UNIVERSIDAD DE CHILE
FACULTAD DE CIENCIAS FÍSICAS Y MATEMÁTICAS
DEPARTAMENTO DE INGENIERÍA DE MINAS

SIMULATION AND OPTIMIZATION TECHNIQUES FOR SHORT-TERM MINE PRODUCTION SCHEDULING

TESIS PARA OPTAR AL GRADO DE
DOCTOR EN INGENIERÍA DE MINAS

FABIÁN ALEJANDRO MANRÍQUEZ LEÓN

PROFESOR GUÍA:
RAÚL CASTRO RUIZ

PROFESOR CO-GUÍA:
NELSON MORALES VARELA

MIEMBROS DE LA COMISIÓN:
HOOMAN ASKARI-NASAB
RAFAEL EPSTEIN NUMHAUSER

Este trabajo ha sido parcialmente financiado por CONICYT/PIA Project AFB180004 and by the CONICYT PFCHA/DOCTORADO BECAS CHILE/2019 - 21190201.

SANTIAGO DE CHILE
2020

RESUMEN DE LA MEMORIA PARA OPTAR
AL TÍTULO DE DOCTOR EN INGENIERÍA DE MINAS
POR: FABIÁN ALEJANDRO MANRÍQUEZ LEÓN
FECHA: 2020
PROF. GUÍA: RAÚL CASTRO RUIZ

SIMULATION AND OPTIMIZATION TECHNIQUES FOR SHORT-TERM MINE PRODUCTION SCHEDULING

Los planes de producción de corto plazo en minas a cielo abierto tienen como objetivo cumplir con las metas previamente definidas por el plan de producción de largo plazo. Desafortunadamente, la complejidad y la incertidumbre de la operación minera provocan desviaciones de los planes de corto plazo. Una desviación de un plan corresponde a cualquier diferencia entre este plan y su ejecución. Por ejemplo, desviaciones en el movimiento de material, en el movimiento de mineral enviado a la planta o en la ley del mineral enviado a la planta.

Uno de los desafíos que enfrentan los planificadores de minas es la consideración de la incertidumbre operacional en la generación de planes de producción de corto plazo que puedan reproducirse en la realidad. Un concepto que cuantifica las desviaciones en un plan de producción de corto plazo y su ejecución se conoce como adherencia, que corresponde a la capacidad del plan para ser reproducido en la realidad.

El objetivo de esta tesis es desarrollar una metodología de simulación-optimización con el objetivo de generar planes de producción mineros de corto plazo con alta adherencia a través de un enfoque iterativo. Los principales aportes de esta investigación son: (i) desarrollo de una metodología genérica que integra técnicas de optimización y simulación en un esquema iterativo, validado por un estudio de caso real; (ii) formalización del concepto de adherencia de un plan, proponiendo varios indicadores de adherencia para medir la desviación entre un plan y su correspondiente simulación; (iii) evaluación del cumplimiento de un plan mediante simulaciones considerando la incertidumbre operacional; (iv) desarrollo de un modelo de optimización para generar planes de producción mineros de corto plazo en minas a cielo abierto, considerando múltiples objetivos, utilizando el método de la suma ponderada y el método jerárquico. El modelo también contempla la asignación de palas a frentes de carga y stockpiles.

Los resultados del trabajo presentado en esta tesis demuestran que la metodología propuesta mejoró los indicadores de cumplimiento del plan minero con respecto a las iteraciones y simultáneamente mantuvo el Valor Actual Neto del plan. Los resultados también muestran que el método de la suma ponderada y el método jerárquico son capaces de generar planes mineros a cielo abierto en minas a cielo abierto optimizando los diversos objetivos de corto plazo. Finalmente, los resultados revelan que los planes con stockpiles obtienen mejores indicadores de adherencia en comparación con los que no tienen stockpiles. También demuestran que los planes con una flota de palas móviles obtienen mejores indicadores de adherencia que los planes mineros con una flota de palas fijas. Esta tesis revela la importancia y el impacto de los métodos de optimización de múltiples objetivos para la generación de planes de producción mineros a corto plazo en minas a cielo abierto. También demuestra que la simulación proporciona una mejor comprensión de los impactos de la incertidumbre operacional en los planes de producción de corto plazo en minas a cielo abierto.

Abstract

Short-term mine production schedules aim to meet goals previously defined by the long-term mine production schedule. Unfortunately, the complexity and uncertainty of a mine operation cause deviations from the short-term schedules. A deviation from a schedule corresponds to any difference between this schedule and its execution; for example, deviations in the movement of material, ore sent to the plant, or in the ore grade sent to the plant.

One of the challenges that mine planners face is the consideration of operational uncertainty in the generation of short-term mine production schedules that can be reproduced in reality. A concept that quantifies the deviations in a short-term production schedule and its execution is known as adherence, which corresponds to the schedule capability to be reproduced in reality.

The objective of the thesis is develop a simulation-optimization framework with the objective of generating short-term mine production schedules with high adherence through an iterative approach. The main contributions of this research are: (i) development of a generic methodology which integrates optimization and simulation techniques in an iterative scheme, that generates short-term mine production schedules with high adherence, validated by a real case study; (ii) formalization of the concept of adherence of a schedule, proposing several adherence indicators to measure the deviation between a schedule and its corresponding simulation; (iii) evaluation of the adherence of a schedule through simulations considering operational uncertainty; (iv) development of an optimization model to perform short-term mine production schedules in open-pit mines, considering multiples objectives, using the weighted sum and the hierarchical method. The model also support the allocation of shovels to mining fronts and stockpiles.

The outcomes of the work presented in this thesis demonstrate that the proposed framework improved the mine schedule adherence indicators over iterations and simultaneously maintained the Net Present Value of the mine schedule. The results also show that the weighted sum and the hierarchical method are capable of generating short-term mine schedules by optimizing the various short-term objectives. Finally, the results reveal that schedules with a stockpile obtain higher schedule indicators compared to the ones with no stockpile. They also demonstrate that schedules with a mobile shovel fleet obtain higher schedules' adherence indicators than the ones with a fixed shovel fleet.

This thesis reveals the importance and impact of multiple objective optimization methods for the generation of short-term mine production schedules in open-pit mines. It also demonstrates that the simulation provides a better understanding of the impacts of the operational uncertainty in short-term mine production schedules.

A mis padres, Ximena y Adolfo.

Agradecimientos

A mis padres, Ximena y Adolfo. Ustedes son la razón de lo que soy. Los amo. A mis hermanos Oscar y Valeria, por todo su cariño y paciencia conmigo. A mis abuelos, tíos y primos, por todo su cariño. A las mascotas históricas de nuestra familia, Lers y Watson.

A la chica con la que quiero pasar el resto de mi vida, Cacau. A Dom, por haberme adoptado.

A mis amigos de la vida Matías, Felipe y Javier. A mis amigos de la universidad José, Alexis e Isaac. También a Kevin, Felipe L. y Erick. Gracias por brindarme su amistad. También, a todos mis amigos de squash de la facultad. Mención especial a Nicolás, Javier, Andrés y Leo. También a la Sra. Asunción Jorquera y al Área de Deportes de la facultad, por haberme apoyado a reservar los múltiples horarios de la cancha de squash de la facultad.

A mis profesores de matemáticas del colegio, Rafael Arenas y Antonio Aedo. A la Escuela de Verano de la U. de Chile, en particular al profesor Jorge San Martín.

A la Universidad de Chile, por haberme formado en el aspecto profesional, académico y humano. Al profesor Nelson Morales, por su confianza, paciencia y apoyo a lo largo de mi estancia en el Laboratorio Delphos. A Enrique Jélvez, por motivarme a comenzar a dirigir memorias de pregrado e invitarme a formar parte del equipo de profesores de la Escuela de Verano del Laboratorio Delphos. Al profesor Carlos Arroyo, por el apoyo brindado durante mi pasantía de investigación en la Universidad Federal de Ouro Preto. También a mis amigos Alisson, Renato y Rubén. A Carol, Renée, Paulet, Felipe y Carlo, por ayudarme en los diversos trámites necesarios del programa de doctorado.

A todos los compañeros que conocí en el Laboratorio Delphos, por crear un excelente ambiente para la investigación, trabajo y recreo. Mención especial para el equipo histórico: Gerson, Pierre, Consuelo, Gonzalo, Nelson, Max, Nelson E. y Diego.

A los profesores de mi comisión: Raúl Castro, Hooman Askari-Nasab y Rafael Epstein, y también al profesor Xavier Emery; por sus observaciones, correcciones, indicaciones y consejos para el desarrollo de esta tesis.

Esta tesis fue parcialmente financiada por CONICYT/PIA AFB180004 y por CONICYT PFCHA/DOCTORADO BECAS CHILE/2019 - 21190201.

Contents

List of Tables

List of Figures

Chapter 1

Introduction

1.1 Introduction

Mine planning is the discipline of mining engineering that transforms the mineral resource into the most profitable business for the owner. The scheduling sequence of mining operations is usually divided into three levels: strategic (long-term), tactical (medium-term) and operational (short-term) L'Heureux et al (2013). Strategic scheduling defines the portions of the ore body to extract, the life of the mine, the production rate and the amount of investment. A long-term mine production schedule defines the portions of waste and ore to mine from the ore body in each year. This schedule seeks to maximize the net present value (NPV) over the life of the mine. Tactical scheduling determines the sequence of mining for up to five years by considering constraints on the production rate. Finally, operational scheduling (short-term scheduling) seeks to make the long-term mine production schedule operationally feasible Smith (1998). This thesis focuses on short-term scheduling. The horizon of interest for a short-term schedule spans several weeks to months (and typically no more than one to two years) (Blom et al, 2018). Therefore, this work focuses on scheduling horizons of no more than one and a half years.

1.2 Motivation

One of the challenges that mine planners face is the consideration of the different types of uncertainty (geological, market, operational) in the generation of long and short-term mine production schedules. Short-term mine production schedules aim to meet the movement of material previously defined by the long-term mine production schedule. A particular problem is the consideration of operational uncertainty in the process of generating short-term mine production schedules. These schedules have to consider in detail how to execute all of the mining activities in a mine operation in order to meet the required production rates. Each of these activities is carried out by specific mining equipment. These activities should also respect the mining activities precedence, which is the sequential order in which the activities must be completed.

1.2.1 Deviation from a mine production schedule

A deviation from a mine production schedule corresponds to any difference between this schedule and its execution; for example, deviations in the movement of material, ore sent to the ore processing plant, or in the ore grade sent to the ore processing plant. Unfortunately, the complexity and uncertainty of a mine operation cause deviations from the short-term schedules Upadhyay and Askari-Nasab (2017). These uncertainties are: (i) market uncertainty, which has to do with unknown future commodity prices; (ii) geological uncertainty, associated with the unknown characteristics of the deposit in terms of grades, rock types, and mineralization; and (iii) operational uncertainty, which has to do with the unknown characteristics of the behaviour of the mining operation associated with the mining equipment. This third uncertainty is the main focus of this paper. The relevance of deviations in the short-term production schedules is crucial in the mining industry. According to Upadhyay and Askari-Nasab (2018), deviations in short-term mining schedules make it difficult to achieve the goals defined by the long-term schedules.

1.2.2 Adherence from a mine production schedule

A concept that quantifies the deviations in a short-term production schedule and its execution is known as adherence. More precisely, the adherence of a mine production schedule corresponds to its capability to be reproduced in reality. Unfortunately, the adherence of a mine schedule is not assessed before it is executed. This issue could lead to implementing schedules whose goals are difficult or even impossible to accomplish. One of the problems that affect mining is the deviation between production schedules and the results of the operation Dimitrakopoulos et al (2002). Due to the sources of uncertainty and the complexity of mining operations, deviations from the production schedule Albor Consuegra and Dimitrakopoulos (2009); Upadhyay and Askari-Nasab (2016, 2017) occur. Compliance schedule is a crucial aspect for mining since deviations in short-term schedules make it difficult to meet the goals defined by long-term schedules Upadhyay and Askari-Nasab (2018). Therefore, it is desirable to generate production schedules with high adherence, which is defined as the ability of the schedule to reproduce in reality

1.2.3 Key Performace Indicator

The efficiency of the mining equipment is related to the achievement of the objectives of a short-term schedule. A key performance indicator (KPI) is a measure of the level of efficiency of a process. A mine operation uses the following KPIs to measure the level of efficiency in each piece of equipment: availability and utilization. These indicators depend on the state of the equipment. A piece of equipment is in the available state if it is enabled to perform the tasks for which it was designed. On the other hand, a piece of equipment is in the effective state if it is performing the tasks for which it was designed. In this way, the availability of equipment corresponds to the quotient between the time in the available state and the total time. Similarly, the utilization of equipment corresponds to the quotient between the time in the effective state and the total time

1.3 Aims and scope of the thesis

This section presents the objectives of this work, classified between general objectives and specific objectives. In addition, the main aspects that represent the scope of the thesis are listed.

1.3.1 Overall objective

The main objective of the thesis is develop a simulation-optimization framework with the objective of generating short-term mine production schedules with high adherence through an iterative approach.

1.3.2 Specific objectives

The specific objectives of the thesis are the following:

- Integrate optimization and simulation techniques in an iterative scheme.
- Formalize of the concept of adherence of a short-term mine schedule.
- Evaluate of the adherence of a mining plan through simulations considering operational uncertainty.
- Propose of a generic methodology for generating short-term mine production schedules with high adherence, validated by a real case study.
- Generate an optimization model to perform short-term mine production schedules in open-pit mines, considering multiples objectives.

1.3.3 Scope

This thesis proposal focuses on the following aspects:

- Short-term mine production scheduling, with a scheduling horizon of one year and monthly periods.
- For the generation of short-term mine production schedules, we develop and solve optimization problems based on mixed-integer linear programming.
- We apply discrete event simulation as a tool for evaluating the adherence of short-term production mine schedules.
- We consider case studies of open-pit and underground mines.
- We do not consider market neither geological uncertainty.

1.4 Thesis organization

Chapter 2 provides a literature review about long- and short-term open-pit mine production scheduling. It also review articles that integrates simulation and optimisation in the context of open-pit mine scheduling. Finally, we briefly explain the optimisation methods used to optimise multi-objective optimisation problems.

Each of the three subsequent chapters (Chapter 3, 4 and 5) has been published in, or submitted to, international scientific journals or international conferences.

Chapter 3 propose a generic simulation-optimization framework to generate short-term production schedules for improving the schedule adherence using an iterative approach. In each iteration of this framework, a short-term schedule is generated using a mixed-integer linear programming model that is simulated later using a discrete-event simulation model. This chapter has been published in *Optimization and Engineering* (Manríquez et al, 2020b).

Chapter 4 proposes an optimization methodology to generate a short-term open-pit mine production schedule optimizing multiple hierarchical objectives. For the generation of mine schedules, we propose an optimization model based on mixed-integer linear programming. In order to optimize the multiple hierarchical short-term objectives, we apply the hierarchical and weighted sum methods in the proposed optimization model. This chapter has been published in the *Proceedings of the 39th international symposium on Application of Computers and Operations Research in the Mineral Industry, APCOM 2019* (Manríquez et al, 2019).

Chapter 5 proposes an optimisation problem to generate short-term open-pit schedules, optimising multiple objectives. The problem is based on mixed-integer linear programming. It allocates shovels to different mining faces, including stockpiles. It considers constraints of plant capacity, ore blending, precedences between mining faces, shovels throughput, and movement of shovel between mining faces. We also propose a set of indicators used to assess and compare different short-term schedules. To optimise the multiple short-term objectives, we apply the single optimisation and the hierarchical method in a real-scale open-pit case study. This chapter has been submitted to *Engineering Optimization* (Manríquez et al, 2020a).

A general discussion and conclusions of each article follow in Chapter 6, while Chapter 7 provides the references mentioned throughout the thesis.

Chapter 2

Literature review

In this section, we review the works used to generate short-term production schedules in open-pit mines. After that, we provide a brief review of the different methodologies that are commonly used to address the simulation-optimization problem. Then, we review the works that combine the simulation by discrete events with the optimization in the mining industry

2.1 Short-term production schedules

A useful tool to generate production schedules is mathematical optimization. Both open-pit and underground mining apply these techniques. In the particular case of short-term underground mine scheduling, this generally consists of mixed-integer linear programming (MILP) in which binary variables address long-term block-extraction decisions and continuous variables address the related short-term decisions of how much ore should be extracted from a block (Newman et al, 2010a). A review of optimization techniques applied to underground mines can be found in Musingwini (2016). Researchers have mainly applied mathematical optimization to generate short-term production schedules in open-pit mines considering the optimization of multiples objectives. An excellent review of works related to short-term production scheduling in open-pit mines can be found in (Blom et al, 2018). Smith (1998) describe a model to maximize ore production subject to ore quality constraints.

2.1.1 Long-term open-pit scheduling with stockpiles

The mineral deposit is partitioned into several blocks. The open-pit mine production scheduling problem (OPMPSP) consists of scheduling the extraction of these blocks to maximise the Net Present Value (Samavati et al, 2018). OPMPSP has been subsequently formulated as an integer linear programming problem by Johnson (Johnson, 1968). Similarly, the OPMPSP+S consists of scheduling the extraction of blocks of the mineral deposit segments to maximise the Net Present Value of the operation, considering stockpiles in their formulation.

There are different ways to model the blending in the stockpiles. Hoerger et al (1999) assume that material is removed from the stockpile, its grade is considered to be the minimum of the associated grade range. Akaike and Dagdelen (1999) model the stockpiles as an infinite number of stockpiles, meaning that every block has its associated stockpile. That is to say, there is no blending in the stockpile. Fu et al (2019) models the stockpiles using a series of grade bins, allowing the model to allocate material with a different grade.

Moreno et al (2017) proposed several linear integer optimisation problems to schedule open-pit mines considering stockpiling. Rezakhah et al (2020) modified on of the problems developed by Moreno et al (2017). They apply it to an operational poly-metallic (gold and copper) open pit mine, in which the stockpile is used to blend materials based on multiple block characteristics. Rezakhah and Newman (2020) extend the OPMPSP+S, considering degradation of stockpiles, which is the deterioration of material when it is exposed to the environment. Rezakhah et al (2020) modified a problem described by Moreno et al (2017) and apply it to an operational poly-metallic (gold and copper) open pit mine, in which the stockpile is used to blend materials based on multiple block characteristics.

For its part, Asad (2005) presents a long-term cut-off grade optimisation algorithm for open-pit mining operations with stockpiling in a deposit with two economic minerals. This algorithm is an extension of the theory of cut-off grades in deposits of two economic minerals presented in Lane et al (1984).

Nonlinear optimisation models have also been proposed to address the open-pit mine production scheduling problem with stockpile (OPMPSP+S). For example, Bley et al (2012) propose two nonlinear optimisation problems to address. They assume that the grade of the material removed from the stockpile is a weighted average of the material inside the stockpile.

Some articles address OPMPSP+S accounting for geological uncertainty in the metal content(Ramazan and Dimitrakopoulos, 2013; Koushavand et al, 2014; Silva et al, 2015; Lamghari and Dimitrakopoulos, 2016; Levinson and Dimitrakopoulos, 2019)

Ramazan and Dimitrakopoulos (2013) consider uncertainty in the geological and economic input data and use a stochastic framework to solve the OPMPS+S problem. The authors do not consider mixing material in the stockpile, meaning that when the material leaves the stockpile, it has the same characteristics as when it enters. Koushavand et al (2014) present a linear programming long-term open-pit mine production scheduling model, considering grade uncertainty and stockpiling. The model considers one stockpile. The modelling of stockpiling and re-handling of this stockpile is based on the over production of ore in certain periods. Silva et al (2015) resolve a OPMPSP, considering metal uncertainty, multiple processing destinations, and a stockpile. They apply one of the heuristics described in Lamghari and Dimitrakopoulos (2016) to solve a gold deposit case study. Lamghari and Dimitrakopoulos (2016) propose a novel formulation of the OPMPSP, considering metal uncertainty, multiple processing destinations, and stockpiles. They also propose and compare four heuristics for the problem. Levinson and Dimitrakopoulos (2019) apply a simultaneous stochastic optimisation model, that integrates waste management into the long- term production schedule, to a gold mining complex. The model aims to define the extraction sequence, destination policy, and processing stream decisions while simultaneously managing the targets and capacities at waste, processing, and stockpile facilities. Geological uncertainty is considered by simulating geological attributes for each block.

2.1.2 Short-term open-pit scheduling with stockpiles

There are fewer articles that address the short-term open-pit scheduling considering stockpiles than in the long-term open-pit scheduling.

Huang et al (2009) describe the MineSight Scheduling Optimiser (MSSO). They propose an optimisation model based on MILP to generate short-term schedules. The optimisation problem considers multiple destinations and blending requirements. Unfortunately, the authors do not provide the formal mathematical model employed. Rehman and Asad (2010) propose a MILP-based optimisation model to define the short-term sequence mining of blocks of a limestone quarry to meet plant quantity and quality requirements at the lowest possible cost. The proposed model does not consider shovel allocation in quarry blocks. Eivazy and Askari-Nasab (2012) develop an MILP-based optimisation model to generate short-term schedules. The model minimises the cost of open-pit mines, considering multiple destinations. Buffer and blending stockpiles, horizontal directional mining and decisions on-ramps are incorporated. The model has blending constraints, mining and processing capacities, and mining precedence. It is assumed that stockpiles are homogeneous and the ore reclaimed from each stockpile has a specific grade equivalent to the average grade of the stockpile material. The stockpiles are explicitly modelled, but the allocation of shovels in stockpiles are not considered. Shovel allocation in mining blocks is not considered either.

Mousavi et al (2016) propose an MILP-based optimisation model that considers including precedence relationship, machine capacity, grade requirements, processing demands and stockpile management. The objective function is the minimisation of the total cost, which includes rehandling and holding costs, misclassification and drop-cut costs. Shovel allocation in mining blocks is considered. The stockpiles are explicitly modelled, but the allocation of shovels in stockpiles are not considered.

Matamoros and Dimitrakopoulos (2016) propose a formulation based on stochastic mixed-integer programming to address the short-term open-pit scheduling. The model considers uncertainty in both orebody metal quantity and quality. It also takes into account fleet parameters and equipment availability. The model allocates shovels to mine sectors and number of truck trips to shovels. The objective function considers operating fleet cost and mining cost. It also considers a deviation cost from the production target. In a given period, the overproduction of ore may be considered as the material that goes to stockpile, and its tonnage is penalised by the corresponding re-handling cost. That is to say, stockpiles are not explicitly modelled.

Blom et al (2017) generate multiple, diverse short-term schedules while optimising for a customisable, prioritised sequence of objectives. They use a rolling horizon-based algorithm to resolve instances. Unfortunately, the stockpiles are not explicitly modelled. That is to say, the movement of material to and from each stockpile is not scheduled.

Table 2.1 summarizes and compares the features of the short-term open-pit mine scheduling articles here reviewed.

Feature \ Article	Shovel allocation in mine faces	Shovel allocation in stockpiles	Explicit modelling of stockpiles
(Rehman and Asad, 2010)	✗	✗	✓
(Eivazy and Askari-Nasab, 2012)	✗	✗	✓
(Mousavi et al, 2016)	✓	✗	✓
(Matamoros and Dimitrakopoulos, 2016)	✓	✗	✗
(Blom et al, 2017)	✓	✗	✗

Table 2.1: Comparison of short-term open-pit mine scheduling articles considering stockpiles.

2.2 Multi-objective optimisation

The weighted sum and hierarchical are methods commonly used to optimise multiple objectives. We describe these methods in detail below

2.2.1 Weighted sum method

The weighted sum method allows the multiple-objective optimization problem to be cast as a single-objective mathematical optimization problem. This single objective function is constructed as a sum of objective functions f_{i} multiplied by weighting coefficients w_{i}, hence the name. The coefficients w_{i} are computed as $w_{\mathrm{i}} = u_{\mathrm{i}}\theta_{\mathrm{i}}$, where u_{i} are the weights assigned by the decisor-maker based on the hierarchy of the objectives and θ_{i} are the normalization factors. In this work, the normalization factors are computed as $\theta_{\mathrm{i}} = z_{\mathrm{i}}^{-1}$, where z_{i} is the value of the objective function of the optimization problem when solving for the single objective function f_{i} (Grodzevich and Romanko, 2006).

In this way, the weighted sum method consists of two steps. The first one solves i optimization problems (corresponding to the i short-term objectives) in order to obtain the normalization coefficients θ_{i}. Then, an optimization problem is solved whose objective function corresponds to the weighted sum of all the short-term objectives considered. The coefficients w_{i} that multiply each of the objectives are calculated using the normalization coefficients θ_{i} obtained from the first stage and the weights u_{i} assigned by the decision maker based on the prioritization of the objectives.

2.2.2 Hierarchical method

In the hierarchical method, the decision-maker sorts the objective functions. A decision-maker sorts these objectives in a descending order of importance. In this method, we resolve as many optimisation problems as objective functions. We resolve the optimisation problems following the order of the objectives previously defined. Each problem optimises its corresponding objective, adding an extra set of constraints. These constraints impose an upper bound to the objectives functions previously solved. The upper bound corresponds to the objective function values already obtained.

Below, we explain the hierarchical procedure in detail. Figure 2.1 provides a schematic view of this procedure. First, we sort the N objectives in a descending order of importance. Without losing generality of the procedure, we can assume all these objectives to be minimised. We denote the j-th objective as A_j. We denote $\overline{A}_j$ as the value of the objective function j obtained in the j-the problem. The set of general constraints of the problem are represented as $g_k \leq C_k \quad \forall k$.

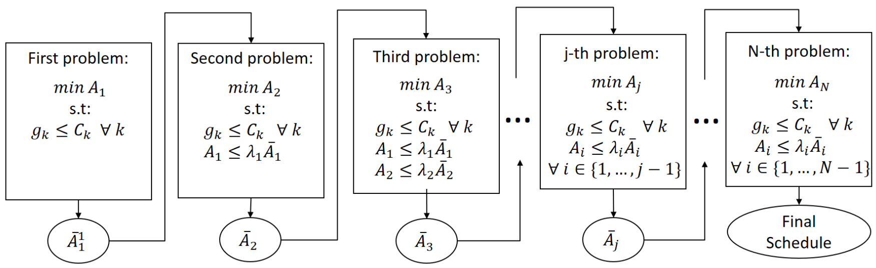

Figure 2.1: Scheme of the hierarchical method considering N objectives

We solve the first problem, which minimises A_1. This problem obtains the first objective function value $\overline{A}_1$. The second problem minimise A_2 subject to an extra constraint. This constraint imposes a limit to the first objective of the second schedule. That limit must not exceed the first objective function value multiplied with certain tolerance λ_1. This tolerance must be equal or greater than 1 to avoid infeasibility of the optimisation problems. Mathematically, this extra constant is $A_1 \leq \lambda_1 \cdot \overline{A}_1$.

After solving the second problem, we obtain the second objective function value $\overline{A}_2$. Next, we solve this problem, which minimises A_3. This problem incorporates two additional constraints. The first one limits the objective function of the first objective $A_1 \leq \lambda_1 \cdot \overline{A}_1$. The second constraint limits the objective function of the second objective $A_2 \leq \lambda_2 \cdot \overline{A}_2$. We follow this procedure until the last optimisation problem is solved. The last schedule corresponds to the output of the procedure.

2.3 Discrete event simulation

One way to assess the likelihood of compliance with a mine schedule is to develop discrete event simulation (DES). In this regard, Panagiotou (1999) states that due to the dynamic and

stochastic nature that characterizes virtually any mining system, simulation is the only reliable method that allows the evaluation of such systems. The DES was used to evaluate the performance of open-pit mining systems in many studies. The application of simulation in the mining sector can be traced back to the 1940s, according to and Moradi Afrapoli and Askari-Nasab (2017). The first application of DES to fleet management can be found in Rist (1961), where the Monte Carlo simulation technique was used to solve hauling problems in mining operations. The DES was applied for evaluating dispatch policies in open-pit mining systems. Lizotte and Bonates (1987) studied the impact of dispatching rules for assigning trucks and shovels, such as minimizing shovel idle time, maximizing truck use and assigning trucks to shovels to meet specific production objectives. Forsman et al (1993) also studied the impact of several dispatching rules, such as fixed trucks, maximum trucks and maximum loaders. Simulation was used to assist the management of the Aitik mine in decision-making regarding the purchase of new trucks, installation of in-pit crushers and route selection for efficient ore and waste transportation. Çetin (2014) developed a simulation model to compare different dispatch policies in terms of total production. Other applications of the DES tool have been reported. Sturgul (1987) discusses the construction of simulation models using a general-purpose simulation system. The authors mention that a separate program has to be written for each system to be simulated as illustrated by three case studies of actual open pit mines located in Australia. Rasche and Sturgul (1991) illustrates the use of DES for determining the optimum number of trucks in the mine, the number of repair crews and the number of spare trucks required. Meech and Parreira (2011) used a deterministic-stochastic model to compare an automated system with a manual to predict the benefits of an autonomous haulage system. Askari-Nasab et al (2007) developed a simulator called the open-pit production simulator. They report that artificial intelligent simulators can be very efficient and helpful for modelling the dynamicity of processes and randomness of input parameters.

2.4 Simulation-optimization approaches

Simulation optimization is the area of research that attempts to optimize a simulation model. The primary objective is to find the values of controllable parameters that optimize a performance function from a simulation model. Chen et al (2008a) reviewed three approaches to address the simulation-optimization problem in a general engineering setting

- The efficient simulation budget approach (Chen et al, 1997, 2000; Chick and Inoue, 2001b,a; Lee et al, 2004; Chen and Yücesan, 2005; Kim and Nelson, 2006; Fu et al, 2007; Chen et al, 2008b) aims to select the optimum simulation design from a set of scenarios given in advance, which differs from the approach in this paper because ours enumerates schedules as a result of the iterative process, i.e., they are not pre-defined by the user.
- The nested partitions method is an approach that solves global optimization problems, for which it requires to partition the space into subregions. Each subregion is evaluated using sampling and the most promising one is used on the next iteration, or the method backtracks to a larger region if the each one of the subregions turns out to be worse than the incumbent region (Chen et al, 2008a). This approach is therefore different from the one proposed in this work, because we do not have such hierarchical structure defined on the schedules.
- The stochastic gradient estimation method(Ho and Cao, 1991; Glasserman, 1991; Fu and Hu, 1997; Glynn, 1987; Rubenstein and Shapiro, 1993; Pflug, 1989, 1996); is an enumeration method based on local search that adjusts parameters, which are required to be continuous, to generate alternate scenarios. The method proposed here also enumerates scenarios (in our case, short-term schedules); however, they are not based on derivatives of certain parameters but on new estimations of key performance indicators

provided by the simulation process. In particular, the search is not local.

2.5 Combination of simulation and optimization in mining

Previous works which combine optimization with DES have mainly been applied to the truck-shovel transportation system in open-pit mines. A conventional approach is to integrate both tools as a combined run. That is, the simulation model calls the optimization model whenever the state of the mine operation changes in order to assist the simulation model to allocate the available pieces of equipment in the mine operation according to the new state. The mine operation state changes whenever there is a maintenance or a failure of a piece of equipment or when a shovel finishes the extraction of all the material at its mining face. The approach mentioned above has been applied in the following reports in the context of open-pit mine operations.

Fioroni et al (2008) presented a MILP model to allocate shovels to mine faces and the number of trips that each type of truck fleet have to carry out to these faces, subject to production and blending constraints. The models are called by an open-pit truck-shovel simulation model, whenever a change in the mine operation system occurs, in order to update the allocation of shovels to the new operation state.

Mena et al (2013) proposed a MILP model to allocate trucks to transportation routes. The simulation model considers density probability distribution in order to model: (i) the uncertainty of the operational parameters and (ii) the times between failures and the time taken to repair load and haulage equipment.

Upadhyay and Askari-Nasab (2016, 2017, 2018). described a MILP model to allocate shovels to mine faces in order to: maximize production, meet desired head grade and tonnage at crushers and minimize shovel movements. This model is called by an open-pit truck-shovel simulation model, similarly than the work of Fioroni et al (2008).

Others authors have integrated optimization and simulation differently. Bodon et al (2011) and Sandeman et al (2010) proposed a linear programming model which determines the quantity of ore extracted from each mining face transported to mine stockpiles, the ore transported from each mine stockpile to the port stockpile, and the ore transported from each port stockpile to ships, thus maximizing the throughput of material from pit to ship. An initial schedule is generated for the first two weeks of a one-year horizon and subsequently this schedule is simulated. Then, a new schedule is generated for the next two weeks given the current state of the system. There is only limited literature regarding underground mining that combines optimization with DES, due to the complex nature of generating mine production schedules in underground mines compared to open-pit mines. Musingwini (2016). Chanda (1990) presented a MILP model to perform short-term production scheduling of a sector of a continuous block caving mine, minimizing the difference in average grade between successive periods, and by considering constraints such as the availability of drawpoints and limits on production and ore quality. This model was combined with a simulation to generate short-term production schedules for six consecutive work shifts. Winkler (1998) described an optimization model of a sub-level caving mine, which defines the amount of ore to be extracted in each block and each period, minimizing deviations from production goals, and by considering constraints such as ore quality, the minimum amount of extraction of each block, and the capacity of the available ore. The model is solved to generate a schedule for a single period, which is subsequently simulated. This same procedure is repeated for successive periods. Salama et al (2014) compared different mineral haulage systems using simulation in order to estimate the mining costs in a sub-level stoping mine. This cost serves as an input to a mixed-integer optimization model which generates a long-term schedule that maximizes the net present value (NPV). As it turns out, the reviewed reports, which combine optimization and simulation applied in mining, use optimization and then simulation in a sequential way, or use optimization as a subroutine of simulation. In none of these studies is there feedback between the optimization and the simulation model, as proposed in this work.

Chapter 3

A simulation-optimization framework for short-term underground mine production scheduling

Abstract

Mine operations are supported by a short-term production schedule, which defines where and when mining activities are performed. However, deviations can be observed in this short-term production schedule because of several sources of uncertainty and their inherent complexity. Therefore, schedules that are more likely to be reproduced in reality should be generated so that they will have a high adherence when executed. Unfortunately, prior estimation of the schedule adherence is difficult. To overcome this problem, we propose a generic simulation-optimization framework to generate short-term production schedules for improving the schedule adherence using an iterative approach. In each iteration of this framework, a short-term schedule is generated using a mixed-integer linear programming model that is simulated later using a discrete-event simulation model. As a case study, we apply this approach to a real Bench & Fill mine, wherein we measure the discrepancies among the level of movement of material with respect to the schedule obtained from the optimization model and the average of the simulated schedule using the mine schedule material's adherence index. The values of this index decreased with the iterations, from 13.1% in the first iteration to 4.8% in the last iteration. This improvement is explained because the effects of the operational uncertainty within the optimization model can be considered by integrating the simulation. As a conclusion, the proposed framework increases the adherence of the short-term schedules generated over iterations. Moreover, these increases in the adherence of schedules are not obtained at the expense of the Net Present Value.

3.1 Introduction

Adherence is a concept that quantifies the deviations in a short-term production schedule and its execution. More precisely, the adherence of a mine production schedule corresponds to its capability to be reproduced in reality. Unfortunately, the adherence of a mine schedule is not usually assessed before its execution. This could result in the implementation of schedules whose objectives are difficult or even impossible to accomplish. The application of discrete-event simulation (DES) is an approach to evaluate the adherence of a short-term schedule before its execution. This approach simulates a given short-term schedule by considering all the operational uncertainties associated with mine operation.

Further, we incorporate the operational uncertainty associated with the operational parameters of the equipment (velocities, capacities, maneuver times, failures times, and maintenance times), which are modeled using the probability density functions based on the historical data. DES has been extensively applied to the model mine operations in which the deterministic models failed to accurately predict the uncertain behavior (Upadhyay and Askari-Nasab, 2017). This approach is extensively used to assess the performance of mine operations because it helps to incorporate the inherent variability and complexity of operational uncertainty (Torkamani and Askari-Nassab, 2015).

Based on the evaluation of the adherence to short-term production schedules using DES, it is desirable to generate short-term production schedules exhibiting high adherence. Mathematical optimization is a useful tool to generate production schedules in both open-pit mining and underground mining. In the case of short-term underground mine scheduling, mixed-integer linear programming (MILP) is generally considered in which the binary variables address long-term block-extraction decisions and continuous variables address the related short-term decisions of how much ore should be extracted from a block (Newman et al, 2010b). A review of the optimization techniques applied to underground mines can be found in Musingwini (2016).

One common approach for optimization under uncertainty is the utilization of stochastic programming, which allows optimization problems with respect to the random variable parameters in the goal function or constraints to be solved (see Birge and Louveaux (2011) for details about stochastic programming). However, the problem that we address in this study, such an approach would be very difficult to implement and most likely impractical to use because the KPIs are not only random but depend upon the schedule; conversely, the feasibility of a schedule is highly dependent on KPIs.

Therefore, we propose a framework that combines a deterministic optimization model with DES. In this framework, the optimization part of the framework generates short-term mine production schedules, whereas the simulation part evaluates these schedules and provides useful feedback to generate a new and better schedule in future iterations. Therefore, the contributions of the paper are (i) the development of a simulation-optimization framework to generate short-term mine production schedules, (ii) providing a set of indicators to measure the adherence to a schedule, and (iii) the application of the proposed framework to a specific case in which a mathematical model, a DES model, and the mechanisms to integrate them are implemented to denote that the proposed methodology provides schedules with high adherence using an iterative approach.

Section 3.2 provides a complete description of the proposed framework, Section 3.3 introduces several adherence indices used to quantify the adherence of a schedule and its corresponding simulation, and Section 3.4 describes the application of the proposed framework to a Bench and Fill (B&F) mine operation by introducing the optimization model used to generate short-term mine production schedules and the simulation model developed to simulate them. In Section 3.5, we apply the proposed framework to a real-world data of a B&F mine. Section 3.6 reports and discusses the results of the case study. Finally, Section 3.7 concludes the present study and outlines future work.

3.2 Framework description

In this section, we describe the proposed framework, which combines simulation and optimization via an iterative approach, to improve adherence to short-term production schedules. This concept was first explored by Pérez et al (2017). In each iteration, we generate a short-term mine production schedule by solving an optimization problem; subsequently, we simulate this schedule using a DES model of mine operation. The steps can be given as follows (also presented in Figure 3.1):

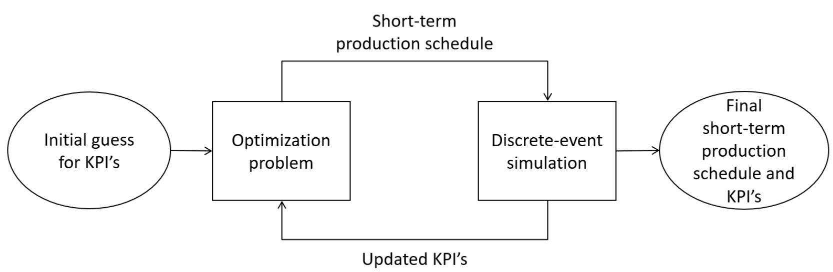

Figure 3.1: Simulation-optimization iterative framework diagram

1. Obtain initial KPIs through benchmarking or deterministic estimation.
2. Generate an initial short-term production schedule by solving a mathematical optimization problem for the current KPI values. This short-term schedule considers a material that has been extracted according to the long-term schedule.
3. Simulate the short-term production schedule generated in Step 2.
4. Update the KPIs per period for each equipment obtained from the simulation of the short-term production schedule.
5. Calculate the actual adherence of the corresponding simulation to the schedule.
6. Whenever any termination criteria is satisfied, e.g., when the schedule adherence index is less than or equal to a specific critical value or a maximum computation time is reached, the procedure is terminated; otherwise, go to Step 2.

The fundamental concept of the proposed framework is that in each iteration, better estimations are obtained for the equipment's KPIs based on the simulation results of the short-term mine production schedule.

Considering the operational uncertainty in the mining operation, the role of the replications is to represent all the potential results obtained when performing the simulation of a given mine schedule as accurately as possible.

In the subsequent iteration, we use new estimations of the equipment KPIs as inputs for the optimization model to generate an updated short-term mine production schedule.

This procedure is repeated until a specific stop criterion is reached. The simulation of the short-term mine production schedule allows us to consider majority of the complexities and operational uncertainties associated with the mine operation, which are difficult and cumbersome to incorporate into a mathematical optimization model. Thus, the estimation of all the equipment KPIs can be improved. In other words, the simulation of a particular short-term mine production schedule allows us to obtain an explicit quantification of the maintenance equipment times, equipment failures, travel time between the locations at which the equipment is used to perform mining activities, and equipment times. The backup time refers to the equipment that is available for operation but is not operating because of a specific condition of mine operation. Furthermore, the simulation considers the exact dispatching routines that assign equipment to mine faces and the on-site specific operational rules for a particular mine. It is noteworthy that the described framework is general because it can be used to address the operational uncertainty in many other situations in which a deterministic model is used; this kind of uncertainty has to be handled. However, its specifics are certainly dependent on the application. The optimization and simulation models consider all the relevant equipment and tasks to reliably emulate the mine operation. The selection of the type of KPIs to be estimated in each iteration is critical for the application of the framework. Usually, the utilization of the pieces of equipment, which is the ratio of the effective time and the nominal time, is the selected KPI.

The optimization feedback to the simulation is presented in Figure 3.1. At the beginning of each iteration, we generate a short-term production schedule by solving an optimization problem. Based on this schedule, a list of priority tasks is created. This list is the input into the simulation model. Thus, the simulation model follows a short-term production schedule. The process by which the tasks are sorted to create a list of priority tasks is based on the start and completion periods of each activity obtained based on the short-term production schedule. If there is a tie in the order of two or more tasks, it can be broken using ad hoc criteria that depend on the application.

The simulation feedback to the optimization process is described here. At the beginning of each iteration, we generate a short-term production schedule by solving an optimization problem, which is further simulated. Subsequently, we compute the mean KPIs of all the replications of the corresponding simulation based on the simulation data for each piece of equipment.

3.3 Adherence of a schedule

We propose several indices to evaluate the adherence to a short-term production schedule. Some of these indices are related to the start and completion periods with respect to the schedule and its corresponding activities. Other indices are related to the material movement in case of the schedule and its corresponding simulation. We summarize the notation related to the optimization model and the simulation model in Tables 3.1 and 3.2.

It is important to note that all the adherence indices defined in this section are associated with a given mine schedule. During the application of the proposed framework, one mine schedule is generated per iteration. Thus, the values of the adherence indices will vary during each iteration.

Symbol	Description
S^{i}_a	Start period of the activity a, according to the mine schedule, in iteration i.
C^{i}_a	Completion period of the activity a, as in the mine schedule, in iteration i.
M	Total material movement, according to the mine schedule.
MP^{i}_t	Total material moved in the mine schedule in period t, in iteration i.

Table 3.1: Optimization problem notation

Symbol	Description
$S^{\mathrm{i}}_{a,r}$	Start period of the activity a, in replication r, in iteration i.
$C^{\mathrm{i}}_{a,r}$	Completion period of the activity a, in replication r, in iteration i.
$M^{\mathrm{i}}_{t,r}$	Total material moved in the mine in period t, in replication r, in iteration i.
MS^{i}_t	Mean material simulated over all the replications in period t, in iteration i.
$y^{\mathrm{i}}_{a,r}$	Equal to 1 if $S^{\mathrm{i}}_{a,r} \leq S^{\mathrm{i}}_a$; otherwise, 0.
$z^{\mathrm{i}}_{a,r}$	Equal to 1 if $C^{\mathrm{i}}_{a,r} \leq C^{\mathrm{i}}_a$; otherwise, 0.

Table 3.2: Simulation problem notation

3.3.1 Mean lateness, earliness and tardiness

The initially proposed indices are related to the concepts of lateness, tardiness, and earliness, as obtained from the literature (Baker and Trietsch, 2009). Given an activity a, its lateness corresponds to the difference between its completion time and deadline, which can be either positive or negative. The tardiness of the activity corresponds to the positive difference between its completion time and deadline, whereas its earliness corresponds to the negative deviation between its completion time and deadline. For details, refer to the second column of Table 3.3.

We extend these concepts to a setting in which multiple activities and replications are present. Let A the set of activities and R the set of replications. $|A|$ represents the quantity of elements of set A. $|R|$ represents the quantity of elements of set R. First, we compute the corresponding activity index for each activity $a \in A$ and replica $r \in R$. Second, we add all the activity indices and subsequently average them based on the number of replications to generate representative indices for the schedule, including the mean lateness ($\bar{L}^{\mathrm{i}}$), mean tardiness ($\bar{D}^{\mathrm{i}}$), and mean earliness ($\bar{E}^{\mathrm{i}}$) of the given schedule. For details, refer to the third column of Table 3.3.

The interpretation of the indices in Table 3.3 is explained below. A mean lateness value of greater than zero indicates that the schedule is late when compared with its simulation on an average. A mean lateness value of zero indicates that the schedule is on time relative to its simulation on an average. A mean lateness value of less than zero indicates that the schedule is ahead of its simulated schedule on an average. Based on the mathematical definition in Table 3.3, the mean tardiness and mean earliness are observed to be greater than or equal to

Index name	Activity expression	Schedule expression
Lateness	$L^{\text{i}}_{a,r} = C^{\text{i}}_{a,r} - C^{\text{i}}_{a}$	$\bar{L}^{\text{i}} = \dfrac{1}{\lvert A\rvert\lvert R\rvert}\sum_{a\in A}\sum_{r\in R} L^{\text{i}}_{a,r}$
Tardiness	$D^{\text{i}}_{a,r} = \max\{0; L^{\text{i}}_{a,r}\}$	$\bar{D}^{\text{i}} = \dfrac{1}{\lvert A\rvert\lvert R\rvert}\sum_{a\in A}\sum_{r\in R} D^{\text{i}}_{a,r}$
Earliness	$E^{\text{i}}_{a,r} = \max\{0; -L^{\text{i}}_{a,r}\}$	$\bar{E}^{\text{i}} = \dfrac{1}{\lvert A\rvert\lvert R\rvert}\sum_{a\in A}\sum_{r\in R} E^{\text{i}}_{a,r}$

Table 3.3: Comparison of the activity and schedule lateness, tardiness, and earliness

zero. A mean tardiness value higher than zero indicates that the schedule is late relative to its simulation on an average. A mean tardiness value of zero indicates that the schedule is on time relative to its simulation on an average. Similarly, a mean earliness value of higher than zero indicates that the schedule is ahead of its simulation on an average. A mean earliness value of zero indicates that the schedule is on time relative to its simulation on an average.

3.3.2 Start and completion period adherence indices

We also define the start period and completion period adherence indices. The start period adherence index is the fraction of activities over all the replications that began in a period equal to or before the short-term mine schedule period. Similarly, the completion period adherence index is the fraction of activities over all the replications that were completed in a period equal to or before its mine schedule completion period. Refer to Table 3.4.

Index adherence name	Index adherence expression
Start period adherence index	$SAI^{\text{i}} = \dfrac{1}{\lvert A\rvert \cdot \lvert R\rvert}\sum_{a\in A}\sum_{r\in R} y^{\text{i}}_{a,r}$
Completion period adherence index	$CAI^{\text{i}} = \dfrac{1}{\lvert A\rvert \cdot \lvert R\rvert}\sum_{a\in A}\sum_{r\in R} z^{\text{i}}_{a,r}$

Table 3.4: Start and completion period adherence index

3.3.3 Production and material movement adherence

We introduce adherence indices related to material movement. Therefore, we introduce the material adherence index, which measures the deviation of the material movement with respect to the mine plan and simulation. The material adherence curve is the ratio of the accumulated material of the simulations up to period t and the accumulated material of the mine plan up to period t. For example, if this index is greater than one at a certain period, the simulation produced more material when compared with that produced by the mine schedule on an average, as presented in Table 3.5.

The adherence indices presented in Table 3.5 can be adapted depending on the application. For instance, the adherence can be evaluated by only considering a subset of the total scheduled activities (for example, development and production activities) instead of considering all the scheduled activities. With respect to the material adherence indices, it is possible

Index adherence name	Index adherence expression
Material adherence index	$MAI^{\mathrm{i}} = \dfrac{1}{M}\sum_{t\in T} \lvert MP_t^{\mathrm{i}} - MS_t^{\mathrm{i}} \rvert$
Material adherence curve	$AT^{\mathrm{i}}(t) = \dfrac{\sum_{t'=1}^{t} MS_{t'}^{\mathrm{i}}}{\sum_{t'=1}^{t} MP_{t'}^{\mathrm{i}}}$

Table 3.5: Material adherence indices

to consider the type of material instead of the total material transported when evaluating the adherence index (for example, to distinguish the production material from different mine operation sectors).

3.4 Application to the operation of a Bench & Fill mine

We apply the proposed framework to B&F mine planning. Therefore, we develop an optimization model that can be used to generate the mine production schedule of a B&F mine. We also develop a simulation model that supports the simulation of the B&F mine production schedule generated by the optimization model. This section explains the B&F mining method and describes the optimization and simulation model, including the manner in which these models interacted to obtain the proposed framework.

3.4.1 Description of the Bench & Fill mining method

The B&F method is an underground mining method, which is applied to ore bodies exhibiting vertical or sub-vertical geometry. Drift and stope are the two types of mining workings that have been used in this study. A stope is the basic mine production unit, which exhibits a tabular or semi-tabular form and contains the ore that is to be extracted. To access each stope, it is necessary to develop two drifts, i.e., the production drift (lower drift) and the drilling drift (the upper drift), in the upper and lower parts of the stope in advance. After the extraction of ore from the stope, it is necessary to backfill the empty portion of the stope from the drilling drift to maintain the stability of the walls and roof of the stope. Figure 3.2 depicts the side view of a stope in a B&F mine.

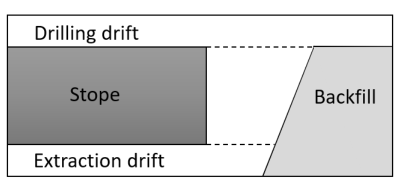

Figure 3.2: Side view of a stope in a B&F mine

To completely develop the drifts and stopes associated with this mining method, a set of sequential mining activities should be performed using specific mining equipment. In order to complete each portion of drift, the following activities should be sequentially performed: drilling, charging, blasting, mucking, scaling, shotcreting, and bolting. The first activity is drilling, which consists of drilling boreholes in a certain drill pattern in the rock face of the drift using a face drill rig. Subsequently, the drills in the drift face are charged with explosives using an explosive charger, and that portion of the drift is then blasted. Next, a load-haul-dump vehicle (LHD), which is a machine similar to a conventional front-end loader used in underground mining, mucked the blasted material, and a scaler was used to eliminate loose rock from the roof or walls of the drift. Shotcreting projects a mixture of concrete, water, sand, and gravel onto the drift walls using a compressed air device mounted on a piece of shotcrete equipment to ensure drift stability. Finally, a piece of bolter equipment is used to install bolts in the drift walls to achieve drift stability.

The extraction of a portion of stope requires benching, explosive charging, blasting, extraction, and backfilling. Benching comprises drilling boreholes in the drilling drift using a production drill rig. Further, the benching boreholes in the stope are charged with explosives using an explosive charger. Subsequently, the portion of the stope charged using explosives is blasted, and the blasted ore is extracted using the LHD equipment. Finally, the empty portion of the stope is backfilled using a backfill truck and non-cemented rock fill.

Globally, the extraction of each stope is ascendant. The progress of ore extraction and the backfilling of the portions of the stope are conducted in the opposite direction when compared with that of the extraction and drilling drifts. Figure 3.3 illustrates the sequencing of three stopes (S1, S2, and S3). The arrows indicate the advance direction of each mine working. The extraction order of the stopes is ascendant and follows the order of S1, S2, and S3. The extraction sequencing of stope S1 is the extraction of drift D1 and the benching of drift D2 as well as the extraction and backfilling of the stope portions S1.1, S1.2, S1.3, and S1.4

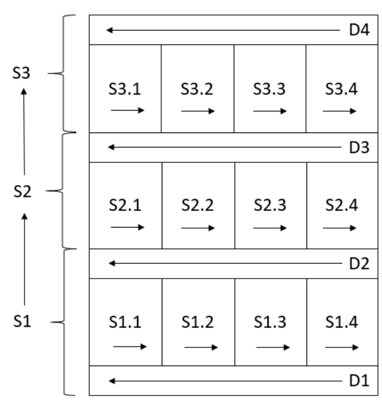

Figure 3.3: Mine sequencing in a B&F mining method

3.4.2 Optimization model

We propose an optimization model based on MILP to generate short-term production schedules in B&F mines. This model schedules activities that result in profit (income minus costs) and that demand resources (e.g., effective time, which is the time interval in which the equipment is performing an productive task) for their completion. The activities that result in a profit lower than zero are also included because they must be completed to access activities resulting in a profit of greater than zero. The optimization model maximizes the NPV over the planning horizon, subject to an activity's precedence and resource constraints.

The solution of the optimization model can be observed as a Gantt chart, where the fraction of progress of each activity is specified in each period of the planning horizon. This optimization model is embedded in a software called UDESS (Morales et al, 2015), which provides utilities for generating, resolving, and analyzing the general scheduling MILP optimization problems. The software is implemented in Python (Van Rossum and Drake Jr, 1995). UDESS can be used through scripts or a graphical user interface.

We present the sets, parameters, and variables of the optimization model in Tables 3.6, 3.7, and 3.8, respectively.

Symbol	Description
$\mathcal{A}$	Set of activities.
$\mathcal{Q}$	Set of stope-type activities.
$\mathcal{P}_a$	Set of activities precedence of activity a.
$\mathcal{T}$	Set of periods, $\mathcal{T} = \{1, \ldots, T\}$.
$\mathcal{E}$	Set of equipment fleet types.
$\mathcal{S}$	Set of mine sectors.
$\mathcal{H}_a^1$	Set containing the stope activity a and its corresponding production drift activity.
$\mathcal{H}_a^2$	Set containing the stope activity a and its corresponding drilling drift activity.
$\mathcal{H}_a^3$	Set containing the stope activity a and its corresponding upper stope activity, if it exists.

Table 3.6: Sets of the optimization problem

Symbol	Description
α	Discount rate per period.
C_a	Value of activity a.
N_{e}	Quantity of equipment type e.
T	Number of periods.
NT_t	Nominal time in period t.
T_a	Time to perform the activity a.
$T_{a,\mathrm{e}}$	Time to perform the activity a with mining equipment of type e. If activity a does not require equipment e, $T_{a,\mathrm{e}}$ is equal to zero.
$UT_{\mathrm{e},t}$	Utilization of equipment e at period t.
$UT_{\mathrm{e},t,s}$	Utilization of equipment e at period t in mine sector s. If activity a does not require equipment e or activity a does not belong to sector s, $T_{a,\mathrm{e},s}$ is equal to zero.

Table 3.7: Parameters of the optimization problem

Activity modeling

Here, we describe activity modeling in UDESS to generate a short-term production schedule for a B&F mine. First, we describe the types of workings in a B&F mine and subsequently explain the slice discretization process of these workings.

Symbol	Description
$s_{a,t} \in \{0,1\}$	If activity a has started in period t or before; otherwise, 0.
$\mathrm{e}_{a,t} \in \{0,1\}$	If the activity a is not finished at the beginning of the period t; otherwise, 0.
$x_{a,t} \in [0,1]$	The fraction of progress made by the activity a in period t.

Table 3.8: Variables of the optimization problem

The B&F mining method has two types of mine workings, i.e., drifts and stopes. Each stope has two drifts, i.e., the production drift (below the stope) and the drilling drift (above the stope). Before beginning a stope activity, the production and drilling drifts should be completely developed. To completely develop a mine working, different activities should be conducted. Thus, to completely develop a drift, the following activities must be sequentially performed: drilling, explosive charging, blasting, hauling, and backfilling. Similarly, to completely develop a stope, the following tasks must be sequentially performed: benching, explosive charging, blasting, hauling, and backfilling.

The slice discretization process is performed to reflect the actual progress of the exploitation of a B&F mine. This process involves the discretization of both types of workings (drift and stope) in equal length slices so that each slice represents one activity in the UDESS model. Figure 4 represents the slice discretization process of two stopes (in gray) and three stopes (in white), including the precedence of the activities. In this figure, instead of considering drift and stope activities of length Δ, we work with several activities of length δ.

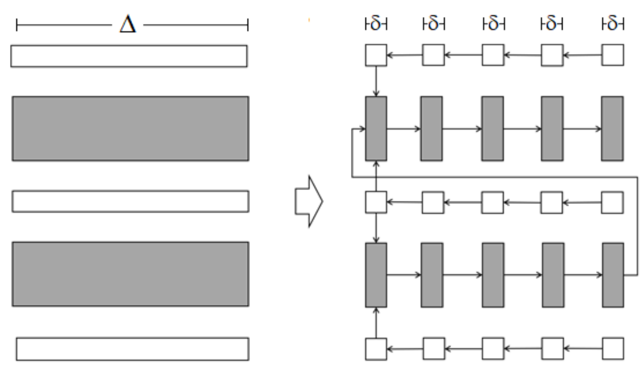

Figure 3.4: The slice discretization process of B&F three drifts (in white) and two stopes (in grey) in UDESS

Objective function

The objective function of the optimization problem is presented in (3.1).

$$\max \sum_{t\in\mathcal{T}}\sum_{a\in\mathcal{A}} \frac{1}{(1+\alpha)^t} \cdot C_a \cdot x_{a,t} \tag{3.1}$$

Constraints

In this section, we explain the constraints of the optimization problem.

$$
\begin{aligned}
s_{a,t+1} &\geq s_{a,t} && \forall a \in \mathcal{A}, \forall t \in \mathcal{T} \setminus \{T\} && (3.2)\\
\mathrm{e}_{a,t} &\geq \mathrm{e}_{a,t+1} && \forall a \in \mathcal{A}, \forall t \in \mathcal{T} && (3.3)\\
x_{a,t} &\leq s_{a,t} && \forall a \in \mathcal{A}, \forall t \in \mathcal{T} && (3.4)\\
\sum_{t \in \mathcal{T}} x_{a,t} &\leq 1 && \forall a \in \mathcal{A}, \forall t \in \mathcal{T} && (3.5)\\
1 - \mathrm{e}_{a,t+1} &\leq \sum_{t'=1}^{t} x_{a,t'} && \forall a \in \mathcal{A}, \forall t \in \mathcal{T} && (3.6)\\
s_{a,t} &\leq 1 - \mathrm{e}_{a',t+1} && \forall a \in \mathcal{A}, \forall a' \in \mathcal{P}_a, \forall t \in \mathcal{T} && (3.7)\\
0 \leq x_{a,t} &\leq 1 && \forall a \in \mathcal{A}, \forall t \in \mathcal{T} && (3.8)\\
\mathrm{e}_{a,1} &\geq 1 && \forall a \in \mathcal{A} && (3.9)\\
\mathrm{e}_{a,t} &\in \{0,1\} && \forall a \in \mathcal{A}, \forall t \in \mathcal{T} \cup \{T+1\} && (3.10)\\
s_{a,t} &\in \{0,1\} && \forall a \in \mathcal{A}, \forall t \in \mathcal{T} && (3.11)\\
\sum_{a' \in \mathcal{H}_a^{\mathrm{i}}} T_{a'} \cdot x_{a',t} &\leq NT_t && \forall t \in \mathcal{T}, \forall a \in \mathcal{Q}, \forall \mathrm{i} \in \{1,2,3\} && (3.12)\\
\sum_{a \in \mathcal{A}} T_{a,\mathrm{e}} \cdot x_{a,t} &\leq NT_t \cdot UT_{\mathrm{e},t} \cdot N_{\mathrm{e}} && \forall t \in \mathcal{T}, \forall \mathrm{e} \in \mathcal{E} && (3.13)\\
\sum_{a \in \mathcal{A}} T_{a,\mathrm{e},s} \cdot x_{a,t} &\leq NT_t \cdot UT_{\mathrm{e},t,s} \cdot N_{\mathrm{e},s} && \forall t \in \mathcal{T}, \forall \mathrm{e} \in \mathcal{E}, \forall s \in \mathcal{S} && (3.14)
\end{aligned}
$$

Constraints (3.2) and (3.3) define the progress of the variables $s_{a,t}$ and $\mathrm{e}_{a,t}$ over time. Constraint (3.4) prevents that an activity a from progressing if it has not started. Constraint (3.5) imposes that the maximum fraction of progress over the scheduling horizon of an activity a is less than or equal to 1. Constraint (3.6) sets activity a as finished when has completed its progress. The activity has been fully completed in period t when the sum of the progress of that activity from period 1 to t is equal to 1.

Constraint (3.7) imposes that an activity a can only start when all its precedence activities $j \in \mathcal{P}_a$ are completed. This constraint models the logical order in which the drift and stopes activities are developed. There are five types of activity precedence constraints (Figure 3.5) Type 1 precedence restricts the sequential advance between the portions of drifts, whereas Type 2 precedence restricts the sequential advance between the portions of stopes. Type 3 precedence ensures that the production drift of the stope must be entirely developed before the operation of the stope itself is initiated. Type 4 precedence ensures that the drilling drift of the stope must be entirely developed before the operation of the stope itself is initiated. Finally, Type 5 precedence ensures that the operation of an upper stope cannot be initiated before the lower stope (if any) is finished.

Constraint (3.8) sets the range of variables $x_{a,t}$; constraint (3.9) sets the activity a in period $t = 1$ as unfinished. Constraints (3.10) and (3.11) set the range of variables $\mathrm{e}_{a,t}$ and $s_{a,t}$,

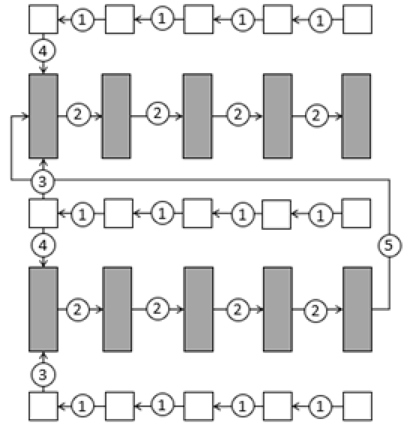

Figure 3.5: Mine precedence between drift activities (in white) and stope activities (in gray) in a B&F mine.

respectively. Constraint (3.12) ensures that the nominal time between different neighboring activities is not exceeded in each period. Constraint (3.13) requires that the sum of the effective time of the type of the mining equipment fleet e on all the activities performed in period t must be less than or equal to the maximum effective time during that period. Finally, constraint (3.14) ensures that each type of mining equipment fleet can operate only in specific mine sectors. This constraint requires that in each mine sector s, the sum of the effective time of each type of mining equipment fleet e on all the activities performed in period t must be less than or equal to the maximum effective time during that period.

3.4.3 Simulation model

The integration of a simulation model with an optimization model can be a considerably challenging task. Although some DES simulation commercial software (ARENA® and ProModel®) have been applied to study mine operations (Torkamani and Askari-Nassab, 2015; Hashemi and Sattarvand, 2015; Ataeepour and Baafi, 1999), they exhibit limited capabilities to efficiently and simply interact with an optimization model. Therefore, we have developed a simulation software called Delphos Simulator (DSIM).

DSIM is a DES software used to simulate mine operations, including material handling systems in open-pit mines and production and preparation in underground mines. It is coded via Python using a specific simulation library called SimPy. DSIM implements (a) a set of functions that allow easy definition of a layout and the modeling of equipment movements, (b) several pieces of equipment that can be used with or without extension to model considerably complex situations, and (c) reports details specially to mine operations (cycle times and production).

Specifically, we use the B&F simulation model based on the model described in Pérez et al (2017). The simulation model implements all the required tasks for the development of a B&F mine. The inputs of the simulation model include (i) the activities to be performed, (ii) the mining equipment, (iii) the mine layout, and (iv) the list of priority tasks. For the simulation, it is assumed that one day comprises three operating shifts, a shift change lasting one hour, and one hour for meals per shift. In the simulation model, the pieces of equipment vary based on their operational states, which can be given as follows: program delays (time interval in which the equipment is not in operation because the operators are changing shift or on meal time), operational losses (the equipment waiting time because other equipment travel through the same drift), backup (time interval in which the equipment is available for operation but is not in operation either (i) it has pending tasks however is unable to complete them because of other tasks needs to be completed before or (ii) there are no pending tasks to be completed for this equipment), non-available time (time interval in which the equipment is not available owing to failure or maintenance), and effective time (time interval in which the equipment is performing productive tasks).

Here, we describe the operation details of the simulation model. The mine layout contains two types of elements, i.e., transport routes and fronts. Transport routes are the roads that are used by the pieces of equipment to reach different fronts. A front is a physical location at which the mining equipment conducts its activities. A front can be either a drift or stope. The type of front determines the performed activities and the type of equipment assigned to the front. Each front has an attribute called "current activity," which indicates the activity that should be performed next. Further, each piece of equipment has a list of priority activities that should be conducted as input. The order of these activities is based on the start and end periods obtained from a given short-term mine production schedule.

At the beginning of the simulation, the drift- and stope-type fronts begin with the states of "drilling" and "benching," respectively. Throughout the simulation, the pieces of equipment travel to different fronts to execute the activities in the order based on the list of priority activities. The activities are performed by respecting the activity precedence given using the B&F method, and only one item of equipment is allowed to perform an activity at any given time. When a piece of mining equipment completes its assigned activity, the activity transitions from the current activity of the corresponding front to the next activity based on the precedence of activities.

The general flowchart of the B&F simulation model, which involves development, production, and backfill of stopes, is presented in Figure 3.6. It is important to mention that the simulation model does not consider the construction of the main ramp to access the ore body because this study focuses on short-term scheduling.

3.4.4 Optimization and simulation feedback

In this section, we explain the interaction of optimization with simulation and vice versa in the B&F application.

Optimization feedback to simulation

A short-term production schedule is generated at the beginning of each iteration by solving an optimization problem. For each type of activity (drift and stope activities), a list of priority tasks is created based on this schedule. Each of these lists is provided as input to the simulation model. Thus, the simulation model follows a short-term production schedule.

To create the list of drift-type activities, each drift activity is sorted in an ascending order

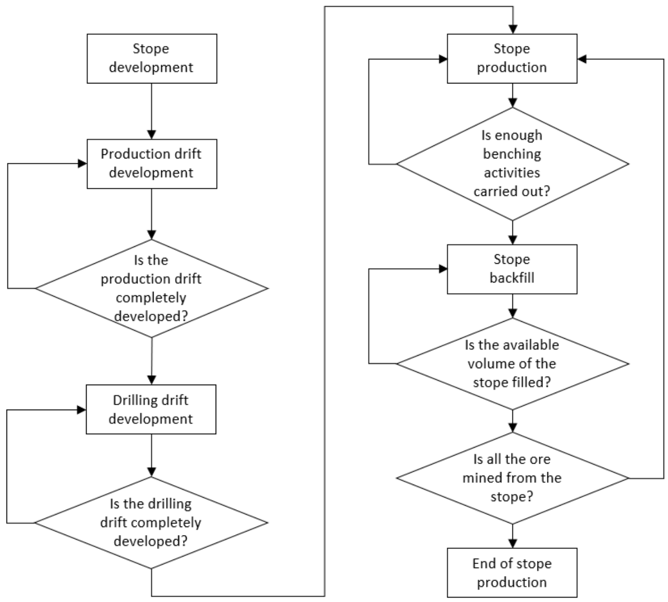

Figure 3.6: General flowchart of the B&F simulation model

according to the following criteria: (1) minimum start period of the first portion of the drift; (2) minimum completion period of the final portion of the drift; (3) minimum start period of the first portion of the stope associated with the drift; and (4) minimum completion period of the first portion of the stope associated with the drift. In the B&F method, a drift can be located between two stopes (the upper and lower stopes) or on one stope (the lower stope). The stope associated with the drift corresponds to the upper stope, if it exists. Otherwise, the associated stope corresponds to the lower stope.

If there is a tie with respect to a particular criterion, it is broken by applying the immediately following criterion and so on. For example, while sorting the drift types of activities, if two or more drift activities have equal minimum start periods with respect to the first portion of the drift (first criterion), the drift activities are sorted using the minimum completion period of the final portion of the drift (second criterion). If these activities have equal minimum completion periods with respect to the final portion of the drift, the criterion used to sort the drifts is the minimum start period of the first portion of the stope associated with the drift (third criterion) and so on.

Similarly, to create a list of the stope-type activities, the stope activities are sorted according to the following criteria, which are applied sequentially until there is no tie: (1) minimum start period of the first portion of the stope; (2) minimum completion period of the first portion of the stope; (3) minimum start period of the final portion of the stope; and (4) minimum completion period of the final portion of the stope.

Simulation feedback to optimization

A short-term production schedule is generated at the beginning of each iteration by solving an optimization problem. Subsequently, this schedule is simulated using the DES model to obtain the average utilization for each piece of equipment for each period over all the replications $UT_{\text{e},t}$ at the end of the simulation process. The mean effective time over the replications for each period is subsequently calculated in (3.15).

$$SET_{\text{e},t} = \frac{1}{|R|} \sum_{r \in R} SET^{r}_{\text{e},t} \qquad \forall \text{e} \in \mathcal{E}, \forall t \in \mathcal{T} \tag{3.15}$$

Where $SET_{\text{e},t}$ is the mean of simulated effective time of the equipment e in period t, and $SET^{r}_{\text{e},t}$ is the mean of effective time of the equipment e in period t in the replication r.

Before feeding this data to the next optimization problem during the next iteration, precise adjustment of the mean effective time is necessary for each equipment. Thus, the total sum of the simulated effective time SET_{e} must be equal to the sum of the planned effective time PET_{e} used in the optimization problem to ensure the feasibility of the optimization problem.

Therefore, for all the periods t, the quantity $SET_{\text{e},t}$ is multiplied with $\frac{PET_{\text{e}}}{SET_{\text{e}}}$ to obtain the modified simulated effective time $S\hat{E}T^{\text{e}}_{t}$. Refer to (3.16).

$$S\hat{E}T_{\text{e},t} = \left(\frac{PET_{\text{e}}}{SET_{\text{e}}}\right) \cdot SET_{\text{e},t} \quad \forall \text{e} \in \mathcal{E}, \forall t \in \mathcal{T} \tag{3.16}$$

Thus, the sum over all periods of the modified simulated effective time $S\hat{E}T_{\text{e},t}$ is equal to PET_{e}. Refer to (3.17).

$$\sum_{t \in T} S\hat{E}T_{\text{e},t} = PET_{\text{e}} \quad \forall \text{e} \in \mathcal{E} \tag{3.17}$$

Finally, the average utilization of each piece of equipment for each period over all the replications $UT_{\text{e},t}$ is calculated as the ratio of the modified simulated effective time $S\hat{E}T_{\text{e},t}$ and the total time per period in hours. As each time period comprises a month (30 days), each time period has $24 \cdot 30$ hours in total. Refer to (3.18).

$$UT_{\text{e},t} = \frac{S\hat{E}T_{\text{e},t}}{24 \cdot 30} \quad \forall \text{e} \in \mathcal{E}, \forall t \in \mathcal{T} \tag{3.18}$$

In the next iteration, the average utilization of each equipment for each period $UT_{\text{e},t}$ is fed into constraint (3.13) to generate a new short-term mine production schedule. For an equipment working in specific mine sector, the procedure to calculate the average utilization per equipment for each period per mine sector $UT_{\text{e},t,s}$ is similar to the procedure used to calculate $UT_{\text{e},t}$. This quantity is fed into constraint (3.14) to generate a new short-term mine production schedule.

3.5 Case study

A case study of a real-world data of a B&F mine is considered for understanding the application of the simulation-optimization framework. The mine is comprised of two exploitation zones (East and West), and each contain three levels. Figure 3.7(a) shows the isometric view of the mine, whereas Figure 3.7(b) depicts the plan view. Figure 3.8(a) illustrates the North-South side view of the mine whereas the Figure 3.8(b) represents the East-West side view.

Figure 3.7 and 3.8 illustrate the mine workings in the mine: stopes (in brown), crosscuts (in green), main drifts (in yellow), access ramps (in red), and the access drifts (in blue). Crosscuts connect the different stopes with the main drifts. For their part, main drifts connect the different crosscuts with the corresponding access ramp. Finally, access drifts connect the West and East sectors with other sectors of the mine.

The ore deposit is of the epithermal type of gold and silver, comprising of veins with an average width of 2.1 m. Table 3.9 summarizes the number of activities by activity type (drift, stope and backfill). The total number of activities to be scheduled differs from the total number of tasks, because the slice discretization process explained in Section 3.4.2 is conducted using a slice discretization length of 9.0 m for each mining task. This length corresponds to the length of the portion of the ore extracted from the stope subsequent to which the backfill of the stopes in the real mining operation begins. Table 3.10 shows the mining equipment involved in the B&F mine. In each zone, one LHD and production drill rig work exclusively; hence there are four pieces of equipment in total. Table 3.11 describes the relation between the tasks and the mining equipment used to perform the mining activities.

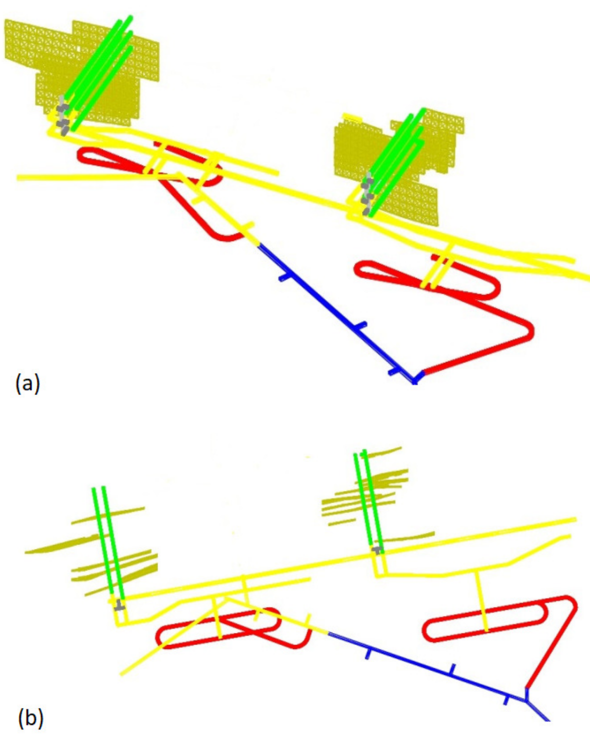

Figure 3.7: Isometric (a) and plan view (b) of the B&F mine case study

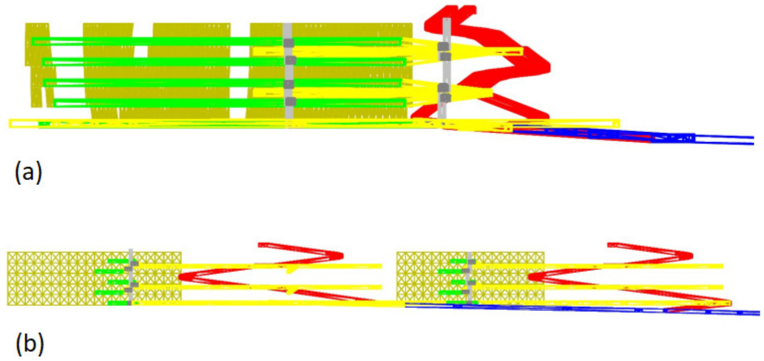

Figure 3.8: North-South side view (a) and West-East side view (b) of the B&F mine case study

Activity type	Number of mine workings	Total activities to be scheduled	Total length [m]	Material [kt]
Drift	89	568	3430	226.57
Stope	67	428	4568	192.26
Backfill	67	428	4568	362.41

Table 3.9: Summary of the B&F mine case study activity type

Equipment	Number	Availability [%]
Face drill rig	1	68.2
Explosives charger	1	78.5
LHD	2	65.1
Scaler	1	75.8
Shotcrete	1	79.4
Bolter	1	82.4
Production drill rig	2	69.8
Backfill truck	1	79.0

Table 3.10: Mining equipment used in the B&F mine case study

The mining equipment distribution parameters used in the simulation are presented in Table 3.12. In this table, $U(a, b)$ represents a uniform distribution, $W(k)$ represents a 1-parameter Weibull distribution, and $N(\mu, \sigma)$ represents a normal distribution. The types of probability distributions used are obtained based on the best fit obtained from the historical data. For further details of the probability density distribution, please consult Oliphant (1995).

All the computational experiments presented in this study were performed on a 2.60 GHz Intel® Xeon® CPU with 256 GB RAM, operating on the Windows 8® operating system. The optimization model is solved using Gurobi (Gurobi Optimization, 2019). The proposed framework considers a stop criterion for the iteration procedure when the value of one adherence index is less than or equal to a particular critical value. In the B&F mine case study,

	Tasks	
Mining Equipment	Drift	Stope
Face drill rig	Drilling	
Explosives charger	Charging	Charging
Explosives	Blasting	Blasting
LHD	Mucking	Hauling
Scaler	Wedging	
Shotcrete	Shotcreting	Shotcreting
Bolter	Bolting	
Production drill rig		Drilling
Backfill Truck		Backfilling

Table 3.11: Relation between the mining equipment and tasks in the B&F mine case study

Parameter	Probability density distribution
Time between failures [min]	$\frac{1}{4} \cdot 46 \cdot W(1.7) \cdot 1440$
Time to repair [min]	$\frac{1}{4} \cdot 1.35 \cdot \ln(1.1) \cdot 1440$
LHD maintenance [min]	$120 + U(-20; 60)$
Jumbo maintenance time [min]	$180 + U(-20; 60)$
Simba maintenance time [min]	$180 + U(-20; 60)$
LHD bucket load [t]	$5 + U(-1.5 + 1.5)$
Drift length [m]	$3 + U(0; 0.5)$
Drift drilling time [min]	$DriftLength \cdot \frac{0.4}{2 \cdot N(1;0.02)} + 15$
Stop Benching time [min]	$\frac{96}{N(1;0.02)} + N(30; 5) \cdot 12$
Load/Dump LHD time [s]	$15 + U(-2; 12)$
Explosive charger time [min]	$30 + U(0; 60)$
Backfill time [min]	$20 + U(-5; 5)$
Wedging time [min]	$30 + U(0; 30)$
Shotcreting time [min]	$60 + U(0; 30)$
Bolting time [min]	$90 + U(0; 30)$

Table 3.12: Mining equipment's probability density distributions

the iteration procedure stops when the value of the material adherence index in a given iteration is less or equal to 5%. We select this value because additional iterations to improve it was not considered to be worth the computational time; however, a different value could be set if necessary. The optimization model considers periods of one month, with a scheduling horizon of approximately a year and a half. The mine schedule assumes that the mine workings required to access the production and drilling drifts have been already developed. The initial utilization value for the mine equipment corresponds to the availability reported in Table 3.10. The annual discount rate observed with respect to the objective function of the optimization model is 10%.

We analyze the cumulative mean of the steady monthly production rates for the drift, stope, and backfill over replications to determine the number of simulation replications. The number of replications is selected such that the cumulative average of the production rates becomes stabilized. Based on this criterion, the number of simulation replications is concluded to be 100. We did not consider a warm-up period at the beginning of the simulation because the conducted simulation considers a mine from the beginning of its production that attains a steady production rate after some months.

To validate the simulation model, we use the confidence interval procedure. The drift, stope, and backfill steady monthly production rates are selected as the response variables of the simulation model. We run a total of 100 replications to obtain the sample mean and standard deviation of the model response variables from the simulation replications. A student's t-distribution of the response variables is conducted (because the standard deviation of the response variables is unknown), and a confidence level of 95% is assumed for calculating the confidence intervals. Based on the short-term mine production model generated from the optimization model, the steady annual production rates for a drift, stope, and backfill are used to verify whether these values are within the corresponding intervals. It is then verified that the response variables are within the confidence intervals, verifying the validity of the model for the considered response variables.

3.6 Results and discussion

In this section, the results and discussion are presented based on the application of the simulation-optimization framework to the B&F case study. The procedure stops when the material adherence index is 4.8%, which is lower than the specified critical value of 5%. With this criterion, we performed a total of five iterations. In this way, using the optimization problem we have generated a total of five schedules. Each schedule requires an average of 23 min to be resolved. The simulation of each schedules, containing 100 replications, requires approximately 5 h for completion in average.

Figures 3.9 and 3.10 show the short-term mine production schedule obtained from the resolution of the optimization model (a), and the average of the mine production schedule obtained from the simulation (b), for the first and fifth iterations, respectively.

In the first iteration (Figure 3.9), discrepancies can be observed between the schedule obtained from the optimization model and the average of the simulated schedule with respect to the level of movement of material in the early periods. This discrepancy affects the number of periods required to complete the extraction at the mine. The schedule needs 14 periods, and the average simulated schedule needs 16 periods. This result is not desired but it is expected because the optimization model alone fails to consider the operational uncertainty of the mine's operation.

However, in the fifth iteration (Figure 3.10), the discrepancies among the level of movement of material in the early periods with respect to the schedule obtained from the optimization model and the average of the simulated schedule are observed to be minor in comparison with those obtained from the first iteration. In the fifth iteration, the number of periods necessary to complete the extraction at the mine with respect to the schedule and the average of the simulated schedule is 17. This is expected because the effects of operational uncertainty within the optimization model can be considered by integrating the simulation. Thus, it is

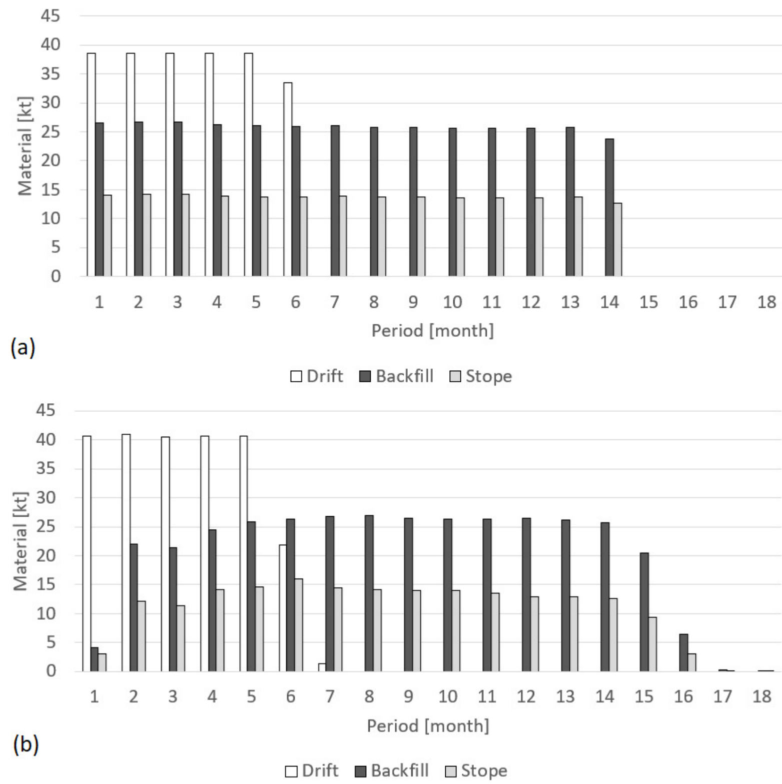

Figure 3.9: Schedule obtained from the optimization model in the 1^{st} iteration (a) and average of the schedule obtained from its corresponding simulation (b)

possible to generate a schedule with smaller discrepancies with respect to the movement of material when compared with the schedule obtained in the first iteration.

In the following paragraphs, we report each mine schedule's adherence indices generated over the iterations of the B&F case study to assess the level of adherence between the schedule generated by the optimization problem and the corresponding simulation.

In Table 3.13, we report the material adherence indices of the mine schedules generated over iterations. In general, the material adherence index do not always decrease over iterations for every type of material. However, when considering all types of activities, the material adherence index always decreased with each iteration. This result implies that the levels of material movement in case of the schedule and the average of the simulated schedule become increasingly similar with iterations.

In Figures 3.11 and 3.12, we report the material adherence curve for each mine schedule generated over the iterations considering different material types (drift, stope, and backfill). In the first two iterations (Figures 3.11(a) and 3.11(b)), the material adherence curves of all the materials were lower than those in the first period. This result indicates that the material movement in the simulation was late based on the mine schedule. In the subsequent periods, the adherence curve was approximately one. This result indicates that the total material movement in the simulation was synchronized with the mine schedule. In the three subsequent iterations (Figures 3.11(c), 3.12(a), and 3.12(b)), the material adherence curve of all the materials was greater than that in the first period. This result indicates that

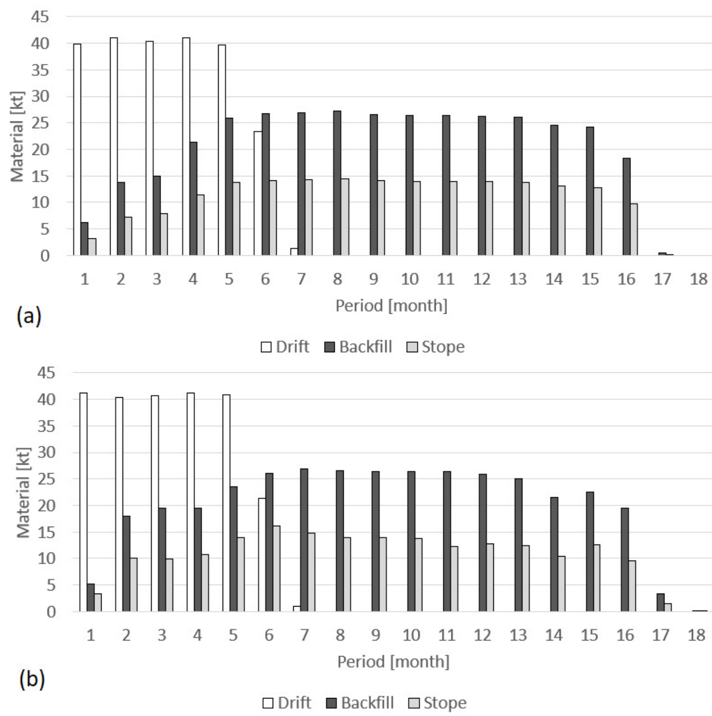

Figure 3.10: Schedule obtained from the optimization model in the 5^{th} iteration (a) and average of the schedule obtained from its corresponding simulation (b)

	Iteration				
Material type	1	2	3	4	5
Drift	10.18	3.83	3.00	3.20	2.63
Stope	18.13	11.29	9.46	9.60	8.80
Backfill	19.00	9.76	6.52	6.00	7.24
Drift & Stope	9.92	6.13	4.98	5.73	4.02
Drift & Stope & Backfill	13.11	7.11	5.48	5.26	4.86

Table 3.13: A mine schedule's material adherence index (in percentage) over the iterations, considering different types of materials

the material movement in the simulation was ahead of the mine schedule. However, in the subsequent periods, the material adherence curve of all the materials was near to that. This result indicates that the material movement in the simulation was synchronized with the mine schedule.

In Tables 3.14 and 3.15, we report the start adherence index and completion adherence index of the mine schedule generated over iterations, respectively. As can be observed, the start and completion adherence indices do not always increase over iterations for every type of material. However, when considering all types of activities, the start adherence index and completion adherence index with respect to a given schedule are higher than those of the immediately previous iteration. This result indicates that the number of simulated activities that start/end in a period less or equal to the period given by the schedule increases in each

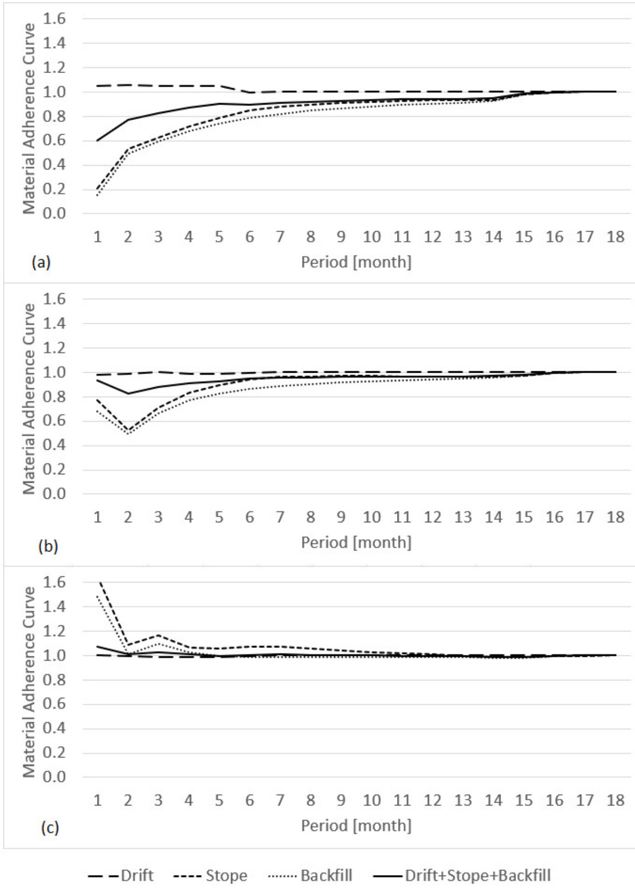

Figure 3.11: Material adherence curve for the 1^{st} (a), 2^{nd} (b) and 3^{rd} mine schedules

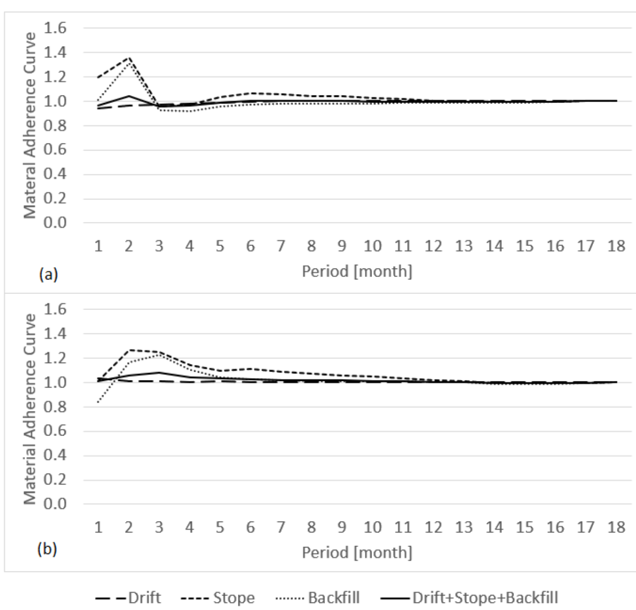

Figure 3.12: Material adherence curve for the 4^{th} (a) and 5^{th} (b) mine schedules

iteration. In other words, the number of simulated activities that start / end in a period greater than the period defined by the schedule decreases in each iteration.

	Iteration				
Activity type	1	2	3	4	5
Drift	99.2	96.6	98.2	96.1	97.3
Stope	47.5	58.9	75.2	77.2	79.2
All	77.0	80.4	88.3	88.0	89.5

Table 3.14: Summary of the start period adherence index (in percentage)

	Iteration				
Activity type	1	2	3	4	5
Drift	85.9	73.5	74.5	72.9	82.1
Stope	26.3	45.1	63.7	66.3	69.9
All	60.3	61.3	69.9	70.1	76.9

Table 3.15: Summary of the completion period adherence index (in percentage)

In Table 3.16 we report the mean lateness, mean tardiness and mean earliness of the schedules generated over iterations.

The mean lateness of the schedule is negative and decreases over iterations. This result implies that the difference between the completion period of simulated activities and the completion period of scheduled activities is negative and increases over the iterations.

Generally, the tardiness of the schedule decreases with each iteration. This result implies that when we consider the activities which are completed after the expected period given in the schedule, the difference between the completion period of simulated activities and the completion period of scheduled activities over the iterations decreases over the iterations.

Generally, the mean earliness of the schedule increases with each iteration. This result implies that when we consider the activities which are completed before the expected period given the schedule, the difference between the completion period of scheduled activities and the completion period of simulated activities over the iterations increases over the iterations.

The results obtained of the adherence indices between of the schedule and its corresponding simulation over the iterations can be summarized as follows: (a) material adherence index decreased from 13.11% in the first iteration to 4.8% in the final iteration, (b) shape of the material adherence curve showed a trend to a horizontal line of unit value over iterations, (c) start adherence index increased from 77.0% in the first iteration to 89.5% in the final iteration, (d) completion adherence index increased from 60.3% in the first iteration to 76.9% in the final iteration, (e) mean lateness varied from -0.002 months in the first iteration to -1.128 months in the final iteration, (f) mean tardiness varied from 0.479 months in the first iteration to 0.138 months in the final iteration, and (g) mean lateness varied from 0.480 months in the first iteration to 1.000 months in the final iteration.

	Iteration				
Index	1	2	3	4	5
Mean lateness	-0.002	0.127	-0.534	-0.603	-0.863
Mean tardiness	0.479	0.283	0.176	0.199	0.138
Mean earliness	0.480	0.701	0.710	0.802	1.000

Table 3.16: Mean lateness, tardiness and earliness (in months) for all the activities over iterations

The results presented in the previous paragraphs and presented in Tables 3.13 to 3.16 and Figures 3.11 and 3.12 demonstrate, in general, that the adherence indices with respect to a given schedule and its corresponding simulation are higher than those of the immediately previous iteration. These results demonstrate that in each iteration, the optimization problem uses continuous improvement in the estimation of the utilization KPIs of each mining equipment provided by the simulations. These estimations imply a better quantification of the maintenance equipment times, equipment failures, travel time between locations where the equipment is used to perform mining activities, and equipment backup times (time during which the equipment is available for operation even though the equipment is not operative for the specific mine operation condition). Furthermore, these utilization KPIs estimations consider the real mine operation behavior that is difficult to consider in an optimization problem, such as the dispatching rules for transporting equipment to mine faces and the specific rules of mine operations.

Finally, we compared the NPV and the material adherence index with respect to all the short-term schedules generated over iterations in Table 3.17.

	Iteration				
Index	1	2	3	4	5
NPV [MUSD]	721.7	718.1	716.2	716.2	715.9
% Difference NPV c/r 1^{st} iteration	0.0%	-0.5%	-0.8%	-0.8%	-0.8%
Material adherence index [%]	13.11	7.11	5.48	5.26	4.86

Table 3.17: Comparison between the NPV and the material adherence index short-term schedules generated over iterations

Based on the results in Table 3.17, we can state that the improvements in the adherence of mine schedules over iterations are not obtained at the expense of NPV. The results denote that the NPV remained constant, whereas the material adherence index of the mine schedules decreased (Table 3.17). In other words, the proposed framework can effectively generate mine schedules over iterations and simultaneously maintain the NPV.

3.7 Conclusion and future work

The deviation between mine schedules and the mine operation results are crucial problems that affect the mining industry. Therefore, the mine engineers should generate a mine production schedule that can be reproduced in reality. Hence, they should develop mine production schedules that exhibit high adherence.

In this study, we proposed a generic framework to increase adherence to a short-term mine production schedule by combining optimization and simulation using an iterative approach. This framework comprises the following steps. First, an initial mine schedule is generated based on the resolution of a mixed-integer linear optimization problem. Second, this schedule is simulated using a DES model. Third, a new short-term mine schedule is created using the optimization model by considering the new utilization KPIs of each equipment, obtained from the simulations performed in the previous step, as inputs for the mine operation. Finally, iterations of the second step are performed. In each iteration, adherence to each mine schedule is evaluated with respect to the corresponding simulations by evaluating several adherence indices.

The proposed framework was applied to a real-scale B&F mine. The mine planning horizon was more than a year and a half, and each period lasted for one month. A total of five iterations were performed.

We measure the discrepancies among the level of movement of material with respect to the schedule obtained from the optimization model and the average of the simulated schedule using the mine schedule material's adherence index. The values of this index decreased with the iterations, from 13.1% in the first iteration to 4.8% in the last iteration. This improvement is explained because the effects of the operational uncertainty within the optimization model can be considered by integrating the simulation.

The outcomes of the work presented in this study demonstrate that the proposed framework improved the mine schedule adherence indices over iterations and simultaneously maintained the NPV of the mine schedule. The results demonstrate that the simulation provides a better understanding of the impacts of uncertainty in short-term mine production schedules.

As future research, the proposed framework will be applied to massive and selective underground mining methods as well as open-pit mines.

Acknowledgements

This article is part of F. Manríquez's PhD thesis. This article was partially funded by the CONICYT/PIA Project AFB180004 and by the CONICYT PFCHA/DOCTORADO BECAS CHILE/2019 - 21190201.

Chapter 4

Short-term open-pit mine production scheduling with hierarchical objectives

Abstract

Short-term mine scheduling in open-pit mines consists of meeting the objectives defined by the long-term mine production schedule. Short-term scheduling in open-pit mines has multiple hierarchical objectives to optimize. This work proposes an optimization methodology to generate a short-term open-pit mine production schedule optimizing multiple hierarchical objectives. For the generation of mine schedules, we propose an optimization model based on mixed-integer linear programming. This model considers the usual restrictions of mine sequencing and also takes into account both time and cost of movement between phases of each shovel. In order to optimize the multiple hierarchical short-term objectives, we apply the hierarchical and weighted sum methods in the proposed optimization model. We verify this methodology in a real open-pit mine case study. The results show that both methods generate short-term mine schedules optimizing the different short-term objectives.

4.1 Introduction

Short-term mine scheduling in open-pit mines consists of meeting the objectives defined by the long-term mine production schedule. Short-term scheduling in open-pit mines has multiple objectives to optimize. In some mine operations, these objectives are ranked in a descending order which are: (i) the minimization of the deviation between the ore sent to the ore processing plant and the ore processing capacity of the plant per period, (ii) the minimization of the deviation between the metal fines sent to the plant and the fines expected by the ore processing plant, and (iii) the overall minimization of the movement time/cost of the shovel fleet. This work proposes an optimization methodology to generate a short-term open-pit mine production schedule optimizing multiple hierarchical objectives. For the generation of mine schedules, we propose an optimization model based on mixed-integer linear programming (MILP). This model considers the usual restrictions of mine sequencing and also takes into account both time and cost of movement between phases of each shovel. In order to optimize the multiple hierarchical short-term objectives, we apply the hierarchical and weighted sum methods (Grodzevich and Romanko, 2006; Nehring et al, 2018) in the proposed optimization model. We verify this methodology in a real open-pit mine case study. The remainder of this paper is organized as follows. Section 4.2 presents the proposed optimization model and explains the procedure to perform various objective optimization using both hierarchical and weighted sum methods. In section 4.3, we describe the open-pit mine case study conducted to generate short-term production schedules optimizing multiple objectives, applying both hierarchical and weighted sum methods. Section 4.4 reports the results and discussion of the case study. Finally, section 4.5 presents the conclusions of the work.

4.2 Materials and methods

In this section, we present the optimization model and describe the methodology to generate short-term mine production schedules optimizing various objective optimizations using the hierarchical and weighted sum methods.

4.2.1 Optimization model

The optimization model proposed is a shovel allocation problem based on MILP. The objective of the model is to provide shovel allocations to phases and benches over a one year scheduling horizon. We present the sets & indexes, variables and parameters of the optimization model in Tables 4.1, 4.2, and 4.3, respectively.

Symbol	Description
P, p	Set and index for shovels.
F, f	Set and index for phases.
$B(f), b$	Set for benches of phase f and index for benches.
T, t	Set and index for periods.
R, r	Set and index for routes.
$R(f)$	Set of routes that contains phase f .
$FR(r)$	Set of routes which the first phase is equal to the last phase of route r.
FC	Set of consecutive phases (f, f')

Table 4.1: Optimisation model indices and sets

4.2.2 Multiple objective optimization methods

In this work, we optimize the short-term objectives in the following hierarchical order:

1. Minimize the maximum deviation between ore sent to the plant and plant capacity per period: $f_1 = \min D$. This objective seeks to use the plant capacity in all periods.
2. Minimize the maximum deviation between metal fines sent to the plant and the expected metal fines by the plant per period: $f_2 = \min G$. This objective seeks to feed the plant with the expected metal grade by the plant.

Symbol	Description
$x_{p,f,b,t} \in [0,1]$	Time percentage of the period $t \in T$ where shovel $p \in P$ is operative in bench $b \in B$ of the phase $f \in F$.
$\bar{x}_{p,f,b,t} \in \{0,1\}$	Equal to 1 if shovel $p \in P$ is allocated in bench $b \in B$ of the phase $f \in F$ in period $t \in T$, 0 otherwise.
$z_{f,b,t} \in \{0,1\}$	Equal to 1 if bench $b \in B$ of phase $f \in F$ is inactive at period $t \in T$, 0 otherwise.
$\bar{z}_{f,b,t} \in \{0,1\}$	Equal to 1 if bench $b \in B$ of phase $f \in F$ finishes its exploitation in period $t \in T$ or later, 0 otherwise.
$\bar{w}_{p,f,t} \in \{0,1\}$	Equal to 1 if shovel $p \in P$ is allocated on phase $f \in F$ in period $t \in T$, 0 otherwise.
$v_{p,r,t} \in \{0,1\}$	Equal to 1 if shovel p goes through route r in period t, zero otherwise.
$D \in \mathbb{R}^+ \cup \{0\}$	Maximum deviation between the processing plant capacity and the ore send to processing plant.
$\mathrm{d}_t^- \in \mathbb{R}^+ \cup \{0\}$	Positive deviation between the processing plant capacity and the ore send to processing plant.
$G \in \mathbb{R}^+ \cup \{0\}$	Maximum deviation between the metal fines sent to processing plant and the metal fines expected by the plant.
$g_t^+ \in \mathbb{R}^+ \cup \{0\}$	Negative deviation between the metal fines sent to processing plant and the metal fines expected by the plant.
$g_t^- \in \mathbb{R}^+ \cup \{0\}$	Positive deviation between the metal fines sent to processing plant and the metal fines expected by the plant.

Table 4.2: Optimization model's variables

3. Minimize overall shovel movement cost: $f_3 = \min \sum_{p \in P, r \in R, t \in T} CT_{p,r} \cdot v_{p,r,t}$. This objective seeks to minimize the movement cost of the fleet of shovels between sectors of the mine.

We apply the weighted sum method and the hierarchical method, described in the Section 2.2, to generate short-term mine production schedules optimizing multiple objectives.

The following problems need to be solved in the context of the weighted sum method:

- mMDP: Minimize the maximum deviation between ore sent to the processing plant and ore processing capacity per period.
- mMDF: Minimize the maximum deviation between metal fines sent to the processing plant and expected metal fines by the ore processing plant per period.
- mD: Minimize the overall shovel fleet movement cost between phases.
- WS(1,2,3): Minimize the weighted sum of the three short-term objectives considered in this work.

In the proposed optimization model, the sets of constraints that impose that the maximum deviation of ore processing capacity per period does not take another value than the obtained in the previously solved problem are 4.19 and 4.20. Analogously, the set of constraints that enforce that the maximum deviation of fines per period does not take another value than the

Symbol	Unit	Description
TT	[h]	Total time per period.
$TI_{p,r}$	[%]	Percentage over the nominal time of the period where shovel p needs to move between the phases of route r.
$AV_{p,t}$	[%]	Availability of shovel p in period t.
$CT_{p,r}$	[USD]	Movement cost of shovel p along the phases of route r.
$RM_{p,f,b}$	[t/h]	Maximum throughput of mined material by the shovel p, in the phase f, in the bench b.
$TM_{f,b}$	[t]	Total material to be mined in bench b in phase f.
$OF_{f,b}$	[%]	Percentage of ore material in the bench b of the phase f.
PC	$[t]$	Total capacity of the ore processing plant.
EG	[%]	Expected grade of the metal by the ore processing plant.
SG	$[t/m^3]$	Rock density.
BH	[m]	Bench height.
$AD_{f,b}$	[m]	Initial operative area of the bench b of the phase f where shovels can operate.
AS_p	$[m^2]$	Minimum operative area of shovel p.
$ML_{f,g}$	-	Maximum number of benches between consecutive phases f and g.
$mL_{f,g}$	-	Minimum number of benches between consecutive phases f and g.

Table 4.3: Optimization model time and cost parameters

obtained in the previously solved problem are 4.21 to 4.23.

The following problems need to be solved in the context of the hierarchical method:

- mMDP: Minimize the maximum deviation between ore sent to the ore processing plant and the ore processing capacity per period.
- mMDF(mMDP): Minimize the maximum deviation between metal fines sent to the ore processing plant and the expected metal fines by ore processing plant per period, subject to the minimum maximum deviation between ore sent to the processing plant and the ore processing capacity per period.
- mD(mMDF(mMDP)): Minimize overall shovel movement between phases subject to: (i) minimum maximum deviation between ore sent to the processing plant and the ore processing capacity per period, and (ii) the minimum maximum deviation between metal fines sent to the processing plant and the expected metal fines by the ore processing capacity per period.

4.2.3 Optimization model constraints

Here we describe the constraints of the proposed optimization model. The material extracted by the fleet of shovels of the bench b of the phase f along the planning horizon must be equal to the total material available (Equation 4.1). The total ore sent to the ore processing plant must be lower than the ore processing capacity (Equation 4.2). The shovel's operative time plus the shovel's movement time between the phases are lower or equal to the availability of the shovel (Equation 4.3). In order to assign operative time, a shovel p must be assigned to the bench b of the phase f (Equation 4.4). When a bench is finished, it cannot be assigned with any shovels (Equation 4.5). To assign a shovel p in a bench b of a phase f, that shovel must be allocated in that phase f (Equation 4.6). To assign operative time in a bench b of a phase f in a shovel p, it must be allocated in that phase f (Equation 4.7). Bench b of phase f is not finished in period t until all the material are extracted (Equation 4.8). If a bench b of a phase f is finished in a period t, it remains finished in period $t+1$. Constraints 4.10 controls the precedence between benches of the same phase. Thus, to a bench b of a certain phase f can be active, the upper bench must be finished. To assign operative time, the phase-bench must be active (Equation 4.11). Constraints 4.12 and 4.13 controls the precedence between benches of consecutive phases. Constraints 4.14 to 4.15 control the area available in the phase-bench in order to allocate shovels. Constraints 4.16 to 4.18 models the shovel movements between phases. At most just one route is performed by shovel p in period t (Equation 4.16). Constraint 4.17 links the binary variable $\bar{w}_{p,f,t}$ with the variable $v_{p,r,t}$. For each shovel p, the constraint 4.18 ensures that the route r of period t and the route s of period $t+1$ are coherent, that is, the last phase f of the r route and the first phase of the s route is the same. The set $FR(t)$ represents the set of all the routes consistent with the r route. That is, the set of routes whose first phase is the same as the first phase of the r route. Constraints 4.19 and 4.20 model the deviation between the ore sent to the ore processing plant and the ore processing capacity per period. Constraints 4.21 to 4.23 model the deviation between the metal fines sent to ore processing capacity and the expected fines by the ore processing plant per period.

$$\sum_{p\in P, f\in F, b\in B} RM_{p,f,b}\cdot x_{p,f,b,t}\cdot TT = TM_{f,b} \qquad \forall f\in F, b\in B(f) \tag{4.1}$$

$$\sum_{p\in P, f\in F, b\in B} RM_{p,f,b}\cdot x_{p,f,b,t}\cdot TT \leq PC \qquad \forall t\in T \tag{4.2}$$

$$\sum_{f\in F, b\in B(f)} x_{p,f,b,t} + \sum_{r\in R} TI_{p,r}\cdot v_{p,r,t} \leq AV_{p,t} \quad \forall p\in P, t\in T \tag{4.3}$$

$$\bar{x}_{p,f,b,t} \leq x_{p,f,b,t} \qquad \forall p\in P, f\in F, b\in B(f), t\in T \tag{4.4}$$

$$\bar{x}_{p,f,b,t} + \bar{z}_{f,b,t-1} \leq 1 \qquad \forall p\in P, f\in F, b\in B(f), t\in T \tag{4.5}$$

$$\bar{w}_{p,f,t} \geq \bar{x}_{p,f,b,t} \qquad \forall t\in T, f\in F, b\in B(f) \tag{4.6}$$

$$\bar{w}_{p,f,t} \geq x_{p,f,b,t} \qquad \forall p\in P, f\in F, b\in B(f) \tag{4.7}$$

$$\bar{z}_{f,b,t} \leq \sum_{p\in P, \tau\in\{1,..,t\}} RM_{p,f,b}\cdot x_{p,f,b,\tau}\cdot TT \qquad \forall f\in F, b\in B(f), t\in T \tag{4.8}$$

$$\bar{z}_{f,b,t} \geq z_{f,b,t-1} \qquad \forall f\in F \tag{4.9}$$

$$z_{f,b,t} \leq \bar{z}_{f,b-1,t} \qquad \forall f\in F, b\in B(f), t\in T \tag{4.10}$$

$$x_{p,f,b,t} \leq z_{f,b,t} \qquad \forall p\in P, f\in F, b\in B(f), t\in T \tag{4.11}$$

$$z_{f,b,t} \geq \bar{z}_{f',(b'-ML_{f,f'}),t} \qquad \forall (f,f')\in FC, b\in B(f), b'\in B(f'), t\in T \tag{4.12}$$

$$z_{f,b,t} \leq \bar{z}_{f',(b'-mL_{f,f'}),t} \qquad \forall (f,f')\in FC, b\in B(f), b'\in B(f'), t\in T \tag{4.13}$$

$$\sum_{p\in P} AS_p \cdot \bar{x}_{p,f,b,t} \leq AD_{f,b} \qquad \forall f \in F, b \in B(f), t = 1 \tag{4.14}$$

$$\sum_{p\in P} AS_p \cdot \bar{x}_{p,f,b,t} \leq AD_{f,b} + \sum_{p\in P,\tau\in\{1,\dots,t\}} \frac{RM_{p,f,b} \cdot x_{p,f,b,\tau} \cdot TT}{SG \cdot BH} \forall f \in F, b \in B(f), t \in T \setminus \{1\} \tag{4.15}$$

$$\sum_{r\in R} v_{p,r,t} \leq 1 \qquad \forall p \in P, t \in T \tag{4.16}$$

$$\bar{w}_{p,f,t} \leq \sum_{r\in R(f)} v_{p,r,t} \qquad \forall p \in P, t \in T, f \in F \tag{4.17}$$

$$\sum_{s\in FR(r)} v_{p,s,t} \geq v_{p,r,t-1} \qquad \forall t \in T \setminus \{1\}, p \in P, r \in R \tag{4.18}$$

$$\mathrm{d}_t^- \leq D \qquad \forall t \in T \tag{4.19}$$

$$\sum_{p\in P,f\in F,b\in B} RM_{p,f,b} \cdot x_{p,f,b,t} \cdot TT + \mathrm{d}_t^- = PC \qquad \forall t \in T \tag{4.20}$$

$$g_t^+ \leq G \qquad \forall t \in T \tag{4.21}$$

$$g_t^- \leq G \qquad \forall t \in T \tag{4.22}$$

$$\sum_{p\in P,f\in F,b\in B} RM_{p,f,b} \cdot x_{p,f,b,t} \cdot TT + g_t^- - g_t^+ = \sum_{p\in P,f\in F,b\in B} RM_{p,f,b} \cdot x_{p,f,b,t} \cdot OF_{f,b} \cdot EG \qquad \forall t \in T \tag{4.23}$$

4.3 Case study

A case study of an open-pit copper mine is considered to verify the model. Year 4 is selected as the short-term schedule for the case study. The schedule requires 63.49 [Mt] of ore and 29.55 [Mt] of waste to be mined in year 4 with four phases (1, 2, 3 and 4). Figure 4.1 depicts the mine layout in year 4 with the road network, one plant crusher and two waste dumps. The plant has an annual ore capacity of 2.51 [Mt]. Plant crusher is desired to have ore with a copper grade of 1.1 [%]. Mine production operations are carried out in two shifts of 12 [h] daily and seven days a week. The mine employs a total of 2 electric shovels. Shovel 1 has a throughput of 4,114 [t/h] and shovel 2 has a throughput of 5,486 [t/h]. Both shovels have an availability of 80.0 %. Shovel 1 requires a minimum operational area of 1,239 [m^2], whereas shovel 2 requires 1,491 [m^2]. The shovel average movement velocity is estimated at 0.24 [km/h], due to numerous curves and slopes of the road network for the mine layout case study. The operational cost of shovel movement is estimated at 1.00 [USD/m]. Table 4.4 shows the distances each shovel needs to travel between phases.

In this case study, the mine operation prioritizes the short-term objectives as shown in section 4.2.2. We generate a short-term mine production schedules. Comparing the hierarchical method with the weighted sum method in terms of: (i) minimum maximum deviation between ore and ore processing capacity per period, (ii) upper and lower maximum deviation between copper fines and expected copper fines in the ore processing capacity per period, (iii) Overall movement cost and time of shovel fleet , and finally (iv) resolution time. In the weighted sum method, the weights u_i assigned by the decision maker for each objective in decreasing order take the values of $10,000$; 100 and 10.

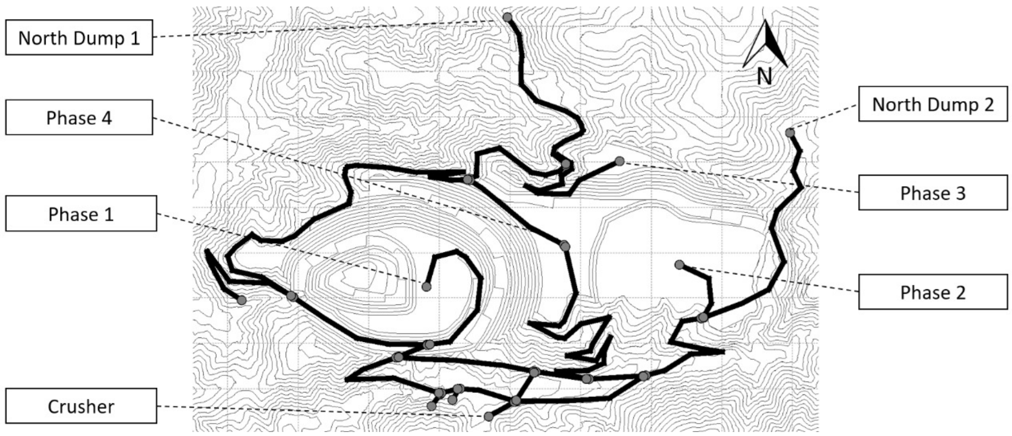

Figure 4.1: Mine layout with ramps and road network in year 4.

Phases	1,2	1,3	1,4	2,3	2,4	3,4
Distance [m]	2,800	4,276	3,273	4,813	2,968	1,845

Table 4.4: Distance between phases.

4.4 Results and discussion

In the case study, the weighted sum method needs 22.11 [h] to be carried out (resolution of mMDP, mMDF, mD and WS(1,2,3)), while the hierarchical method needs 11.85 [h] (resolution of mMDP, mMDF(mMDP), mD(mMDP(mMDF)). Table 4.5 shows the results of the mD, mMDP, and mMDF problems.

Figure 4.2 shows in the final production schedule of the case study, whereas Table 4.6 compares deviations of plant capacity, deviations of copper fines and overall movements of the shovel fleet for problems associated with the hierarchical method (mMDP, mMDF(mMDP), mD(mMDF(mMDP))) and the problem associated with the weighted sum method (WS(1,2,3)).

From the Table 4.6 it is verified that each problem of the hierarchical method maintains the objectives imposed by the previous problems and at the same time diminishes or maintains the short-term objective that it wishes to minimize. In effect, it is observed that the mMDP problem obtained a maximum deviation of the plant processing capacity of 1.8 %, whose level is maintained in the following problems mMDF(mMDP) and mD(mMDF(mMDP)). On the other hand, the problem mMDF(mMDP) decreased the positive and negative deviation of the copper fines concerning the mMDP problem. This deviation is maintained

Problem	mMDP	mMDF	mD
Maximum ore processing capacity deviation [%]	1.864%	8.237%	14.500%
Maximum positive deviation of copper fines [t]	6,462	4,967	6,734
Maximum negative deviation of copper fines [t]	6,754	4,967	6,945
Shovel fleet movement time [days]	12.5	9.5	4.0
Shovel fleet movement cost [kUSD]	71.9	54.6	22.8

Table 4.5: Optimization of single optimization problem objectives.

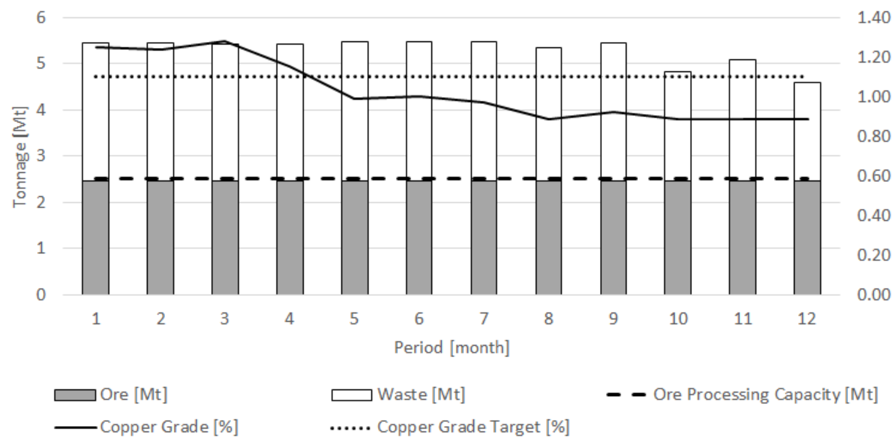

Figure 4.2: Mine short-term production schedule for the case study.

by the post problem mD(mMDF(mMDP)). Also, the problem mMDF(mMDP)) decreased the total days of blade movement obtained by the mMDP problem from 12.5 days to 10.2 days. Finally, problem mD(mMDF(mMDP)) maintained the level of deviations of plant capacity, deviations of copper fines and movement costs and movement times of the shovel fleet obtained by the mMDF problem (mMDP). On the other hand, it is verified that the application of the hierarchical method obtains the same value of the short-term objectives (problem mD(mMDF(mMDP))) than those obtained by the weighted sum method (problem WS(1,2,3)).

4.5 Conclusions

Short-term scheduling in open-pit mines needs several objectives to be optimized jointly. In some open-pit mine operations, these objectives are ranked in a descending order of importance: (i) the minimization of the deviation between the ore sent to the ore processing plant and the ore processing capacity of the plant, (ii) the minimization of the deviation between the metal fines sent to the ore processing plant and the metal fines expected by the ore processing plant and finally (iii) the overall minimization of the movement of the shovel fleet.

	Hierarchical			Weighted Sum
Problem	mMDP	mMDF (mMDP)	mD (mMDF (mMDP))	WS(1,2,3)
Maximum ore processing capacity deviation [%]	1.864%	1.864%	1.864%	1.864%
Maximum positive deviation of copper fines [t]	6,462	4,484	4,484	4,484
Maximum negative deviation of copper fines [t]	6,754	5,202	5,202	5,202
Shovel fleet movement time [days]	12.5	10.2	10.2	10.2
Shovel fleet movement cost [kUSD]	71.9	58.9	58.9	58.9

Table 4.6: Short-term objectives deviations of case study.

This work proposes an optimization methodology to generate a short-term open-pit mine production schedule optimizing multiple hierarchical objectives. For the generation of mine schedules, we propose an optimization model based on mixed-integer linear programming. This model considers mining sequencing constraints, and also takes into account both time and cost of shovels movement between phases. For the optimization of the various short-term objectives, we apply the hierarchical method and the weighted sum method to a real open-pit mine case study. The results of the case study show that both methods are capable of generating short-term mine schedules by optimizing the various short-term objectives. Additionally, we verified that both methods obtain the same mine production schedule. This article shows the importance and impact of multiple objective optimization methods for the generation of short-term mine production schedules in open-pit mines.

Acknowledgements

This article is part of F. Manríquez's PhD thesis. This article was partially funded by the CONICYT/PIA Project AFB180004 and by the CONICYT PFCHA/DOCTORADO BECAS CHILE/2019 - 21190201.

Chapter 5

Short-term open-pit production scheduling optimising multiple objectives accounting for shovel allocation in stockpiles

Abstract

Short-term open-pit mine production scheduling deals with several objectives. Unfortunately, there are trade-offs between them. To overcome this problem, we propose a methodology based on mathematical programming to generate schedules, considering multiple objectives using single optimisation and the hierarchical optimisation method. The model allocates shovels to different mining faces, including stockpiles. It considers constraints of plant capacity, ore blending, precedences between mining faces, shovels throughput, and movement of shovel between mining faces. We propose a set of indicators to compare different schedules. We apply the proposed model to a real iron open-pit mine. The results show that when using the hierarchical method, both the objectives and the order of optimisation have a significant impact on the indicators. The results also show that the schedules with a stockpile obtain higher adherence indicators compared to the ones with no stockpile.

5.1 Introduction

Open-pit mining is a technique for the extraction of ore deposits located near to the surface of the Earth. The scheduling sequence of mining operations is usually divided into strategic (long-term), tactical (medium- term), and operational (short-term) levels (L'Heureux et al, 2013). Strategic scheduling defines the portions of the ore body that can be extracted, the life of the mine, the production rate, and the amount of investment. A long-term production schedule defines the tonnage of waste and ore to be mined from each bench-phase for each year over the scheduling horizon. This schedule seeks to maximise the Net Present Value (NPV) over the life of the mine. Tactical scheduling determines the mining sequence for up to a typical period of 5 years based on the production rate constraints. Finally, operational scheduling seeks to ensure the operational feasibility of the long-term mine production schedule (Smith, 1998). Unfortunately, to date, the short-term planning problem for open-pit mines has not been as widely considered as that for the medium- and long-term horizons (Blom et al, 2018). The reason for that is probably since short-term scheduling requires to account for more complex operational constraints, making it challenging to model by means of mathematical models like optimisation problems.

Short-term open-pit scheduling is required to meet the objectives defined by the long-term mine schedule by delivering the budgeted ore tonnes and grade to the ore processing plant while following the long-term mine schedule Chanda (1990). A short-term schedule has multiple criteria to take into account, such as: maximisation of the plant utilisation, maximisation of ore extraction, and maximisation of waste extraction. In this paper, short-term refers to a production schedule time horizon, expanding from one week to one year with a daily up to monthly fidelity.

Another very relevant aspect of short-term scheduling is the availability and utilization of stockpiles. Indeed, they are key because they allow to keep the balance between the ore extracted from the mine and the processing capacity, First, stockpiles act as buffers or storing, so that processes before them and processes after they can operate without being constrained by each other. Second, stockpiles can be used to control the blending of material to be processed, reducing their variability. Finally, stockpiles can be used to sort material by grade or other properties. (See Jupp et al (2014) Robinson (2004).)

In this study, we propose an optimisation model to generate short-term open-pit schedules, optimising multiple objectives and accounting for the movement of shovel between different locations (faces) and stockpiles. The model is based on mixed-integer linear programming (MILP) (Bertsimas and Tsitsiklis, 1997) and its main goal is to allocate shovels to different mining faces, which can be located in the mine or stockpiles. The model takes as input a targets for extraction from the mine (ore and waste) and to be sent to the mill, and aims to achieve these targets while also considering the following constraints: plant capacity, ore blending requirements, precedences between mining faces, shovels throughput, mine sequencing and movement of shovels fleet between sectors of the mine.

We apply the single-optimisation and the hierarchical method in the proposed optimisation model to optimise multiple short-term objectives (Grodzevich and Romanko, 2006; Nehring et al, 2018) (see Section 2.2). In this way, we generate various schedules in a real open-pit mine case study. These schedules study the impact of the proposed indicators considering the following scenarios: mobile or fixed shovel fleet between sectors of the mine, presence or absence of a stockpile, and optimisation approach.

The contributions of the paper are the following: (i) a MILP-based optimisation problem to generate a short-term schedule for open-pit mines considering loading equipment allocation, stockpiles and multi-objectives, (ii) a set of indicators to assess and compare the different short-term schedules, and (iii) an application of the model in an iron open-pit mine case study showing the validity of the model in different scenarios.

The remainder of this paper is organised as follows. Section 5.2 states the problem to solve by the optimisation model. Section 5.3 presents the proposed optimisation model. Section 5.4 describes the real-scale open-pit mine case study. It also outlines the different schedules to solve. Section 5.5 reports and discusses the results of the case study. Finally, Section 5.6 concludes the study and outlines future work.

5.2 Problem statement

This section provides a complete description of the problem solved by the optimisation problem that is mathematically described in Section 5.3.

5.2.1 Sectors and mining faces

In the proposed model, we consider that shovels can be assigned to different mining faces, which can be either in the interior of the pit or at the stockpiles. Each mining face known in advance and it is characterized by a tonnage, a type (ore or waste), the grade of the metal of interest and its type (extraction or stockpile). The difference between extraction and stockpiles is that the former only allow extraction, while the latter type also allow accumulation of material extracted at the mine. Finally, the sequence at which extraction faces can be mined is determined by precedences, which force the depletion of other faces before starting extraction at a given one.

Because in short-term the movement of shovels can be significant with regards to the total time for scheduling, the proposed model considers that mining faces are grouped into sectors, in such a way that the travel time between faces at the same sector can be neglected, while the time travel between faces located in different sectors cannot be used for production.

Each component of a mining face can refer to the element or compound of interest to extract (for example, the metal produces the mine). A component of a mining face can also refer to the components that condition the extraction of elements or compound of interest (for example, impurities).

5.2.2 Stockpiles modelling

A stockpile is a type of mining face that can accumulate ore extracted from the extraction mining faces and also supply ore to the plant. In the optimisation problem, the stockpiles are explicitly modelled. That is to say, the movement of material to and from each stockpile is scheduled. As in Hoerger et al (1999), it is assumed that the stockpiles are homogeneous and the ore reclaimed from each stockpile has a grade equivalent to the average grade of the stockpile, which is constant over the scheduled horizon.

5.2.3 Shovel modelling

A shovel can perform different tasks. Each shovel can be either available or not available. We consider that the shovel is available if it is operating (extracting material from a mining face), moving between mining faces, or in stand-by position. Conversely, the shovel is not available during meals, shift changes, maintenance and failures (Table 5.1). The proposed model will allocate the shovels utilizing only the available time, however it will decide how much of it is spent operating, moving, or in stand-by.

Available time (assigned by model)			Non-available time (input)			
Operating	Moving between sectors	Stand-by	Meals	Shift changes	Maintenances	Failures

Figure 5.1: Distribution of time of a shovel

Shovels are also characterized by an operative throughput. This parameter quantifies the extracted material per unit of operating time that a shovel can extract (in tonnes per hour) during its operating period. This parameter depends on the mining face in which the shovel is allocated, and is an input of the model.

Finally, each shovel is characterized the time it needs to move between any pair of sectors in the mine. Such parameter is an input to the model that can be determined using the shovel speed and the distance between said sectors. However, we also consider a maximum number of sector change for each shovel within the scheduling horizon.

A route is a list of sectors in which a shovel travels sequentially in a given period. The maximum number of sectors by which a route is composed is not limited by the optimization model. However, in practice, routes up to two sectors are used in the case studies of this article.

5.2.4 Schedule performance and deviations from targets

To compare and evaluate the performance of the results beyond the optimisation targets, several performance indicators are considered. These indicators aim to measure the compliance between the short-term schedule and material movement goals and the results from the schedule, and are based on the material flows depicted in Figure 5.2, namely: O the tonnage of ore sent directly from the mine to the processing plants; R the tonnage of rehandling, that is, material sent from stockpiles to the processing plants; S the tonnage of material sent from the mine to stockpiles; and W the tonnage of material sent to waste dumps.

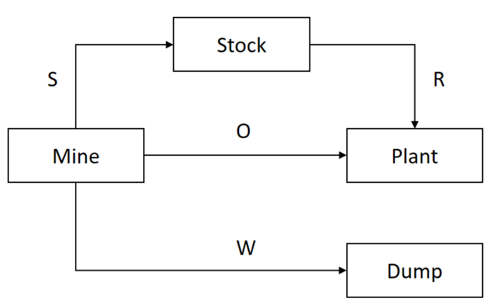

Figure 5.2: An example of material flows in an open-pit mine operation.

Table 5.1 reports the flow material targets. Table 5.2 reports the scheduled indicators associated to flow material targets.

Target	Description
P_0	Ore to be sent to the plant.
M_0	Ore to be extracted from the mine.
W_0	Waste to be extracted from the mine.

Table 5.1: Targets of the short-term scheduling

Indicator	Formula	Description
$C(W)$	$\frac{W}{W_0}$	Waste extraction.
$C(P)$	$\frac{O+R}{P_0}$	Plant utilisation.
$C(O)$	$\frac{O}{P_0}$	Plant utilisation due to ore directly sent from mine.
$C(R)$	$\frac{R}{P_0}$	Plant utilisation due to ore sent from stockpile.
$C(S)$	$\frac{S}{P_0}$	Ore sent to stockpile.
$C(M)$	$\frac{O+S}{M_0}$	Ore extraction.

Table 5.2: Schedule indicators

5.3 Optimisation model

5.3.1 Sets, indexes, decision variables and objectives functions

The sets and indexes are reported in Table 5.3. The decision variables are reported in Tables 5.4 (for the main decision variables) and Table 5.5 (for auxiliary variables associated to deviations from targets). Table 5.6 lists the objectives considered by the optimisation model. The optimisation model parameters are reported in the Appendix (Section 5.7).

Symbol	Description
$\mathcal{P}, p$	Set and index for shovels.
$\mathcal{S}, s$	Set and index for sectors.
$\mathcal{F}, f$	Set and index for mining faces.
$\mathcal{F}_s$	Set for mining faces belonging to sector s.
$\mathcal{T}, t$	Set and index for periods.
$\mathcal{J}, j$	Set and index for component.
$\mathcal{R}, r$	Set and index for routes.
$\mathcal{R}_f$	Set of routes that contains sector f .
$\mathcal{H}_r$	Set of routes whose first sector is equal to the last sector of route r.
$\mathcal{Q}$	Set of precedence between the mining faces b. $(f, g) \in Q$ means that the mining face f is a predecessor of mining face d.

Table 5.3: Optimisation model sets and indexes

Variable	Description
$\bar{w}_{p,s,t} \in \{0,1\}$	Equal to 1 if shovel $p \in P$ is allocated on sector $s \in S$ in period $t \in T$, 0 otherwise.
$\bar{x}_{p,f,t} \in \{0,1\}$	Equal to 1 if shovel $p \in P$ is allocated in mining face $f \in F$ in period $t \in T$, 0 otherwise.
$x_{p,f,t} \in [0,1]$	Time percentage of the period $t \in T$ where shovel $p \in P$ is operative in mining face $f \in F$
$y_{p,f,t} \in [0,1]$	Time percentage of the period $t \in T$ where shovel $p \in P$ is operative in mining face $f \in F$ sending material to dump or plant (according of material type).
$\bar{y}_{p,f,t} \in [0,1]$	Time percentage of the period $t \in T$ where shovel $p \in P$ is operative in mining face $f \in F$ sending material to the stockpile.
$z_{f,t} \in \{0,1\}$	Equal to 1 if mining face $f \in F$ is active at period $t \in T$, 0 otherwise.
$\bar{z}_{f,t} \in \{0,1\}$	Equal to 1 if mining face $f \in F$ finishes its exploitation in period $t \in T$ or later, 0 otherwise.
$v_{p,r,t} \in \{0,1\}$	Equal to 1 if shovel p goes through route r in period t, zero otherwise.
$u_{f,t} \in$	Tonnage of the mining face f at the end of the period t.

Table 5.4: Optimisation model decision variables

Variable	Description
$\Delta O \in \mathbb{R}_{\geq 0}$	Deviation between the ore sent to the plant from mine and the total ore capacity over the scheduled horizon.
$\Delta P \in \mathbb{R}_{\geq 0}$	Deviation between the ore sent to plant and plant capacity.
$\Delta W \in \mathbb{R}_{\geq 0}$	Deviation between the waste sent to the waste dump and the total waste content in the mine.
$\Delta \mathrm{d}_t \in \mathbb{R}_{\geq 0}$	Deviation between the plant capacity and the ore send to processing plant.
$\Delta D \in \mathbb{R}_{\geq 0}$	Minimum maximum deviation between the plant capacity and the ore send to plant.
$\Delta g^+_{j,t} \in \mathbb{R}_{\geq 0}$	Positive deviation of the content j between plant capacity and the ore content j send to plant.
$\Delta g^-_{j,t} \in \mathbb{R}_{\geq 0}$	Negative deviation of the content j between plant capacity and the ore content j send to plant.
$\Delta G_j \in \mathbb{R}_{\geq 0}$	Maximum deviation of the content j between plant capacity and the ore content j send to plant.

Table 5.5: Optimisation model decision variables of deviation

Single-objective function	Description
ΔO	Minimisation of the deviation between ore sent to the plant from the mine and ore expected to be sent to plant.
ΔP	Minimisation of the deviation between ore sent to the plant from the mine and stockpile and ore expected to be sent to plant.
ΔW	Minimisation of the deviation between waste hauled and the waste expected to be hauled.
ΔD	Minimisation of the deviation between ore sent to plant and the ore plant capacity per period.
ΔG_j	Minimisation of the deviation between the content of component j sent to plant and the content of component j expected by plant.
$\sum_{p\in\mathcal{P},r\in\mathcal{R},t\in\mathcal{T}} MT_{p,r}\cdot v_{p,r,t}$	Minimisation of the movement time of the shovel fleet.
$\sum_{p\in\mathcal{P},r\in\mathcal{R},t\in\mathcal{T}} MC_{p,r}\cdot v_{p,r,t}$	Minimisation of the movement cost of the shovel fleet.
$\sum_{p\in\mathcal{P},r\in\mathcal{R},t\in\mathcal{T}} NM_{r}\cdot v_{p,r,t}$	Minimisation of the number of movement of the shovel fleet.

Table 5.6: Single-objective functions considered in the optimisation problem

5.3.2 Constraints

First, we present the mathematical expressions corresponding to the constraints of the optimisation model. Second, we describe them in groups associated to different concepts.

$$\sum_{p\in\mathcal{P},t\in\mathcal{T}} TT_t\cdot RM_{p,f}\cdot(1-ST_f)\cdot x_{p,f,t}\leq TM_f \qquad \forall f\in\mathcal{F} \tag{5.1}$$

$$PC_t^-\leq\sum_{p\in\mathcal{P},f\in\mathcal{F}} TT_t\cdot RM_{p,f}\cdot y_{p,f,t}\leq PC_t^+ \qquad \forall t\in\mathcal{T} \tag{5.2}$$

$$\begin{aligned}\sum_{p\in\mathcal{P},f\in\mathcal{F}} TT_t\cdot RM_{p,f}\cdot OM_f\cdot mG_j\cdot y_{p,f,t}\leq&\\ \sum_{p\in\mathcal{P},f\in\mathcal{F}} TT_t\cdot RM_{p,f}\cdot OM_f\cdot OG_{j,f}\cdot y_{p,f,t}\leq&\\ \sum_{p\in\mathcal{P},f\in\mathcal{F}} TT_t\cdot RM_{p,f}\cdot OM_f\cdot MG_j\cdot y_{p,f,t}& \qquad \forall t\in\mathcal{T},j\in\mathcal{J}\end{aligned} \tag{5.3}$$

$$\sum_{f\in\mathcal{F}} TT_t\cdot x_{p,f,t}+\sum_{r\in\mathcal{R}} MT_{p,r}\cdot v_{p,r,t}\leq TT_t\cdot UT_{p,t} \quad \forall p\in\mathcal{P},t\in\mathcal{T} \tag{5.4}$$

$$\sum_{p\in\mathcal{P},f\in\mathcal{F},t\in\mathcal{T}} TT_t \cdot RM_{p,f} \cdot OM_f \cdot (1 - ST_f) \cdot y_{p,f,t} + \Delta O = \sum_{t\in\mathcal{T}} PC_t^+ \tag{5.5}$$

$$\sum_{p\in\mathcal{P},f\in\mathcal{F},t\in\mathcal{T}} TT_t \cdot RM_{p,f} \cdot OM_f \cdot y_{p,f,t} + \Delta P = \sum_{t\in\mathcal{T}} PC_t^+ \tag{5.6}$$

$$\sum_{p\in\mathcal{P},f\in\mathcal{F},t\in\mathcal{T}} TT_t \cdot RM_{p,f} \cdot (1 - OM_f) \cdot y_{p,f,t} + \Delta W = \sum_{f\in\mathcal{F}} TM_f \cdot (1 - OM_f) \tag{5.7}$$

$$\sum_{p\in\mathcal{P},f\in\mathcal{F}} TT_t \cdot RM_{p,f} \cdot OM_f \cdot y_{p,f,t} + \Delta \mathrm{d}_t = PC_t^+ \qquad \forall t \in \mathcal{T} \tag{5.8}$$

$$\Delta \mathrm{d}_t \leq \Delta D \qquad \forall t \in \mathcal{T} \tag{5.9}$$

$$\sum_{p\in\mathcal{P},f\in\mathcal{F}} TT_t \cdot RM_{p,f} \cdot OM_f \cdot OG_{j,f} \cdot y_{p,f,t} + \Delta g_{j,t}^- - \Delta g_{j,t}^+ = \sum_{p\in\mathcal{P},f\in\mathcal{F}} TT_t \cdot RM_{p,f} \cdot OM_f \cdot EG_j \cdot y_{p,f,t} \qquad \forall j \in \mathcal{J}, t \in \mathcal{T} \tag{5.10}$$

$$\Delta g_{j,t}^+ \leq \Delta G_j \qquad \forall j \in \mathcal{J}, t \in \mathcal{T} \tag{5.11}$$

$$\Delta g_{j,t}^- \leq \Delta G_j \qquad \forall j \in \mathcal{J}, t \in \mathcal{T} \tag{5.12}$$

$$z_{g,t} \leq \bar{z}_{f,t} \qquad \forall (f,g) \in \mathcal{Q} \tag{5.13}$$

$$\bar{x}_{p,f,t} \leq \bar{w}_{p,s,t} \qquad \forall s \in S, f \in \mathcal{F}_s, t \in \mathcal{T} \tag{5.14}$$

$$x_{p,f,t} \leq \bar{x}_{p,f,t} \qquad \forall p \in \mathcal{P}, f \in \mathcal{F}, t \in \mathcal{T} \tag{5.15}$$

$$\bar{x}_{p,f,t} \leq 1 - \bar{z}_{f,t-1} \qquad \forall p \in \mathcal{P}, f \in \mathcal{F}, t \in \mathcal{T} \setminus \{1\} \tag{5.16}$$

$$\bar{z}_{f,t} \leq \sum_{p\in P,\tau\in\{1,..,t\}} \frac{TT_\tau \cdot RM_{p,f} \cdot x_{p,f,\tau}}{TM_f} \qquad \forall f \in \mathcal{F}, t \in \mathcal{T} \tag{5.17}$$

$$\bar{z}_{f,t} \geq \bar{z}_{f,t-1} \qquad \forall f \in \mathcal{F}, t \in \mathcal{T} \setminus \{1\} \tag{5.18}$$

$$x_{p,f,t} \leq z_{f,t} \qquad \forall p \in \mathcal{P}, f \in \mathcal{F}, t \in \mathcal{T} \tag{5.19}$$

$$\bar{x}_{p,f,t} \leq FA_{p,f} \qquad \forall p \in \mathcal{P}, f \in \mathcal{F}, t \in \mathcal{T} \tag{5.20}$$

$$\sum_{r\in\mathcal{R},t\in\mathcal{T}} NM_r \cdot v_{p,r,t} \leq MM_p \qquad \forall p \in \mathcal{P} \tag{5.21}$$

$$\sum_{r\in\mathcal{R}} v_{p,r,t} \leq 1 \qquad \forall p \in \mathcal{P}, t \in \mathcal{T} \tag{5.22}$$

$$\bar{w}_{p,f,t} \leq \sum_{r\in\mathcal{R}_f} v_{p,r,t} \qquad \forall p \in \mathcal{P}, f \in \mathcal{F}, t \in \mathcal{T} \tag{5.23}$$

$$\sum_{s \in \mathcal{H}_r} v_{p,s,t} \geq v_{p,r,t-1} \qquad \forall p \in \mathcal{P}, r \in \mathcal{R}, t \in \mathcal{T} \setminus \{1\} \tag{5.24}$$

$$y_{p,f,t} + \bar{y}_{p,f,t} = x_{p,f,t} \qquad \forall p \in \mathcal{P}, \in \mathcal{F}, t \in \mathcal{T} \tag{5.25}$$

$$\bar{y}_{p,f,t} \leq OM_f \qquad \forall p \in \mathcal{P}, \in \mathcal{F}, t \in \mathcal{T} \tag{5.26}$$

$$\bar{y}_{p,f,t} \leq 1 - ST_f \qquad \forall p \in \mathcal{P}, \in \mathcal{F}, t \in \mathcal{T} \tag{5.27}$$

$$u_{f,0} = TM_f \cdot ST_f \qquad \forall f \in \mathcal{F} \tag{5.28}$$

$$\begin{aligned} \sum_{p \in \mathcal{P}, f \in \mathcal{F}} TT_t \cdot RM_{p,f} \cdot OM_f \cdot (1 - ST_f) \cdot \bar{y}_{p,f,t} + u_{f,t-1} = \\ \sum_{p \in \mathcal{P}, f \in \mathcal{F}} TT_t \cdot RM_{p,f} \cdot OM_f \cdot ST_f \cdot y_{p,f,t} + u_{f,t} \\ \forall p \in \mathcal{P}, \in \mathcal{F}, t \in \mathcal{T} \end{aligned} \tag{5.29}$$

$$\Delta O \leq \Delta O^* \cdot (1 + O^\varepsilon) + (1 - O^\mu) \cdot MM \tag{5.30}$$

$$\Delta W \leq \Delta W^* \cdot (1 + W^\varepsilon) + (1 - W^\mu) \cdot MM \tag{5.31}$$

$$\Delta D \leq \Delta D^* \cdot (1 + D^\varepsilon) + (1 - D^\mu) \cdot MM \tag{5.32}$$

$$\Delta P \leq \Delta P^* \cdot (1 + P^\varepsilon) + (1 - P^\mu) \cdot MM \tag{5.33}$$

$$\Delta g_j^+ \leq \Delta G_j^* \cdot (1 + G_j^\varepsilon) + (1 - G_j^\mu) \cdot MM \qquad \forall j \in \mathcal{J} \tag{5.34}$$

$$\Delta g_j^- \leq \Delta G_j^* \cdot (1 + G_j^\varepsilon) + (1 - G_j^\mu) \cdot MM \qquad \forall j \in \mathcal{J} \tag{5.35}$$

$$\sum_{p \in \mathcal{P}, r \in \mathcal{R}, t \in \mathcal{T}} MT_{p,r} \cdot v_{p,r,t} \leq \Delta MT^* \cdot (1 + MT^\varepsilon) + (1 - MT^\mu) \cdot MM \tag{5.36}$$

$$\sum_{p \in \mathcal{P}, r \in \mathcal{R}, t \in \mathcal{T}} MC_{p,r} \cdot v_{p,r,t} \leq \Delta MC^* \cdot (1 + MC^\varepsilon) + (1 - MC^\mu) \cdot MM \tag{5.37}$$

$$\sum_{p \in \mathcal{P}, r \in \mathcal{R}, t \in \mathcal{T}} NM_{p,r} \cdot v_{p,r,t} \leq \Delta NM^* \cdot (1 + NM^\varepsilon) + (1 - NM^\mu) \cdot MM \tag{5.38}$$

Operational constraints

Constraints (5.1)-(5.4) impose the ore plant capacity and models the distribution of time of each shovel. Constraint (5.1) imposes that the total material extracted in each mining face along the planning horizon must be less or equal to the total material contained in that mining face. Constraint (5.2) sets the minimum and maximum ore tonnages sent to the ore processing plant. Constraint (5.3) limits the minimum and maximum contents of component j sent to the ore processing plant. Constraint (5.4) models the shovel time. The effective shovel time plus the shovel movement time between sectors are less than or equal to the maximum shovel utilisation.

Deviation constraints

Deviation constraints (5.5)-(5.12) are related to the deviations between the objectives of the schedule and the objectives desired to meet. Constraint (5.5) models the deviation between the ore from the mine sent to the ore processing plant and the ore processing plant along the scheduled horizon. Constraint (5.6) models the deviation between the ore from the mine and the stockpile sent to the ore processing plant and the ore processing plant along the scheduled horizon. Constraint (5.7) models deviation between the waste sent to the waste dump and the waste content in the mine along the scheduled horizon. Constraint (5.8) models deviations per period between the ore sent to the plant and the capacity of the plant. Constraint (5.9) limit, with a maximum plant deviation variable, each one of the deviations per period between the ore sent to the plant and the capacity of the plant. Constraint (5.10) models each one of the upper and lower deviations per period between the content of component j sent to the ore processing plant and content of component j expected by the ore processing plant. Constraint (5.11) to (5.12) limit, with a maximum deviation variable of component j, each one of the upper and lower deviations per period between the content of component j sent to the ore processing plant and content of component j expected by the ore processing plant.

Shovel allocation constraints

Shovel allocation constraints (5.13)- (5.20) models the shovel allocation to mine faces. Constraint (5.13) sets the precedences between sector-benches. The element $(f, g) \in \mathcal{Q}$ indicates that the mining face f is a predecessor of the mining face g. Constraint (5.14) imposes that to allocate a shovel p in a mining face f in a period t, the shovel must be allocated in this sector s in this period t. Constraint (5.15) imposes that the shovel that has effective time in the mining face b of sector f, must be allocated in that mining face. Constraint (5.16) ensures that when a mining face is finished, it cannot be assigned with any shovel. Constraint (5.17) imposes that a mining face b of sector f is not finished in period t until all the material of that mining face is completely extracted. Constraint (5.18) ensures that if a mining face b of a sector f is finished in a period t, it remains in that state in period $t+1$. Constraint (5.19) imposes that, to assign effective time in a shovel allocated in a mining face, that mining face must be active. Constraint (5.20) restricts the allocation of shovel to certain mining faces.

Route constraints

Constraints (5.21)-(5.24) models the movement of each shovel between mine faces of different sectors. Constraint (5.21) sets the maximum number of movements between sectors along the scheduled horizon for each shovel. Constraint (5.22) ensures that at most just one route is performed by a shovel p in period t. Constraint (5.23) makes sure that when a shovel p in a period t is allocated in sector f, just one of the variables $v_{p,r,t}$ with indexes of sector f and period t must be equal to 1. Constraint (5.24) ensures that for each shovel p, route r of period t and route s of period $t+1$ are consistent. That is to say, the last sector of the route r and the first sector of the s route must be equal. Set $\mathcal{H}_r$ contains of all the routes consistent with the route r. In other words, this set contains all the routes whose first sector is equal to the last sector of route r.

Stockpile constraints

Constraints (5.25)-(5.29) models the stockpiles. Constraint (5.25) imposes that the sum of the fractions of the effective time in primary and secondary destinations of a shovel p at mining face b of sector,f must be equal to the fraction of the effective time of the shovel p in the same mining sector. Constraint (5.26) imposes that the fraction of the effective time in the second destination of mining sector that contains waste is zero. Constraint (5.27) imposes that the fraction of the effective time in the second destination of mining faces that are stockpiles is zero. Constraint (5.28) sets the initial ore tonnage of the mining faces that are of type stockpile. Constraint (5.29) defines the inventory of ore tonnage in the mining sector of the stockpile type. The ore sent to the stockpile from the mining faces in a period t plus the ore contained in the mining face of stockpile in the final of period $t-1$ is equal to the ore sent to the plant from the mining face of the stockpile type in period t plus the remaining ore tonnage in the mining face of the stockpile type in the final of period t.

Additional deviation constraints

Additional deviation constraints (5.30)-(5.38) impose an upper bound for each one of the corresponding deviations. These are the additional constraints used to perform the hierarchical optimisation explained in Section 2.2 For example, in constraint (5.30), it is imposed that the deviation ΔO must be less than or equal to the upper bound $\Delta O^* \cdot (1+O^\varepsilon)+(1-O^\mu)\cdot MM$. When $O^\mu = 1$, the upper bound is a very large number $O^* \cdot (1+O^\varepsilon)+M$, while when $O^\mu = 0$, the bound higher is equal to $\Delta O^* \cdot (1+O^\varepsilon)$. The ΔO^* parameter is a known value of the constraint deviation (5.30), while the O^ε parameter corresponds to a number greater than or equal to zero, which represents the fraction of parameter tolerance ΔO^*. Constraints (5.31)-(5.38) work analogously.

5.4 Case study

In this Section, we describe the real-scale open-pit mine case study. It also outlines the different schedules to solve.

5.4.1 Mine operation description

A real-scale open-pit mine case study is used for generating different schedules using the proposed optimisation model. The mine comprises of six sectors. The mine planning horizon is one month, considering ten periods. Each period lasts between one and four days. The main parameters related to the mine operation are summarized in Tables 5.7 to 5.10. Table 5.7 summarises the period length and ore plant capacity in each period. Table 5.8 shows the shovel parameters involved in the open-pit mine. Table 5.9 presents the tonnage of ore and waste contained in each mining face. In this case study, the mining face 6_1380 is the unique mining face of stockpile type, that have 0 tonnages of ore at the beginning of the scheduling horizon. Finally, Table 5.10 reports the distance to travel of each shovel to move from one sector to another. At the begining of the scheduling period, the stockpiles are empty.

Period	Period length [days]	Ore plant capacity [kt]
1	1	0
2	4	42
3	3	0
4	4	42
5	3	0
6	4	42
7	3	0
8	4	39
9	3	0
10	2	15
Total	31	180

Table 5.7: Length of periods and ore plant capacity

Shovel	Effective throughput [t/h]	Maximum utilisation [%]	Speed [km/h]
1	1,350	43	2
2	1,300	35	2
3	1,200	47	15
4	1,150	63	7
5	1,150	45	7
6	1,200	40	15

Table 5.8: Shovel fleet parameters

5.4.2 Experimental design

The model is used under different configurations to analyze how these aspects affect the performance indicators described in Section 5.2.4. There are a total of 28 configurations, which are obtaining consider the following possibilities:

- Optimization criteria. In this case the following objectives are considered (a) maximisation of plant utilisation ($\min \Delta P$), (b) maximisation of waste extraction ($\min \Delta W$), and (c) maximisation of plant utilisation due to ore directly sent from mine ($\min \Delta O$) (see Section 5.3). Each of these criteria is evaluated using a single step optimization and then several two steps configurations are tested. Table 5.11 reports the criteria considered for the single optimisation and hierarchical optimisation in each case, so for example $\Delta P(\Delta W)$ means that ΔP is minimized subject to ΔW, that is, first the waste deviation ΔW is minimized and then plant deviation ΔP is minimized, subject to the waste deviation, hierarchically.
- Presence or absence of a stockpile. These two options are labeled as "Yes" and "No", respectively, in the corresponding results.
- Fixed or mobile shovel fleet. For this, two possible configurations are considered: a fixed fleet, meaning that shovels cannot change sector, and a mobile fleet, in which case

Mining face	Sector	Waste [kt]	Ore [kt]
1_ 1380	1	335	4
2_1330	2	40	2
2_1320	2	299	41
3_1280	3	21	2
3_1270	3	102	20
3_1260	3	131	10
3_1250	3	22	2
3_1240	3	618	94
4_1230	4	85	4
5_1240	5	59	0
5_1230	5	288	1
6_1380	6	0	0
Total tonnage		1,999	180

Table 5.9: Ore and waste tonnage of mining faces (in [kt])

Sectors	1	2	3	4	5	6
1	-	1.7	2.4	2.8	3.2	2.4
2	1.7	-	0.7	1	1.5	1.6
3	2.4	0.7	-	0.4	0.5	1.3
4	2.8	1	0.4	-	0.1	1.8
5	3.2	1.5	0.5	0.1	-	2.1
6	2.4	1.6	1.3	1.8	2.1	-

Table 5.10: Distance between sectors (in km)

shovels are allowed changing sector at most once during the planning horizon.

5.4.3 Computational resources

All the schedules presented in this study were performed on a 2.60 GHz Intel® Xeon® CPU, with 256 GB RAM, running Windows 8®. The optimisation model is solved using Gurobi Optimizer version 8.1 (Gurobi Optimization, 2019). We impose a minimum MIP gap of 5.0% to resolve the optimisation problems. We believe that this value is an adequate trade-off between the resolution time and the objective function value.

5.5 Results and discussion

In this section, we present the results and discussion based on the application of the optimisation model proposed to the open-pit mine case study.

Optimisation method	Notation	Description
Single	ΔW	Minimise waste deviation.
	ΔP	Minimise ore sent to plant deviation.
	ΔO	Minimise ore deviation.
Hierarchical	$\Delta P(\Delta W)$	Minimise plant deviation s.t. the minimum waste deviation.
	$\Delta W(\Delta O)$	Minimise waste deviation s.t. the minimum ore deviation.
	$\Delta W(\Delta P)$	Minimise waste deviation s.t. the minimum plant deviation.
	$\Delta O(\Delta W)$	Minimise ore deviation s.t. the minimum waste deviation.

Table 5.11: Different optimisation of short-term objectives considered in case study

5.5.1 Computational aspects

The resolution time, MIP gap, and objective function of the solved schedules are given in Tables 5.12, 5.13, and 5.14, respectively.

Table 5.12 reports the resolution time of the resolved schedules. In general, the resolution times of the schedules with a mobile shovel fleet are higher than the ones generated with fixed shovel fleet. This could be explained because the number of decision variables in the optimisation problem of schedules with a mobile shovel fleet is higher than the one with a fixed shovel fleet.

Type of shovel fleet	Fixed		Mobile	
Stockpile	No	Yes	No	Yes
ΔW	0.5	0.9	30.8	0.4
$\Delta W(\Delta O)$	1.3	0.7	7.0	23.3
$\Delta W(\Delta P)$	1.5	22.5	10.2	102.9
ΔO	0.3	0.6	6.3	15.5
$\Delta O(\Delta W)$	0.4	0.7	116.3	31.7
ΔP	0.3	0.5	4.9	34.4
$\Delta P(\Delta W)$	0.5	16.0	45.9	518.2

Table 5.12: Resolution time of scenarios (in [min]).

5.5.2 Hierarchical schedules with a mobile shovel fleet

In the Appendix Section (5.8), Tables 5.20 to 5.23 summarize the indicators of all the generated schedules that considers: (i) different single and hierarchical objectives, (ii) considering and not considering a stockpile in its generation and (iii) considering a mobile shovel fleet mobile (each shovel can move at most once between sectors) or fixed (a shovel can be allocated

Type of shovel fleet	Fixed		Mobile	
Stockpile	No	Yes	No	Yes
ΔW	0.0	0.0	0.0	0.0
$\Delta W(\Delta O)$	0.0	0.0	3.1	0.0
$\Delta W(\Delta P)$	0.0	0.0	0.0	4.1
ΔO	0.0	0.0	0.0	0.0
$\Delta O(\Delta W)$	0.0	0.0	0.2	3.5
$\Delta P(\Delta W)$	0.0	0.0	0.0	0.0
ΔP	0.0	0.0	0.0	4.7

Table 5.13: Scenarios MIP gap (in percentage)

Type of shovel fleet	Fixed		Mobile	
Stockpile	No	Yes	No	Yes
ΔW	85	85	35	0
$\Delta W(\Delta O)$	767	767	111	767
$\Delta W(\Delta P)$	767	767	112	767
ΔO	49	49	18	11
$\Delta O(\Delta W)$	85	85	49	26
ΔP	49	49	18	10
$\Delta P(\Delta W)$	85	85	49	26

Table 5.14: Objective function values of scenarios (in [kt])

in mining face of just one sector).

In this section, we summarize the adherence indicators obtained by hierarchical schedules with a mobile fleet movement, considering stockpile (Table 5.15) and without stockpile (Table 5.16).

Indicator	$\Delta P(\Delta W)$	$\Delta O(\Delta W)$	$\Delta W(\Delta P)$	$\Delta W(\Delta O)$
$C(W)$	100	100	62	62
$C(P)$	86	86	95	94
$C(O)$	86	86	50	94
$C(R)$	0	0	45	0
$C(S)$	14	14	45	1
$C(M)$	100	100	95	95

Table 5.15: Adherence indicators (in percentage) of hierarchical schedules with a mobile shovel fleet, considering stockpile

From the analysis of Tables 5.15 and 5.16 we conclude that the hierarchical schedules that consider stockpile obtain higher indicators than the ones of the schedules that do not consider stockpile.

Indicator	$\Delta P(\Delta W)$	$\Delta O(\Delta W)$	$\Delta W(\Delta P)$	$\Delta W(\Delta O)$
$C(W)$	98	98	94	94
$C(P)$	73	73	90	90
$C(O)$	73	73	90	90
$C(R)$	0	0	0	0
$C(S)$	0	0	0	0
$C(M)$	73	73	90	90

Table 5.16: Adherence indicators (in percentage) of hierarchical schedules with a mobile shovel fleet, not considering stockpile

Below, we analyse the primary objectives of the hierarchical schedules: ΔW, ΔP and ΔO.

For the primary objective ΔW, the associated indicator is $C(W)$. In the $\Delta P(\Delta W)$ scenario with stockpile gets a value of $C(W)$ higher than the same scenario without stockpile ($C(W) = 100\%$ against $C(W) = 98\%$). On the other hand, the scenario $\Delta O(\Delta W)$ with stockpile gets a value of $C(W)$ greater than the same scenario generated without stockpile ($C(W) = 100\%$ against $C(W) = 98\%$).

For the primary objective ΔP, the associated indicator is $C(P)$. The $\Delta W(\Delta P)$ scenario with stockpile gets a higher $C(W)$ than the same scenario without stockpile ($C(P) = 95\%$ against $C(P) = 94\%$).

For the primary objective ΔP, the associated indicator is $C(P)$. The $\Delta W(\Delta O)$ scenario with stockpile gets $C(O) = 94\%$, greater than $C(O) = 90\%$.

Below, we analyse the global maximums of the scenarios generated under the $C(O)$, $C(W)$ and $C(P)$ indicators.

The schedule that obtain the maximum extraction of ore sent directly to the plant ($C(O)$) is the schedule $\Delta W(\Delta O)$ with stockpile, with a value of $C(O) = 94\%$. However, its waste extraction is low, with a value of $C(W) = 62\%$.

The schedule that obtain the maximum extraction of waste ($C(W)$) corresponds to the $\Delta W(\Delta P)$ and $\Delta W(\Delta O)$ schedules, both generated from stockpile. These schedules get a value of $C(W) = 100\%$. However, its extraction of ore to the plant of these schedules is not the highest of the schedules generated, with a value of $C(O) = 86\%$.

The schedule that extracts the maximum mineral sent to the plant, from mine and stockpile ($C(P)$) is the $\Delta W(\Delta P)$ schedule with stockpile, with a value of $C(P) = 95\%$. However, this schedule contemplates an intensive shipment of mine ore to stockpile ($C(S) = 45\%$) and intensive use of rehandling ($C(R) = 45\%$).

5.5.3 Discussion

From the results of the case study, we conclude that when applying the hierarchical optimisation method, both the objective and the order of optimisation of these have a significant impact on the values of the adherence indicators.

A schedule with a mobile shovel fleet gets higher adherence indicators than the ones of a schedule with a fixed shovel fleet. This is because of a schedule with a mobile shovel fleet allows higher shovels allocations to mining faces, compared to a schedule of fixed shovel fleet.

A schedule with stockpiles allows operational continuity of the ore and waste extraction. To illustrate this, consider a shovel allocated in sector f in period t, which extracts ore to the plant. Also, suppose that this loading equipment cannot move from the actual sector. This assumption occurs because either (a) the schedule is generated with a fixed fleet of shovels or (b) the shovel was already allocated in another sector in a period before t. In both scenarios, the shovel cannot be assigned to a different sector in the following periods after period t. In this manner, in period after period t, this shovel faces one of the following scenarios: zero plant capacity and saturated plant capacity. In both scenarios, when there is a stockpile, the shovel can continue with the extraction of ore in the mining face. It simply has to send ore to the stockpile instead of the plant. However, in a scenario with no stockpile, the shovel cannot continue with the ore extraction in the actual period.

5.6 Conclusions and future work

In this study, we propose a MILP-based optimisation model to generate open-pit short-term mine production schedules. This model allocates shovels to different mining faces, including stockpiles. In the proposed model, the stockpiles are explicitly modelled. That is to say, the movement of material to and from each stockpile is scheduled. The model considers the following constraints: plant capacity, ore blending, mining faces precedences, and movement of shovels between mining faces.

The model can optimise different objectives; namely, maximisation of the plant utilisation, maximisation of ore extraction, maximisation of waste extraction, maximisation of the minimum plant utilisation per period, minimisation of the overall cost/time/number of movement of the shovel fleet and minimisation of metal grade deviation in the plant.

We also propose schedule indicators, which assess some objectives of the short-term schedules. Thus, mine planners can use these indicators to evaluate and compare multiple short-term schedules.

We apply the proposed optimisation model to a real-scale open-pit mine case study, comprising six exploitation sectors. The extracted material can be ore or waste. The mine operation has one ore processing plant and uses six shovels. The case study has a scheduled horizon of one month, considering ten periods. Each period has a duration between one and four days.

The schedules are generated under different scenarios; namely, single-optimisation or hierarchical optimisation of different short-term objectives, presence or absence of a stockpile, a mobile or a fixed shovel fleet. The objective is to study the impact of the different scenarios on the schedule indicators.

The schedules to generate consider the following short-term objectives: maximisation of plant utilisation, maximisation of waste extraction, and maximisation of plant utilisation for ore sent directly from the mine. We apply the single-optimisation method and the hierarchical method to optimise them.

The results of the case study show that: (a) the hierarchical method can generate short-term mine production schedules optimising the considered objectives, (b) when applying the hierarchical optimisation method, both the objectives and the order of optimisation of these have a great impact on the values of the different schedule indicators, (c) in general, schedules with a stockpile obtain higher schedule indicators compared to the ones with no stockpile, and (d) schedules with a mobile shovel fleet obtain higher schedules' indicators than the ones with a fixed shovel fleet.

As future work, we want to incorporate more aspects of the mining operation in the optimisation model such as: scheduled shovel maintenance, allocation of drilling rigs to mining faces, multiple ore processing plants, multiple stockpiles, and grade blending in the stockpiles. We also intend to simulate the short-term mine production schedule generated by the optimisation model. We plan to apply discrete-event simulation to assess the probability of compliance of the schedule.

Acknowledgements

This article is part of F. Manríquez's PhD thesis. The authors would like to thank the following people for their support in this article: To Dr Carlos Arroyo for asking the consideration of stockpiles in the optimisation model proposed. To Luis Navea for verifying the results of the experiments conducted in this article. To Daniel Mariño for providing some of the articles reviewed in the Related Work section and for reviewing the optimisation model. To Dr Xavier Emery for proofreading this paper. Finally, to Vilson Reis and Wesley Almeida for providing the real-data of the case study.

Disclure statement

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this article.

Funding

This article was partially funded by the CONICYT/PIA Project AFB180004 and by the CONICYT PFCHA/DOCTORADO BECAS CHILE/2019 - 21190201.

5.7 Appendix: Optimisation model parameters

Parameter	Description
TM_f	Total material to be mined in mining face f (in tonnes).
OM_f	Equal to 1 if the material in the mining face f is ore, 0 otherwise.
ST_f	Equal to 1 if the mining face f is modelled as a stockpile, 0 otherwise.
$OG_{j,f}$	Fraction of the content j in the mining face f.
$FA_{p,f}$	Equal to 1 if shovel p can be allocated in the mining face f, 0 otherwise.
TT_t	Total time in period t (in hours).
$UT_{p,t}$	Maximum utilisation of shovel p in period t.
$RM_{p,f}$	Maximum throughput of mined material by the shovel p in the mining face f (in tonnes per hour).
PC_t^+	Total capacity of the ore processing plant in period t (in tonnes).
PC_t^-	Minimum desired tonnage to be sent to the ore processing plant in period t (in tonnes).
MG_j	Maximum grade of the content j to be sent to the ore processing plant in period t.
mG_j	Minimum grade of content j to be sent to the ore processing plant in period t.
EG_j	Expected grade of the content j by the ore processing plant (in percentage).
$MT_{p,r}$	Movement time of shovel p along the sectors of route r (in hours).
$MC_{p,r}$	Movement cost of shovel p along the sectors of route r (in US dollars).
NM_r	Number of movements between sectors of route r.
MM_p	Maximum number of movements between sector of shovel p over the scheduling horizon.
MM	A very large number.

Table 5.17: Optimisation model parameters (1)

Parameter	Description
ΔO^*	Deviation of ore sent from the mine to the ore processing plant (in tonnes).
ΔP^*	Deviation of ore sent from the mine and stockpile to the ore processing plant (in tonnes).
ΔW^*	Deviation of waste sent to the waste dump and the waste content in the mine (in tonnes).
ΔD^*	Deviation between the processing plant capacity and the ore send to processing plant (in tonnes).
ΔG_j^*	Deviation of the content j between the processing plant capacity and the ore content j send to processing plant.
ΔMT^*	Movement time of shovel p along the sectors of route r (in hours).
ΔMC^*	Movement cost of shovel p along the sectors of route r (in US dollars).
ΔNM^*	Maximum number of movements between sector of shovel p over the scheduling horizon.

Table 5.18: Optimisation model parameters (2)

Parameter	Description
O^{ε}	Tolerance of the deviation ΔO^{*}.
P^{ε}	Tolerance to the deviation ΔP^{*}.
W^{ε}	Tolerance to the deviation ΔW^{*}.
D^{ε}	Tolerance to the deviation ΔD^{*}.
G_j^{ε}	Tolerance to the deviation ΔG_j^{*}.
MT^{ε}	Tolerance to the deviation ΔMT^{*}.
MC^{ε}	Tolerance to the deviation ΔMC^{*}.
NM^{ε}	Tolerance to the deviation ΔNM^{*}.
O^{μ}	Equal to 1 if the upper bound of the deviation ΔO is considered; otherwise, 0.
P^{μ}	Equal to 1 if the upper bound of the deviation ΔP is considered; otherwise, 0.
W^{μ}	Equal to 1 if the upper bound of the deviation ΔW is considered; otherwise, 0.
D^{μ}	Equal to 1 if the upper bound of the deviation ΔD is considered; otherwise, 0.
G_j^{μ}	Equal to 1 if the upper bound of the deviation ΔG_j is considered; otherwise, 0.
MT^{μ}	Equal to 1 if the upper bound of the deviation ΔMT is considered; otherwise, 0.
MC^{μ}	Equal to 1 if the upper bound of the deviation ΔMC is considered; otherwise, 0.
NM^{μ}	Equal to 1 if the upper bound of the deviation ΔNM is considered; otherwise, 0.

Table 5.19: Optimisation model parameters (3)

5.8 Appendix: Single and hierarchical schedules

		Fixed fleet		Mobile fleet	
Indicator	Stockpile	ΔP	$\Delta W(\Delta P)$	ΔP	$\Delta W(\Delta P)$
$C(W)$	No	62	62	65	94
	Yes	62	62	62	62
$C(P)$	No	73	73	90	90
	Yes	73	73	95	95
$C(O)$	No	73	73	90	90
	Yes	44	73	68	50
$C(R)$	No	0	0	0	0
	Yes	29	0	27	45
$C(S)$	No	0	0	0	0
	Yes	29	7	27	45
$C(M)$	No	73	73	90	90
	Yes	73	80	95	95

Table 5.20: Schedule indicators (in percentage) of scenarios generated using $\min \Delta P$ and $\min \Delta W(\Delta P)$ optimisation scheme

		Fixed fleet		Mobile fleet	
Indicator	Stockpile	ΔW	$\Delta P(\Delta W)$	ΔW	$\Delta P(\Delta W)$
$C(W)$	No	96	96	98	98
	Yes	96	96	100	100
$C(P)$	No	20	53	72	73
	Yes	0	53	0	86
$C(O)$	No	20	53	72	73
	Yes	0	53	0	86
$C(R)$	No	0	0	0	0
	Yes	0	0	0	0
$C(S)$	No	0	0	0	0
	Yes	20	0	72	14
$C(M)$	No	20	53	72	73
	Yes	20	53	72	100

Table 5.21: Schedule indicators (in percentage) of scenarios generated using $\min \Delta W$ and $\min \Delta P(\Delta W)$ optimisation scheme

		Fixed fleet		Mobile fleet	
Indicator	Stockpile	ΔO	$\Delta W(\Delta O)$	ΔO	$\Delta W(\Delta O)$
$C(W)$	No	62	62	65	94
	Yes	62	62	62	62
$C(P)$	No	73	73	90	90
	Yes	73	73	94	94
$C(O)$	No	73	73	90	90
	Yes	73	73	94	94
$C(R)$	No	0	0	0	0
	Yes	0	0	0	0
$C(S)$	No	0	0	0	0
	Yes	0	7	1	1
$C(M)$	No	73	73	90	90
	Yes	73	80	95	95

Table 5.22: Schedule indicators (in percentage) of scenarios generated using $\min \Delta O$ and $\min \Delta W(\Delta O)$ optimisation scheme

		Fixed fleet		Mobile fleet	
Indicator	Stockpile	ΔW	$\Delta O(\Delta W)$	ΔW	$\Delta O(\Delta W)$
$C(W)$	No	96	96	98	98
	Yes	96	96	100	100
$C(P)$	No	20	53	72	73
	Yes	0	53	0	86
$C(O)$	No	20	53	72	73
	Yes	0	53	0	86
$C(R)$	No	0	0	0	0
	Yes	0	0	0	0
$C(S)$	No	0	0	0	0
	Yes	20	7	72	14
$C(M)$	No	20	53	72	73
	Yes	20	60	72	100

Table 5.23: Schedule indicators (in percentage) of scenarios generated using $\min \Delta W$ and $\min \Delta O(\Delta W)$ optimisation scheme

Chapter 6

Conclusions

In this section we presented the conclusions and the future work of each one of the articles of this thesis.

6.1 A simulation-optimization framework for short-term underground mine production scheduling

The deviation between mine schedules and the mine operation results are crucial problems that affect the mining industry. Therefore, the mine engineers should generate a mine production schedule that can be reproduced in reality. Hence, they should develop mine production schedules that exhibit high adherence.

In this study, we proposed a generic framework to increase adherence to a short-term mine production schedule by combining optimization and simulation using an iterative approach. This framework comprises the following steps. First, an initial mine schedule is generated based on the resolution of a mixed-integer linear optimization problem. Second, this schedule is simulated using a DES model. Third, a new short-term mine schedule is created using the optimization model by considering the new utilization KPIs of each equipment, obtained from the simulations performed in the previous step, as inputs for the mine operation. Finally, iterations of the second step are performed. In each iteration, adherence to each mine schedule is evaluated with respect to the corresponding simulations by evaluating several adherence indices.

The proposed framework was applied to a real-scale B&F mine. The mine planning horizon was more than a year and a half, and each period lasted for one month. A total of five iterations were performed.

We measure the discrepancies among the level of movement of material with respect to the schedule obtained from the optimization model and the average of the simulated schedule using the mine schedule material's adherence index. The values of this index decreased with the iterations, from 13.1% in the first iteration to 4.8% in the last iteration. This improvement is explained because the effects of the operational uncertainty within the optimization model can be considered by integrating the simulation.

The outcomes of the work presented in this study demonstrate that the proposed framework improved the mine schedule adherence indices over iterations and simultaneously maintained the NPV of the mine schedule. The results demonstrate that the simulation provides a better understanding of the impacts of uncertainty in short-term mine production schedules.

As future research, the proposed framework will be applied to massive and selective underground mining methods as well as open-pit mines.

6.2 Short-term open-pit mine production scheduling with hierarchical objectives

Short-term scheduling in open-pit mines needs several objectives to be optimized jointly. In some open-pit mine operations, these objectives are ranked in a descending order of importance: (i) the minimization of the deviation between the ore sent to the ore processing plant and the ore processing capacity of the plant, (ii) the minimization of the deviation between the metal fines sent to the ore processing plant and the metal fines expected by the ore processing plant and finally (iii) the overall minimization of the movement of the shovel fleet. This work proposes an optimization methodology to generate a short-term open-pit mine production schedule optimizing multiple hierarchical objectives. For the generation of mine schedules, we propose an optimization model based on mixed-integer linear programming. This model considers mining sequencing constraints, and also takes into account both time and cost of shovels movement between phases. For the optimization of the various short-term objectives, we apply the hierarchical method and the weighted sum method to a real open-pit mine case study.

The results of the case study show that both methods are capable of generating short-term mine schedules by optimizing the various short-term objectives. Additionally, we verified that both methods obtain the same mine production schedule. This article shows the importance and impact of multiple objective optimization methods for the generation of short-term mine production schedules in open-pit mines.

6.3 Short-term open-pit production scheduling optimising multiple objectives accounting for shovel allocation in stockpiles

In this study, we propose a MILP-based optimisation model to generate open-pit short-term mine production schedules. This model allocates shovels to different mining faces, including stockpiles. In the proposed model, the stockpiles are explicitly modelled. That is to say, the movement of material to and from each stockpile is scheduled. The model considers the following constraints: plant capacity, ore blending, mining faces precedences, and movement of shovels between mining faces.

The model can optimise different objectives; namely, maximisation of the plant utilisation, maximisation of ore extraction, maximisation of waste extraction, maximisation of the minimum plant utilisation per period, minimisation of the overall cost/time/number of movement of the shovel fleet and minimisation of metal grade deviation in the plant.

We also propose schedule indicators, which assess some objectives of the short-term schedules. Thus, mine planners can use these indicators to evaluate and compare multiple short-term schedules.

We apply the proposed optimisation model to a real-scale open-pit mine case study, comprising six exploitation sectors. The extracted material can be ore or waste. The mine operation has one ore processing plant and uses six shovels. The case study has a scheduled horizon of one month, considering ten periods. Each period has a duration between one and four days.

The schedules are generated under different scenarios; namely, single-optimisation or hierarchical optimisation of different short-term objectives, presence or absence of a stockpile, a mobile or a fixed shovel fleet. The objective is to study the impact of the different scenarios on the schedule indicators.

The schedules to generate consider the following short-term objectives: maximisation of plant utilisation, maximisation of waste extraction, and maximisation of plant utilisation for ore sent directly from the mine. We apply the single-optimisation method and the hierarchical method to optimise them.

The results of the case study show that: (a) the hierarchical method can generate short-term mine production schedules optimising the considered objectives, (b) when applying the hierarchical optimisation method, both the objectives and the order of optimisation of these have a great impact on the values of the different schedule indicators, (c) in general, schedules with a stockpile obtain higher schedule indicators compared to the ones with no stockpile, and (d) schedules with a mobile shovel fleet obtain higher schedules' indicators than the ones with a fixed shovel fleet.

As future work, we want to incorporate more aspects of the mining operation in the optimisation model such as: scheduled shovel maintenance, allocation of drilling rigs to bench-sectors, multiple ore processing plants, multiple stockpiles, and grade blending in the stockpiles. We also intend to simulate the short-term mine production schedule generated by the optimisation model. We plan to apply discrete-event simulation to assess the probability of compliance of the schedule. Finally, we also plan to develop a stochastic integer model that incorporates uncertainty in geological and operational parameters based on the optimisation model described in this article.

6.4 Funding

This thesis was partially funded by the CONICYT/PIA Project AFB180004 and by the CONICYT PFCHA/DOCTORADO BECAS CHILE/2019 - 21190201.

Chapter 7

Bibliography

Akaike A, Dagdelen K (1999) A strategic production scheduling method for an open pit mine. Proceedings of the 28th Application of Computers and Operation Research in the Mineral Industry pp 729–738

Albor Consuegra FR, Dimitrakopoulos R (2009) Stochastic mine design optimisation based on simulated annealing: pit limits, production schedules, multiple orebody scenarios and sensitivity analysis. Mining Technology 118(2):79–90, DOI 10.1179/037178409X12541250836860

Asad MW (2005) Cutoff grade optimization algorithm with stockpiling option for open pit mining operations of two economic minerals. International Journal of Surface Mining, Reclamation and Environment 19(3):176–187, DOI 10.1080/13895260500258661

Askari-Nasab H, Frimpong S, Szymanski J (2007) Modelling open pit dynamics using discrete simulation. International Journal of Mining, Reclamation and Environment 21(1):35–49, DOI 10.1080/17480930600720206

Ataeepour N, Baafi E (1999) Arena simulation model for truck-shovel operation in despatching and non-despatching modes. International Journal of Surface Mining, Reclamation and Environment 13(3):125–129, DOI 10.1080/09208119908944228

Baker K, Trietsch D (2009) Principles of Sequencing and Scheduling. John Wiley and Sons

Bertsimas D, Tsitsiklis JN (1997) Introduction to linear optimization, vol 6. Athena Scientific Belmont, MA

Birge J, Louveaux F (2011) Introduction to stochastic programming. Springer Series in Operations Research and Financial Engineering, Springer New York, DOI 10.1007/978-1-4614-0237-4

Bley A, Boland N, Froyland G, Zuckerberg M (2012) Solving mixed integer nonlinear programming problems for mine production planning with stockpiling. Optimization Online (http://www optimization-online org/DB_HTML/2012/11/3674 html)

Blom M, Pearce AR, Stuckey PJ (2017) Short-term scheduling of an open-pit mine with multiple objectives. Engineering Optimization 49(5):777–795, DOI 10.1080/0305215X.2016.1218002

Blom M, Pearce AR, Stuckey PJ (2018) Short-term planning for open pit mines: a review. International Journal of Mining, Reclamation and Environment 0930:1–22, DOI 10.1080/17480930.2018.1448248, URL http://doi.org/10.1080/17480930.2018.1448248

Bodon P, Fricke C, Sandeman T, Stanford C (2011) Modeling the mining supply chain from mine to port: A combined optimization and simulation approach. Journal of Mining Science 47(2):202–211, DOI 10.1017/CBO9781107415324.004, URL http://www.scopus.com/inward/record.url?eid=2-s2.0-80855138623{&}partnerID=40{&}md5=e66f32bcdd24b335980ce233123e111f, arXiv:1011.1669v3

Çetin N (2014) Open-pit truck/shovel haulage system simulation. PhD thesis, Middle East Technical University

Chanda EC (1990) An application of integer programming and simulation to production planning for a stratiform ore body. Mining Science and Technology 11(2):165–172, DOI 10.1016/0167-9031(90)90318-M, URL https://www.sciencedirect.com/science/article/pii/016790319090318M

Chen CH, Yücesan E (2005) An alternative simulation budget allocation scheme for efficient simulation. International Journal of Simulation and Process Modeling 1(1/2):49–57

Chen CH, Lin J, Yücesan E, Chick SE (2000) Simulation budget allocation for further enhancing the efficiency of ordinal optimization. Discrete Event Dynamic Systems: Theory and Applications 10:251–270

Chen CH, Fu M, Shi L (2008a) Simulation and optimization. In: Chen Z, Raghavan S (eds) Tutorials in Operations Research INFORMS 2008, INFORMS, Hanover, MD, pp 247–260, DOI 10.1287/educ.1080.0050

Chen CH, He D, Fu M, Lee LH (2008b) Efficient simulation budget allocation for selecting an optimal subset. INFORMS Journal on Computing 20(4):579–595

Chen HC, Dai L, Chen Ch, Yücesan E (1997) New development of optimal computing budget allocation for discrete event simulation. In: Proceedings of the 29th conference on Winter simulation, Institute of Electrical and Electronics Engineers, Piscataway, NJ, pp 334–341, DOI 10.1145/268437.268501

Chick SE, Inoue K (2001a) New procedures to select the best simulated system using common random numbers. Manage Sci 47(8):1133–1149, DOI 10.1287/mnsc.47.8.1133.10229

Chick SE, Inoue K (2001b) New two-stage and sequential procedures for selecting the best simulated system. Operations Research 49(5):732–743

Dimitrakopoulos R, Farrelly C, Godoy M (2002) Moving forward from traditional optimization: grade uncertainty and risk effects in open-pit design. Mining Technology 111(1):82–

88, DOI 10.1179/mnt.2002.111.1.82

Eivazy H, Askari-Nasab H (2012) A mixed integer linear programming model for short-term open pit mine production scheduling. Mining Technology 121(2):97–108, DOI 10.1179/1743286312Y.0000000006

Fioroni M, Franzese L, Bianchi T, Ezawa L, Pinto L, de Miranda G (2008) Concurrent simulation and optimization models for mining planning. In: 2008 Winter Simulation Conference, pp 759–767, DOI 10.1109/WSC.2008.4736138

Forsman B, Rönnkvist E, Vagenas N (1993) Truck dispatch computer simulation in Aitik open pit mine. International Journal of Surface Mining, Reclamation and Environment 7(3):117–120, DOI 10.1080/17480939308547571, URL https://doi.org/10.1080/17480939308547571

Fu MC, Hu JQ (1997) Conditional Monte Carlo: gradient estimation and optimization applications. Springer US, Boston, MA, DOI 10.1007/978-1-4615-6293-1_1

Fu MC, Hu JQ, Chen Ch, Xiong X (2007) Simulation allocation for determining the best design in the presence of correlated sampling. INFORMS Journal on Computing 19(1):101–111, DOI 10.1287/ijoc.1050.0141

Fu Z, Asad MWA, Topal E (2019) A new model for open-pit production and waste-dump scheduling. Engineering Optimization 51(4):718–732, DOI 10.1080/0305215X.2018.1476501, URL https://doi.org/0305215X.2018.1476501

Glasserman P (1991) Gradient estimation via perturbation analysis. Kluwer Academic Publishers

Glynn PW (1987) Likelihood ratio gradient estimation: an overview. In: Proceedings of the 1987 Winter Simulation Conference, Institute of Electrical and Electronics Engineers (IEEE), pp 366–375, DOI http://doi.acm.org/10.1145/318371.318612

Grodzevich O, Romanko O (2006) Normalization and other topics in multi-objective optimization. In: Fields-MITACS Industrial Problems Workshop, Toronto, Canada, pp 89–101

Gurobi Optimization L (2019) Gurobi optimizer reference manual. URL http://www.gurobi.com

Hashemi A, Sattarvand J (2015) Application of arena simulation software for evaluation of open pit mining transportation systems – a case study. In: Niemann-Delius C (ed) Proceedings of the 12th International Symposium Continuous Surface Mining - Aachen 2014, Springer International Publishing, Cham, pp 213–224

Ho YC, Cao XR (1991) Perturbation analysis of discrete event dynamic systems. Springer US, Boston, MA, DOI 10.1007/978-1-4615-4024-3

Hoerger S, Seymour F, Hoffman L (1999) Mine planning at Newmont's Nevada operations. Mining engineering 51(10):26–30

Huang Z, Cai W, Banfield A (2009) A new short- and medium-term production planning tool - Minesight Schedule Optimizer (MSSO). In: SME annual meeting & exhibit, Denver, CO, pp 491–495

Johnson TB (1968) Optimum open pit mine production scheduling. Tech. rep., California Univ Berkeley Operations Research Center

Jupp K, Howard TJ, Everett JE (2014) Role of pre-crusher stockpiling for grade control in iron ore mining. Transactions of the Institutions of Mining and Metallurgy, Section B: Applied Earth Science 122(4):242–255, DOI 10.1179/1743275814Y.0000000045

Kim SH, Nelson BL (2006) Chapter 17 selecting the best system. In: Henderson SG, Nelson BL (eds) Simulation, Handbooks in Operations Research and Management Science, vol 13, Elsevier, pp 501 – 534, DOI https://doi.org/10.1016/S0927-0507(06)13017-0

Koushavand B, Askari-Nasab H, Deutsch CV (2014) A linear programming model for long-term mine planning in the presence of grade uncertainty and a stockpile. International Journal of Mining Science and Technology 24(4):451–459, DOI 10.1016/j.ijmst.2014.05.006, URL `http://dx.doi.org/10.1016/j.ijmst.2014.05.006`

Lamghari A, Dimitrakopoulos R (2016) Network-flow based algorithms for scheduling production in multi-processor open-pit mines accounting for metal uncertainty. European Journal of Operational Research 250(1):273–290, DOI 10.1016/j.ejor.2015.08.051, URL `http://dx.doi.org/10.1016/j.ejor.2015.08.051`

Lane KF, Hamilton DJ, Parker JJB (1984) Cutoff grades for two minerals. In: Application of Computers and Mathematics in the Minerals Industries. International symposium. 18, pp 485–491

Lee LH, Chew EP, Teng S, Goldsman D (2004) Optimal computing budget allocation for multi-objective simulation models. In: Ingalls RG, Rossetti MD, Smith JS, Peters BA (eds) Procedings of 2004 Winter Simulation Conference, IEEE, Piscataway, NJ, pp 586–594, DOI 10.1109/WSC.2004.1371365

Levinson Z, Dimitrakopoulos R (2019) Simultaneous stochastic optimisation of an open-pit gold mining complex with waste management. International Journal of Mining, Reclamation and Environment 00(00):1–15, DOI 10.1080/17480930.2019.1621441, URL `https://doi.org/10.1080/17480930.2019.1621441`

L'Heureux G, Gamache M, Soumis F (2013) Mixed integer programming model for short term planning in open-pit mines. Mining Technology 122(2):101–109, DOI 10.1179/1743286313Y.0000000037

Lizotte Y, Bonates E (1987) Truck and shovel dispatching rules assessment using simulation. Mining Science and Technology 5(1):45–58, DOI 10.1016/S0167-9031(87)90910-8, URL `https://www.sciencedirect.com/science/article/pii/S0167903187909108`

Manríquez F, González H, Morales N (2019) Short-term open-pit mine production scheduling with hierarchical objectives. In: Mining Goes Digital - Proceedings of the 39th international symposium on Application of Computers and Operations Research in the Mineral Industry, APCOM 2019, pp 443–451, DOI 10.1201/9780429320774-51

Manríquez F, González H, Morales N (2020a) Short-term open-pit production scheduling optimising multiple objectives accounting for shovel allocation in stockpiles. Engineering Optimization (submitted)

Manríquez F, Pérez J, Morales N (2020b) A simulation–optimization framework for short-term underground mine production scheduling. Optimization and Engineering DOI 10.1007/s11081-020-09496-w, URL http://link.springer.com/10.1007/s11081-020-09496-w

Matamoros ME, Dimitrakopoulos R (2016) Stochastic short-term mine production schedule accounting for fleet allocation, operational considerations and blending restrictions. European Journal of Operational Research 255(3):911–921, DOI 10.1016/j.ejor.2016.05.050, URL http://dx.doi.org/10.1016/j.ejor.2016.05.050

Meech J, Parreira J (2011) An interactive simulation model of human drivers to study autonomous haulage trucks. Procedia Computer Science 6:118–123, DOI 10.1016/j.procs.2011.08.023, URL http://dx.doi.org/10.1016/j.procs.2011.08.023

Mena R, Zio E, Kristjanpoller F, Arata A (2013) Availability-based simulation and optimization modeling framework for open-pit mine truck allocation under dynamic constraints. International Journal of Mining Science and Technology 23(1):113 – 119, DOI https://doi.org/10.1016/j.ijmst.2013.01.017

Moradi Afrapoli A, Askari-Nasab H (2017) Mining fleet management systems: a review of models and algorithms. International Journal of Mining, Reclamation and Environment 0(0):1–19, DOI 10.1080/17480930.2017.1336607

Morales N, Nelis G, Quiroz J, Rocher R (2015) UDESS - A multipurpose scheduling problem for mine planning. In: 2015 CORS/INFORMS International Conference

Moreno E, Rezakhah M, Newman A, Ferreira F (2017) Linear models for stockpiling in open-pit mine production scheduling problems. European Journal of Operational Research 260(1):212–221, DOI 10.1016/j.ejor.2016.12.014, URL http://dx.doi.org/10.1016/j.ejor.2016.12.014

Mousavi A, Kozan E, Liu SQ (2016) Open-pit block sequencing optimization: A mathematical model and solution technique. Engineering Optimization 48(11):1932–1950, DOI 10.1080/0305215X.2016.1142080, URL https://doi.org/10.1080/0305215X.2016.1142080

Musingwini C (2016) Optimization in underground mine planning- developments and opportunities. Journal of the Southern African Institute of Mining and Metallurgy 116(9):809–820, DOI 10.17159/2411-9717/2016/v116n9a1

Nehring M, Knights PF, Kizil MS, Hay E (2018) A comparison of strategic mine planning approaches for in-pit crushing and conveying, and truck/shovel systems. International Journal of Mining Science and Technology 28(2), DOI 10.1016/j.ijmst.2017.12.026

Newman A, Rubio E, Caro R, Weintraub A, Eurek K (2010a) A review of operations research in mine planning. Interfaces 40(3):222–245, DOI 10.1287/inte.1090.0492

Newman A, Rubio E, Caro R, Weintraub A, Eurek K (2010b) A review of operations research in mine planning. Interfaces 40(3):222–245, DOI 10.1287/inte.1090.0492

Oliphant T (1995) *NumPy: Random sampling.* https://docs.scipy.org/doc/numpy-1.14.1/reference/routines.random.html

Panagiotou GN (1999) Discrete mine system simulation in Europe. International Journal of Surface Mining, Reclamation and Environment 13(2):43–46, DOI 10.1080/09208119908944208, URL http://www.tandfonline.com/doi/abs/10.1080/09208119908944208

Pérez J, Morales N, Widzyk-Capehart E, Rojas V, Vallejos J (2017) Short and medium term mine planning in selective underground mine considering equipment performance. In: Behzad Ghodrati UK, Schunneson H (eds) Proceedings of the 26th International Symposium on Mine Planning and Equipment Selection, Luleå, Sweden, pp 281–286

Pflug GC (1989) Sampling derivatives of probabilities. Computing 42(4):315–328, DOI 10.1007/BF02243227

Pflug GC (1996) Optimization of stochastic models. Springer US, Boston, MA

Ramazan S, Dimitrakopoulos R (2013) Production scheduling with uncertain supply: A new solution to the open pit mining problem. Optimization and Engineering DOI 10.1007/s11081-012-9186-2

Rasche T, Sturgul JR (1991) A simulation to assist a small mine: A case study. International Journal of Surface Mining, Reclamation and Environment 5(3):123–128, DOI 10.1080/09208119108944297, URL https://doi.org/10.1080/09208119108944297

Rehman SU, Asad MWA (2010) A mixed-integer linear programming (MILP) model for short-range production scheduling of cement quarry operations. Asia-Pacific Journal of Operational Research 27(3):315–333, DOI 10.1142/S0217595910002727

Rezakhah M, Newman A (2020) Open pit mine planning with degradation due to stockpiling. Computers and Operations Research 115, DOI 10.1016/j.cor.2018.11.009, URL https://doi.org/10.1016/j.cor.2018.11.009

Rezakhah M, Moreno E, Newman A (2020) Practical performance of an open pit mine scheduling model considering blending and stockpiling. Computers and Operations Research 115:104638, DOI 10.1016/j.cor.2019.02.001, URL https://doi.org/10.1016/j.cor.2019.02.001

Rist K (1961) The solution of a transportation problem by use of a monte carlo technique. In: APCOM 1961 - 1st International Symposium on Application of Computers and Operations Research in the Minerals Industries, pp L2.1–L2.15

Robinson GK (2004) How much would a blending stockpile reduce variation? In: Chemometrics and Intelligent Laboratory Systems, Elsevier, vol 74, pp 121–133, DOI 10.1016/j.chemolab.2004.03.010

Rubenstein RY, Shapiro A (1993) Discrete event systems: sensitivity analysis and stochastic optimization by the score function method. Wiley, New York, USA

Salama A, Nehring M, Greberg J (2014) Operating value optimisation using simulation and mixed integer programming. International Journal of Mining, Reclamation and Environment 28(1):25–46, DOI 10.1080/17480930.2013.768019

Samavati M, Essam D, Nehring M, Sarker R (2018) A new methodology for the open-pit mine production scheduling problem. Omega (United Kingdom) 81(August 2018):169–182, DOI 10.1016/j.omega.2017.10.008, URL `https://doi.org/10.1016/j.omega.2017.10.008`

Sandeman T, Fricke C, Bodon P, Stanford C (2010) Integrating optimization and simulation - a comparison of two case studies in mine planning. In: Winter Simulation Conference, Baltimore, MD, pp 1898–1910

Silva MF, Dimitrakopoulos R, Lamghari A (2015) Solving a large SIP model for production scheduling at a gold mine with multiple processing streams and uncertain geology. Transactions of the Institutions of Mining and Metallurgy, Section A: Mining Technology 124(1):24–33, DOI 10.1179/1743286314Y.0000000075

Smith ML (1998) Optimizing short-term production schedules in surface mining: Integrating mine modeling software with ampl/cplex. International Journal of Surface Mining, Reclamation and Environment 12(4):149–155, DOI 10.1080/09208118908944038

Sturgul JR (1987) Simulating mining engineering problems using the GPSS computer language. Bulletin of the Proceedings of the Australian Institute of Mining and Metallurgy 292(4):75–78

Torkamani E, Askari-Nassab H (2015) A linkage of truck-and-shovel operations to short-term mine plans using discrete-event simulation. International Journal of Mining and Mineral Engineering 6:97

Upadhyay SP, Askari-Nasab H (2016) Truck-shovel allocation optimisation: a goal programming approach. Mining Technology 9009:1–11, DOI 10.1179/1743286315Y.0000000024

Upadhyay SP, Askari-Nasab H (2017) Dynamic shovel allocation approach to short-term production planning in open-pit mines. International Journal of Mining, Reclamation and Environment 0(0):1–20, DOI 10.1080/17480930.2017.1315524

Upadhyay SP, Askari-Nasab H (2018) Simulation and optimization approach for uncertainty-based short-term planning in open pit mines. International Journal of Mining Science and Technology 28(2):153–166, DOI 10.1016/j.ijmst.2017.12.003

Van Rossum G, Drake Jr FL (1995) Python tutorial. Centrum voor Wiskunde en Informatica Amsterdam, The Netherlands

Winkler B (1998) A system for quality oriented mine production planning with molp. In: APCOM 98: Computer Applications in the Mineral Industries International Symposium, pp 53–59