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A new look at the value of leisure in two-worker households

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A R T I C L E I N F O

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A B S T R A C T

We depart from the individual view behind those time allocation models aimed at the calculation of the value of leisure (VoL) when dealing with two-worker households, as this (prevailing) view requires approximations and assumptions that do not hold well. As the existing theories that consider the household deals with time allocation but not with time values, we propose an integration of some elements of both theories, generating a model with a collective view from which the VoL can be obtained. We argue theoretically that this model yields larger VoL and contains elements that should diminish the possible gap in VoL between the workers. The improved model is applied using Chilean data verifying its theoretical properties at various levels and correcting counterintuitive results obtained with the individual model.

1. Introduction

Following the analysis of DeSerpa (1971) and Oort (1969), Jara-Díaz and Guevara (2003), showed that the willingness to pay to reduce travel time or SVTTS (subjective value of travel time savings) usually obtained from travel choice models, hides two components: the value of reallocating the time liberated to another activity (that is, the value of time as a resource or value of leisure) and the value of time assigned to travel or VTAT (which reflects the conditions of the trip and the possible discomfort that it causes). Disentangling these two components is necessary to make a correct analysis of the impacts of a change in the transport system and to guide the way investments in passenger transport should be made whenever a conflict between speed and comfort arises (Jara-Diaz, 2020).

Advancements in the microeconomic theory of time use during this century have made it possible to formulate and estimate the value of leisure (VoL) of those individuals represented in travel choice models, which permits the calculation of both components. Although the evolution of travel choice models yields increasingly reliable estimates of SVTTS, there is still a wide space for improvements in the estimation of the VoL in order to obtain the VTAT. As models to estimate VoL have been conceived and applied looking at individuals only, in this paper we want to improve the theory behind those time use models from which the VoL can be obtained by changing the view on observed behavior from the individual to the household, and to verify these improvements empirically.

Although Becker (1965) established a microeconomic framework where time use was integrated into consumer theory in a formal way. The fact that his model lacked time at work as an argument in utility induced a single value of time for all activities considered by Becker (consumption, work, and leisure), and this was the wage rate. As explained in the next section, this rather myopic approach was enriched by DeSerpa (1971), who justified rigorously a series of different values of time and their relationships. It was this much richer framework the one that served to link SVTTS - as extracted from travel choice models - with the VoL, as suggested by Truong and Hensher (1985) and corrected by Bates (1987). The VoL, however, remained hidden behind SVTTS until Jara-Díaz and Guevara (2003) managed to establish the theoretical basis for its empirical estimation, which was first fully applied in Jara-Díaz et al. (2008) using data from various cities and countries, and has been enriched with the simultaneous estimation with travel choice models in Munizaga et al. (2008) and Jokubauskaite et al. (2019).

All these advances have been made under an individual approach, such that the extensive applied work has required either the consideration of single-worker households or some assumptions regarding the distribution of expenses within multi-worker households. In this article, we examine the possible influence that the individual-unitary approach taken so far might have on the value of leisure in the case of two-worker households. We depart from this approach by integrating elements from the household economic theory which are used to build a theoretical framework that expands the scope of the calculation of the value of leisure. This makes it possible to estimate VoL for the first time

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considering consumption and time externalities at a household level. Most importantly, theoretical properties that can be tested empirically are found; they relate to the marginal utilities of both income and leisure, to the VoL, and to the differences between workers regarding these values.

To illustrate this, we create a weekly time use database of two-worker (of a different gender) Chilean households, built from the Chilean National Time Use Survey (NTUS; INE , 2015). The household time-use model yields empirical results that are consistent with the theoretical findings regarding marginal utilities, VoL, and gender differences which are shown to diminish with the new household model for the high income segment.

In Section 2 we present a synthesis of microeconomic time use models, reviewing those individual-based theoretical models that allow estimating empirically the values of leisure and/or work, as well as household models. In Section 3 we present the microeconomic formulation of a household time use model, analyzing its properties and the resulting theoretical values of time, and we obtain a system of equations that can be estimated econometrically. In Section 4 we apply the theoretical model experimentally, verifying the theoretical improvements when compared against parameters and time values obtained from the individual model of Jara-Díaz and Guerra (2003) applied to the same sample of workers. In Section 5 we summarize the approach, the main conclusions, and the new avenues for further research.

2. Microeconomic time use models and values of time

Becker (1965) was the first to propose a general theory of time allocation by including consumption time in the utility function plus budget and time constraints. As Becker considered paid work only as a source of income but not of utility, a single value of time equal to the wage rate (the opportunity cost of allocating time to consumption instead of work) was obtained. Johnson (1966) incorporated working time into the utility function, obtaining that the value of leisure was equal both to the total value of work (wage rate plus value of time assigned to work) and to the value of travel time, asserting that diminishing travel time was like expanding the day; this was corrected by Oort (1969) by proposing a model that incorporates travel time into utility, showing that the value of travel time savings is equal to the VoL minus the marginal value of time assigned to travel (usually considered to be negative, therefore adding willingness to pay for its reduction). Evans (1972) considered a utility function that depended exclusively on the time allocated to all activities, which in turn require market goods to be carried out, such that each activity has an implicit cost that comes from the consumption necessary to actually perform it.

DeSerpa (1971) made a long-lasting theoretical contribution to the concept and analysis of the VoL. He postulated a utility function dependent on all goods and activities (including work) and proposed technical relations reflecting a minimum time needed for each activity imposed by the consumption of goods. DeSerpa defined three different concepts of value of time: i) the *value of time as a resource* μ/λ, where μ and λ are the Lagrange multipliers associated with time and budget constraints respectively (i.e. λ is the marginal utility of income); ii) *the value of time as a commodity*, which is the money value of the marginal utility of assigning time (T_j) to an activity *j*, i.e. the ratio between $\partial U/\partial T_j$ and λ; and iii) *the value of saving time* in an activity *j*: κ_j/λ, where κ_j is the Lagrange multiplier associated to the technical constraint of activity *j*. He found two important relations among values of time. First, the value of saving time in one activity equals the value of time as a resource minus the value of time assigned to that activity; for *leisure* activities, i.e. those to which more time than the necessary minimum is allocated, the multiplier κ_j is zero, making $\mu/\lambda = \text{VoL}$. Second, time is assigned in such a way that the VoL equals the total value of work. So DeSerpa confirmed Oort's result: the willingness to pay to reduce travel time has two components: the value of liberated time (leisure or work) and (minus) the value of time assigned to travel (which depends on the conditions of travel).

Jara-Díaz and Guevara (2003) formally demonstrated the equivalence between SVTTS (obtained with discrete choice models) and the value of saving time in the transport activity κ_j/λ. From their model inspired by DeSerpa, they deduced that the optimal working time (conditional on preference parameters) was a function of the wage rate, and the transport time and cost. By estimating the preference parameters and a discrete choice model, they obtained empirical values of not only leisure and work, but also of the two components of SVTSS.

This model was generalized by Jara-Díaz (2003) showing that there are two families of technical relations: goods consumption imposes minimum duration of associated activities, and activities impose minimum consumption of associated goods. Jara-Díaz and Guerra (2003) considered simplified expressions for these technical relationships in the form of exogenous minima for consumption and time, obtaining an explicit system of equations for unrestricted goods consumption, unrestricted activities time allocation, and working time, as functions of committed time and committed expenses (i.e. time and money that the individual cannot assign freely), and the wage rate. The explicit equations obtained allowed the econometric estimation of the utility parameters, which permitted the calculation of the values of both leisure and time assigned to work. This will be referred here as the *basic model*. Jara-Díaz et al. (2008) applied this framework to estimate the first general time use model that obtained values of leisure and work empirically. The basic model was also used by Jara-Díaz et al. (2013) to estimate these values for different segments in Santiago; and by Jara-Díaz and Candia (2017) for Chilean workers.

The first simultaneous time-use travel-choice model was estimated by Munizaga et al. (2008) obtaining all SVTTS components. Advancements were made by Jara-Díaz et al. (2016) by improving the treatment of technical relations between goods and activities, and by Castro et al. (2012) using the multiple discrete–continuous extreme value (MDCEV) choice model.[2] Rosales-Salas and Jara-Díaz (2017) incorporated domestic work as a special activity that can also be performed by external (paid) suppliers. Jokubauskaite et al. (2019) estimated the basic model simultaneously with consumption and mode choice, using for the first time expenditure data directly collected and mode choice for all trips made by the individual in the period (a week), The basic model and its derivatives constitute the only existing microeconomic framework from which the VoL can be obtained by observing revealed preferences regarding time use and goods consumption.[3] It requires, however, the identification of committed time and committed expenses as exogenous variables. This presents no big problem when single-worker households are considered, but in the case of multiple-worker households some assumptions have to be done regarding individual committed expenses, as in Jokubauskaite et al. (2019) or Hössinger et al. (2020), where two adjustments were tried: making free expenses proportional to individual earnings or distributing free expenses evenly across workers. As the results were indeed sensitive to these adjustments, they concluded that "the VoL is influenced by how resources are allocated among household members. Given the importance of this aspect, we consider household models (…) as an avenue for future work". This motivates the examination of household models as useful sources to improve the theoretical

[2] This model has also been applied in time use related research that is not aimed at the calculation of values of leisure. See Bernardo et al. (2015), who study activity patterns of dual-earner couples; and Calastri et al. (2020), who study the correlation between activities within and across different days. A nested version developed by Pinjari and Bhat (2010) has been used by Calastri et al. (2017) to model activity type and duration.

[3] One possible exception is the Structural Equations Model by Konduri et al. (2011), who obtained a revealed willingness to pay for a leisure activity as one of the results; this was shown to be different from the VoL (Jara-Díaz and Astroza, 2013).

framework of time use models to obtain the VoL.

Household time use models are those that represent the preferences or decisions of all providers within a household, and have been developed mainly to improve the understanding of time assignment *per se*, including prominently paid and domestic work time. Such models see the household as a community that benefits from avoiding transaction costs in the market, economies of scale through the joint consumption of public goods, insurance against risks such as illness or unemployment, and specialization in different activities (Ben-Porath, 1980; Pollak, 1985; Beblo, 2001; Sen, 1989). They can be classified into two categories: unitary and bargaining models. In the former, the main implicit assumption is that households behave as a unit of decision (Chiappori and Mazzocco, 2017), which is equivalent to represent the household's preferences using a single utility function that does not depend on individual but on household's aggregate consumption (e.g., Samuelson, 1956; Becker, 1974); Bargaining models take elements of game theory, looking at each member as an autonomous decision unit with individual but interdependent preferences, focusing on the distribution of resources and internal power (Beblo, 2001; Chiappori and Mazzocco, 2017); they can be cooperative or non-cooperative.

In non-cooperative models each household member decides individually but is influenced by the decisions of the other members (Beblo, 2001). This was pioneered by Leuthold (1968), who considered an individual utility function which depends on the leisure time of the individual and the joint goods consumption; each individual acts as if working time of the other individual is given (see also Bergstrom et al., 1986; Browning et al., 2009; or Chen and Woolley, 2001). Cooperative models assume free communication between members, symmetry of information and access to bargaining, which leads to a Pareto optimum time allocation and distribution of resources (Beblo, 2001). Manser and Brown (1980) and McElroy and Horney (1981) proposed a cooperative model to analyze the benefits of a marriage who shares income but is subject to dissolution due to individual options outside the household. Lundberg and Pollak (1993) proposed a similar approach, but where the household is subject not to dissolution, but to a non-cooperative equilibrium achieved by a distribution of roles according to social norms. Cooperative models require the identification of a bargaining source, which limits their empirical application (Vermeulen, 2002). Chiappori (1988; 1992) and Apps and Rees (1988) took care of this limitation in the so-called collective models, where no mechanism of interaction within the household is specified and the so-called sharing rule emerges (how members distribute income). Later developments along this line include Apps and Rees (1996) and Chiappori (1997), who captured the importance of incorporating domestic production into household models.

For summary, consumer theory including time provides a good basis to investigate time use and values, but only a segment of that literature helps obtaining theoretically and empirically the VoL beyond the simplistic idea that this is well represented by the individual wage rate. The main empirical limitations caused by the consideration of single individuals when analyzing multiple-worker households are twofold: all individual models consider that time assigned to work does not depend on the income (or work) of the other household worker; and committed expenses have to be arbitrarily assigned to each individual. These limitations make it necessary to adopt a collective view in order to consider their interaction regarding activities and expenses. Household literature, however, has not dealt with values of time. The fusion is presented in the next section.

3. From individual to household time use models and values

3.1. The individual worker model: synthesis

The basic (individual) time use model of Jara-Díaz and Guerra (2003) described earlier is represented by Eqs. (1), where $X$is a vector of goods consumption, and each good i has a market price P_i; T is a vector of time assigned to all activities j different from paid work, and T_w is the time assigned to paid work, which has a wage rate w. I is the fixed non-labor income, τ is the total time available, and X_i^{min} and T_J^{min} are exogenous minimum consumption and time for each good and activity respectively.

$$Max\ U\ (X, T,\ T_w) \tag{1}$$

$$s.t.$$

$$I + wT_w - \sum_i P_i X_i \geq 0 \quad (\lambda) \tag{1.2}$$

$$\tau - T_w - \sum_j T_j = 0 \quad (\mu) \tag{1.3}$$

$$X_i - X_i^{min} \geq 0 \quad \forall i(\eta_i) \tag{1.4}$$

$$T_j - T_J^{min} \geq 0 \quad \forall j \quad (\kappa_j) \tag{1.5}$$

From this formulation, and considering a Cobb-Douglas utility form such as $U = \Omega T_w^{\theta_w} \prod_{j\in J} T_j^{\theta_j} \prod_{i\in I} X_i^{\phi_i}$, explicit closed-form equations for time at work (Eq. (2)), for time at unrestricted activities (Eq. 3), and for unrestricted goods consumption (Eq. (4)) were obtained by the authors: (the detailed derivation of these equations is shown is Appendix C)

$$T_w^* = \frac{(\Phi + \theta_w)(\tau - T_c) + (\Theta + \theta_w)\frac{\widehat{E}_c}{w} + \sqrt{\left((\Phi + \theta_w)(\tau - T_c) + (\Theta + \theta_w)\frac{\widehat{E}_c}{w}\right)^2 - 4\theta_w(\tau - T_c)\frac{\widehat{E}_c}{w}}}{2(\Phi + \Theta + \ \theta_w)} \tag{2}$$

$$T_j^* = \frac{\theta_j}{\Theta}\left(\tau - T_w^* - T_c\right) \quad \forall j \in A^f \tag{3}$$

$$X_i^* = \frac{\phi_i}{P_i\Phi}\left(wT_w^* - \widehat{E}_c\right) \quad \forall i \in G^f \tag{4}$$

where A^f and G^f are the sets of unrestricted activities and goods, respectively (those activities - or goods - that are assigned more time - or consumption - than the minimum). Θ is the summation of the exponents θ_j over A^f and Φ is the summation of the exponents η_i over G^f. T_c is the summation of time allocated to restricted activities and $\widehat{E}_c$ is the summation of individual expenses on restricted gods minus non-working income. It is important to note that Eq. (2) is a labor supply model that depends on the wage rate w and also on committed time T_c and individual committed expenses $\widehat{E}_c$, which reduces total time and available income, respectively.

From this model the value of leisure can be obtained as (for details see Appendix C):

$$VoL = \frac{\mu}{\lambda} = \frac{\Theta}{\Phi}\frac{\left(wT_w^* - \widehat{E}_c\right)}{\left(\tau - T_w^* - T_c\right)} \tag{5}$$

As stated earlier, the model is perfectly suited for single-worker households. When it is applied to a sample of workers that includes multiple-worker households, some *a priori* adjustments have to be made and some assumptions are required. Among the main adjustments, the assignment of committed expenses observed at a household level has to be distributed across earners (workers). An important assumption is that individuals make decisions on their own labor supply that is a function of their own wage rate only (and T_c and $\widehat{E}_c$), irrespective of the earnings of the other(s). Also, individual time assignment does not interact with other workers' decisions in the household. These are key elements to apply this framework and to estimate the parameters and the values of leisure and work empirically.

3.2. The household time use model

In the household time use model (HTUM, Eqs. (6.1) - (6.6), we consider a household with two workers of class $n = a, b$ (e.g. man, woman) with a household utility function (Eq. (6.1)) whose arguments are individual time assigned to activities and market and domestic goods consumed by the household, inspired by the household unitary approach and looking like the utility of a collective model with 'caring' preferences as defined by Chiappori and Mazzocco (2017).

The assumption of a common consumption follows Chiappori (1988; 1992) where the individual consumption is not observable. This is particularly useful for the formulation of the HTUM and the estimation of values of time, since in previous (individual) estimations it has been necessary to make assumptions about how the observable household consumption is distributed between the different members.

The household utility depends on individual time allocation following Evans (1972); in this case, however, observed individual time allocation will depend on what the other member of the household does. We distinguish three types of activities: paid work T_w^n, a vector of domestic work activities T_D^n and a vector of other activities T^n. The total time available for each individual is τ.

Following most unitary, collective and cooperative household models (e.g. Manser and Brown, 1980; McElroy and Horney, 1981; Lundberg and Pollak, 1993) we consider the budget constraint at household level with the household income being the sum of the total individual incomes as shown in Eq. (6.2), where w^n is the individual's n wage rate, I is the fixed household income that comes from sources other than paid work and P_i is the market price per unit of the good i. As in all household models, the time constraint (Eq. (6.3)) holds for each individual.

Following Rosales-Salas and Jara-Díaz (2017) we consider intermediate goods X_d necessary to produce each final domestic good Z_d. The intermediate goods are related technically with the time assigned to produce them (T_d^n) in the form $X_d = o_d^a T_d^a + o_d^b T_d^b$ where o_d^n is the amount of intermediate domestic good needed by unit of time allocated to domestic activity d by the individual n, so the total expenses on intermediate goods are expressed by $\sum_d P_d(o_d^a T_d^a + o_d^b T_d^b)$.

Following Jara-Díaz and Guerra (2003), Eqs. (6.4) and (6.5) state that there is a minimum exogenous consumption X_i^{min} for each good at household level (which can be zero), and a minimum time $T_j^{n\,min}$ for each activity, which may be different for each individual (and may also be zero). The production of domestic good is represented by the function h_d (Eq. (6.6)), which states that each final domestic good depends on the time allocated by each individual.[4] From Eqs. (6.4) and (6.5), we define the set of restricted activities A^r as those in which the time assigned is equal to $T_j^{n\,min}$, and its complement the set of unrestricted or free activities A^f. For simplicity, from now on we consider these sets equal for both members. In addition, we define the set of restricted goods X^r as those in which the household consumes the minimum X_i^{min}, and its complement the set of unrestricted or free goods X^f.

$$Max\ U = U\left[T_w^a,\ T^a,\ T_D^a, T_w^b,\ T^b,\ T_D^b,\ X,\ Z_D\right] \tag{6.1}$$

s.t

$$\sum_i P_i X_i + \sum_d P_d\left(o_d^a T_d^a + o_d^b T_d^b\right) = w^a T_w^a + w^b T_w^b + I \quad \leftarrow \lambda \tag{6.2}$$

$$T_w^n + \sum_j T_j^n + \sum_d T_d^n = \tau \quad \leftarrow \mu^n \quad n = a, b \tag{6.3}$$

$$X_i \geq X_i^{min} \quad \forall i \quad \leftarrow \eta_i \tag{6.4}$$

$$T_j^n \geq T_j^{n\,min} \quad \forall j \quad \leftarrow \kappa_j^n \quad n = a, b \tag{6.5}$$

$$h_d\left(T_d^a,\ T_d^b\right) - Z_d = 0 \quad \forall d \quad \leftarrow \gamma_d \tag{6.6}$$

From the first order conditions with respect to individual paid work time for each individual (see Appendix A) we obtain the relation between the value that the household assigns to paid work and leisure of each individual. Eq. (7) shows that the household total value of paid work for an individual n (term on the left) and the household value of leisure for that individual are related in a similar way to that obtained by DeSerpa (1971) with two key differences: the marginal utilities of both income and work time are at household level. Note that this means that λ is the same for both individuals a and b.

$$\frac{\partial U / \partial T_w^n}{\lambda} + w^n = \frac{\mu^n}{\lambda} \quad n = a, b \tag{7}$$

In the HTUM the value of leisure represents the valuation that the household assigns to an additional unit of time available for one of its members. As income is totally shared, its marginal utility at the household level (λ) should be lower than under an individual approach. In addition, increasing the leisure of an individual increases the utility of the household (positive crossed externalities). Both effects suggest that the values of leisure and of time assigned to work will be larger than those obtained with the basic model and rather similar across individuals.

From the first-order condition of the problem with respect to an activity j of the individual n, we obtain the household willingness to pay for saving time in a restricted activity (κ_j^n / λ) of the individual n (see Appendix A), as it is shown in Eq. (8), where the same two key differences that showed up in Eq. (7) can be seen.

$$\frac{\kappa_j^n}{\lambda} = \frac{\mu^n}{\lambda} - \frac{\partial U / \partial T_j^n}{\lambda} \quad \forall j \in A^r\ n = a, b \tag{8}$$

Although in the empirical work we will assume that domestic work is a restricted activity, it is worth establishing the first order condition with respect to T_d^n for future work. Following the derivations shown in Appendix A we obtain Eq. (9), which states that, in equilibrium, the household value assigned to individual domestic work time (left hand) equals the household value of leisure for that individual (similar to that obtained by Rosales-Salas and Jara-Díaz, 2017, for the single-worker case).

[4] The amount of final domestic good depends on intermediate goods to be produced; as these are a function of domestic work, the time allocated to this type of work is what determines the level of the final domestic good.

$$\frac{\partial U/\partial T_d^n}{\lambda} - P_d o_d^n + \frac{\gamma_d}{\lambda}\frac{\partial h_d}{\partial T_d^n} = \frac{\mu^n}{\lambda} \quad \forall d \quad n = a, b \tag{9}$$

The household total value of domestic work for an individual has three components: i) the value of the time allocated to the domestic work of the individual n, which considers the monetary effect of domestic work on the utility of the household; ii) the effect on income, since one extra unit of domestic work induces an increase on expenses on intermediate domestic goods in an amount $P_d o_d^n$ per unit of time; and iii) the variation induced by domestic work in the production of final domestic good $(\partial h_d/\partial T_d^n)$ multiplied by γ_d, which is the variation in household utility that an increase in domestic good causes.

In order to obtain a system of stochastic equations from the first-order conditions, we assume a Cobb-Douglas functional form for utility in (Eq. (6.1))[5]

$$U = \Omega T_w^{a\,\theta_w^a}\prod_j T_j^{a\theta_j^a}\prod_d T_d^{a\theta_d^a}T_w^{b\,\theta_w^b}\prod_j T_j^{b\theta_j^b}\prod_d T_d^{b\theta_d^b}\prod_i X_i^{\phi_i}\prod_d Z_d^{\phi_d} \tag{10}$$

As advanced above, intermediate domestic goods are considered restricted and the final domestic good is not a decision variable in this model. With this assumption, we obtain from the first order conditions a system of $2 + 2|A^f| + |X^f|$ equations (Eqs. (11)–(13)) with explanatory variables w^a, w^b., E_c, T_c^a, T_c^b and P_i, where $T_c^n$is the committed time (sum of observed time assigned to restricted activities) of individual n, and E_c are household expenses on restricted goods minus the fixed income from sources other than work. Φ is the summation of the parameters ϕ_i of each unrestricted good and Θ^n is the summation of the parameters θ_j^n of each unrestricted activity. See Appendix A for a detailed derivation of this system.

$$\frac{\theta_w^n}{T_w^{n*}} + \frac{\Phi}{w^a T_w^{a*} + w^b T_w^{b*} - E_c}w^n - \frac{\Theta^n}{(\tau - T_w^{n*} - T_c^n)} = 0 \quad n = a, b \tag{11}$$

$$\frac{\theta_j^n}{T_l^{n*}} - \frac{\Theta^n}{(\tau - T_w^{n*} - T_c^n)} = 0 \quad \forall j \in A^f,\ n = a, b \tag{12}$$

$$\frac{\phi_i}{P_i X_i^*} - \frac{\Phi}{w^a T_w^{a*} + w^b T_w^{b*} - E_c} = 0 \quad \forall i \in X^f \tag{13}$$

From this system an analytical solution can be obtained for T_w^n, T_j^n and X_i(Eqs. (14)–(16)).

$$T_w^{n*} = \frac{-B^n + \sqrt{B^{n2} - 4A^nC^n}}{2A^n} \quad n = a, b \tag{14}$$

$$T_j^{n*} = \frac{\tilde{\theta}_l^n}{\Theta^n}\left(\tau - T_w^{n*} - T_c^n\right) \quad \forall j \in A^f,\ n = a, b \tag{15}$$

$$X_i^* = \frac{\phi_i}{\Phi}\frac{\left(w^n T_w^{a*} + w^b T_w^{b*} - E_c\right)}{P_i} \quad \forall i \in X^f \tag{16}$$

where:

$$A^n = \theta_w^n + \Phi + \Theta^n \quad n = a, b \tag{17}$$

$$B^n = -\left(\frac{E_c}{w^n} - \frac{w^{-n}}{w^n}T_w^{-n*}\right)\left(\theta_w^n + \Theta^n\right) - \left(\theta_w^n + \Phi\right)\left(\tau - T_c^n\right) \; n = a, b \tag{18}$$

$$C^n = \theta_w^n\left(\frac{E_c}{w^n} - \frac{w^{-n}}{w^n}T_w^{-n*}\right)\left(\tau - T_c^n\right) \quad n = a, b \tag{19}$$

Eq. (14) is a solution of a quadratic equation for T_w^n that has originally two roots, but only the positive root is correct, as can be verified by solving Eq. (11) when $\theta_w^n = 0$.

It is important to recall that in this model the unit of observation is a household and, within a household, the modeler has to distinguish between individuals belonging to classes a and b. Estimating such a model, then, reveals the preference parameters of each of these types of individuals; although gender is a natural (and interesting) class candidate, it is not the only one that could be used (consider, for example, age or education).

As formulated, the HTUM borrows from the household unitary and collective approaches but leads to the calculation of values of time. With the assumptions described before, the household value of leisure for an individual (VoL_n) is given by Eq. (20), and the household value of time assigned to work (VoW_n, Eq. (21)) can be calculated either subtracting the respective wage rate from VoL_n or - equivalently – using the estimated parameters:

$$VoL_n = \frac{\Theta^n}{\Phi}\frac{w^a T_w^{a*} + w^b T_w^{b*} - E_c}{\left(\tau - T_w^{n*} - T_c^n\right)} \tag{20}$$

$$VoW_n = \frac{\theta_w^n}{\Phi}\frac{w^a T_w^{a*} + w^b T_w^{b*} - E_c}{T_w^{n*}} = VoL_n - w_n \quad n = a, b \tag{21}$$

Eq. (20) for the value of leisure of individual type n assigned by the household (i.e. by the collective of workers) is quite interesting theoretically, as it involves two preference parameters and two quantities. The parameter in the numerator represents the individual preference for leisure time, linked to the marginal utility of leisure time (expected to be larger for the individual with less uncommitted time), and the one in the denominator (Φ) is the (household) preference for goods consumption, linked to the marginal utility of income λ. Regarding quantities, the numerator is (household) income available to spend on goods and the denominator is the individual time that can be assigned for freely chosen activities. A similar analysis can be done for the value of work. This will prove useful for the analysis of results, and suggests that the HTUM should be estimated by income group because λ- and therefore Φ- is expected to differ across groups, diminishing with income.

The HTUM is indeed a better representation of two-worker households with the aim of obtaining values of time, overcoming the limitations of the individual model, but is certainly not the only possible formulation from the viewpoint of time assignment to activities under other household arrangements. The problem described by Eqs. (6.1) - (6.6) could be used to formulate the case of alternative organizations at a household level as, for example, that of one member doing only unpaid domestic work while the other brings the money; producing such a model and the associated time equations and time values is likely to be a fruitful avenue for research.

3.3. Formulation of the stochastic equations systems

Let us consider leisure (l) and sleep (s) as unconstrained activities, with leisure represented by Eq. (15) (sleep is obtained from the time constraint). We apply a normalization of the parameters to obtain an econometrically identifiable system of equations; accordingly, Eqs. (11) and (12) are divided by θ_s^a. In what follows we do not consider Eq. (13) or 16 for unrestricted goods, as they will not be used in the empirical section due to lack of data regarding these variables. As each individual satisfies the equilibrium condition for paid work (Eq. (14)), we use the observed working time of the other household member ($T_w^{a\,obs}$ and $T_w^{b\,obs}$) in the respective resulting equations, borrowing from the non-cooperative models as described in Section 2. By adding stochastic error terms u_w^n and u_l^n to Eqs. (14) and (15), we obtain a system of stochastic equations (Eqs (22)–(24)). As stated by Jara-Díaz et al. (2008), the error sources to be considered in the definition of the error structure are: measurement errors in all the observed variables, differences among individuals, specification errors and the randomness inherent to human nature.

[5] This permits comparison with the basic individual model of Jara-Díaz et al. (2008) both alytically and empirically. It prevents, however, the use of observations where one potential worker actually does not undertake paid work.

$$T_w^{n^*} = \frac{-\tilde{B}^n + \sqrt{\tilde{B}^{n2} - 4\tilde{A}^n\tilde{C}^n}}{2\tilde{A}^n} + u_w^n \quad n = a, b \tag{22}$$

$$T_l^{a^*} = \frac{\tilde{\theta}_l^a\left(\tau - T_w^{a^*} - T_c^a\right)}{\left(\tilde{\theta}_l^a + 1\right)} + u_l^a \tag{23}$$

$$T_l^{b^*} = \frac{\tilde{\theta}_l^b\left(\tau - T_w^{b^*} - T_c^b\right)}{\left(\tilde{\theta}_l^n + \tilde{\theta}_s^n\right)} + u_l^b \tag{24}$$

with

$$\tilde{A}^a = \tilde{\theta}_w^a + \tilde{\Phi} + \tilde{\theta}_l^a + 1 \tag{25}$$

$$\tilde{A}^b = \tilde{\theta}_w^b + \tilde{\Phi} + \tilde{\theta}_l^b + \tilde{\theta}_s^b \tag{26}$$

$$\tilde{B}^a = -\left(\frac{E_c}{w^a} - \frac{w^b}{w^a}T_w^{b\,obs}\right)\left(\tilde{\theta}_w^a + \tilde{\theta}_l^a + 1\right) - \left(\tilde{\theta}_w^a + \tilde{\Phi}\right)\left(\tau - T_c^a\right) \tag{27}$$

$$\tilde{B}^b = -\left(\frac{E_c}{w^b} - \frac{w^a}{w^b}T_w^{a\,obs}\right)\left(\tilde{\theta}_w^b + \tilde{\theta}_l^b + \tilde{\theta}_s^b\right) - \left(\tilde{\theta}_w^b + \tilde{\Phi}\right)\left(\tau - T_c^b\right) \tag{28}$$

$$\overline{C}^n = \tilde{\theta}_w^n\left(\frac{E_c}{w^n} - \frac{w^{-n}}{w^n}T_w^{-n\,obs}\right)\left(\tau - T_c^n\right) \; n = a, b \tag{29}$$

where $\tilde{\theta}_w^n = \frac{\theta_w^n}{\theta_s^a}$, $\tilde{\theta}_l^n = \frac{\theta_l^n}{\theta_s^a}$and $\tilde{\Phi} = \frac{\Phi}{\theta_s^a}$.

The values of leisure and of time assigned to work (the counterparts of Eqs. (20) and (21), respectively) can be calculated using the estimated parameters as:

$$VoL_a = \frac{\tilde{\theta}_l^a + 1}{\tilde{\Phi}}\frac{w^aT_w^{a^*} + w^bT_w^{b\,obs} - E_c}{\left(\tau - T_w^{a^*} - T_c^a\right)} \tag{30}$$

$$VoL_b = \frac{\tilde{\theta}_l^b + \tilde{\theta}_s^b}{\tilde{\Phi}}\frac{w^aT_w^{a\,obs} + w^bT_w^{b^*} - E_c}{\left(\tau - T_w^{b^*} - T_c^b\right)} \tag{31}$$

$$VoW_n = \frac{\tilde{\theta}_w^n}{\tilde{\Phi}}\frac{w^nT_w^{n^*} + w^{-n}T_w^{-nobs} - E_c}{T_w^{n^*}} = VoL_n - w_n \; n = a, b. \tag{32}$$

As shown in Section 3.2, parameter $\tilde{\Phi}$. Is linked to the marginal utility of income which is expected to vary (diminish) with income and calls for segmentation to estimate the models. This is why it is advisable to split the sample into high and low income leaving a comparable number of observations in each group.

We assume that for each individual the vector of stochastic errors follows a multivariate normal distribution, with zero mean and covariance matrix Σ^n, which includes terms of correlation $\rho_{w,l}^n$ between the error terms of paid work and leisure equations (Eqs (23) and (24)) for an individual, and also a correlation $\rho_w^{a,b}$ between the errors of paid work equations of the two household members.

Let us now consider the basic (individual) time use model of Jara-Díaz and Guerra (2003) synthesized in Eq. (1), which will be applied to the same set of workers used to estimate the HTUM, in order to make the comparisons of parameters a meaningful tool to verify the theoretical propties of HTUM that the individual model cannot capture.

Calling the basic model presented in Section 3.1, we have to impose some conditions in order to obtain comparable parameters: we consider the same two unrestricted activities, leisure and sleep, and we normalize each equation system (there is one independent system for each type of individual, Eqs. (2) and (3)) by θ_s. Then, we add stochastic error terms to the equations and assume two normal multivariate distributions (one for each individual).

$$T_w^* = \frac{-B + \sqrt{B^2 - 4AC}}{2A} + u_w \tag{33}$$

$$T_l^* = \frac{\tilde{\theta}_l\left(\tau - T_w^* - T_c\right)}{\left(\tilde{\theta}_l + 1\right)} + u_l \tag{34}$$

where $A = \tilde{\theta}_w + \widehat{\Phi} + \tilde{\theta}_l + 1$, $B = -\frac{\widehat{E_c}}{w}(\tilde{\theta}_w + \tilde{\theta}_l + 1) - (\tilde{\theta}_w + \widehat{\Phi})(\tau - T_c)$ and $C = \tilde{\theta}_w\frac{\widehat{E_c}}{w}(\tau - T_c)$, with $\tilde{\theta}_w = \frac{\theta_w}{\theta_s}$, $\tilde{\theta}_l = \frac{\theta_w}{\theta_s}$, and $\widehat{\Phi} = \frac{\Phi}{\theta_s}$. $\widehat{E}_c$ are the individual expenses on restricted goods minus the individual fixed income from sources other than work.

As mentioned earlier, the basic model requires making assumptions regarding the assignment of the committed expenses within the household. In Section 4 we will assume that each individual pays committed expenses in the same proportion his/her individual income represents at the household level, as in Jara-Díaz and Candia (2017). With these adjustments, the calculation of the values of leisure and time assigned to work using the estimated parameters can be made for each individual as:

$$VoL = \frac{\tilde{\theta}_l + 1}{\widehat{\Phi}}\frac{wT_w - \widehat{E}_c}{\left(\tau - T_w - T_c\right)} \tag{35}$$

$$VoW = \frac{\tilde{\theta}_w}{\widehat{\Phi}}\frac{wT_w - \widehat{E}_c}{T_w} \tag{36}$$

For synthesis, in the next Section the HTUM system has to be estimated following Eqs. (22)–(24) while the predicted VoL and VoW are obtained using the estimated parameters in Eqs. (30)–(32). The individual models follow Eqs. (33) and (34) and the VoL and VoW are calculated from Eqs. (35) and (36). As deduced theoretically, it will be particularly important to look at those parameters and values of time with expected theoretical properties as now summarized:

- Parameter $\widehat{\Phi}$ represents λ (the marginal utility of income), and is expected to decrease with the HTUM for both income segments regarding the individual counterpart, and is expected to be lower for the high income segment.
- The normalized utility parameter for leisure activities ($\tilde{\theta}_l^n$) is expected to be larger for those in the class that exhibit less free time.
- The VoL and the VoW are expected to increase in the HTUM because of two reasons: λdiminishes and the marginal utilities of leisure are expected to increase with leisure of the other individual. Both elements suggest that VoL and VoW will be closer between individuals in the HTUM than in the basic individual model.

4. Empirical examination

4.1. Data

We generated a sample based on the Chilean National Time Use Survey (NTUS)[6] carried out by the Chilean National Institute for Statistics (INE , 2015), following a procedure described below. The NTUS contains time use and socioeconomic data on 21,690 individuals (10, 706 households) out of which 10,224 are workers (7 250 households). Individuals have to declare how many hours they assigned to 105 different activities on two previously selected days (one working day and one weekend day). Using this survey and applying some filters the individual time use model described in section 3.1 has been estimated by Jara-Díaz and Candia (2017) obtaining results on the values of work and leisure for various segments that are within the range of previous

[6] Available at https://www.ine.cl/estadisticas/sociales/genero/uso-del-tiempo as of July 2020.

empirical analysis using other type of time use samples in Chile.

We selected households that contain two workers that can be identified by a certain characteristic, which we chose to be gender. The procedure to obtain the sample of Chilean two-worker households was the following: first, we selected the 1 529 households from the NTUS that contain exactly two workers of different gender. Then, we considered those households where both workers declared a positive monthly income and a total number of hours allocated to activities that are between 17 and 38 h in both working and weekend days (the distribution of total hours shown in Appendix B exhibits a much larger tail to the right). We also checked that the household satisfy the condition $w^aT_w^a + w^bT_w^b - Ec > 0$. And that the size of the household was not larger than five (including the workers). This led to a sample of 770 households containing two workers of different gender - roughly 50% of the total - with an average size of 3.4 members (with a very small variation coefficient, 0.29).

We adjusted proportionally the declared hours except paid work and sleep, activities that were remembered quite precisely by the individuals because: i) these are the ones with the largest proportion of time assigned; ii) each was declared as a single category; iii) they distribute around commonly known values (see distribution in Appendix B). Following this procedure, the sum for each individual is exactly 24 on each surveyed day. We aggregated the 105 activities into 7: the two large single ones - paid work and sleep -, leisure, education, transportation, personal care, and unpaid work. Committed time was constructed as the addition of the last four. In the case of transportation, the NTUS asks to consider only the travel time with three specific purposes: paid work, healthcare and education. In leisure activities, the corresponding transport time is added to the time declared. More details on the aggregation is shown in Appendix B.

The shortest period to capture the leisure-work cycle is a week, a construction of the missing data was necessary. Following Jara-Díaz and Rosales-Salas (2015) we multiplied the activities declared on a working day by 5. For the missing weekend day, we applied a matching method considering six socioeconomic characteristics. Finally, we imputed committed expenses from the Chilean Budget Survey (INE (Instituto Nacional de Estadísticas), 2015), considering a geographical differentiation (Santiago Metropolitan Area and the other Chilean regions) and calculating the proportion in the household income of committed expenses by income quintile, and then applying the same proportion to our sample.

The final sample of 770 households with two workers of different gender was divided into two household income segments by grouping those households in the two largest income quintiles (high income), and the rest (low income). Data is described in Table 1 (means and standard deviations), where important gender differences can be seen, particularly regarding average wages and committed time (lower and larger for women, respectively); also, the coefficients of variation of wages within each gender are relatively high. In terms of time allocation, leisure and paid work are larger for men on average, while the opposite occurs with committed time and sleep. The differences in committed time are because it contains unpaid work, to which men assign less time.

For synthesis, the data for the estimation of the HTUM considers 770 observations (households), and for each observation the endogenous variables are the time assigned by each individual belonging to class $a$and b. To work and non-committed activities, while the explanatory variables are the wage rates, committed time by each worker a and b in the household, and total committed expenses.

4.2. Results and comparison of estimated parameters

As established in Section 3.3, for estimation purposes, we consider leisure (l) and sleep (s) as unrestricted activities. Due to the presence of the time constraint (Eq. (6.3)) the estimation should involve only one unrestricted activity for each individual, and we choose leisure for that. The likelihood maximization was made with the statistical software R. In Table 2 we show the results for both models and income segments: the estimated maximum-likelihood normalized utility parameters $\tilde{\Phi}$, $\tilde{\theta}_w^n$, $\tilde{\theta}_l^n$ and $\tilde{\theta}_s^b$; the standard deviation parameters of the stochastic errors of equationsor paid work and leisure, $\sigma_w^n$and σ_l^n; the correlation parameters $\rho_{w,l}^n$ and $\rho_w^{a,b}$; and the average values of leisure ($\overline{VoL}_n$) and time assigned to work ($\overline{VoW}_n$) calculated for each individual according to Eqs. (30)–(32) (HTUM) and Eqs. (35) and (36) (individual); t-statistics are in parenthesis. The basic (individual) models for men and women were estimated as a single model using dummy variables for gender.

All parameters and values of time obtained are statistically significant. Note that HTUMs yield slightly lower log-likelihoods (LL) regarding the individual models; LL diminishes by 1.8% and 1% with one extra parameter which suggests that the fit is somewhat inferior. As the individual model is not a reduced form of the HTUM (think of committed expenses) a test for nested models is inadequate. According to (Vuong, 1989) a strictly non-nested test seems also inadequate because the two models share common explanatory variables. However, as one could see them as overlapping, we run the sequential tests proposed by Vuong (1989) for this type of models. First, by computing the log-likelihoods for each observation a variance test indicated that we can reject the hypothesis that the models cannot be discriminated given the data; then we applied a model selection test which would support what the difference in LL suggested.

Let us look at the parameters to see whether the theoretical properties advanced earlier are fulfilled. The results show that the values of leisure and of time assigned to work are larger with the HTUM model as theoretically argued in the previous Section[7]: in the HTUM, income at the household level is the sum of its members', and a larger income available makes the household a richer modeling entity with a lower marginal utility of income; in addition, an extra unit of time for one individual would also benefit the other member (positive externality), through the possibility of assigning more time to work and increase the household income, and also because increasing the utility of one individual increases the utility of the household. So the values of leisure are larger at the household level, which makes the value of time assigned to work also larger with the HTUM, as the value of leisure increases maintaining the wage rate.

Behind this general result there are a number of elements that confirm the theoretical superiority of the HTUM specification and its microeconomic interpretation. As discussed earlier, the parameter $\tilde{\Phi}$ is indicative of the marginal utility of income and is expected to decrease with income. This is indeed what happens with the HTUM where $\tilde{\Phi}$ decreases by 29% from the low income to the high income segment; the opposite happens with the individual view of household members, for whom $\tilde{\Phi}$ increases by 13% for men and by 16% for women. Regarding leisure, women exhibit less time assigned to unrestricted activities (sleep plus leisure) as observed in Table 1, so one expects the normalized utility parameters for these activities to be larger for women. This does not happen in the basic models when comparing $\tilde{\theta}_l^n$, larger for men than for women in both segments (13% and 14%); it is, however, correctly captured in the HTUM where $\tilde{\theta}_l^b$ plus $\tilde{\theta}_s^b$ for women is larger than $\tilde{\theta}_l^a$ plus one for men (which is the correct comparison according to Eqs. (30) and

[7] Previous estimates of the VoL and VoW with the basic model in Chile have only been obtained for Santiago. In Jara-Díaz et al. (2013) time use data inferred from the 2001 Origin-Destination survey was used; the resulting VoL exhibited a range between 1 000 and 4 300 CLP depending on location (strongly correlated with income) and gender, while the VoW was very small, negative for men and positive for women. The first basic model reported in Jara-Díaz et al. (2008) used data from a South East corridor, obtaining an overall VoL of 2000 CLP and a small negative VoW. See Jara-Díaz and Candia (2017) for details.

Table 1
Mean (standard deviation) of time use and socioeconomic variables, household sample.

	Total sample		High income		Low income	
	Men	Women	Men	Women	Men	Women
Observations	770	770	413	413	357	357
Age (years)	43.60 (13.39)	41.71 (12.42)	44.63 (13.25)	42.02 (12.22)	42.4 (13.48)	41.34 (12.66)
Leisure [a]	35.85 (12.22)	31.98 (12.69)	37.54 (12.24)	33.44 (12.76)	33.90 (11.91)	30.28 (12.40)
Paid work	50.24 (13.22)	42.28 (13.83)	49.54 (13.47)	43.12 (13.51)	51.05 (12.90)	41.31 (14.15)
Sleep	49.72 (8.52)	50.88 (8.62)	48.72 (8.48)	50.01 (8.75)	50.88 (8.44)	51.87 (8.38)
T_c.	31.19 (12.31)	42.87 (15.28)	32.20 (12.06)	41.42 (15.17)	32.17 (12.61)	44.53 (15.26)
Wage rate [b] (1000 CLP/h)	3.37 (4.25)	2.50 (2.55)	4.80 (5.36)	3.443.10)	1.71 (0.81)	1.42 (0.89)
$\widehat{E}_c$ (ind.)	70.62 (75.80)	43.39 (41.96)	93.54 (95.84)	56.96 (51.61)	44.11 (21.58)	27.69 (16.08)
E_c	114.01 (98.76)		150.50 (120.50)		71.80 (30.57)	

[a] All time units are in weekly hours.
[b] All monetary units are in Chilean pesos of 2015.1 US$ = 691.7 CLP.

Table 2
Household and individual time use models by income segments.

	Low income				High income			
	HTUM		Basic		HTUM			
	Men	Women	Men	Women	Men	Women	Men	Women
$\tilde{\Phi}$	0.241 (6.28)		0.263 (8.66)	0.252 (10.32)	0.171 (4.110)		0.296 (6.67)	0.292 (8.27)
$\tilde{\theta}_w^n$	0.691 (14.36)	0.896 (3.13)	0.430 (7.19)	0.263 (5.82)	0.841 (18.58)	0.866 (2.44)	0.496 (6.71)	0.353 (6.16)
$\tilde{\theta}_l^n$	0.674 (45.09)	0.818 (3.61)	0.675 (45.28)	0.598 (42.89)	0.777 (52.25)	0.773 (2.63)	0.779 (52.08)	0.681 (47.82)
$\tilde{\theta}_s^n$		1.382 (3.62)				1.142 (2.64)		
σ_w^n	10.287 (26.78)	11.743 (26.64)	9.240 (26.70)	9.627 (26.45)	12.042 (28.67)	11.902 (28.89)	10.984 (28.60)	10.515 (28.62)
σ_l^n	10.836 (26.66)	11.318 (26.68)	10.502 (26.70)	10.579 (26.56)	11.399 (28.67)	11.535 (28.68)	10.984 (28.57)	10.983 (28.63)
$\rho_{w,l}^n$	−0.680 (−23.79)	−0.750 (−32.21)	−0.651 (−21.24)	−0.679 (−23.05)	−0.742 (−33.85)	−0.737 (−33.01)	−0.705 (−27.91)	−0.689 (−26.23)
$\rho_w^{a,b}$	0.054 (2.084)				0.085 (3.76)			
$\overline{VoL_n}$	5 740.8 (2.08)	8 053.4 (2.62)	3 105.0 (7.18)	2 223.1 (8.80)	27529.3 (25.60)	30327.5 (2.15)	9 690.1 (7.39)	6 014.4 (8.83)
$\overline{VoW_n}$	4 030.1 (17.87)	6 629.9 (3.24)	1 394.3 (5.72)	799.5 (6.22)	22729.0 (24.78)	26892.2 (2.35)	4 889.7 (6.58)	2 579.2 (6.73)
w^n	1710.7	1 423.6	1710.7	1 423.6	4 800.3	3 435.3	4 800.3	3 435.3
OBS.	357		357		413		413	
LL	−5 193.9		−5 101.4		−6 074.2		−6 009.9	

[a]US$ = 691.7 CLP (September 2015).

(31)) by 31% in low income and by 8% in the high income segment. Applying the individual model to two-worker households is both conceptually and empirically inadequate.

As a result, the HTUM values of leisure are larger for women than for men, reversing the order obtained with the basic model for both income segments. A most interesting result is achieved in the high income segment, namely the notable decrease in the relative difference in both the values of leisure (from 38% to 9.2%) and of time assigned to work (from 47% to 15%) between the household members, which is intuitively reasonable when considering a household model.

5. Synthesis, conclusions and further research

In this research we have re-examined the prevailing approach to calculate the value of leisure from a utilitarian framework in the case of two-worker households. The individual-unitary approach taken so far has been enriched by borrowing elements from the household economic theory in order to look at observed individual workers' decisions regarding time use and goods consumption as a collective decision made by all workers in the unit. In the resulting household time use model (HTUM) a two-worker household is modeled as if it maximizes a utility that depends on the time allocated to activities of each individual, and on the consumption of domestic and market goods. The household income is the sum of the total individual incomes but time constraints remain - of course - individual. We argue that this improved approach should yield larger values of leisure for workers in the household mainly because of two reasons: a lower marginal utility of income due to a larger (common) income, and a positive (consumption and time) externality (the other person matters). This is illustrated empirically using Chilean data, obtaining as predicted larger values of leisure and closer values across genders.

We derived new analytical expressions for the values of leisure, paid work and domestic (unpaid) work, and for the household willingness to pay for saving time in a restricted activity of one of its members. Assuming a Cobb-Douglas functional form and considering domestic work as committed time, we obtained a system of equations for the variables of the model and explicit expressions for the household values of leisure and of time assigned to work, which are shown to depend on the utility parameters associated to each of these activities, on the household uncommitted income, and on the individually uncommitted time. This allows calculation of these values from the econometric estimation of the normalized utility parameters.

For an exploratory empirical analysis, we constructed a sample of households with two workers (one woman and one man) extracted from a Chilean database collected nationally in 2015. Then we estimated the HTUM and an individual time use model by gender considering two income groups - low and high - which permits different marginal utilities of income. Results show that the values of leisure obtained with the HTUM specification are substantially larger than those obtained with the basic individual model and closer between men and women, as theoretically expected. Behind these (improved) values we found utility parameters that represent much better the marginal utilities of money -

across income segments - and time - across genders.

This is quite relevant for decision making in the transport sector, as it implies that the importance of the willingness to reduce travel time due to gains in time devoted to other activities increases relative to the importance of travel conditions. Another interesting result is the notable decrease in the relative difference in both the values of leisure and of time assigned to work between the household members in the high income segment, which is intuitively reasonable when considering a household model.

This formulation opens up new perspectives on the values of time. By treating the household as the unit of observation where two types of workers are identified, any improvement in the allocation of time perceived by one of the members benefits the household as a whole (and not just the member him/herself). Capturing this effect is a step forward in understanding the time and consumption decisions of individuals and their monetary valuation. This can be seen as a change of perspective, from an individual to a collective view which seems to capture in a better way the welfare impact of a reduction in committed time, something that might be worth analyzing with a link to the field of happiness as well.

The econometric formulation presented here admits the inclusion of time allocated to domestic work as a decision variable, which is a potentially fruitful avenue for research. The theoretical formulation was aimed in that direction and could be used to incorporate other activities that could be covered by external providers or by a member of the household, as proposed in Rosales-Salas and Jara-Díaz (2017). A complementary line of work is to consider an implicit wage attached to unpaid labor which, if done by a member, means a larger disposable money income in the household, as done in Jokubauskaite et al. (2020). By introducing these elements in the HTUM model presented here, a richer framework for time values should result; households with one worker could then depart from the individual basic model and be treated in comparable terms with two-worker households as done here. On the other hand, a larger sample of households would allow segmentation by other variables (household size and presence of children, or employment status) to analyze the differences in time values between types of households. Finally, there is one interesting challenge regarding the formulation of the model itself: the consideration of leisure activities carried out together.

Author statement

Sergio Jara-Díaz: conceptualization, methodology, formal analysis, investigation, resources, writing original draft, writing-review-editing, visualization, supervision, project administration, funding acquisition. Diego Candia: methodology, software, formal analysis, investigation, data curation, writing original draft, writing-review-editing, visualization.

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APPENDIX A HTUM, values of time and resulting equations

The first order conditions of the model described by Eqs. (6.1) - (6.6) are:

$$\frac{\partial U}{\partial X_i} - P_i\lambda = 0 \quad \forall i \in X^f \tag{A.1}$$

$$\frac{\partial U}{\partial X_i} - P_i\lambda + \eta_i = 0 \quad \forall i \in X^r \tag{A.2}$$

$$\frac{\partial U}{\partial T_w^n} + \lambda w^n - \mu^n = 0 \quad n = a, b \tag{A.3}$$

$$\frac{\partial U}{\partial T_d^n} - \lambda P_d o_d^n - \mu^n + \gamma_d \frac{\partial h_d}{\partial T_d^n} = 0 \quad \forall\, d\; n = a, b \tag{A.4}$$

$$\frac{\partial U}{\partial T_j^n} - \mu^n = 0 \quad \forall j \in A^f \quad n = a, b \tag{A.5}$$

$$\frac{\partial U}{\partial T_j^n} - \mu^n + \kappa_j^n = 0\, \forall j \in A^r \quad n = a, b \tag{A.6}$$

$$\frac{\partial U}{\partial Z_d} - \gamma_d = 0 \quad \forall\, d \tag{A.7}$$

From Eq. A.3 a relation between the household value of leisure and the household total value of paid work of an individual is obtained:

$$\frac{\partial U / \partial T_w^n}{\lambda} + w^n = \frac{\mu^n}{\lambda} \quad n = a, b \tag{A.8}$$

From Eq. A.6, the household willingness to pay for saving time in a restricted activity is:

$$\frac{\kappa_j^n}{\lambda} = \frac{\mu^n}{\lambda} - \frac{\partial U / \partial T_j^n}{\lambda} \quad \forall\, j \in A^r\; n = a, b \tag{A.9}$$

From Eq. A.4, a relation between the household value of leisure and the household total value of domestic work is obtained:

$$\frac{\partial U / \partial T_d^n}{\lambda} - P_d o_d^n + \frac{\gamma_d}{\lambda}\frac{\partial h_d}{\partial T_d^n} = \frac{\mu^n}{\lambda} \quad \forall\, d \quad n = a, b \tag{A.10}$$

When considering a Cobb-Douglas utility form (Eq. (10)) and domestic work as a restricted activity, the following first order conditions are obtained:

$$\frac{\phi_i U}{X_i} - P_i\lambda = 0 \quad \forall\, i \in X^f \tag{A.11}$$

$$\frac{\phi_i U}{X_i} - P_i\lambda + \eta_i = 0 \quad \forall\, i \in X^r \tag{A.12}$$

$$\frac{\theta_w^n U}{T_w^n} + \lambda w^n - \mu^n = 0 \quad n = a, b \tag{A.13}$$

$$\frac{\theta_j^n U}{T_j^n} - \mu^n = 0 \quad \forall\, j \in A^f \quad n = a, b \tag{A.14}$$

$$\frac{\theta_j^n U}{T_j^n} - \mu^n + \kappa_j^n = 0\, \forall j \in A^r \quad n = a, b \tag{A.15}$$

From Eq. A.14 we have:

$$U\,\theta_j^n = \mu^n T_j^n \quad \forall\, j \in A^f \tag{A.16}$$

We define committed time as _{$T_c^n = \sum_{j \in A^r} T_j^{nmin}$}. Adding for all activity in the set A^f and considering that $\sum_{j \in A^f} T_j^n = \tau - T_w^n - T_c^n$, and we can obtain:

$$\frac{\mu^n}{U} = \frac{\Theta^n}{\left(\tau - T_w^n - T_c^n\right)} \quad n = a, b \tag{A.17}$$

where Θ^n is the summation of the parameters θ_j^n of each unrestricted activity.

From Eq. A.11 we have:

$$\phi_i U = \lambda P_i X_i \quad \forall\, i \in X^f \tag{A.18}$$

We define committed expenses as $E_c = \sum_{i \in X^r} P_i X_i - I$. Adding for all goods in the set X^f and considering that $\sum_{i \in X^f} P_i X_i = w^a T_w^a + w^b T_w^b - E_c$ we can obtain:

$$\frac{\lambda}{U} = \frac{\Phi}{w^a T_w^a + w^b T_w^b - E_c} \tag{A.19}$$

where Φ is the summation of the parameters ϕ_i of each unrestricted good.

Using Eq. A.13 we have:

$$\frac{\theta_w^n}{T_w^n} + \frac{\lambda}{U} w^n - \frac{\mu^n}{U} = 0 \quad n = a, b. \tag{A.20}$$

Combining Eqs. A.20, A.17 and A.19 yields:

$$\frac{\theta_w^n}{T_w^n} + \frac{\Phi}{w^a T_w^a + w^b T_w^b - E_c} w^n - \frac{\Theta^n}{\left(\tau - T_w^n - T_c^n\right)} = 0\; n = a, b. \tag{A.21}$$

From Eq. A.14 we have:

$$\frac{\theta_j^n}{T_j^n} - \frac{\mu^n}{U} = 0 \quad \forall\, j \in A^f \quad n = a, b \tag{A.22}$$

Combining Eqs. A.22 and A.17 yields:

$$\frac{\theta_j^n}{T_j^n} - \frac{\sum_{j \in A^f} \theta_j^n}{\left(\tau - T_w^n - T_c^n\right)} = 0 \quad \forall\, j \in A^f \quad n = a, b \qquad .23)$$

From Eq. A.11 we have:

Using Eqs. A.24 and A.19 we obtain:

$$\frac{\phi_i}{P_i X_i} - \frac{\Phi}{w^a T_w^a + w^b T_w^b - E_c} = 0 \quad \forall\, i \in X^f \tag{A.29}$$

In addition, using Eqs. A.17 and A.19 we obtain an expression for the household value of leisure:

$$\frac{\mu^n}{\lambda} = \frac{\Theta^n}{\Phi} \frac{w^a T_w^{a^*} + w^b T_w^{b^*} - E_c}{\left(\tau - T_w^{n^*} - T_c^n\right)} \quad n = a, b \tag{A.30}$$

Taking the derivative of the utility function (Eq. (10)) with respect to T_w^n yields:

$$\frac{\partial U}{\partial T_w^n} = \frac{\theta_w^n U}{T_w^n} \quad n = a, b \tag{A.31}$$

Dividing Eq. A.31 by $\lambda$and using Eq. A.19, we obtain an equation for the household value of time assigned to work:

$$\frac{\partial U / \partial T_w^n}{\lambda} = \frac{\theta_w^n}{\Phi} \frac{w^a T_w^{a^*} + w^b T_w^{b^*} - E_c}{T_w^{n^*}} = VoL_n - w_n \quad n = a, b \tag{A.32}$$

APPENDIX B. Aggregation and distribution of activities

In case of paid work and sleep time, the NTUS contains a single activity data for each one, so no aggregation is needed. For **leisure**, we consider the following categories defined in the NTUS, where in each case we provide examples of the activities that the NTUS category considers:

1. Social life: Spending time with family or friends.
2. Attending to events: Going to the cinema/theater.
3. Games and hobbies: Playing music, playing videogames.
4. Sports: Practicing sports or doing physical activity.
5. Communication media: Watching TV, reading a book.

For **committed time**, we consider four aggregated activities: unpaid work, transportation, education and personal care. **Unpaid work** considers the following NTUS categories (with its correspondent examples of activities or sub-categories in the NTUS respective category):

1. Taking care of other people: helping children with homework, taking elders to health centers.
2. Domestic work: house cleaning, preparation of meals.
3. Unpaid aid to other households or to the community: domestic (unpaid) work for other households, volunteering.

In the case of **transportation**, as mentioned in the main text, the NTUS asks to consider only the travel time with three specific purposes: paid work, healthcare and education.

For **education**, we consider attendance to educational activities and other learning activities (such as homework or studying at home).

For **personal care**, we consider the correspondent category in the NTUS, that contains activities such as eating and medical or health care (where we exclude the transportation time to considering it in the transportation time, as mentioned above).

For a full detail description of the sub-categories of the NTUS, see Jara-Díaz and Candia (2017) and INE (2015).

In Figure B.1 we show the distribution of total declared hours for the 1 529 households of two workers with different gender.

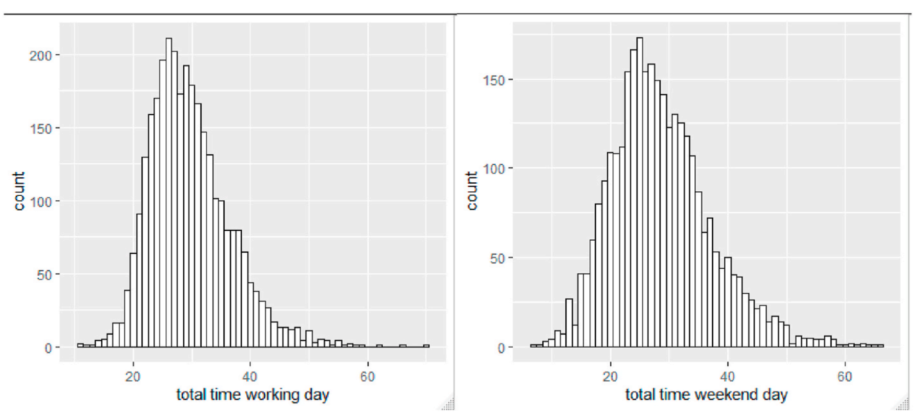

Figure B.1. Distribution of total declared hours, households with two workers of different gender.

In Figure B.2 we show the distribution of declared working and sleep hours on a working day for the 1 540 individuals in the 770 households in the final sample. They show that more than 50% declare 8–9 h of work, and some 70% declare 6–9 h of sleep.

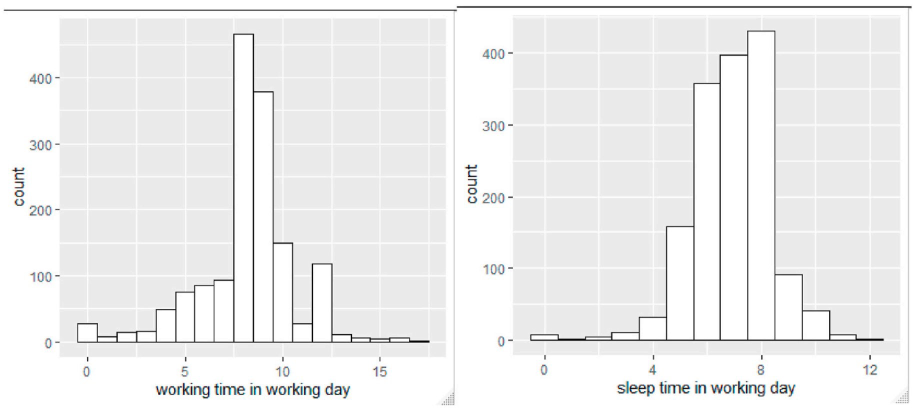

Figure B.2. Distribution of declared working and sleep hours on working day, household sample.

APPENDIX C. The individual time use model

The first order conditions of the model by Jara-Díaz and Guerra (2003) are:

$$\frac{\theta_j U}{T_j} - \mu = 0 \; \forall j \in A^f \tag{C.1}$$

$$\frac{\phi_i U}{X_i} - \lambda P_i = 0 \quad \forall i \in X^f \tag{C.2}$$

$$\frac{\theta_w U}{T_w} + \lambda w - \mu = 0 \tag{C.3}$$

Eqs. C.1 and C.2 for all unrestricted activities and goods plus budget and time constraints yield:

$$\frac{\mu}{U} = \frac{\Theta}{(\tau - T_w - T_c)} \tag{C.4}$$

$$\frac{\lambda}{U} = \frac{\Phi}{(wT_w - E_c)} \tag{C.5}$$

where $\Theta$is the summation of the exponents $\theta_j$over A^f. And $\Phi$is the summation of the exponents η_i. over X^f.

Using Eqs. C.3 – C5, we obtain:

$$\frac{\theta_w}{T_w} + \frac{\Phi}{(wT_w - E_c)} w - \frac{\Theta}{(\tau - T_w - T_c)} = 0. \tag{C.6}$$

Using Eqs. C.1 and C.4 we obtain:

$$\frac{\theta_j}{T_j} - \frac{\Theta}{(\tau - T_w - T_c)} = 0 \; \forall j \in A^f$$

And using Eqs. C.2 and C.5 we obtain:

$$\frac{\phi_i}{X_i} - \frac{\Phi}{(wT_w - E_c)} P_i = 0 \quad \forall i \in X^f \qquad .8)$$

Solving Eqs. C.6 – C.8 for T_w, $T_j$and $X_i$respectively, we obtain Eqs. (2)–(4), where in case of T_w,only the solution with the positive root is considered, as is the only correct solution, as shown by Jara-Díaz and Guerra (2003) analyzing when $\theta_w$approaches to zero. In addition, combining Eqs. C.4 and C.5 yields to the equation for the value of leisure (Eq. (5)).

Then, for proper comparison with the household model, we consider two unrestricted activities: leisure and sleep (which implies that $\Theta = \theta_s + \theta_l$). We divide the system of Eqs. C.6 – C.7 by θ_s, and then solve the resulting equations for $T_w$and T_l, obtaining the system of Eqs. 30 and 31, and a normalized equation for the value of leisure (Eq. (35)). Taking the derivative of the utility function with respect to T_w^n, dividing by $\lambda$and using Eq. C.5, allows to obtain an equation for the value of time assigned to work (Eq. (36)).

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