survive in Π, after the Selection). The Crossover step was developed in a way that only feasible

Procedure GENERATE INITIAL POPULATION
  for $j = 1, \ldots, L$ **do**
    $\Rightarrow check \longleftarrow 0$
    while $check = 0$
      $\Rightarrow t_p \longleftarrow \lfloor random[1, H] \rfloor$
      $\Rightarrow s \longleftarrow \left\lfloor random\left[\left\lfloor \frac{H}{t_p + R_P} \right\rfloor, \left\lfloor \frac{B}{C_S} \right\rfloor\right]\right\rfloor$
      $\Rightarrow$ calculate fitness for $\langle t_p, s\rangle$
      if $F3 \leq \Theta$ **then**
        $\Rightarrow check \longleftarrow 1$
    $\Pi[j] \longleftarrow \langle t_p, s\rangle$
End Procedure

Fig. 7. Pseudocode for the proposed procedure to generate the initial population.

Procedure CROSSOVER
  for $j = 1, \ldots, L$ **do**
    $\Rightarrow select \longleftarrow random[0,1]$
    if $select \leq p_{cr}$ **then**
      $\Rightarrow \Pi[j]$ is inserted into $Cross$
  $\Rightarrow$ a pair of individuals is randomly chosen from $Cross$
  $\Rightarrow$ calculate fitness for $\langle t_p$ of $parent[1] | s$ of $parent[2]\rangle$
  if $\langle t_p$ of $parent[1] | s$ of $parent[2]\rangle$ is feasible **then**
    $\Rightarrow children[1] \longleftarrow \langle t_p$ of $parent[1] | s$ of $parent[2]\rangle$
  else
    $\Rightarrow children[1] \longleftarrow parent[1]$
  $\Rightarrow$ calculate fitness for $\langle t_p$ of $parent[2] | s$ of $parent[1]\rangle$
  if $\langle t_p$ of $parent[2] | s$ of $parent[1]\rangle$ is feasible **then**
    $\Rightarrow children[2] \longleftarrow \langle t_p$ of $parent[2] | s$ of $parent[1]\rangle$
  else
    $\Rightarrow children[2] \longleftarrow parent[2]$
End Procedure

Fig. 8. Pseudocode for the proposed Crossover procedure.

individuals are generated. First, it is decided which individuals in Π takes part in the crossover. For every $\Pi[j]$, a random number in [0, 1] is generated, and if this number is lower than the crossover probability p_{cr}, the individual participates in the crossover (a copy of this individual is inserted into the set *Cross*). Then, a pair of individuals is randomly chosen (the parents) and 2 individuals are generated as copies of them, but having the contents exchanged for one position of the phenotype (randomly defined). These 2 other individuals are the children if they are feasible. Fig. 8 shows the pseudocode for the crossover procedure.

To maintain the population with L individuals, the Replacement step takes place after Crossover. The adopted strategy is "children replace parents", i.e. the parents are discarded, and the children take their places in Π. Note that the children generated in the crossover step may be an identical copy of the parents.

Mutation. To expand the search region of the algorithm, the Mutation step is performed Fig. 9 shows the pseudocode for this procedure). Its objective is "to mutate" some individuals in Π. A number between 0 and 1 is randomly generated for each individual $\Pi[j]$. If this number is lower than the predefined mutation probability p_{mt}, the content of one position randomly selected is substituted by a value randomly generated into the intervals that satisfy Eqs. (8) and ((9). The fitness is calculated for the "mutated" individual and, if the constraint (7) is met, it is inserted into Π, in place of $\Pi[j]$.

Stopping criterion. With an exception for the Generation of the Initial Population, the steps are repeated for N_{gen} times, where each iteration is a MOGA generation. After this, the Selection and Π_{aux} update step is performed for the last time and the algorithm provides the nondominated feasible individuals from Π_{aux}.

Procedure MUTATION
  for $j = 1, \dots, L$ **do**
    $\Rightarrow select \longleftarrow random[0,1]$
    if $select \leq p_{mt}$ **then**
      $\Rightarrow pos \longleftarrow \lfloor random[1,2] \rfloor$
      if $pos = 1$ **then**
        $\Rightarrow t_p' \longleftarrow \left\lfloor random\left[\left(\frac{H}{s}\right) - R_P, H\right]\right\rfloor$
        $\Rightarrow$ calculate fitness for $\langle t_p' | s \rangle$
        if $F3 \leq \Theta$ **then**
          $\Rightarrow \Pi[j] \longleftarrow \langle t_p' | s \rangle$
      if $pos = 2$ **then**
        $\Rightarrow s' \longleftarrow \left\lfloor random\left[\left\lfloor\frac{H}{t_p + R_P}\right\rfloor, \left\lfloor\frac{B}{C_S}\right\rfloor\right]\right\rfloor$

Fig. 9. Pseudocode for the proposed procedure to Mutation.

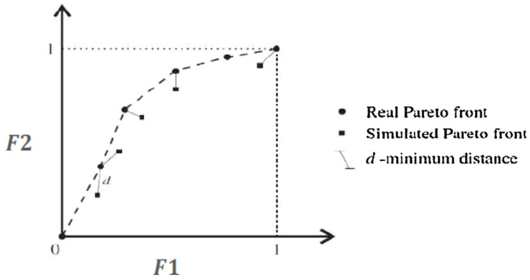


Fig. 10. Point-to-point distance for a bi-dimensional Pareto front.

Table 3
Parameters for the multi-objective problem.

Parameter	Value	Parameter	Value
C_{p1}	$ 20,000	R_r	26h
C_{p2}	$ 45,000	R_{p1}	134.4 h
C_{c1}	$ 40,000	R_{p2}	158.4 h
C_{c2}	$ 65,000	R_{c1}	141.1 h
B	$ 150,000	R_{c1}	165.1 h
Θ	0.1		

6. Application Examples

6.1. Model validation

6.1.1. Point-to-point distance: a metric for comparing real and simulated Pareto fronts

The MOGA procedure is applied as an alternative to the exhaustive method. Therefore, comparing the performance of the MOGA in obtaining simulated Pareto fronts against the exact one is useful to validate the proposed model. Such comparison requires the use of metrics that may, even heuristically, represent the convergence of the simulated Pareto front towards the exact one.

In this context, the point-to-point distance metric [17] is here considered. For each point in the obtained front, we compute the minimum Euclidean distance from it to one of the points in the real front. From one simulated front, the mean of all minimum distances (d, see Fig. 10) is calculated, representing the entire front ($\bar{d}_k$, $k = 1, \dots, nsFronts$). Then, the metric $\bar{d}_k$ summarizes the convergence of the obtained front in one single number, and the following weighted mean may be calculated:

$$D = \frac{\sum_k \bar{d}_k \times ns_k}{\sum_k ns_k}, \;\; k = 1, \dots, nsFronts \tag{26}$$

where ns_k is the number of obtained nondominated solutions in the kth simulated front.

The four objectives to be minimized vary over different scales, and then we normalized all of them within the interval [0, 1]. The normalization factors are the minimum and maximum exact values of each objective function.

Table 4
MOGA + DES parameters for the application example.

Parameter	Value
L	200
N_{gen}	200
p_{cr}	0.95
p_{mt}	0.1
NI	1200

Table 5
Summary of results for validation example.

	Number of solutions	Number of obtained exact Pareto solutions	$\bar{d}_k$
Min.	144	96	0.0262
Max.	172	146	0.0280
Mean	168.8	129.5	0.0269
Std. Dev.	13.55	15.83	0.0004

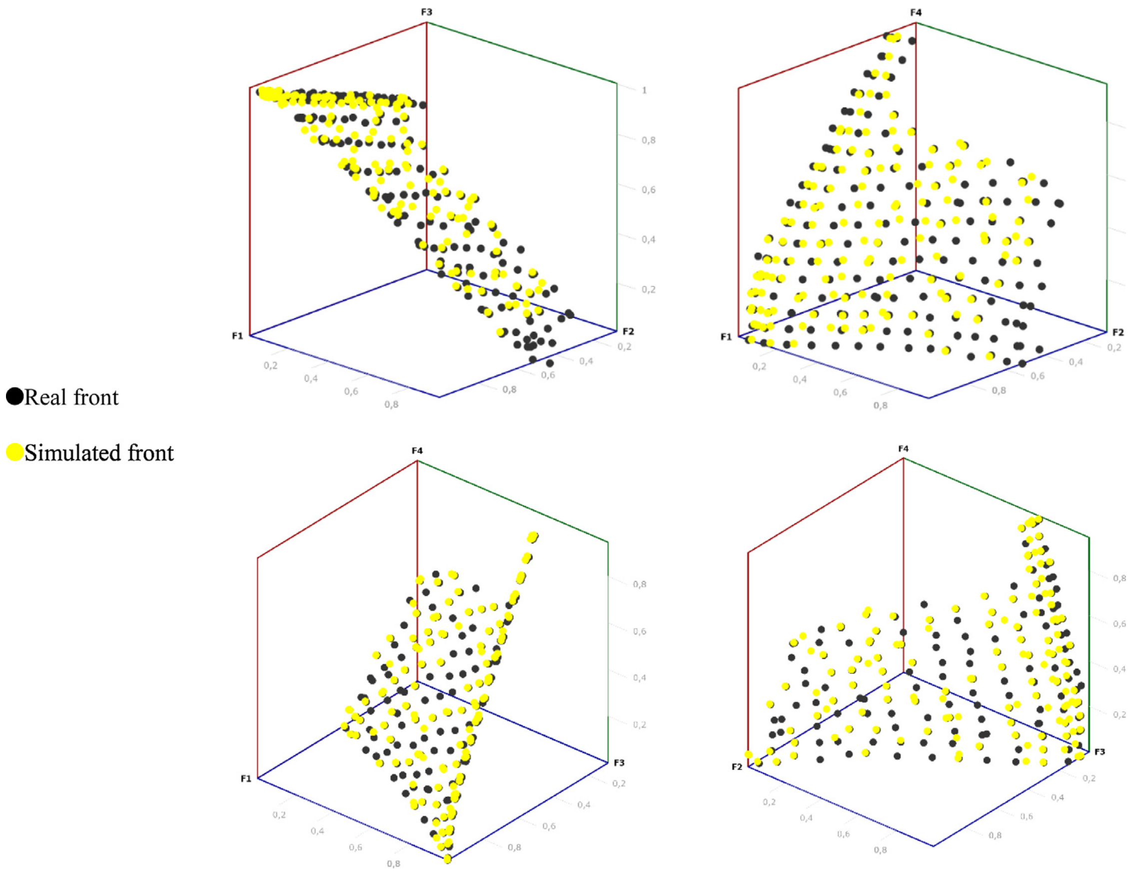


Fig. 11. Exact and simulated (with minimum exact Pareto solutions) Pareto fronts for the validation example.

6.1.2. An example comparing real and simulated Pareto fronts

To validate the proposed MOGA, an example is devised to allow for the comparison between the exact Pareto-optimal set and the results obtained via MOGA. To that end, suppose $\alpha_1 = 3,072$, $\beta_1 = 1.62$, $\alpha_2 = 1,828$, $\beta_2 = 2.02$ and $q = 0.7$. A planning horizon of 43,800 hours ($H = 5$ years) was considered and the mean cost of performing n imperfect repairs is given by $C_r(n) = 3,000 \times n^{1.2}$; the other parameters of the problem are seen in Table 3.

This example has 832,200 possible combinations (t_p,s) and was solved by using an exhaustive method to obtain the exact Pareto set; 670,353 out of 832,200 are feasible, and the dominance relationship was evaluated among all of them. A set of 198 nondominated solutions outlines the exact Pareto front. Then, 30 trials of the MOGA + DES were executed; Table 4 shows the parameters used to feed the MOGA + DES model. Table 5 presents some descriptive statistics regarding

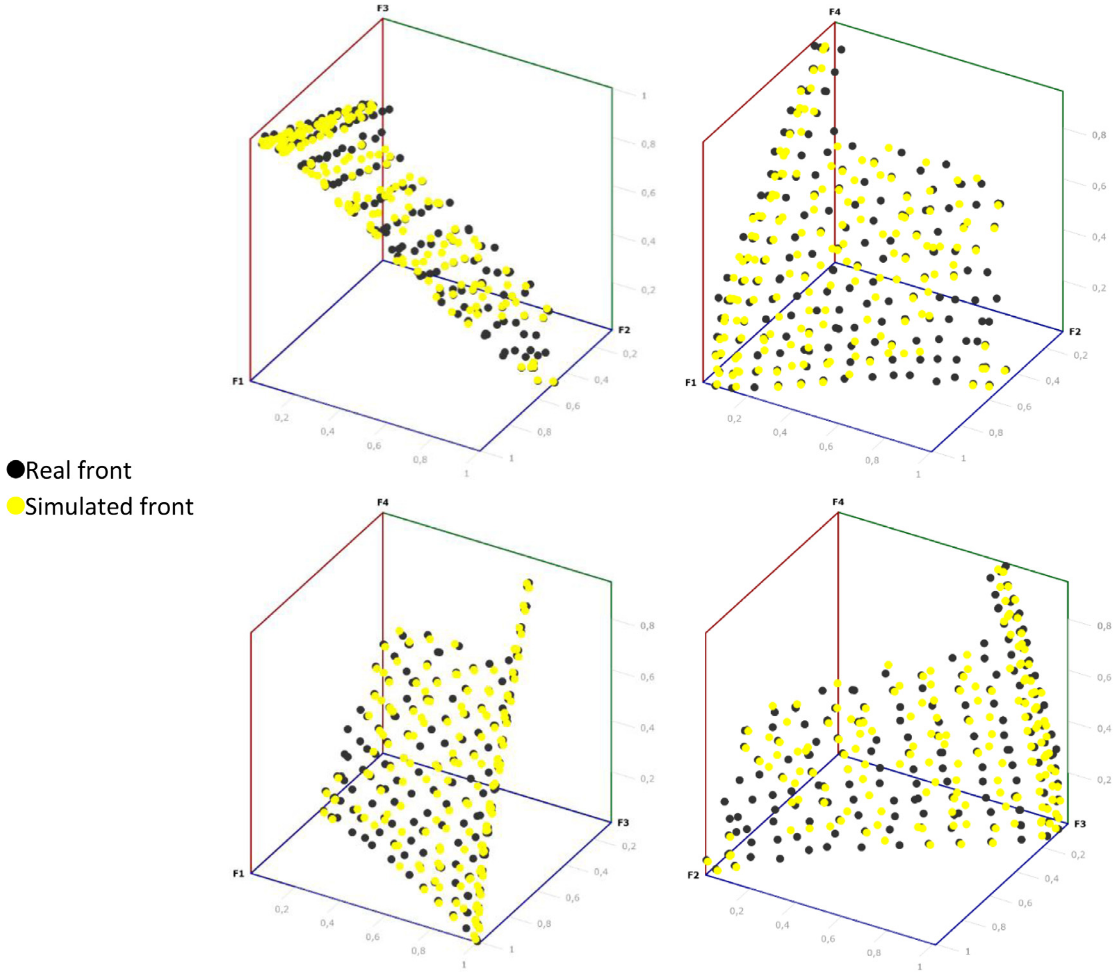


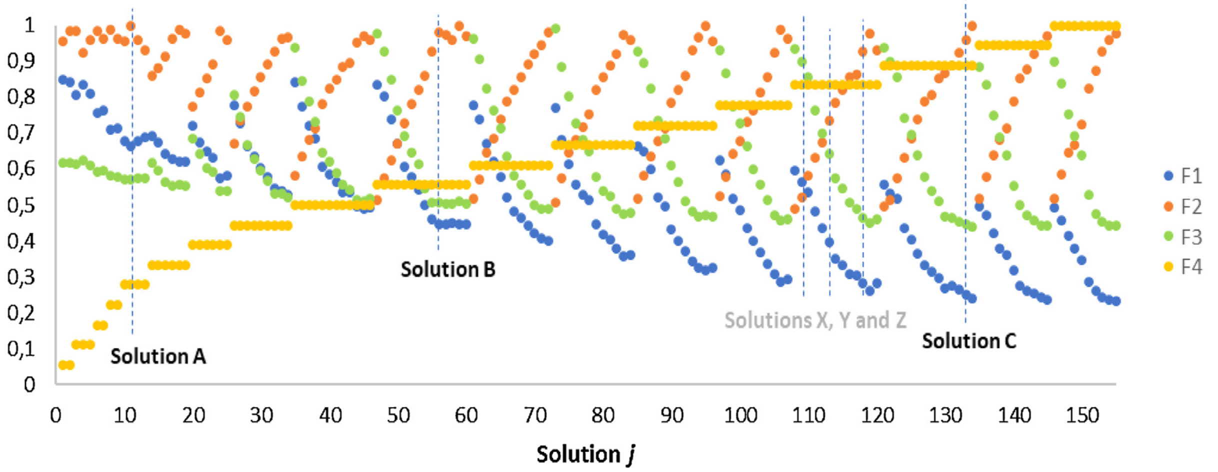
Fig. 12. Exact and simulated (with maximum exact solutions) Pareto fronts for the validation example.

the number of solutions, the number of exact Pareto solutions and the point-to-point distance metrics obtained in each of the 30 simulated Pareto fronts. The weighted metric (Eq. 22) was $D = 0.0272$.

In the point-to-point procedure, it could occur some loss of information since the solution is the entire front and not only one single point. Thus, we specifically examined the simulated Pareto fronts with the maximum and minimum numbers of exact Pareto solutions, for which the mean point-to-point distances were $\bar{d}_k = 0.0280$, and $\bar{d}_k = 0.0262$ respectively. The maximum values of point-to-point distance (max[d]) for both fronts were 0.0800 and 0.0807 respectively for the simulated Pareto fronts with the maximum and the minimum number of exact Pareto solutions (the minimum values of point-to-point distance were 0, corresponding to the exact Pareto solutions obtained). Then, for those solutions corresponding to the maximum point-to-point distance, we analyzed the four coordinate distances on a real scale, which were 0.222 (for $F1$), 0.00011 (for $F2$), 0.00064 (for $F3$) and 0 (for $F4$). From the front with minimum exact Pareto solutions, these values were 0.610 (for $F1$), 0.000074 (for $F2$), 0.0015 (for $F3$) and 0 (for $F4$). We can notice that for both situations these distances do not represent significant differences, which means solutions from simulated fronts show close agreement with the real Pareto set.

Figs. 11 and 12 depict a comparison between the real and simulated Pareto fronts (the ones with the maximum and the minimum number of exact solutions) over tri-dimensional graphs. It can be noticed that the points in simulated Pareto front are either on or very close to the points in the exact one. Then, the MOGA + DES model is able to find solutions comparable to the exhaustive method.

Yet, comparing the computational effort, the application of the exhaustive method lasts about 505 computer hours, while the proposed model required, on average, about 6.4 hours (~1.3% of the computational effort by brute force). Some tests were performed with other MOGA + DES parameters. The values shown in Table 4 correspond to the best trade-off between the method performance (in simulating the Pareto front) and the computational effort (time). As the problem addressed in this paper is a tactical planning decision, which must be done only once at the beginning of the planning horizon, the



Solution	t_p	s	$F1$	$F2$	$F3$	$F4$
A	26,486	5	23.4792	4.34157	0.08264	40,000
B	26,486	10	20.7191	4.29593	0.08008	80,000
C	26,486	16	18.2598	4.23416	0.07771	128,000
X	3,983	15	22.2040	2.98279	0.09567	120,000
Y	5,491	15	20.0785	3.59465	0.08554	120,000
Z	24,535	15	18.4121	4.28213	0.07783	120,000

Fig. 13. Results for application example considering minimal repairs.

computational time result is feasible and suitable for the method justification. The experiments were executed in a PC with Windowsu© operating system, 2.0 GHz processor and 4 GB of RAM.

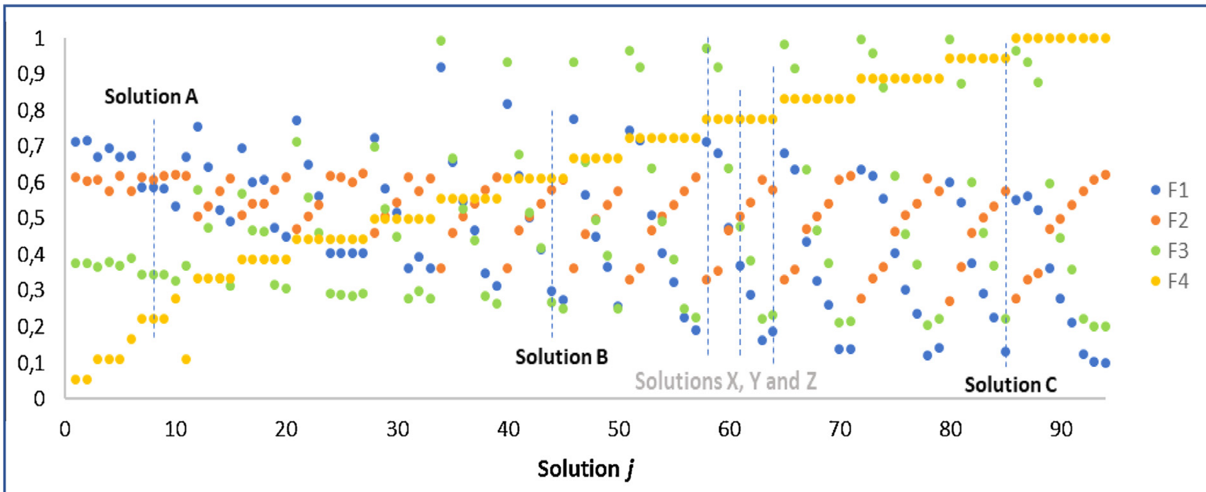
6.2. Trade-off analysis and the repair effectiveness assessment

In the previous example, we considered $q = 0.7$, which corresponds to imperfect repairs to treat non-critical failures. To analyze the impacts caused by different types of corrective maintenance actions (minimal, imperfect and perfect repairs) in the replacement policy, the same example was solved for $q = 0.0$, 0.5 and 1.0. In this manner, the results in this section show the effect of varying the repair effectiveness in obtaining the Pareto front. This sensitivity analysis is useful because it allows evaluating the difference in system performance, measured by the four objectives, as the maintenance effectiveness varies, enabling the assessment of how much it is worth investing in improving the performance of the maintenance team.

Figs. 13-15 show the normalized fitness value for the obtained Pareto sets, and some selected solutions, for the assumption of minimal, imperfect and perfect repairs respectively. For purposes of tradeoff analysis, the solutions A, B, and C have the same value for t_p, while solutions X, Y and Z have the same s. Thus, by analyzing the fitness values of solutions A, B, and C, it can be seen that when more spare parts purchased at the beginning of the planning horizon, the mean maintenance cost rate ($F1$) and the mean unavailability ($F3$) get better. No change is significant on the mean failure rate ($F2$) and, naturally, the amount spent on purchasing these spare parts ($F4$) is higher for solution C. On the other hand, from solutions X, Y and Z, it is stated that the best performances for the objectives 1 and 3 are obtained for the longest preventive replacement intervals, though the mean failure rate ($F2$) increases, which illustrates the compromise among the objectives. These conflicting behaviors were expected, as discussed in Section 3.2.6.

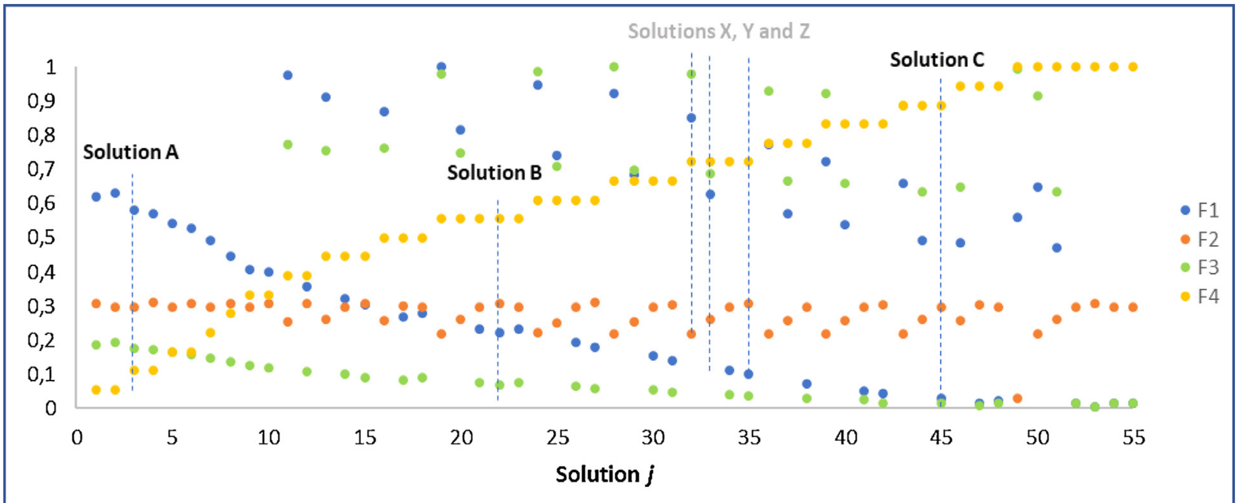
Note that all the Pareto solutions are optimal in a multi-objective perspective and the benefit brought for them is the trade-off between cost-savings ($F1$ and $F4$) and reliability growths ($F2$ and $F3$). The manager could, then, choose a solution in this set which corresponds to the best trade-off relation for him. For example, in Fig. 14, solution C corresponds to an additional investment of $24,000 in spare parts, from solution Z (25.43% more), and provides a maintenance cost savings of only 0.68 $/h with no significant difference in performance in reliability (failure rate) and unavailability. A decision-maker could define whether it is worth investing $24,000 to obtain a small reduction in maintenance costs and, then, discard any of these solutions.

In another example, the only advantage of solution Y over solution B is a small reduction in the expected number of failures in a cycle (only 0.21 cycle^{-1}). In all remaining objectives, solution B is better than solution Y, including a difference of 24,000 in investment in spare parts. Some decision-makers could discard, thus, the solution Y. These are some examples of analyzes that can be performed to aid the decision-maker in choosing a solution within the obtained Pareto set.



Solution	t_p	s	$F1$	$F2$	$F3$	$F4$
A	27,850	4	21.4925	3.23219	0.07366	32,000
B	27,850	11	18.8918	3.14984	0.07064	88,000
C	27,850	17	16.7609	3.14543	0.06885	136,000
X	4,077	15	24.0793	2.44771	0.09862	112,000
Y	6,363	15	19.7400	2.94196	0.07898	112,000
Z	37,815	15	17.4455	3.14980	0.06928	112,000

Fig. 14. Results for application example considering imperfect repairs.



Solution	t_p	s	$F1$	$F2$	$F3$	$F4$
A	29,500	2	22.4179	2.34344	0.0670	16,000
B	29,500	10	17.8941	2.37373	0.0627	80,000
C	29,500	16	15.4928	2.34860	0.06063	128,000
X	4,904	13	26.1042	2.10083	0.09927	104,000
Y	6,785	13	23.0578	2.19385	0.08245	104,000
Z	38,812	13	16.1291	2.38429	0.06183	104,000

Fig. 15. Results for application example considering perfect repairs.

Table 6
Results considering different types of repair.

Repair type	Number of non-dominated solutions	Average fitness vector			
		*F*1	*F*2	*F*3	*F*4
Minimal	1550	0.443	0.950	0.474	0.649
Imperfect	94	0.405	0.477	0.378	0.636
Perfect	55	0.396	0.028	0.191	0.620

Table 6 shows the number of nondominated solutions in each situation and the average normalized fitness vector (i.e. the mean values, between the non-dominated solutions, of the *F*1, *F*2, *F*3 and *F*4 functions). It is noted that all positions of the average fitness vector from the perfect repair situation are lower than the other two. Yet, similar behavior is observed as we compare the average fitness vector from imperfect repair with the average fitness vector of the minimal repair. Therefore, solutions for the perfect repair assumption, in most cases, dominate the solutions obtained as we consider imperfect repairs, which, in general, dominate minimal repair solutions. This behavior is expected since the item subjected to imperfect repairs has an intermediate performance between devices that undergo either minimal or perfect repairs.

7. Conclusions

This paper presented a novel multi-objective method for a simultaneous decision about the interval for preventive replacements and the number of spare parts to be purchased at the beginning of a planning horizon for equipment subject to imperfect repairs. The proposed approach applies to equipment that undergoes failures with different levels of severity. For instance, failures repaired either through a full replacement or via imperfect repairs. Thus, we solved a multi-objective problem, where imperfect repairs were modeled according to a GRP model.

Four objectives were considered: maintenance cost, rate of occurrence of failures, unavailability and the investment for buying spare parts at H_0. Yet, we evaluated the effects of s on the probability of having spare parts available when the replacement is required; this probability affects the objective functions.

The analytical handling of the objective functions is not possible, and then a DES algorithm was developed for this purpose, which avoids the use of exhaustive methods to obtain the Pareto front. Moreover, a MOGA was coupled with DES for obtaining the solutions. As it was demonstrated through the validation case, the proposed MOGA + DES model could provide solutions on or very near the exact Pareto fronts with a lower computational cost. A point-to-point distance metric and a graphic view analysis were used to evaluate the agreement between exact and simulated fronts. We also solved the example considering three different assumptions regarding repair effectiveness. As expected, the solutions related to perfect repairs dominated the ones associated with either imperfect or minimal repairs.

The proposed model is applicable to plan the maintenance policy of critical equipment, module or component, with a predetermined period of use (for example, equipment used in oil fields), which may undergo critical and non-critical failures. The model does not consider the dependency between these types of failure and maintenance actions (replacement and repair). Historical data must be available for estimating failures and maintenance time distributions and there is no possibility to update the replacement policy from the equipment's operating data (new failure data and monitoring sensors data).

Therefore, in future research, a competing risk approach [39] can be used to model the dependence relation between the probabilities of occurrence of the critical and non-critical failures, for instance, by inserting a rejuvenation parameter q_{nc}, which models the impact of the repair action performed to recover the equipment from non-critical failure on the time to the critical failure. Also, note that the proposed GRP based approach solely relies on censored and failure times. However, one can extend the approach to make use of the wealth of data that might be available from industrial IoT, i.e., massive and multi-sensors monitoring systems (also known as Big Machinery Data [40]). Indeed, given that this source of data is available, then one can think of extending the proposed approach by replacing the GRP by a Deep Learning based model [40–42] for the system's fault diagnosis and prognosis based on the fusion of both process sensors (e.g., pressure, temperature, flow sensors) as well as maintenance-related sensors such as vibration and acoustic emission under uncertainty [39,43,44]. In this context, the use of DES would no longer be required and the MOGA method would then be integrated with the Deep Learning model for obtaining a replacement policy of equipment under imperfect repairs.

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