Risk Aversion, Downside Risk Aversion, and the Transition to Entrepreneurship*

Claudio A. Bonilla
School of Economics and Business, University of Chile†
Marcos Vergara
School of Economics and Business, Universidad del Desarrollo

Abstract

In this paper we discuss the transition from secure employment to risky self-employment (entrepreneurship) caused by a small increase in wealth. Building on the apportioning risk literature, we prove that the transition from secure employment to risky entrepreneurship is based on a measure of the difference between the strength of downside risk aversion and the strength of risk aversion. This result highlights the idea that using the behavioral approach of risky lotteries to study entrepreneurship can produce different results from the traditional economic theory of entrepreneurship, which can have policy implications that must be considered with caution.

Keywords: Apportioning Risk, Entrepreneurship, Downside Risk Aversion.
JEL Classification: D81; L26

*We would like to thank the useful comments of the Editor and the Coordinating Editor. The usual disclaimer applies. Marcos Vergara thanks the support of FONDECYT Project Number 11170052.
†Corresponding author Claudio Bonilla, University of Chile, Diagonal Paraguay 257, Santiago, Chile, Tel.+56 22 9783331. Email: cbonilla@fen.uchile.cl

1 Introduction

The connection between entrepreneurship and risk aversion is an old idea, initially discussed by Knight (1921) and later formalized by Kihlstrom and Laffont (1979). The main idea behind this theory is that the wealthy are, on average, **less risk averse** than the poor because well-behaved utility functions present decreasing absolute risk aversion (DARA) and, therefore, the wealthy are more prone to starting risky ventures. Also, most of the recent literature that uses microeconomic models to study the transition from secure employment to entrepreneurship builds either on the DARA assumption (Cressy, 2000, Ahn, 2009 or Hvide and Panos, 2014) or on prudent behavior (Bonilla and Vergara, 2013), which is also induced by DARA. This paper uses results from the apportioning risk literature to dive deeper into the problem of self-selection of occupations and entrepreneurship. In particular, we argue that the choice of self-selection of occupations can be interpreted as the decision of choosing between lotteries.

In the real world an individual decision maker may have a portfolio of different potential entrepreneurial ventures to undertake, each representing a different lottery. However, for the purposes of this paper, we will study the case in which the individual compares only two alternatives: continuing to be employed, with no risk —this lottery is simply a degenerate probability distribution that collapses at some point—; and transitioning to his best entrepreneurial option, which represents the risky lottery that we will analyze in section 3.

We think that our theory can be tested in a laboratory experiment, given that our result is connected with the difference between the strength of downside risk aversion (prudence) and the strength of risk aversion. Moreover, since there are accepted experimental methodologies to test risk aversion and prudence, our result could also, in principle, be tested in the laboratory. Examples of the experimental literature that deals with risk aversion and prudence are the works of Deck and Schlesinger (2014), Ebert and Wiesen (2014) and Krieger and Mayrhofer (2017). Running experiments is beyond the scope of our own paper but we believe that, starting from the abovementioned experimental papers, a specific experiment can be designed to analyze the occupational self-selection decision expressed as a behavioral choice between lotteries.

Almost all previous literature on occupational self-selection assumes that when the marginal decision maker experiences a small increase in wealth, he transitions from secure employment to risky entrepreneurship, as long as his utility function exhibits the DARA property (as a sufficient condition). In this paper, we expand on the self-selection occupational decision. From Bonilla and Vergara (2014), we know that there are other cases where prudent agents with CARA or IARA preferences can also transition to entrepreneurship when wealth increases. This is based on the desegregation of the income and substitution effects produced by random risk shocks. The idea was first developed in Dreze and Modigliani (1972) and later explained in Davis (1989) and Snow (2003). In this paper, we delve deeper into the analysis of the transition to entrepreneurship. We use the risk apportionment perspective and the main result in Eeckhoudt, Schlesinger and Tsetlin (2009) (herein, EST (2009)) to show that, as long as the strength of downside risk aversion exceeds the strength of risk aversion, the indifferent decision maker will decide to self-select into entrepreneurship.

Our result highlights the idea that, when using the behavioral approach of lotteries to model entrepreneurship, we have to go beyond the risk aversion and DARA (or prudence) assumptions in order to guarantee the transition from secure employment into risky self-employment. Therefore, even though the apportioning risk literature is a novel and intuitive way to model higher order risk preferences (Deck and Schlesinger, 2014), we have to be cautious in the applications of this method to applied economic problems because differences with the traditional models may occur. These potential differences highlight the fact that public policies promoting entrepreneurship are going to be — at least in part— dependent on the approach that policy-makers use in the study of entrepreneurship. In consequence, empirical and experimental studies become of key importance to clarify in what context one approach or the other is the right one to apply and also what polices better promote entrepreneurship in each context.

In the next section, we present the basic model of occupational self-selection. Section 3 presents the features of increases in risk and the main result of this paper, and section 4 concludes.

2 The Basic Model of Self-Selection of Occupations

An economy is characterized by a single-good stochastic production function $f(L, \theta)$, where L is the labor hired and θ is a random variable indexing the state of the world and representing uncertainty in the model. $f(L, \theta)$ and its first derivative $f_1(L, \theta)$ are increasing in θ, and the stochastic production function satisfies $f_{11} < 0 < f_1$, where f_1 and f_{11} denote the first and second order derivative of $f(L, \theta)$ with respect to L. Inada conditions are assumed to hold and so an interior solution to the problem is expected.

There is a continuum of agents in the unit interval. Agents have identical preferences but differ in their initial level of wealth. They have the utility function $u(y)$, where y is realized income. The utility function satisfies $u'' < 0 < u'$ and prudence ($u''' > 0$), a concept coined by Kimball (1990) and widely used in models of precautionary savings and precautionary effort (Eeckhoudt et al. (2012) or Wang and Li (2014)), and in models showing higher-order risk attitudes like Menegatti (2014), Denuit and Rey (2014), Eeckhoudt and Rey (2011) or Eeckhoudt et al. (2016).

Agents vary in the amount of initial wealth a, the distribution of which is exogenous. The agents have to choose between two occupations. They can become workers and earn a certain wage w (their total income in this case would be $a + w$) or they can become entrepreneurs, hiring L units of labor and earning the residual profit from a stochastic production function which is denoted by $y(\theta) = f(L, \theta) - wL + a$, where the stochastic component is the random variable θ.

Each agent takes w as given and chooses the occupation that offers the highest utility. This result is a competitive equilibrium that translates into a partition of the set of agents into a set of workers and a set of entrepreneurs.

Let $V_E(a) = Eu(f(L, \theta) - wL + a))$ be the expected utility function of the entrepreneur for a given wealth level a and let $V_W(a) = u(w+a)$ be the utility function of the worker for a given wealth level a. In equilibrium, there is a wealth level $\bar{a}$ at which an individual is indifferent to either occupation, i.e., the level of utility is the same whether the individual is a worker or an entrepreneur. We will call this individual the marginal or indifferent entrepreneur. Therefore, at $\bar{a}$ we have:

$$Eu(f(L(w,\bar{a}),\theta) - wL(w,\bar{a}) + \bar{a})) = u(w + \bar{a}) \tag{1}$$

where $L(w,\bar{a})$ comes from the expected utility maximization of the entrepreneur, i.e., $L(w,\bar{a}) = \underset{\{L\}}{Argmax}\,\{Eu(f(L,\theta) - wL + \bar{a}))\}$, which is obtained by the following first-order condition:

$$E[u'(f(L,\theta) - wL + \bar{a})(f_1(L,\theta) - w)] = 0 \tag{2}$$

Note that w in (1) corresponds to the certainty equivalent of the entrepreneur's random income for the marginal agent $\bar{a}$. By the Jensen inequality, we then know that, for any risk-averse agent ($u'' < 0$), the certainty equivalent is smaller than the expected value of the random variable, i.e., $w < E[f(L(w,\bar{a},\theta)) - wL(w,\bar{a},\theta)]$. We can therefore say that, for the marginal or indifferent entrepreneur in our entrepreneurial context, the expected value of his residual profits must be greater than the sure wage received by being an employee.

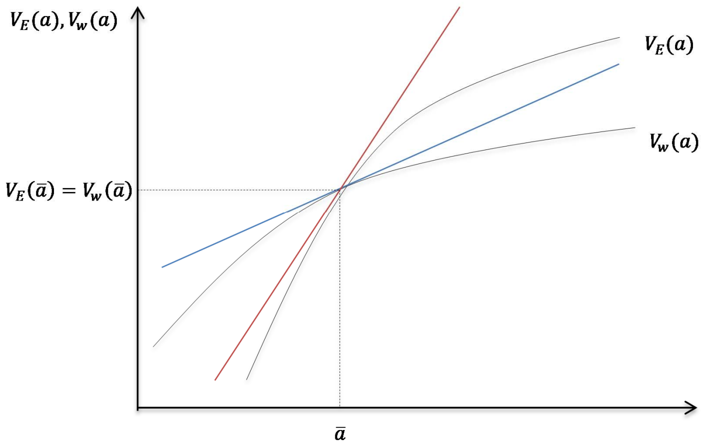
Now, let us assume that the wealth level $\bar{a}$ is unique. As we will see below, risk aversion, prudence and the fact that the strength of downside risk aversion exceeds the strength of risk aversion guarantee that for any other wealth level $a' > \bar{a}$, individuals would prefer to self-select as entrepreneurs. If the above is true, it means that the expected utility function of the entrepreneur $V_E(\bar{a})$ cuts off the utility function of the worker $V_W(\bar{a})$ from below and, therefore, the slopes of the utility functions at $\bar{a}$ satisfy:

$$V_E'(\bar{a}) = Eu'(f(L(w,\bar{a}),\theta) - wL(w,\bar{a}) + \bar{a}) > u'(w + \bar{a}) = V_W'(\bar{a}) \tag{3}$$

The graph below represents equation (3).

Let us take a short detour into ETS (2009) in order to have the tools to present our main result. We will now combine the occupational choice decision about becoming or not an entrepreneur with the apportion risk literature.

Figure 1: Marginal Utilities of Employee and Entrepreneur at $\bar{a}$



3 Apportioning Risk

EST (2009) provide an interesting result from the literature on the economics of risk. In their paper, the authors develop a way to rank lotteries focusing on higher order distributional characteristics. Their paper has two main results: one for the case of stochastic dominance and the other for cases of more nth-degree risk. For our purposes, we will focus on the second result, which is the corollary in EST (2009) and is based on 50-50 lotteries.

Corollary of EST (2009).

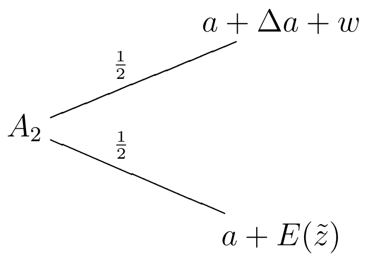
Suppose that $\tilde{Y}_i$ has more ith-degree risk than $\tilde{X}_i$ for $i = M, N$. So the lottery $[\tilde{X}_N+\tilde{X}_M, \tilde{Y}_N+\tilde{Y}_M]$ has more $(N+M)$th-degree risk than the lottery $[\tilde{X}_N+\tilde{Y}_M, \tilde{Y}_N+\tilde{X}_M]$.

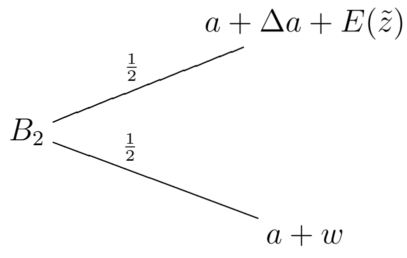
We will develop two applications of this result in our context of entrepreneurship and, since they are based on EST (2009), they are applicable only to 50-50 lotteries, like in EST (2009).

3.1 Mean-preserving increases in risk in the entrepreneurial context

Let $\tilde{X}_1 = a + \Delta a$, $\tilde{Y}_1 = a$, $\tilde{X}_2 = E(\tilde{z})$ and $\tilde{Y}_2 = w$, where a and Δa are the wealth and increment in wealth of the entrepreneur respectively and $w > 0$. Also, $\tilde{z}$ is a lottery (this is the stochastic residual profit of the entrepreneur) with non-zero expected value and let $E(\tilde{z}) > w$. Clearly $\tilde{Y}_1$ has more one-degree risk than $\tilde{X}_1$ and $\tilde{Y}_2$ has also more one-degree risk than $\tilde{X}_2$. Let A_2 be the $50-50$ lottery $[\tilde{X}_1 + \tilde{Y}_2, \tilde{Y}_1 + \tilde{X}_2]$ and B_2 be the $50-50$ lottery $[\tilde{X}_1 + \tilde{X}_2, \tilde{Y}_1 + \tilde{Y}_2]$.

These pairs of lotteries are represented by





Given that $N+M = 1+1 = 2$, EST (2009) guarantee that the lottery B_2 displays more second-degree risk than A_2. That is, B_2 can be obtained from A_2 by a mean-preserving increase in risk. This mean-preserving spread is graphically observed in Figure 2.

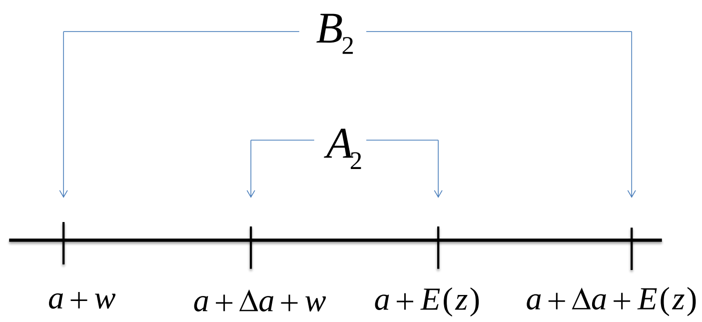
In the expected utility framework, $A_2 \succeq B_2$ if and only if

$$SR = Eu(A_2) - Eu(B_2) \geq 0 \tag{4}$$

Where SR stands for the disutility associated with a mean-preserving increase in risk and represents the **strength of risk aversion** (Menezes and Wang, 2004). Observe that $Eu(A_2) \geq Eu(B_2)$ is equivalent to

$$\frac{1}{2}u(a + \Delta a + w) + \frac{1}{2}u(a + E(\tilde{z})) \geq \frac{1}{2}u(a + w) + \frac{1}{2}u(a + \Delta a + E(\tilde{z}))$$

Figure 2: Mean-preserving spread



and note that if $\Delta a \to 0$, equation (4) implies that

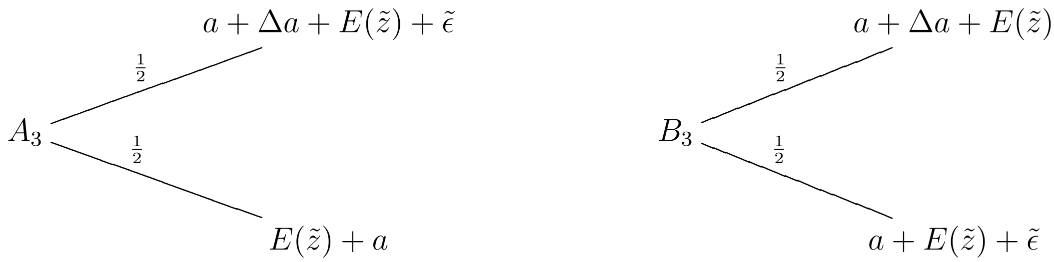
$$u'(a + w) \geq u'(a + E(\tilde{z})) \tag{5}$$

which is true if and only if $u'' < 0$. That is, the individual must be risk averse for (4) to be true.

3.2 Downside risk in the entrepreneurial context

Now, let us assume that $\tilde{X}_1 = a + \Delta a$, $\tilde{Y}_1 = a$, $\tilde{X}_2 = E(\tilde{z})$ and $\tilde{Y}_2 = E(\tilde{z}) + \tilde{\epsilon} = \tilde{z}$, where $\tilde{\epsilon} = \tilde{z} - E(\tilde{z})$ is a zero-mean risk. Note that $\tilde{Y}_1$ has more one-degree risk than $\tilde{X}_1$ and $\tilde{Y}_2$ has more two-degree risk than $\tilde{X}_2$ ($\tilde{Y}_2$ is a second-order increase in risk or a mean-preserving spread of $\tilde{X}_2$). Let A_3 be the $50-50$ lottery $[\tilde{X}_1 + \tilde{Y}_2, \tilde{Y}_1 + \tilde{X}_2]$ and B_3 be the $50-50$ lottery $[\tilde{X}_1 + \tilde{X}_2, \tilde{Y}_1 + \tilde{Y}_2]$.

These pairs of lotteries are represented by



Given that $N + M = 1 + 2 = 3$, EST (2009) guarantee that the lottery B_3 displays more third-degree risk than A_3, which means that B_3 has more downside risk than A_3 (Menezes et al., 1980). In the expected utility framework, $A_3 \succeq B_3$ if and only if

$$SD = Eu(A_3) - Eu(B_3) \geq 0 \tag{6}$$

Where SD stands for the disutility associated with the increase in downside risk and represents the **strength of downside risk aversion** (Menezes and Wang, 2004). Observe that $Eu(A_3) \geq Eu(B_3)$ is equivalent to

$$\frac{1}{2}Eu(a + \Delta a + E(\tilde{z}) + \tilde{\epsilon}) + \frac{1}{2}u(E(\tilde{z}) + a) \geq \frac{1}{2}u(a + \Delta a + E(\tilde{z})) + \frac{1}{2}Eu(a + E(\tilde{z}) + \tilde{\epsilon})$$

and note that, if $\Delta a \to 0$, equation (6) implies that

$$Eu'(a + \tilde{z}) \geq u'(a + E(\tilde{z})) \tag{7}$$

which is true if and only if $u''' > 0$. That is, the individual must be prudent for (6) to be true.

3.3 Risk Aversion, Downside Risk and Entrepreneurship

We now present the main result of this paper.

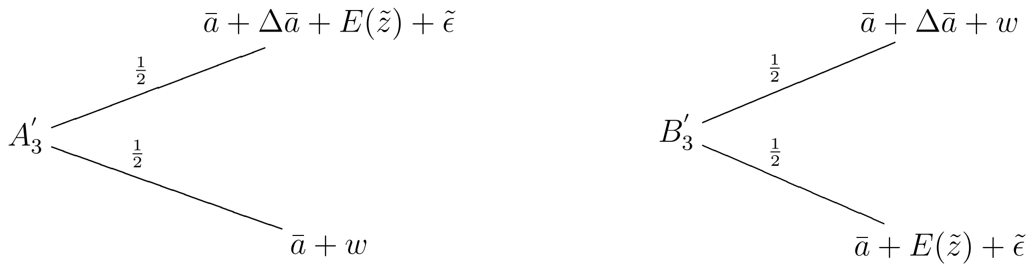
Proposition 1

In 50-50 lotteries, if the agent is risk averse and prudent ($u'' < 0$ and $u''' > 0$), when there is a small increase in wealth for the marginal entrepreneur ($\bar{a}$), i.e., the entrepreneur who is indifferent between choosing secure employment or self-selecting as an entrepreneur, he decides to transition from secure employment to entrepreneurship as long as the strength of downside risk aversion exceeds the strength of risk aversion.

Proof. Let us define the stochastic residual profit for the entrepreneur as $\tilde{z} = f(L, \theta) - wL$ and observe that the lottery $\tilde{z}$ has a non-zero expected payoff that can be decomposed into its expected payoff $E(\tilde{z})$ and a zero-mean lottery $\tilde{\epsilon} = \tilde{z} - E(\tilde{z})$. Also, from the entrepreneurial model above, w represents the certainty equivalent of the entrepreneur's random income from equation (1), which is smaller than the expected residual profit for the case of the marginal entrepreneur. In our context this means that $w < E(\tilde{z}) = E\left[f(L(w, \bar{a}, \theta)) - wL(w, \bar{a}, \theta)\right]$.

Now, let us assume that $\tilde{X}_1 = \bar{a} + \Delta\bar{a}$, $\tilde{Y}_1 = \bar{a}$, $\tilde{X}_2 = w$ and $\tilde{Y}_2 = E(\tilde{z}) + \tilde{\epsilon}$. Let A_3' be the $50-50$ lottery $[\tilde{X}_1 + \tilde{Y}_2, \tilde{Y}_1 + \tilde{X}_2]$ and let B_3' be the $50-50$ lottery $[\tilde{X}_1 + \tilde{X}_2, \tilde{Y}_1 + \tilde{Y}_2]$.

These pairs of lotteries are represented by



Observe that $Eu(A_3') > Eu(B_3')$ is equivalent to

$$\frac{1}{2}Eu(\bar{a} + \Delta\bar{a} + E(\tilde{z}) + \tilde{\epsilon}) + \frac{1}{2}u(\bar{a} + w) > \frac{1}{2}u(\bar{a} + \Delta\bar{a} + w) + \frac{1}{2}Eu(\bar{a} + E(\tilde{z}) + \tilde{\epsilon}) \quad (8)$$

and note that if $\Delta\bar{a} \to 0$, equation (8) implies that

$$Eu'(\bar{a} + E(\tilde{z}) + \tilde{\epsilon}) > u'(\bar{a} + w) \quad (9)$$

which is exactly condition (3) of the previous section, where the difference in the slopes of the utility functions guarantees that the expected utility function of the entrepreneur cuts off the utility function of the worker from below. In consequence, when the marginal (or the indifferent) entrepreneur gets a small increase in a, he prefers to transition to entrepreneurship. But when is this condition met?

Simple algebra over the lotteries shows that

$$Eu(A_3') - Eu(B_3') = [Eu(A_3) - Eu(B_3)] - [Eu(A_2) - Eu(B_2)] = SD - SR \quad (10)$$

Therefore, $Eu(A_3') > Eu(B_3')$ if and only if $SD > SR$. □

Intuition of this result tells us that, for the employee to shift into self-employment, the strength of downside risk aversion must exceed the strength of risk aversion when facing an increase in wealth. Figures 3a and 3b below provide the graphical intuition of this result.

Figure 3a shows the case where $SD > SR$, so a small increase in a for the marginal entrepreneur ($\bar{a}$) translates into a shift from secure employment into risky self-employment or entrepreneurship. Figure 3b, on the other hand, shows the opposite case. As we observe, $u' > 0$, $u'' < 0$ and $u''' > 0$ in both figures but only the first case satisfies our required condition ($SD > SR$) that guarantees the occupational transition.

The strength of risk aversion induces a reaction against risky projects like becoming an entrepreneur. On the contrary, the strength of downside risk is based on prudence, and prudence induces precautionary actions that guarantee a greater future payment under risky situations —which is exactly the case of the residual entrepreneurial profits— that is larger than the employee's wage. In consequence, we find two opposite effects when the indifferent individual $\bar{a}$ has a small increase in wealth a, and the transition to entrepreneurship depends on the relative magnitude of these effects, as has been shown in this article.

Figure 3: the Strength of Downside Risk Aversion versus the Strength of Risk Aversion

Figure 3a

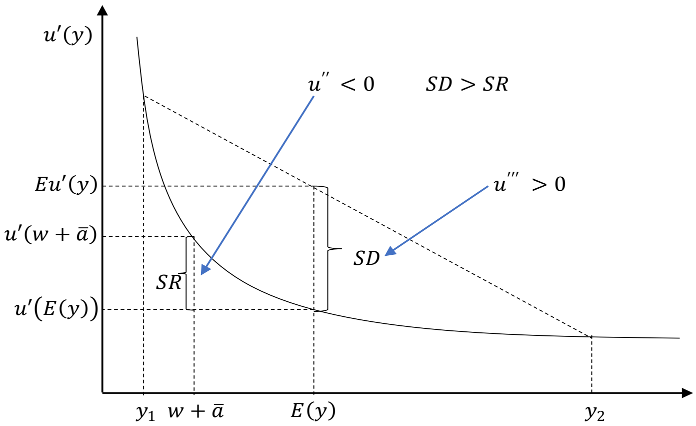
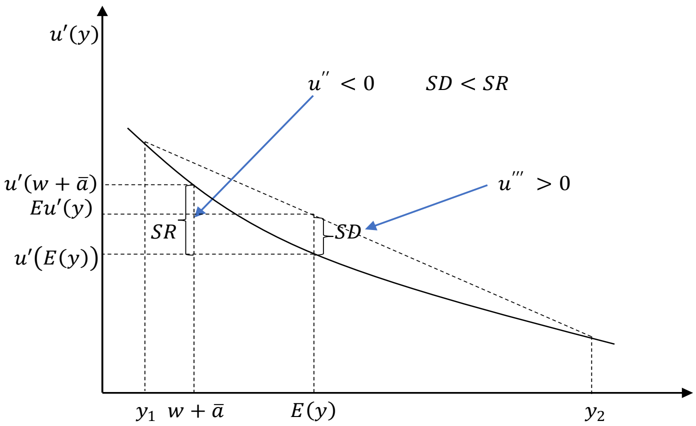


Figure 3b



4 Concluding Remarks

In this paper we have made use of the risk apportionment literature to delve deeper into the economics of occupational self-selection and show that, given risk aversion and prudence, and using the behavioral approach of risky of lotteries, the transition from secure employment to risky entrepreneurship is guaranteed if the strength of downside risk aversion exceeds the strength of risk aversion. The intuition of this result is based on the relative effect between prudence and risk aversion, as was discussed in the previous section.

From a policy perspective, we can say that traditional public policies promoting entrepreneurship can, in many cases, be assimilated to small increases in wealth for the entrepreneur (public subsidies, tax cuts or even loan guarantees). Therefore, the final impact of those policies depends not only on quality of the policy itself but also on the characteristics of preferences like risk aversion and downside risk aversion, and on their respective magnitudes.

We argued that, given that entrepreneurship is immersed in a world of uncertainty, incorporating recent advances from the theory of risk into the economic theory of entrepreneurship can be of importance to clarify —or even, in some cases, to refute— some concepts that had been previously accepted by the literature. Uncertainty and risk are at the heart of entrepreneurship and, therefore, considering these concepts explicitly should enrich the economic perspective of the study of entrepreneurship.

References

[1] Ahn, T. 2009. "Attitudes toward risk and self-employment of young workers". *Labour Economics*, 17, pp. 434-442.

[2] Bonilla C. and M. Vergara. 2013. "Credit Rationing or Entrepreneurial Risk Aversion? A Comment" *Economics Letters*, 120(2), pp. 329 - 331.

[3] Cressy, R. 2000. "Credit rationing or entrepreneurial risk aversion? An alternative explanation for the Evans and Jovanovic finding." *Economics Letters*, 66(2), pp. 235-240.

[4] Davis, G. 1989. Income and Substitution Effects for Mean-Preserving Spreads. *International Economic Review* 30(1): 131-136.

[5] Deck, C. and H. Schlesinger. 2014. Consistency of higher order risk preferences. *Econometrica* 82(5): 1913-1943.

[6] Denuit, M. and B. Rey. 2014. "Benchmarking values for higher order coefficients of relative risk aversion". *Theory and Decision*, 76, pp. 81 - 94

[7] Dreze, J. and F. Modigliani. 1972. Consumption decisions under uncertainty. *Journal of Economic Theory* 5: 308-355.

[8] Ebert, S. and D. Wiesen. 2014. Joint measurement of risk aversion, prudence, and temperance. *Journal of Risk and Uncertainty* 48: 231Â–252

[9] Eeckhoudt, L. and R. Huang. 2012. "Precautionary effort: A new look". *Journal of Risk and Insurance*, 79(2), pp. 585 - 590

[10] Eeckhoudt, L. and B. Rey. 2011. "Risk vulnerability: A graphical interpretation. *Theory and Decision*, 71, pp. 227 - 234

[11] Eeckhoudt, L. and H. Schlesinger. 2006. "Putting risk in its proper place". *The American Economic Review*, 96, pp. 280 - 289.

[12] Eeckhoudt, L., Schlesinger, H. and Tsetlin, I. 2009. Apportioning of risks via stochastic dominance. *Journal of Economic Theory*, 144: 994-1003.

[13] Eeckhoudt, L., Fiori, A. and Gianin, E. (2016). "Loss-averse preferences and portfolio choices: An extension ". *European Journal of Operational Research*, 249: pp. 224-230.

[14] Hvide, H. K., G. A. Panos. 2014. "Risk tolerance and entrepreneurship". *Journal of Financial Economics*, 111, pp. 200-223.

[15] Kihlstrom, R. and J. Laffont. 1979. "A general equilibrium entrepreneurial theory of new firm formation based on risk aversion." *Journal of Political Economy* 87, pp. 304-316.

[16] Kimball, M. S. 1990. "Precautionary saving in the small and in the Large". *Econometrica*, 58(1), pp. 53-73.

[17] Knight, F. 1921. "Risk, Uncertainty, and Profit." Hart, Schaffner, and Marx Prize Essays, no. 31. Boston and New York: Houghton Mifflin.

[18] Krieger, M. and T. Mayrhofer. 2017. Prudence and prevention: an economic laboratory experiment, *Applied Economics Letter*s 24:1, 19-24

[19] Menegatti, M. 2014. "New results on the relation among risk aversion, prudence and temperance ". *European Journal of Operational Research*, 232: pp. 613-617.

[20] Menezes, C., Geiss, C., & Tressler, J. 1980. Increasing downside risk. *The American Economic Review* 70(5):pp 921-932.

[21] Menezes, C. F. and Wang, X. H. 2004. On the risk-downside risk tradeoff. *The Manchester School*, 72(2): pp. 179-187.

[22] Snow, A. 2003. Substitution and Income Effects for Increases in Risks. Economic Letters 79: 313-317.

[23] Wang, J. and J. li. 2014. "Precautionary effort: Another trait for prudence". *Journal of Risk and Insurance*, 82(4), pp. 977 - 983