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The effects of ambiguity on entrepreneurship

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Abstract

We incorporate ambiguity (Knightian uncertainty) into a classic model of entrepreneurship to analyze, among other things, its effects on the optimal level of business startups, the relation between total assets and the size of the entrepreneurial investment, the effects of increasing ambiguity on developing new ventures, and the decision to self-select into entrepreneurship for an indifferent decision maker. We first show that, under the monotone-likelihood ratio property, the introduction of ambiguity negatively affects the optimal entrepreneurial investment, something that is consistent with most experimental evidence about entrepreneurial choice under ambiguity. Then, we show that the classical explanations for the positive correlation between total assets and business startups based on decreasing absolute risk aversion preferences and prudent behavior can be challenged when ambiguity is incorporated into the analysis, and we provide the conditions that guarantee that the traditional comparative static result under risk is replicated under ambiguity. We also show that increases in ambiguity aversion reduce entrepreneurial activities. Finally, we discuss our results under alternative ways of modeling ambiguity.

1 | INTRODUCTION

Entrepreneurship is essential to economic development and growth. Entrepreneurship is the engine by which economic agents look for opportunities to innovate and develop new products and services that intensify competition and increase productivity. Therefore, understanding the factors (like risk and uncertainty) that affect entrepreneurship is important not only to economists but also to policymakers and government officials.

Risk (measurable uncertainty) and uncertainty (unmeasurable uncertainty) have been analyzed in the economics of entrepreneurship since Knight (1921) conceptualized the idea that, in principle, less-risk-averse agents will tend to become entrepreneurs because they can bear the risk of a venture more efficiently than more-risk-averse individuals. More-risk-averse agents will then tend to become employees with a sure wage to avoid exposure to entrepreneurial risk. Some authors have challenged this view about the relationship between entrepreneurship and risk aversion (Kan & Tsai, 2006; Vereshchagina & Hopenhayn, 2009). However, most literature accepts the negative relation between risk aversion and entrepreneurship (or self-employment) as a common feature of the economics-of-entrepreneurship field (Ahn, 2009; Ekelund, Johansson, M, & Lichtermann, 2005; Hsieh & Parker, 2017; Kihlstrom & Laffont, 1979; Parker, 1997, 2006).

An interesting previous empirical work that documented the positive relation between business startups (or entrepreneurship) and total assets, instead of risk characteristics, is the work of Evans and Jovanovic (1989). These authors

[Correction added on 21 September 2020, after first online publication: the name of the co-author Pablo G. Cubillos has been corrected to Pablo A. Gutiérrez Cubillos] argue that as the economy develops, the greater quantity of assets relaxes the credit-rationing constraint for some individuals, increasing their likelihood of obtaining financing to forge new ventures, which makes them switch from paid employment to self-employment (entrepreneurship). Cressy (2000) challenged the intuition of that result by arguing that the positive correlation between assets and the rate of business startups comes from decreasing absolute risk aversion (DARA) preferences. In other words, as the economy develops, some individuals accumulate more assets and, consequently, their level of risk aversion decreases. This leads them to switch from safe employment to risky entrepreneurial activities, thus increasing the rate of business startups. More recently, Bonilla and Vergara (2013) have shown that the positive correlation between assets and entrepreneurship is actually due to a more general characteristic of preferences. In particular, they argue that prudent behavior[1] is the main reason behind the positive correlation between assets and entrepreneurship. Prudent behavior has been previously studied in savings theory as a mechanism that explains the empirical finding that agents desire to use a positive fraction of their wealth in more savings when they face an increase in future risk, which is known as the precautionary savings motive (Baiardi, Magnani, & Menegatti, 2020; Kimball, 1990).

Intuitively, risk emerges when an agent makes a choice that makes him face different states of nature, with a given probability associated with each state. In contrast, ambiguity is present when there are different probability distribution functions in the environment, and we make a choice without knowing which distribution is the one we are going to face. This makes optimal decisions more complex.

Recently, the concept of ambiguity has been incorporated into a wide range of economic modeling since Klibanoff, Marinacci, and Mukerji (KMM) (2005) developed a smooth way to work with ambiguity. The smoothness idea provided a simpler manner to deal with ambiguity and expanded its use to a wider range of applications compared to the traditional approach. The traditional approach involved using the maxmin model of Gilboa and Schmeidler (1989)—where individuals maximize the minimum expected utility over the distributions—and the α-maxmin expected utility model of Ghirardato, Maccheroni, and Marinacci (2004)—in which welfare is measured as a weighted average between the minimum and the maximum expected utility levels, given the set of second-order distributions.

It is easy to justify the inclusion of ambiguity in entrepreneurial decisions. For example, consider the case of an agent who decides to start her own business. In this case, she has two levels of uncertainty. The first level is related to the amount of demand for her product. For instance, assume a situation with two possible outcomes: high and low demand as the first (basic) level of uncertainty. The second level of uncertainty relates to how likely it is that demand will be high or low. In other words, the entrepreneur knows neither the final outcome nor the probability distribution underlying the possible outcomes and, therefore, this entrepreneur is facing ambiguity in her venture. Another example of ambiguity applied to entrepreneurship is the usual robustness analysis in the evaluation of new projects. In this case, the net present value of projects can be estimated for bad, average, or good macroeconomic conditions under different probability distributions representing each scenario. The robustness exercise is a way to deal not only with the risk embedded in a new venture, but also with the ambiguity associated with unknown probability distributions of results.

In this context, some natural questions arise. For instance, what is the effect of ambiguity on entrepreneurship? Is that effect different to what we already know from the traditional economic theory of entrepreneurship under risk? Should we have a separate entrepreneurial theory under ambiguity?

The main contribution of this paper is to extend the theory of entrepreneurship under risk, incorporating the concept of ambiguity. In doing so, we provide new insights about the effects of ambiguity—sometimes challenging previous results in the field, but also offering conditions to restore the intuition found in the previous literature. We first show that under certain conditions (monotone-likelihood ratio property [MLRP])[1] the inclusion of ambiguity reduces the optimal level of the entrepreneurial investment, something that negatively affects economic growth and development. Second, we show that, even in the case where we assume that agents are prudent and ambiguity averse, an additional condition is required to guarantee a positive monotonic relationship between total assets and business startups. This condition tells us that there is an upper bound for the absolute ambiguity aversion coefficient that guarantees the positive monotonic relation between total assets and entrepreneurship. Third, we show that increases in ambiguity aversion reduce the pool of entrepreneurs, and we provide the conditions that guarantee that increases in wealth increase the expected return of the marginal entrepreneur. Fourth, we provide conditions on the priors and preferences that guarantee that increases in wealth increase entrepreneurial investment. Finally, we discuss our results under alternative ways of modeling ambiguity.

Our results highlight that the relevance of the study of ambiguity in entrepreneurship is threefold. First, it brings the theory and practice of entrepreneurship closer, since the cases in which the decision maker knows the "real" probability distribution of results are rare. Ambiguity about the distribution of results and making decisions without all the information are essential parts of entrepreneurship. Incorporating ambiguity into economic models of entrepreneurship reduces the gap between theory and practice.

Second, theoretical models of entrepreneurship help to highlight new aspects of the decision-making process of an individual evaluating shifting occupations from secure employment to entrepreneurship. Formalization of this individual decision-making process under ambiguity contributes new comparative statics providing insights that can be tested experimentally in future research.

Finally, the effect of ambiguity on entrepreneurship has implications for economic growth and development—for instance, in an economy with low social capital and high political polarization where the "rules of the game" change too often, such as a Latin American country governed by a populist leader. In this context, political and economic ambiguity will probably induce very low levels of investment, which in turn imply negative consequences and effects on growth and development. This is a new and interesting way to look at poverty traps as a consequence of ambiguous environments that affect ambiguity-averse entrepreneurs and their desire to start new ventures. Some research in this area is being developed (Brach & Spanjers, 2014).

2 | RELATED LITERATURE

There are different interpretations of the concept of ambiguity in applied economic models. Ranging from the classical idea of unmeasurable uncertainty developed by Knight (1921) and vague probabilities developed by Savage (1954), to the ambiguous probabilities presented in Ellsberg (1961)[1], passing through the subjective expected utility (SEU) explained in detail in Camerer and Weber (1992). Each of these interpretations rests on two main ideas. First, no objective probability distribution is observed by decision makers, and second, individuals develop subjective probability distributions that are personal assessments about the true probability distribution underlying the economy. These two characteristics are typical components found in the world of entrepreneurship and, therefore, incorporating ambiguity into entrepreneurial research seems like a natural next step.

An interesting way to test ambiguity is by using an idea that can be traced back to the seminal work of Kahneman and Tversky (1982) on variants of uncertainty and that in today's risk literature has evolved into what is called the source method. Source preferences (Chew & Sagi, 2008) allow the agents to distinguish between different sources of uncertainty. Abdellaoui, Baillon, Placido, and Wakker (2011) developed an application of this method where they did not commit to any particular ambiguity attitude but instead let the data speak in an effort to capture the richness of ambiguity. Interestingly, they exploit within-source uniformity while allowing between-source heterogeneity, and they conclude that the attitude of individuals toward risk and uncertainty depends not only on their personal characteristics, but also on the source that created the uncertainty.

Recent experimental evidence suggests that the phenomenon of ambiguity has important effects on the behavior of entrepreneurs, and its effects are different from those coming from their attitudes toward risk. Therefore, testing ambiguity becomes an important part of recent experimental literature on risk and uncertainty.

Shyti and Paraschiv (2014) and Shyti (2013) compare the effect of ambiguity between a group of entrepreneurs and nonentrepreneurs. They observe that entrepreneurs give more pessimistic evaluations of ambiguity compared to risk, while nonentrepreneurs do not discriminate between them. Dommock, Kouwenberg, and Wakker (2015) show that ambiguity has real effects on stock market participation as well. These results are important because they highlight the need to have a modified framework to analyze decision-making under ambiguity.

A different perspective is given by Macko and Tyszka (2009) from the applied psychology literature, who provide experimental evidence that entrepreneurs are not more risk tolerant than nonentrepreneurs. Instead, they show that entrepreneurs have two key features: they have a high level of self-confidence, and they are more prone to risky choices in ambiguous environments. Macko and Tyszka (2009) argue that entrepreneurship is mainly based on the psychological characteristics of individuals.

Bengtsson, Sanandaji, and Johannesson (2012) investigate risk and ambiguity for entrepreneurs and nonentrepreneurs in a gender study, finding that entrepreneurs are less risk and ambiguity averse than nonentrepreneurs. Contradicting evidence is provided by Holm, Nee, and Opper (2013), who compare 700 entrepreneurs and 200 nonentrepreneurs from China to conclude that entrepreneurs are more willing to accept strategic uncertainty (competition and trust), but they do not differ from ordinary people when it comes to nonstrategic uncertainty such as risk and ambiguity.

Finally, Koudstaal, Sloof, and van Praag (2015) show that even though entrepreneurs perceive themselves as more risk tolerant than nonentrepreneurs, when such a belief is tested using elicitation methods in experiments, their attitudes toward risk and ambiguity are no different. However, entrepreneurs seems to be less loss averse than the control groups.

In conclusion, all this mixed evidence makes it even more important to try to incorporate ambiguity more formally into the applied economics model of entrepreneurship to highlight the main forces of behavior behind ambiguity.

3 | THE MODEL

3.1 | Cressy's model of entrepreneurship

Following Cressy (2000), consider an agent with a utility function $u(\cdot)$: $\mathbb{R}_+ \to \mathbb{R}$ that is three times differentiable. Let $u(\cdot)$ be increasing ($u' > 0$), concave ($u'' < 0$, i.e., risk averse) and prudent ($u''' > 0$). This agent has assets $z \in \mathbb{R}$, which can be invested at a risk-free rate of return r. The agent must choose between two alternatives: (a) to keep working as an employee for a given market wage (w) at no risk, or (b) to become an entrepreneur with a random payment and risk exposure. If the individual decides to stay employed, her certain income will be

$$y_0 = w + rz$$

On the contrary, if the agent decides to become an entrepreneur, she confronts j possible states of nature $(1, 2, ..., j)$. Hence, her random income is[1]

$$y(\tilde{\alpha}, k) \equiv \tilde{y} = \theta\tilde{\alpha}k - r[k - z] = (\theta\tilde{\alpha} - r)k + rz$$

where θ is a productivity factor, k is the capital used for self-employment (entrepreneurship) and $\tilde{\alpha}$ is a random variable that captures the notion that the entrepreneur may end up with any of the j states of nature. So, the random variable $\tilde{\alpha}$ is distributed as $((\alpha^1, p^1), ..., (\alpha^j, p^j))$ with $p^1 > 0, ..., p^j > 0$ and $\sum_{i=1}^{j} p^i = 1$. We will denote the probability distribution for $\tilde{\alpha}$ by $F = (p^1, ..., p^j)$.

As a consequence, when the agent decides to become a self-employed entrepreneur, her decision problem is reduced to choosing the optimal level of capital k to

$$\max_{\{k\}} E(u) = \sum_{i=1}^{j} p^i u(y_i)$$

subject to

$$y_i = (\theta\alpha^i - r)k + rz, \quad \text{for} \quad i = 1, ..., j.$$

3.2 | Ambiguity and entrepreneurship

In this section, we build from the smooth ambiguity model. Following KMM (2005), we assume that the agent has n subjective second-order distributions of $\tilde{\alpha}$, which will be characterized by $\tilde{\alpha}_\psi$, where $\psi = 1, 2, ..., n$. Let us characterize ambiguity as a collection of probability distributions over the random income $\tilde{y}$. Since this random variable can be described by different distributions, we will denote by $\tilde{y}_\psi$ the random income of the entrepreneur distributed by the cumulative probability function F_ψ.

The agent has first-order preferences $u(\cdot)$ over the random payment, but also second-order preferences $\Phi(\cdot)$ over the first-order utilities. The function $\Phi(\cdot)$ is assumed to be increasing ($\Phi' > 0$) and three times differentiable. Ambiguity aversion implies that the function $\Phi(\cdot)$ is strictly concave ($\Phi'' < 0$) and, as a consequence, a less ambiguous environment is preferred to a more ambiguous one. In addition, the decision maker is ambiguity prudent[1] which implies that $\Phi''' > 0$.[1]

The entrepreneur's second order *subjective probabilities* are $(q_1, q_2, ..., q_n)$ over the set of second-order probability distributions $(F_1, F_2, ..., F_n)$ with $q_\psi \geq 0\ \forall\ q_\psi$, and $\sum_{\psi=1}^{n} q_\psi = 1$. Following Cherbonnier and Gollier (2015), the welfare of the lottery is defined as the certainty equivalent of the conditional expected values $U(k,\psi) \equiv E(u(\tilde{y}_\psi))$, which is just a given number. Then,

$$W(k) = \Phi^{-1}\left[\sum_{\psi=1}^{n} q_\psi \Phi(Eu((\theta\tilde{\alpha}_\psi - r)k + rz))\right] = \Phi^{-1}\left[\sum_{\psi=1}^{n} q_\psi \Phi(U(k,\psi))\right], \tag{1}$$

where $W(k)$ is the certainty equivalent of the conditional expected utility $U(k,\psi)$, and since Φ is increasing in $E(u(\tilde{y}_\psi))$, which implies that $\Phi^{-1}(\cdot)$ is also an increasing function, choosing k to maximize (1) is equivalent to choosing k to maximize

$$\sum_{\psi=1}^{n} q_\psi \Phi(Eu((\theta\tilde{\alpha}_\psi - r)k + rz)). \tag{2}$$

In this part, we will show that the introduction of ambiguity induces a decrease in the optimal size of the entrepreneurial investment. At first sight, this seems intuitive when compared to the case in which only risk aversion is present (ambiguity neutrality). However, as we will see in Proposition 1, such a first impression is incorrect because to achieve our intuitive result, we have to impose an additional restriction on second order distributions to be able to rank such distributions (MLRP) and, therefore, without that assumption, it is not clear what the effect of ambiguity is on the size of the entrepreneurial investment. This result resembles the case in the principal agent model where the absence of the MLRP may induce the agent to sabotage himself, even though it does not initially seem intuitive to do so. To prove our point, we will make use of one of the results in Taboga (2005) that is also used in Gollier (2011): they show that there are some cases in which the introduction of ambiguity can increase the demand for ambiguous assets. For our purposes, it will be enough to assume that the subjective second-order probabilities satisfy the MLRP to obtain the negative effect of incorporating ambiguity on the optimal entrepreneurial investment. As always, the use of the MLRP is not free, and the cost is associated with the fact that our results are restricted to a class of distributions that meet the following definition.

Definition 1. A probability density function $f(x)$ satisfies the MLRP with respect to another distribution $g(x)$ if for every $x_1 \geq x_0$ in the support of the distribution functions we have

$$\frac{f(x_1)}{g(x_1)} \geq \frac{f(x_0)}{g(x_0)}.$$

The intuition behind this property tells us that the higher the observed value of x, the more likely it will be drawn from $f(\cdot)$ rather than from $g(\cdot)$. That is, as x increases, the monotone-likelihood ratio also increases.

In this context, we can show that the presence of ambiguity reduces entrepreneurship if the second-order probability distribution $(F_1, ..., F_n)$ can be ordered using the monotone-likelihood order.

Proposition 1. *The inclusion of ambiguity reduces the optimal size of the entrepreneurial investment as long as the second-order probability distributions* $(F_1, ..., F_n)$ *can be ordered using the monotone-likelihood order.*

Proof. See the appendix. □

The previous proof highlights the damage that ambiguity can potentially cause in any economy by reducing entrepreneurial ventures. Of course, this result is valid when probability distributions can be ordered using the MLRP. Adding variations to the traditional expected utility also causes a nonmarginal increment in the complexity of the analysis. However, this potential damage to the entrepreneurial environment caused by political and economic ambiguity must be taken into consideration by policymakers and political players because of its potential effects on long-term economic growth and development.

4 | ENTREPRENEURSHIP AND TOTAL ASSETS UNDER AMBIGUITY

Since Evans and Jovanovic (1989) empirically documented the positive relation between total assets (z) and entrepreneurship (k), three theoretical explanations have been given for such a relation. The first explanation was given by Evans and Jovanovic (1989) themselves, who argue that the positive relation $\left[\frac{dk}{dz} > 0\right]$ exists because the credit-rationing constraint relaxes when there are more assets, which allows more individuals to obtain financing for entrepreneurial ventures. The second explanation was given by Cressy (2000), who showed that $\left[\frac{dk}{dz}\right]$ is positive because the utility function $u(\cdot)$ is DARA, and as total assets increase, absolute risk aversion decreases and more agents are willing to undertake risky ventures. Finally, Bonilla and Vergara (2013) show that $\left[\frac{dk}{dz}\right]$ is positive because the utility function $u(\cdot)$ is prudent, that is $u''' > 0$, and prudence is consistent with DARA, constant absolute risk aversion, or increasing absolute risk aversion (IARA). However, prudence guarantees that the wealth effect offsets the substitution effect even in the case of IARA, which induces a positive relation between total assets (or wealth) and entrepreneurship.

But what can be said about the relation between total assets (or wealth) and business startups in an ambiguous environment? We have already provided reasons why developing entrepreneurial ventures may be considered not only a decision under uncertainty but also a decision under ambiguity. Do the previous results under uncertainty change in some way by incorporating ambiguity into the analysis? Do we require new conditions in the utility function to guarantee the positive relation between total assets and entrepreneurship? Must we restrict the probability distribution of results? Or should we impose restrictions on the form of the utility functions to be consistent with the empirical findings of Evans and Jovanovic (1989)? The relevance of these questions is not only literature-driven—there are some important underlying policy implications. For example, let us take the case of an improvement of social security programs. We know that this implies an increase in wealth for the majority of individuals, but does this mean an increase in entrepreneurship if we are in an economy with increasing ambiguity? Alternatively, we can look at the case of an important increase in the bequest tax. The question that arises in this case is: are the children of the superrich now less willing to start new ventures in an ambiguous context? We should try to answer these very practical questions.

To show how incorporating ambiguity into the model can distort classical comparative analyses of entrepreneurship under risk, let us start by considering the following example.

Example 1. Consider the entrepreneur's problem under ambiguity with the following parametrization: $u(x) = \sqrt{x}$, $\Phi(x) = -100 \cdot exp(-100 \cdot x)$, productivity θ is normalized to one, and the cost of capital is assumed to be $r = 5\%$. With probability p, the firm fails to recover the cost of capital, earning $\alpha = 2.5\%$. With complementary probability $(1 - p)$, the firm is wildly successful, earning $\alpha = 80\%$. We assume the following structure on ambiguity: with probability $q_0 = 0.9$, failure is more likely than not, $p = 54\%$; with probability $(1 - q_0)$, failure is less likely, $p = 40\%$. After solving (3) for the given parameters, we obtain the following values for the optimal entrepreneurial demand for capital investment.

The analytical solution to the previous problem can be found in the appendix. Clearly, even if $u' > 0, u'' < 0, u''' > 0$ and $\Phi' > 0, \Phi'' < 0$, the sign of $\left[\frac{dk^*}{dz}\right]$ is still undefined. The simulated example highlights the point that even with the optimal self-employment demand for capital, optimal entrepreneurship under ambiguity can locally decrease as an individual's total assets increase, as we can see in Table 1. In other words, there is a locally negative relationship between total assets and entrepreneurship. Considering the analytical results in the appendix and the simulated example provided here, we can conclude that the results of Cressy (2000) and Bonilla and Vergara (2013)

TABLE 1 Optimal capital investment of the self-employment problem for different levels of assets (z)

z	1,070.15	1,070.2	1,070.25	1,070.3	1,070.35
k^*	51.231	51.201	50.912	50.944	51.104

are insufficient to explain the empirical evidence between total assets and entrepreneurship when ambiguity exists. Thus, neither DARA nor prudence guarantee a positive monotonic relationship between total assets and entrepreneurship.[1]

This result indicates that, although there are just two possible realizations of α and, therefore, ambiguity is reduced to only two possible distributions of results in our simulated example (p^1, p^2), the final outcome in the model is endogenously determined by the optimal choice of k. This, in turn, depends on the initial level of assets, risk preferences and ambiguity preferences. Ambiguity is the component that causes the nonmonotonic result. In particular, for different levels of total assets, there are different choices between the two possible distributions. This implies that the ambiguity effect on the choice of the probability distribution generates an opposite effect that is greater in magnitude than the risk effect. In turn, this means that an increase in total assets can discourage an agent's choice to become an entrepreneur in an ambiguous world. Ambiguous rules of the game negatively affect entrepreneurship and, therefore, political, social and economic ambiguity should be of great concern to politically unstable countries. For them, every election represents a possibility "to restart the nation" from scratch with a new economic model and even a new political constitution, instead of marginal improvements to the economy and its institutions that build a nation step by step.

Therefore, we now have the following problem: Is it possible to guarantee the positive relation between total assets and entrepreneurship in an ambiguous context? What are the conditions that guarantee these comparative static results?

Proposition 2. *When ambiguity is included in Cressy's (2000) model of entrepreneurship, an increase in total assets induces an increase in the optimal size of the entrepreneurial investment, as long as the absolute ambiguity aversion coefficient is smaller than an upper bound.*

Proof. First, recall that $U(k,\psi) = Eu((\theta\tilde{\alpha}_\psi - r)k + rz) = Eu(\tilde{y}_\psi)$ is just a given number (an expected value) that depends on k. We know that the decision maker chooses the optimal level of k to maximize $\sum_{\psi=1}^{n} q_\psi \Phi(U(k,\psi))$. The first-order condition of this problem is given by

$$\sum_{\psi=1}^{n} q_\psi \Phi'(U(k^*,\psi))E[(\theta\tilde{\alpha}_\psi - r)u'((\theta\tilde{\alpha}_\psi - r)k^* + rz)] = 0 \tag{3}$$

Then, the second-order condition of the problem is given by

$$\begin{aligned}\sum_{\psi=1}^{n} q_\psi [\Phi''(U(k^*,\psi))[E[(\theta\tilde{\alpha}_\psi - r)u'((\theta\tilde{\alpha}_\psi - r)k^* + rz)]]^2 \\ +\Phi'(U(k^*,\psi))E[(\theta\tilde{\alpha}_\psi - r)^2 u''((\theta\tilde{\alpha}_\psi - r)k^* + rz)]] = 0\end{aligned} \tag{4}$$

which, given the assumptions of the model, is always negative. Let us now re-express the first-order condition (3) as

$$F(k^*, z, \theta, r, \psi) = 0.$$

Then, using the implicit function theorem, we know that $\frac{dk}{dz} = \frac{-F_z}{F_k}$, and since F_k is the second-order condition (negative) just derived above, we only need to know the sign of F_z to infer the sign of $\frac{dk}{dz}$, that is, $sign\left(\frac{dk}{dz}\right) = sign(F_z)$. Let us now determine F_z.

$$\begin{aligned}F_z = \sum_{\psi=1}^{n} q_\psi [\Phi''(U(k^*,\psi))E[u'((\theta\tilde{\alpha}_\psi - r)k^* + rz)]rE[(\theta\tilde{\alpha}_\psi - r)u'((\theta\tilde{\alpha}_\psi - r)k^* + rz)] \\ +\Phi'(U(k^*,\psi))E[(\theta\tilde{\alpha}_\psi - r)ru''((\theta\tilde{\alpha}_\psi - r)k^* + rz)]]\end{aligned} \tag{5}$$

which can be expressed as

$$\sum_{\psi=1}^{n} q_\psi [r\beta_\psi] \tag{6}$$

where

$$\beta_\psi = \Phi''(U(k^*, \psi))E[u'((\theta\tilde{\alpha}_\psi - r)k^* + rz)]E[(\theta\tilde{\alpha}_\psi - r)u'((\theta\tilde{\alpha}_\psi - r)k^* + rz)] \\ +\Phi'(U(k^*, \psi))E[(\theta\tilde{\alpha}_\psi - r)u''((\theta\tilde{\alpha}_\psi - r)k^* + rz)]$$

Now, observe that in (6), r and q_ψ are always positive. Therefore, we only need to look for the conditions that guarantee a positive β_ψ to have $\frac{dk}{dz} > 0$. Then, by defining $((\theta\tilde{\alpha}_\psi - r)k^* + rz) \equiv \tilde{y}_\psi$, we know that $\beta_\psi > 0$ if the following condition is true:

$$\Phi''(\cdot)E[u'(\tilde{y}_\psi)] \cdot E[(\theta\tilde{\alpha}_\psi - r)u'(\tilde{y}_\psi)] + \Phi'(\cdot)E[(\theta\tilde{\alpha}_\psi - r)u''(\tilde{y}_\psi)] > 0$$

which in turns becomes

$$-\frac{\Phi''(\cdot)}{\Phi'(\cdot)} < \frac{E[(\theta\tilde{\alpha}_\psi - r)u''(\tilde{y}_\psi)]}{E[u'(\tilde{y}_\psi)] \cdot E[(\theta\tilde{\alpha}_\psi - r)u'(\tilde{y}_\psi)]}. \tag{7}$$

Now, given $\Lambda \equiv \min_\psi \left\{ \frac{E[(\theta\tilde{\alpha}_\psi - r)u''(\tilde{y}_\psi)]}{E[u'(\tilde{y}_\psi)] \cdot E[(\theta\tilde{\alpha}_\psi - r)u'(\tilde{y}_\psi)]} \right\}$, if $-\frac{\Phi''(\cdot,\psi)}{\Phi'(\cdot,\psi)} < \Lambda \; \forall \; \psi$, the relation between total assets and entrepreneurship is always positive. □

The previous result must be satisfied by each ψ and highlights the fact that absolute ambiguity aversion is a key parameter to guarantee a positive relation between entrepreneurship and initial wealth. Indeed, we can say that ambiguity distorts previous results from the risk and entrepreneurship literature, but when the distortion is not strong enough, the results from the risk literature prevail.[1] This limited degree of distortion no doubt allows the set of possible utility functions that can be used in the analysis of ambiguity to be restricted. And since nobody guarantees that such a condition is met, ambiguity may again cause damage to entrepreneurial activities, negatively affecting economic growth and development in economies with a high degree of political and economic ambiguity.[1]

Therefore, given the importance of the level of ambiguity aversion in entrepreneurship, we present the following result.

Proposition 3. *An increase in ambiguity aversion reduces the pool of entrepreneurs because it makes it less attractive for a marginal entrepreneur to self-select into entrepreneurship.*

Proof. We say that agent A is more ambiguity-averse than agent B if their second-order utilities can be written in the following fashion: $\Phi_A = h(\Phi_B)$, where $h(\cdot)$ is an increasing ($h' > 0$) and concave ($h'' < 0$) transformation function.

Let us define w_A and w_B as the salaries (secure wages) that make individuals A and B indifferent to secure employment or becoming entrepreneurs. That is, given w_A and w_B, A and B are the marginal or the indifferent entrepreneur for each level of ambiguity aversion:

$$\Phi_A^{-1}\left[E_q \Phi_A\left(Eu(\theta\tilde{\alpha}_\psi - r)k_A^* + rz\right)\right] = u(w_A + rz), \tag{8}$$

$$\Phi_B^{-1}\left[E_q \Phi_B\left(Eu(\theta\tilde{\alpha}_\psi - r)k_B^* + rz\right)\right] = u(w_B + rz). \tag{9}$$

Now, since $\Phi_A = h(\Phi_B)$, we know that

$$\Phi_B^{-1}h^{-1} = \Phi_A^{-1} \tag{10}$$

where h^{-1} is a convex transformation function. Then, using (8), (9), and (10) plus the convexity of h^{-1} and the Jensen Inequality, we have the following:

$$\begin{aligned} u(w_A + rz) &= \Phi_A^{-1}\Big[E_q\Phi_A\Big(Eu(\theta\tilde{\alpha}_\psi - r)k_A^* + rz\Big)\Big] = \Phi_B^{-1}h^{-1}\Big[E_q\Phi_A\Big(Eu(\theta\tilde{\alpha}_\psi - r)k_A^* + rz\Big)\Big] \\ &= \Phi_B^{-1}h^{-1}\Big[E_q h\Phi_B\Big(Eu(\theta\tilde{\alpha}_\psi - r)k_A^* + rz\Big)\Big] < \Phi_B^{-1}\Big[E_q\Big(h^{-1}h\Phi_B\Big(Eu(\theta\tilde{\alpha}_\psi - r)k_A^* + rz\Big)\Big)\Big] \\ &= \Phi_B^{-1}\Big[E_q\Big(\Phi_B\Big(Eu(\theta\tilde{\alpha}_\psi - r)k_A^* + rz\Big)\Big)\Big] < \Phi_B^{-1}\Big[E_q\Big(\Phi_B\Big(Eu(\theta\tilde{\alpha}_\psi - r)k_B^* + rz\Big)\Big)\Big] \\ &= u(w_B + rz) \end{aligned} \tag{11}$$

and therefore, $u(w_A + rz) < u(w_B + rz)$.

This last inequality indicates that increases in ambiguity aversion reduce the certainty equivalent (in first-order utility terms) that makes an individual indifferent between secure employment and entrepreneurship. So, to transition into entrepreneurship, the more ambiguity-averse individual needs better lotteries to self-select to be an entrepreneur, reducing the total pool of entrepreneurs. □

In the risk space, a small certainty equivalent means that the risk-averse individual is willing to sacrifice a great amount of resources (risk premium) to get rid of the risk. Therefore, this highly risk-averse individual needs an attractive lottery to accept it. In the ambiguity space, we have the homologous intuition for Proposition 3. The main difference, however, is that the certainty equivalent of the second-order utility is not an amount of monetary resources but instead a level of the first order expected utility.

This proposition is important from a public policy perspective because it shows that policies that help to reduce Knightian uncertainty in the economy may increase entrepreneurial investment. For instance, in the COVID-19 context, policies that help to flatten the curve also help to reduce the economy's recovery time, and this can be seen as a reduction in environmental ambiguous conditions for entrepreneurs.

Now that we know that increases in ambiguity aversion reduce the level of optimal entrepreneurship, we would like to analyze the conditions under which an indifferent (or marginal) decision maker self-selects into entrepreneurship when there is an increase in total assets or wealth z. The indifferent individual is represented by

$$W(k) \equiv \Phi^{-1}\left[\sum_{\psi=1}^{n} q_\psi \Phi(E(u((\theta\tilde{\alpha}_\psi - r))k + rz))\right] = u(w + rz) \tag{12}$$

The agent will self-select into entrepreneurship when wealth increases if

$$\frac{dW}{dz} \geq \frac{du(w + rz)}{dz} \tag{13}$$

that is, if the function W single-crosses the function u from below.

Now, we will prove that if the first-order utility function of the agent has the DARA property, and the second order preferences have the decreasing absolute ambiguity aversion (DAAA) property, then an increase in wealth induces the indifferent individual to become an entrepreneur.

Proposition 4. *Under DARA in u and decreasing absolute ambiguity aversion in Φ, the relative return on entrepreneurship increases wealth, making it more attractive for the marginal entrepreneur to self-select into entrepreneurship.*

Proof. First, notice that in our comparative statics exercise $\frac{dW(k)}{dz} = \frac{\partial W(k)}{\partial z}$, using the envelope theorem. In addition, by applying the derivative of the inverse function to (12), inequality (13) is equivalent to

$$\sum_{\psi}^{n} q_{\psi}[\Phi'(Eu(\cdot))Eu'(\cdot)]r \geq \Phi'(u(w + rz))u'(w + rz)r \tag{14}$$

In consequence, proving (13) is equivalent to proving (14). Now, if we define $\tilde{w}$ as a random variable associated with each second-order probability distribution ψ, such that $w_{\psi} + rz$ is the certainty equivalent of the second-order distribution ψ under utility function u, then

$$E(u((\theta\tilde{\alpha}_{\psi} - r)k + r \cdot z)) = u(w_{\psi} + rz)$$

Also, define $-u'$ as another utility function that is a concave transformation of the initial u. By DARA of u, we know that $(-u')'' < 0$ and also that

$$E(-u'((\theta\tilde{\alpha}_{\psi} - r)k + r \cdot z)) = -u'\left(w_{\psi}^{'} + rz\right)$$

where $w'_{\psi} + rz$ is the certainty equivalent associated with the second order distribution ψ and utility function $-u'$. Given that $-u'$ is more concave than u, the certainty equivalent associated with the second order distribution ψ under u $(w_{\psi} + rz)$ is higher than the certainty equivalent associated with the second-order distribution ψ under $-u'$ $(w'_{\psi} + rz)$. This, and the fact that u' is decreasing in $z (u'' < 0)$, means that:

$$E(u'((\theta\tilde{\alpha}_{\psi} - r)k + r \cdot z)) = u'\left(w_{\psi}^{'} + rz\right) \geq u'(w_{\psi} + rz) \tag{15}$$

Thus, if (15) is plugged into the left-hand side of (14), we get

$$\sum_{\psi}^{n} q_{\psi}(\Phi'(Eu(\cdot))Eu'(\cdot))r \geq \sum_{\psi}^{n} q_{\psi}(\Phi'(u(w_{\psi} + rz)) \cdot u'(w_{\psi} + rz))r \tag{16}$$

Now, we can write the right-hand side of the previous expression as the following composite function:

$$E(\Phi \circ u(\tilde{w} + rz))'.$$

In addition, from (12) we know that

$$E(\Phi \circ u(\tilde{w} + rz)) = \Phi \circ u(w + z)$$

and given that u is DARA and Φ is DAAA, it is true that $-\frac{(\Phi \circ u)''}{(\Phi \circ u)'}$ decreases with z and that

$$\sum_{\psi}^{n} q_{\psi}(\Phi'(Eu(\cdot))Eu'(\cdot))r \geq E(\Phi \circ u((\tilde{w} + rz)))' \geq \Phi'(u(w + rz))u'(w + rz)r$$

—which completes the proof. □

DARA means that an increase in wealth decreases the disutility generated by risk, and decreasing absolute ambiguity aversion means that an increase in wealth decreases the disutility generated by ambiguity. Thus, an increase in the marginal entrepreneur's wealth means that becoming an entrepreneur is now more attractive (less painful) than before. Therefore, the indifferent individual will shift from safe employment into entrepreneurship after an increase in wealth.

In addition, assuming DAAA and DARA, and limiting the second-order distribution (as we do in Proposition 1) by assuming a monotone-likelihood order, we can guarantee that an increase in wealth will increase the size of the venture, as we will see in the next proposition.

Proposition 5. *When ambiguity is included in Cressy's (2000) model of entrepreneurship, an increase in total assets induces an increase in the optimal size of entrepreneurial investment if preferences for ambiguity are DAAA, preferences for risk are DARA and the second-order distribution can be ordered using the monotone-likelihood criterion.*

Proof. See the appendix. □

This proposition implies that if an increase in wealth reduces the disutility generated by ambiguity and risk, and the second-order distribution can be ordered using the standard monotone-likelihood order, then we go back to the world where an increase in wealth unambiguously increases the amount of entrepreneurship. Compared with Proposition 2, this is a more restrictive result. In the former, we only restrict the preferences for ambiguity, but in Proposition 5, we limit the preferences for ambiguity and the structure of second-order distributions.

5 | EXTENSION TO OTHER AMBIGUITY PREFERENCES

There are alternative ways to model ambiguity beyond the KMM model. Assume for a moment that an agent makes a guess about the possible distribution. A positive distribution in our entrepreneurial example is more likely to face a high demand, and a negative distribution is more likely to face a low demand for the product. One familiar rule of thumb is to assume the worst-case scenario before making a decision. This rule represents a broadly used ambiguity model called Maxmin Expected Utility preferences (MEU model, or nonunique prior model of uncertainty), and it was introduced by Gilboa and Schmeidler (1989) as a way to explain the Ellsberg (1961) paradox that challenged the traditional expected utility model. Those preferences can be written as follows:

$$V(k) = \min_{p}\left\{\int u(k)dp\right\}, \tag{17}$$

In addition, it is very common for the agent to make a guess q about the possible distribution. For example, some agents are very confident about the success of their prospective project. However, they also know that the project could fail. So, to formulate a better guess, the agent can assume something intermediate between the guess and the worst-case scenario. In this case, there is another model that represents such ambiguous preferences. This model is called multiplicative preferences (Hansen & Sargent, 2001), and it was developed as a way to link the MEU model with robust-control theory applied in macroeconomics and finance. Those preferences can be written as follows:

$$V(k) = \min_{p}\left\{\int u(k)dp + \theta R(p||q)\right\}, \tag{18}$$

where $\theta \in (0, \infty]$ and the function $R(p||q)$ represent the relative entropy of p with respect to q. This function measures the "distance" between two probability distributions. In this case, the agent suffers a cost of considering another probability distribution different from q.

Now, we can generalize Proposition 2 in our paper to two alternative types of ambiguity preferences.

Proposition 6. *When ambiguity is included in Cressy's (2000) model of entrepreneurship:*

i) *If the ambiguity preferences are MEU, the utility function $u(\cdot)$ satisfies prudence and the second-order distributions can be ordered using the monotone-likelihood order, then an increase in total assets induces an increase in the optimal size of the entrepreneurial investment.*
ii) *If the ambiguity preferences are multiplier preferences, then if θ is high enough, an increase in total assets induces an increase in the optimal size of the entrepreneurial investment.*

Proof.

i) If the second-order distribution in the core beliefs satisfies the monotone-likelihood order (similar to what is assumed in this paper), the worst-case scenario is constant for all investment levels because the monotone-likelihood order implies first-order stochastic dominance. Therefore, with MEU and monotone-likelihood order, the problem collapses to the SEU with the same pessimistic second-order distribution for all investment levels. Thus, from Bonilla and Vergara (2013), prudence is sufficient for the standard wealth effect of entrepreneurship with MEU. □

Proof.

i) From Dupuis and Ellis (1997) and Strzalecki (2011) we know that:

$$V(k) = \min_p\left\{\int u(k)dp + \theta R(p||q)\right\} = \Phi_\theta^{-1}\left(\int \Phi_\theta \circ u(k)dq\right) \tag{19}$$

with

$$\Phi_\theta(u) = \begin{cases} -exp\left(-\dfrac{u}{\theta}\right) & \text{for} \quad \theta < \infty \\ u \quad for \quad \theta = \infty \end{cases} \tag{20}$$

Thus, KMM preferences and multiplier preferences are ordinarily equivalent when Φ is defined as (20) and the absolute ambiguity aversion coefficient is given by $\frac{1}{\theta}$. Thus, if θ is a high enough number, then Proposition 2 holds. □

Those two previous preferences belong to a generalized category called "variational preferences" that are defined by

$$V(k) = \min_p\left\{\int u(k)dp + c(p)\right\} \tag{21}$$

The interpretation of this expression is that the decision maker considers a broader set $p \in P$ of plausible probability distributions. $c(p)$ is the cost of choosing the probability distribution p. In this case, the guess is not necessarily conditioned to q. Notice that $c(p) = 0$ collapses (21) into MEU preferences, and $c(p) = \theta R(p||q)$ collapses (21) into multiplier preferences. In particular, Strzalecki (2011) shows that multiplier preferences are the intersection between KMM and variational preferences.[1] In addition, KMM preferences are more general than multiplier preferences and account for a different type of behavior than variational preferences. We justify the use of the KMM model over variational preferences because, as Grant and Polak (2013) showed, variational preferences are constant absolute ambiguity averse while preferences defined by the KMM framework can be characterized by decreasing and constant ambiguity aversion behavior, particularly considering the recent empirical evidence provided by Baillon and Lætitia (2019), who show that agent behavior is more consistent with DAAA than CAAA.[1] Intuitively, as long as wealth increases, ambiguity aversion should fall and people should become more engaged in entrepreneurship with larger projects, which is consistent with most of the analysis done in this paper.[1]

6 | CONCLUSION

The rapid growth in theoretical developments in the theory of risk and uncertainty make it worthwhile to study how these concepts can be understood and applied in fields like the economics of entrepreneurship. Closing the gap between simplistic and treatable models applied to entrepreneurship, based on the traditional expected utility framework, and real-life decisions in the field of entrepreneurship that consider, for example, ambiguous environments, is the key to better understanding entrepreneurial incentives and the decision-making process in developing new ventures.

In this paper, we have shown that the presence of ambiguity in the economic environment challenges previous results in the field of the economics of entrepreneurship under risk. We first studied the effect of ambiguity on the optimal size of the entrepreneurial investment and found that, as long as the probability distributions can be ordered by the MLRP, the introduction of ambiguity negatively affects the optimal entrepreneurial investment, which is consistent with the effects of entrepreneurship under risk.

Then, we showed that the classical explanations for the positive relation between business startups and total assets documented by Evans and Jovanovic (1989) are no longer valid. Now, we know that the ideas developed in the literature to explain such a relation based on credit constraints, DARA preferences or prudence behavior are not sufficient to explain the empirical regularity between entrepreneurship or business startups and total assets in a context of ambiguity. Consequently, a new condition is added, which states that the absolute ambiguity aversion coefficient should be smaller than a given upper bound. However, this new condition restricts the set of possible utility functions, which is the cost of making sure that the classical results in the economics of entrepreneurship can still be valid.

We also showed the negative effect of increasing ambiguity on the size of the entrepreneurial investment, and studied the conditions under which the indifferent (or marginal) entrepreneur prefers to switch from secure employment into entrepreneurship when total assets increase.

Lastly, we connected our results under the KMM model with alternative ways to model ambiguity, like the Maxmin expected utility framework and multiplier preferences. Intuition tells us that the KMM model and other models can be linked by accommodating the second-order utility function of the KMM model to fit the characteristics of these alternative models.

The effect of ambiguity on entrepreneurship has also recently been tested in experimental research, with some mixed evidence that in most cases favors negative effects. The potential negative effect of ambiguity on entrepreneurship and venture creation also has potentially negative effects on long-term growth and development. This is particularly important for economies with a low level of institutional stability that suffer from a high level of economic and political ambiguity. Future research should further enhance our understanding of the effects of ambiguity on the economy, especially in dynamic models that account for long-term effects.

This paper has several limitations beyond the use of one specific way to treat ambiguity (the KMM model), and these limitations provide potential ideas for future research. First, we do not analyze what happens with necessity entrepreneurs when a huge uncertainty shock hits the economy (like the COVID-19 pandemic that we are currently experiencing). In this case, a GDP fall could be assimilated to a reduction in total assets, and it is highly likely that necessity entrepreneurs may flourish—contrary to the intuition of our theory, which is based on productive entrepreneurship. Another limitation in our paper is the use of homogeneous ambiguity-averse decision makers and, thus, heterogeneity in ambiguity preferences can probably become an interesting possible extension for this paper (see the discussion in Chapman & Polkovnichenko, 2009). Also, the use of kinked preferences has shown to be empirically consistent with the behavior of some investors (Ahn et al., 2014) and, therefore, it is something to keep in mind for future research. Finally, we believe that even though thinking in dynamic contexts under ambiguity does not seem to be an easy task, it can still prove to be a worthwhile endeavor for future research.

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ENDNOTES

[1]Following Eeckhoudt and Schlesinger (2006), under prudent behavior "we are more willing to accept an extra risk when wealth is higher, rather than when wealth is lower." Prudent preferences are characterized by $u''' > 0$.

[2]An intuitive explanation of the MLRP is provided in Definition 1, Section 3.2.

[3]Ellsberg (1961)'s seminal paper shows that individuals dislike ambiguity, which means that in the face of risk, a decision maker will tend to prefer a known probability of results instead of an imprecise probability of results (unmeasurable or ambiguous risks). This idea of ambiguity aversion challenged previous theories of decision-making under risk, based mainly on subjective probabilities (Savage, 1954), and induced the development of new ways to model risk that incorporate ambiguity aversion into the analysis (Gilboa & Schmeidler, 1989; Schmeidler, 1989, and more recently KMM, 2005).

[4]$y(\tilde{\alpha}, k)$ has two parts: a random component that is characterized by $\tilde{\alpha}$, and a nonrandom part that is characterized by the decision variable k.

[5]Following Baillon (2017) "…ambiguity prudence can be defined as the preference for assigning a probability loss to a part of an act in which there is no ambiguity about the probability of winning. In short, the decision maker prefers improving her odds in ambiguous events rather than in unambiguous events."

[6]Berger (2014) and Baillon (2017) proved this.

[7]This example highlights the results shown in Cherbonnier and Gollier (2015), who show that, in the context of KMM preferences, decreasing concavity is not enough to guarantee decreasing aversion (or in their context, the fact that an increase in wealth implies an increase in the demand for the risky asset).

[8]This threshold defines the maximum level of ambiguity aversion that makes our comparative static consistent with the results in the risk space. If we had looked for the conditions that make (5) negative, we would have had another threshold that is determined by an expression analogous to Equation (7) but with the opposite sign and a max function. Therefore, we have three segments for the absolute ambiguity-averse coefficient. The first one is where risk and ambiguity have the same comparative statics. The third one is where risk and ambiguity have opposite comparative statics, and the second one (the one in the middle) is where anything can happen.

[9]If we assume $\Phi(u) = -exp(\lambda \cdot u)$, with λ big enough and $\frac{dk}{dz} < 0$. It is possible to show that $\frac{d^2k}{dzd\lambda} < 0$. That is, an increase in ambiguity aversion means that an increase in total assets decreases entrepreneurship even more.

[10]Obtaining general conditions for variational preferences is outside the scope of this paper. To understand the meaning of absolute ambiguity aversion under variational preferences, see Grant and Polak (2013) and Cerreia-Vioglio, Maccheroni, and Marinacci (2020).

[11]Cubitt, Van De Kuilen, and Mukerji (2020) find that for individuals identified as ambiguity sensitive, the smooth ambiguity model was more representative than MEU. This relative support is stronger among subjects identified as ambiguity averse.

[12]Despite this, Ahn, Choi, Gale, and Kariv (2014) experimentally demonstrate that investor behavior can be represented by kinked preferences, a characteristic that is consistent with MEU and variational preferences but is not well captured by KMM preferences.

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APPENDIX A

A.1. Proof of Proposition 1

Proof. Take the case where two individuals are completely alike, except that individual 2 is more ambiguity averse than individual 1. Second-order utilities of each of the individuals are denoted by $\Phi_1(\cdot)$ and $\Phi_2(\cdot)$, respectively. Following the KMM (2005) smooth ambiguity model, we can say that there is a function $h(\cdot)$ that is increasing ($h' \geq 0$) and concave ($h'' \leq 0$) such that $\Phi_2(U(k, \psi)) = h(\Phi_1(U(k, \psi)))$. The first-order condition of expression (2) with respect to the optimal k for individual 1 is given by

$$\sum_{\psi=1}^{n} q_\psi \Phi'_1\left(U\left(k_1^*, \psi\right)\right) E\left[(\theta\tilde{\alpha}_\psi - r)u'\left((\theta\tilde{\alpha}_\psi - r)k_1^* + rz\right)\right] = 0 \tag{A1}$$

Since individual 2 is more ambiguity averse than individual 1, we would expect individual 2 to have a smaller amount of optimal k^*, and this would be true if

$$\sum_{\psi=1}^{n} q_{\psi} \Phi'_2\left(U\left(k_1^*, \psi\right)\right) E\left[(\theta\widetilde{\alpha}_{\psi} - r)u'\left((\theta\widetilde{\alpha}_{\psi} - r)k_1^* + rz\right)\right] \leq 0 \tag{A2}$$

If more ambiguity aversion implies a reduction in the optimal size of entrepreneurial investment, we can infer that going from ambiguity neutrality (only risk aversion) to any level of positive ambiguity aversion is equivalent to analyzing the inclusion of ambiguity in our entrepreneurship model. Therefore, we only need to look for the conditions that guarantee that (22) $\Rightarrow$ (23) to prove Proposition 1. Fortunately, Taboga (2005) developed an interesting model of portfolio selection under ambiguity, in which he disentangled ambiguity from ambiguity aversion and provided a very tractable way to deal with ambiguous environments. We will apply the results of that paper to our entrepreneurial context.

Note that expression (22) can be rewritten as follows:

$$E\widetilde{x}_1 u'\left((\theta\widetilde{\alpha}_{\psi} - r)k_1^* + rz\right) = 0 \tag{A3}$$

and therefore, (22) $\Rightarrow$ (23) would be equivalent to

$$E\widetilde{x}_1 u'\left((\theta\widetilde{\alpha}_{\psi} - r)k_1^* + rz\right) = 0 \;\Rightarrow\; E\widetilde{x}_2 u'\left((\theta\widetilde{\alpha}_{\psi} - r)k_1^* + rz\right) \leq 0 \tag{A4}$$

where $\widetilde{x}_i$ is a random variable that equals $(\theta\widetilde{\alpha}_{\psi} - r)$ with probability $\widehat{q_{\psi}}^i$, and $\widehat{q_{\psi}}^i$ is the Radon-Nikodym derivative of individual i as follows:

$$\widehat{q_{\psi}}^i = \frac{q_{\psi} \Phi'_i\left(U\left(k_1^*, \psi\right)\right)}{\sum_{t=1}^{n} q_t \Phi'_i\left(U\left(k_1^*, t\right)\right)} \tag{A5}$$

Expression (26), for individual 1, corresponds to her distorted second order belief from $(q_1, q_2, ..., q_n)$ to the observationally equivalent probability distribution $(\widehat{q_1}^1, \widehat{q_2}^1, ..., \widehat{q_n}^1)$ (Taboga, 2005). In consequence, the left-hand side of Equation (A4) can be interpreted as the first-order condition of the following problem: $\max_{\{k\}} Eu((\theta\widetilde{\alpha}_{\psi} - r)k_1^* + rz)$ for an expected utility maximizer entrepreneur whose beliefs are $\widetilde{x}_1 \sim (\widetilde{y}_1, \widehat{q_1}^1, \widetilde{y}_2, \widehat{q_2}^1, ..., \widetilde{y}_n, \widehat{q_n}^1)$.

The right-hand side of Equation (A4) is negative if $\widetilde{x}_1$ satisfies the MLRP with respect to $\widetilde{x}_2 \sim (\widetilde{y}_1, \widehat{q_1}^2, \widetilde{y}_2, \widehat{q_2}^2, ..., \widetilde{y}_n, \widehat{q_n}^2)$. The intuition behind this requirement is based on the fact that any increase in ambiguity aversion deteriorates the observationally equivalent second-order probability distribution, transferring more weight onto the worst prior (Gollier, 2011).

Now, let us assume that $U(k_1^*, 1) \leq U(k_1^*, 2) \leq \cdots \leq U(k_1^*, n)$ and recall that $\Phi_1(\cdot)$ and $\Phi_2(\cdot)$ are increasing in U, and that the function $h(\cdot)$ is increasing and concave. Then, $\Phi_2(U) = h(\Phi_1(U))$ implies that

$$\Phi'_2\left(U\left(k_1^*, \psi\right)\right) = h'\left(\Phi_1\left(U\left(k_1^*, \psi\right)\right)\right)\Phi'_1\left(U\left(k_1^*, \psi\right)\right)$$

Then, using (26) and definition 1 we obtain

$$\frac{\widehat{q_{\psi}}^1}{\widehat{q_{\psi}}^2} = \frac{\dfrac{q_{\psi}\Phi'_1\left(U\left(k_1^*, \psi\right)\right)}{\sum_t^n q_t \Phi'_1\left(U\left(k_1^*, t\right)\right)}}{\dfrac{q_{\psi}\Phi'_2\left(U\left(k_1^*, \psi\right)\right)}{\sum_t^n q_t \Phi'_2\left(U\left(k_1^*, t\right)\right)}} = \left(\frac{1}{h'\left(\Phi_1\left(U\left(k_1^*, \psi\right)\right)\right)}\right)\frac{\sum_t^n q_t \Phi'_2\left(U\left(k_1^*, t\right)\right)}{\sum_t^n q_t \Phi'_1\left(U\left(k_1^*, t\right)\right)} \tag{A6}$$

and observe that given that Φ'_i is positive and h' is decreasing ($h'' \leq 0$), expression (27) is increasing in ψ, something that guarantees the MLRP, which in turn guarantees the right-hand side of (25)—thus, completing the proof. □

A.2. Analytical solution to Example 1

Here, we analyze the solution of Example 1, in which there are 2 second-order distributions p^1 and p^2 for y_1 and y_2, respectively. Using the implicit function theorem on the first order condition (3), in this particular context we get

$$
\begin{aligned}
\frac{dk}{dz} &= -\frac{C+D}{A+B}\\
A &= q_0(\Phi''(\cdot)\cdot(p^1u'(y_1)(\theta\alpha_1 - r) + (1-p^1)u'(y_2)(\theta\alpha_2 - r))^2)\\
&\quad + q_0(\Phi'(\cdot)\cdot(p^1u'(y_1)(\theta\alpha_1 - r)^2 + (1-p^1)u'(y_2)(\theta\alpha_2 - r)^2))\\
B &= (1-q_0)(\Phi''(\cdot)\cdot(p^2u'(y_1)(\theta\alpha_1 - r) + (1-p^2)u'(y_2)(\theta\alpha_2 - r))^2)\\
&\quad + (1-q_0)(\Phi'(\cdot)\cdot(p^2u''(y_1)(\theta\alpha_1 - r)^2 + (1-p^2)u''(y_2)(\theta\alpha_2 - r)^2))\\
C &= q_0\Phi''(\cdot)\cdot(p^1u'(y_1)r + (1-p^1)u'(y_2)r)\cdot(p^1u'(y_1)(\theta\alpha_1 - r) + (1-p^1)u'(y_2)(\theta\alpha_2 - r))\\
&\quad + q_0\Phi'(\cdot)\cdot(p^1u''(y_1)(\theta\alpha_1 - r)r + (1-p^1)u''(y_2)(\theta\alpha_2 - r))r\\
D &= (1-q_0)\Phi''(\cdot)\cdot(p^2u'(y_1)r + (1-p^2)u'(y_2)r)\cdot(p^2u'(y_1)(\theta\alpha_1 - r) + (1-p^2)u'(y_2)(\theta\alpha_2 - r))\\
&\quad + (1-q_0)\Phi'(\cdot)\cdot(p^2u''(y_1)(\theta\alpha_1 - r)r + (1-p^2)u''(y_2)(\theta\alpha_2 - r))r
\end{aligned}
$$

The latter is not always positive when $u''' > 0$ and $\phi''' > 0$, as we have also proved in the simulation example provided in the text.

A.3. Proof of Proposition 5

Proof. Let us start with the following statement: *The demand for k increases with z for any second order preference Φ that is DAAA and first order utility function u that is DARA when, for each $i \neq j$ such that $E((\theta\tilde{\alpha}_i - r)u'((\theta\tilde{\alpha}_i - r)k^*) + rz) > 0 > E((\theta\tilde{\alpha}_j - r)u'((\theta\tilde{\alpha}_j - r)k^*) + rz)$, the following two conditions hold:*

1) $E(u((\theta\tilde{\alpha}_i - r)k^* + rz)) \geq E(u((\theta\tilde{\alpha}_j - r)k^* + rz))$
2) $E(u'((\theta\tilde{\alpha}_j - r)k^* + rz)) \geq E(u'((\theta\tilde{\alpha}_i - r)k^* + rz))$.

To prove this, notice that the condition for k to increase with z comes from Equation (5) in Proposition 2, and is the following:

$$
\begin{aligned}
F_z = \sum_{\psi=1}^{n} q_\psi[&\Phi''(U(k^*,\psi))E[u'((\theta\tilde{\alpha}_\psi - r)k^* + rz)]rE[(\theta\tilde{\alpha}_\psi - r)u'((\theta\tilde{\alpha}_\psi - r)k^* + rz)]\\
&+ \Phi'(U(k^*,\psi))E[(\theta\tilde{\alpha}_\psi - r)ru''((\theta\tilde{\alpha}_\psi - r)k^* + rz)]] \geq 0.
\end{aligned} \tag{A7}
$$

Let us work with the second term of the left-hand side of expression (28), which is

$$
\sum_{\psi=1}^{n} q_\psi\Phi'(U(k^*,\psi))E[(\theta\tilde{\alpha}_\psi - r)ru''((\theta\tilde{\alpha}_\psi - r)k^* + rz)] \tag{A8}
$$

Define $A(c) = -\frac{u''(c)}{u'(c)}$ and plug it into the previous expression. Then, we can write (29) as

$$
-\sum_{\psi=1}^{n} q_\psi\Phi'(U(k^*,\psi))E[(\theta\tilde{\alpha}_\psi - r)rA((\theta\tilde{\alpha}_\psi - r)k^* + rz))u'((\theta\tilde{\alpha}_\psi - r)k^* + rz)]
$$

Now, let us work with a particular realization of $\tilde{\alpha}_\psi$: call it α_ϕ^v. Obverse that if $(\theta\alpha_\phi^v - r)$ is negative, then given that u is DARA, we have that

$$
-\left(\theta\alpha_\psi^v - r\right)rA\left(\left(\theta\alpha_\psi^v - r\right)k^* + rz\right)\Big)u'\left(\left(\theta\alpha_\psi^v - r\right)k^* + rz\right) \geq -\left(\theta\alpha_\psi^v - r\right)rA(rz)u'\left(\left(\theta\alpha_\psi^v - r\right)k^* + rz\right)
$$

If $\left(\theta\alpha_{\psi}^{v}-r\right)$ is positive, then

$$-\left(\theta\alpha_{\psi}^{v}-r\right)rA\left(\left(\theta\alpha_{\psi}^{v}-r\right)k^{*}+rz\right)\right)u'\left(\left(\theta\alpha_{\psi}^{v}-r\right)k^{*}+rz\right)\geq-\left(\theta\alpha_{\psi}^{v}-r\right)rA(rz)u'\left(\left(\theta\alpha_{\psi}^{v}-r\right)k^{*}+rz\right)$$

This is also true because u is DARA. Then, recognizing that $A(rz)$ is just a constant number and using the first-order condition, we get

$$\sum_{\psi=1}^{n}q_{\psi}\Phi'(U(k^{*},\psi))E[(\theta\tilde{\alpha}_{\psi}-r)ru''((\theta\tilde{\alpha}_{\psi}-r)k^{*}+rz)]$$
$$\geq-\sum_{\psi=1}^{n}q_{\psi}\Phi'(U(k^{*},\psi))A(rz)E[(\theta\tilde{\alpha}_{\psi}-r)ru'((\theta\tilde{\alpha}_{\psi}-r)k^{*}+rz)]=0.$$

Now, let us work with the first part of the left-hand side of expression (28)

$$\sum_{\psi=1}^{n}q_{\psi}\Phi''(U(k^{*},\psi))E[u'((\theta\tilde{\alpha}_{\psi}-r)k^{*}+rz)]rE[(\theta\tilde{\alpha}_{\psi}-r)u'((\theta\tilde{\alpha}_{\psi}-r)k^{*}+rz)]$$

Using an analogous logic like before, we can rewrite this expression in the following fashion:

$$-\sum_{\psi=1}^{n}q_{\psi}\eta(\psi)\Phi'(U(k^{*},\psi))rE[(\theta\tilde{\alpha}_{\psi}-r)u'((\theta\tilde{\alpha}_{\psi}-r)k^{*}+rz)]$$

where $\eta(\psi)=-\frac{\Phi''(U(k^{*},\psi))}{\Phi'(U(k^{*},\psi))}E[u'((\theta\tilde{\alpha}_{\psi}-r)k^{*}+rz)]$. Let us rank ψ such that $E[u'((\theta\tilde{\alpha}_{\psi}-r)k^{*}+rz)]$ is positive if and only if $\psi\geq\bar{\psi}$ for a given $\bar{\psi}$. By using the two conditions stated above and the fact that Φ is DAAA, $\eta(\psi)\leq\eta(\bar{\psi})$ if $\psi\geq\bar{\psi}$ and $\eta(\psi)\geq\eta(\bar{\psi})$ if $\psi\leq\bar{\psi}$. Then, using the same method as above and realizing that $\eta(\bar{\psi})$ is just a given number and using the first-order condition, we get

$$\sum_{\psi=1}^{n}q_{\psi}\Phi''(U(k^{*},\psi))E[u'((\theta\tilde{\alpha}_{\psi}-r)k^{*}+rz)]rE[(\theta\tilde{\alpha}_{\psi}-r)u'((\theta\tilde{\alpha}_{\psi}-r)k^{*}+rz)]$$
$$\geq-\eta(\bar{\psi})\sum_{\psi=1}^{n}q_{\psi}\Phi'(U(k^{*},\psi))rE[(\theta\tilde{\alpha}_{\psi}-r)u'((\theta\tilde{\alpha}_{\psi}-r)k^{*}+rz)]=0.$$

Now, when the second-order distributions are ordered using the monotone-likelihood order $\{(\theta\tilde{\alpha}_{1}-r),\ldots,(\theta\tilde{\alpha}_{n}-r)\}$, this order induces first-order stochastic dominance among the second-order distributions, which implies that for each $i>j$

$$Eu((\theta\tilde{\alpha}_{i}-r)k^{*}+rz)\geq Eu((\theta\tilde{\alpha}_{j}-r)k^{*}+rz)$$

And since u is DARA, $u'''>0$, we know that

$$Eu'(rz+(\theta\tilde{\alpha}_{j}-r)k^{*})\geq E(u'(rz+(\theta\tilde{\alpha}_{i}-r)k^{*}))$$

Finally, because the second-order distributions are ordered by the monotone-likelihood order, $E(\theta\tilde{\alpha}_{\psi}-r)u'(zr+(\theta\tilde{\alpha}_{\psi}-r)k^{*})$ has the single crossing property in ψ. Thus, $E((\theta\tilde{\alpha}_{i}-r)u'(zr+(\theta\tilde{\alpha}_{i}-r)k^{*}))>0>E((\theta\tilde{\alpha}_{j}-r)u'(zr+(\theta\tilde{\alpha}_{j}-r)k^{*}))$ implies that $i>j$, which in turn implies the conditions required for k to be increasing in z. □