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OPTIMIZATION OF U-SHAPED ANTI-ROLL TANKS DESIGN FOR SHIPS UNDER STOCHASTIC SEAS

MEMORIA PARA OPTAR AL TÍTULO DE
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RESUMEN:
OPTIMIZACIÓN DEL DISEÑO DE ESTANQUES ANTI-ROLIDO CON FORMA DE "U" PARA BUQUES OPERANDO EN OCÉANOS ESTOCÁSTICOS

El trabajo reportado en el presente informe de tesis trata sobre el proceso de diseño para la selección óptima de las dimensiones de un estanque Anti-Rolido en forma de "U" (ART). La razón de incorporar el ART en un buque es para reducir el giro centrado en el eje longitudinal: el rolido. Para lograr esta meta, se presta especial atención al método de obtención de la reducción de rolido, involucrando una descripción estocástica de un oceano no armónico para obtener excitaciones realistas que ejercen las olas sobre el buque.

El estudio empieza a partir de un modelo dinámico lineal que describe el rolido para excitaciones de olas regulares (armónicas). Luego, toma el modelo y lo expresa en formulación de espacio-estado para así obtener la solución del ángulo de rolido a través de la ecuación de Lyapunov. Esta formulación en espacio estado es acoplada a un filtro de mínimos cuadrados que sirve para simular la densidad de potencia espectral de la pendiente de las olas del océano; de tal forma que, a partir de una excitación de Ruido Blanco Gaussiano, se obtiene la respuesta final de la solución del filtro: la cuantificación del rolido del buque. Finalmente, la reducción, que es una comparación entre el rolido del barco con un estanque estabilizador versus el mismo barco sin el estabilizador, permite realizar la optimización de las dimensiones del diseño del estanque anti-rolido en forma de "U". En este trabajo, se propone y se explica un método riguroso para obtener el diseño óptimo del estanque estabilizador a fin de que la operación del barco sea con un nivel de rolido mínimo considerando todos los escenarios posibles.

Para ejecutar el método propuesto, se realiza la optimización para un caso específico. A partir de esta se obtiene el diseño óptimo del ART, luego se realiza un análisis del comportamiento del buque implementado con el ART óptimo operando en diferentes escenarios; obteniendo que el mejor desempeño de la reducción de rolido ocurre para la operación en los estados más probables del oceano.

Este trabajo de estudio permite inferir que el diseño óptimo de un ART siempre debe tener una frecuencia natural cercana a la frecuencia natural del buque; por otro lado, se infiere que mientras más arriba sea la posición vertical del ART dentro del buque mejor es el desempeño de la reducción del rolido; y finalmente, que el ancho del ART debe ser el máximo permisible por las restricciones de la manga (ancho) del buque.

ABSTRACT:
OPTIMIZATION OF U-SHAPED ANTI-ROLL TANKS DESIGN FOR SHIPS UNDER STOCHASTIC SEAS

The work reported in this thesis study is concerned about design procedures for the selection of a U-shaped Anti-Roll Tank (ART) optimum dimensions. The reason to incorporate the ART into a ship is to reduce the gyration of the ship over its longitudinal axis: the roll motion. For this purpose, special attention is paid to the method to obtain the reduction and it involves a stochastic description of a non-harmonic sea to obtain realistic excitations that the waves exert onto the ship.

The study starts from a linear dynamic system that describes the ship roll for regular waves. Then, it takes this model to express it in state-space formulation and obtain the solution of roll angle through the Lyapunov equation. This last formulation is coupled to a filter of minimum squares that simulates the ocean Power Spectral Density so that, from Gaussian White Noise, is obtained the final output response of the solution of the filter: the ship roll motion. Finally, the reduction, compared between a ship with a U-shaped Tank implemented and the same ship without the tank, allows optimizing the design dimensions of the U-shaped Anti-roll tank. In this work is proposed a thorough method to achieve the obtention of the optimum design of the U-Shaped Anti-Roll tank stabilizer in order to operate at a minimum state of rolling under all possible scenarios.

To apply the proposed method, the optimization is performed over a specific case of study. It is obtained the optimum design of the ART, and then it is performed an analysis of the behavior of the ship implemented with optimum the ART operating at different scenarios, obtaining that the best performance of the roll reduction occurs at the most probable states of the ocean.

This work led to infer that an optimum ART design always has to have almost the same natural frequency as the ship; it also is inferred that the highest the position of the ART inside the ship the better performance in the roll reduction; and finally, that the width of the ART has to be the maximum permissible value inside the ship's breadth.

As you set out for Ithaka...
Hope your road is a long one
May there be many summer mornings...

Ithaka - C. P. Cavafy

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Chapter 1

Introduction

Ship stabilization is important for proper operation. Designing a stabilizer for a ship brings substantial benefits; in order to assure ocean preservation, stabilization guarantees vessel security, crew comfort, and sickness prevention for all passengers. Ships are indispensable for the global economy, and a good operation of ships ensures economic performance. So stabilization comes with profitable and sustainability benefits.

The way the ship will react to the waves depends on the ship's characteristic quantities such as mass, the moment of inertia, stiffness, and metacentric height. It is also involved the characteristics of the waves such as amplitude, frequency, and the distribution of these quantities in irregular wave motion [1]. The aim that characterizes the foundations of this study is that a naval architect desires to dimension stabilizing systems in such a way, that the reduction of the ship motion will be optimum under different scenarios [1].

It is known that for a ship´s design all angular degrees of freedom (DoF), roll, pitch, and yaw, are important for Seakeeping. However, roll motion is the critical one [2]. This is because ship roll is typically lightly damped and the restoring moment of the ship is small in the cross-plane in comparison with the other planes [3].

Roll reduction methods include keels, fin stabilizers[4], rudders, gyrostabilizers, azimuth propellers[5], and Anti-Roll Tanks (ART)[6]; among others. The ART is very attractive because of its simplicity and its favorable shape [1]. Its benefits are that they do not cause highly concentrated loads, like gyrostabilizers do[3], and that do not require complicated systems of control. However, the biggest handicap is the remarkable space required for the installation; space that could potentially, for example, be available for transport cargo[3].

Several studies have been made on ship stabilization. In this study, are used two of these studies as a basis to propose an optimization method to finally analyze an applied practical case for an Offshore Patrol Vessel (OPV).

The mathematical model used in this study was proposed by **Stigter**[1] (1966). It is a linearized two-DoF model for the analysis of the roll of a ship equipped with a U-Shaped ART. Lately, **Alujevic**[3](2019) used this model to perform a study on the quantification of the power absorption of the ART due to the input power of the waves. In the first one, the model was validated experimentally, whereas the second validates the model with a comparison **Stigter** results. In the same way, the present study implemented the same model in computational codes and is validated with a comparison with **Alujevic** results. The model is limited by several assumptions; one of them is that waves are regular motion (harmonic excitations). It is implied then that for a realistic description of the ocean, with irregular waves, it is needed to perform a stochastic analysis that simulates a reliable representation of the behavior of the ocean.

To implement this stochastic model to simulate the ocean characteristics it is used "Control System Theory" techniques to create a "Filter" that allows obtaining the roll response for the realistic ocean. The result of the filter is used in an optimization that compares twofold scenarios of the ship with an ART and the same ship without the ART in order to calculate the reduction of the implementation of the ART in the ship. Thus, the optimization gives the optimum design dimensions that behave with the better roll damping performance for all possible scenarios of the sea.

Finally, it is performed an optimization for a specific ship: an "Offshore Patrol Vessel" (OPV), where the decision of the better design takes into account the location of operation of the OPV, the operational profile (velocities of operation), and all possible sea states that cause irregular wave excitations. This particular example is an applied case for the proposed method of design of ARTs.

In the next chapters it is exposed the scope and objectives; the theoretical frame of the study involving the dynamic model of rolling and the Theory of control applying the Lyapunov solution to the implemented filter; and finally it is exposed the procedure for the complete execution of the study and the procedure of the "Case of Study". The thesis ends with a discussion of the obtained results and a proposal for further work.

Chapter 2

Objectives of the study

2.1. Final objective

Propose a methodology to select the design of U-Shaped Anti-Roll tanks for optimum ship stabilization under stochastic seas.

2.2. Secondary objectives

- Implement a simplified formulation that allows estimating the behavior of the roll in ships under a regular sea excitation.
- Validate the proposed formulation of the rolling behavior for ships under regular sea excitation.
- Implement a probabilistic model that allows modeling the irregular sea excitations of the waves over ships.
- Propose an optimization methodology to select the u-Shaped Anti-Roll tank dimensions in vessels operating in irregular seas.
- Present an illustrative example applying the proposed methodology of the study.

Chapter 3

Scope of the study

1. This is an applied study developed on a linearized mathematical model of ship roll. There are some assumptions on neglections as: the metacentric height does not vary during rolling.

2. The study does not contemplate the acquisition or processing of experimental data.

3. The proposed design method applies only to engineering stages where the characteristics of the vessel have been fixed.

4. The Case of study analyses the behavior of the ship during operation at open seas.

5. The study considers solely the moment centered in the longitudinal axis of the vessel, this means it is assumed that the inclination of the free surfaces of the fluid inside ART is defined solely by the roll motion. Neglecting the movement over the other 5 DoF of the ship: trim, yaw, heave, sway and surge.

6. For the scope of the study, the control parameters of the ART (damping (χ_t) and stiffness (μ_t) coefficients) are not controlled.

7. The optimization is performed over the ship roll and not over the ship angular roll velocity. Neglecting the drawback effects that the angular velocity can cause. For example, human sickness.

8. The tuning of the filter does not produce a PSD that matches exactly with the PSD of the wave slope of the ocean. Thus there are biases in the filter that produces errors in the optimization. To quantify these errors, an uncertainty analysis should be performed, which is proposed for further work.

9. Some parameters that define the problem are uncertain during operation (Ship mass, metacentric height, radius of gyration). So, there are biases associated to the result given the randomness of those parameters.

10. The aim of this study is to hopefully be used in the Chilean Shipbuilding Industry to apply evidence-based designs for U-Shaped ART stabilizers.

Chapter 4

The linearized model of ship's dynamical response

4.1. The dynamic model of ship rolling

It is presented at first the mathematical model of a ship without a U-shaped Anti-roll Tank (ART), to furtherly present the model of a ship with the ART incorporated.

The ship's response to regular waves, which are considered as wave slope excitation, is represented by the solution of the simplified equation:

$$m_s\ddot{\varphi} + c_s\dot{\varphi} + k_s\varphi = m_\theta \tag{4.1}$$

The equilibrium of momenta is defined by the external excitation and ship characteristics, which defines the response in terms of the angle of rolling (φ), as well as the angular acceleration ($\ddot{\varphi}$) and angular velocity ($\dot{\varphi}$). The external momenta (m_θ) depend on the change in the direction of the buoyancy force, which is related to the change in the slope of the waves as it passes through the ship. It is defined by

$$m_\phi = K_s\theta \tag{4.2}$$

where θ represents the slope of the waving water surface.

Terms m_s, c_s, and k_s represent the ship characteristics. Respectively, m_s represents the moment of inertia of the ship. c_s is the coefficient of the linear damping of the ship roll, it is defined by the damping that the ship's hull produces as the resistance with the water dissipates energy. k_s represents the momenta related to the rigidity of the dynamical movement of the ship. Those terms are:

$$m_s = Pr_I^2 \tag{4.3}$$

$$c_s = d_S \tag{4.4}$$

$$k_s = Pmg \tag{4.5}$$

In which:

- P : mass of the ship
- r_I : radius of inertia (radius of gyration). $\overline{MB}$, where is the buoyancy point.
- d_S : coefficient of the linear damping of the ship roll
- m : distance between the metacentre and the center of gravity of the ship, $\overline{MG}$. See Fig. C.1
- g : gravity constant

For an assumption of a simple harmonic motion, the external excitation and the response have the following forms, respectively:

$$\begin{aligned} \theta &= \theta_o e^{i\omega t} \\ \varphi &= \varphi_o e^{i\omega t} \end{aligned} \tag{4.6}$$

Meaning that $\theta = \mathrm{Re}(\Theta e^{i\omega t}))$. Thus, $\varphi = \mathrm{Re}(\Phi e^{i\omega t})$. Where ω is the frequency variable, i is the complex number $i = \sqrt{-1}$ and Re stands for "Real part".

Then, applying equations 4.6 to equation 4.1, it is obtained

$$(-\omega^2 m_s + i\omega c_s + k_s)\varphi_o e^{i\omega t} = \theta_o e^{i\omega t} \tag{4.7}$$

The response φ_o can be isolated as

$$\varphi_o = \frac{\theta_o}{(-\omega^2 m_s + i\omega c_s + k_s)} \tag{4.8}$$

obtaining the frequency response of rolling for a Simple Degree of Freedom (DoF).

The response of the recently described dynamical system behaves as shown in figure 4.1. It can be seen that the response has only one frequency of resonance due to the single DoF. The graph shows the amplitude of rolling (φ_o) normalized by the slope of the wave (θ_o), against the wave frequency (ω) normalized by the natural frequency of the ship (ω_s).

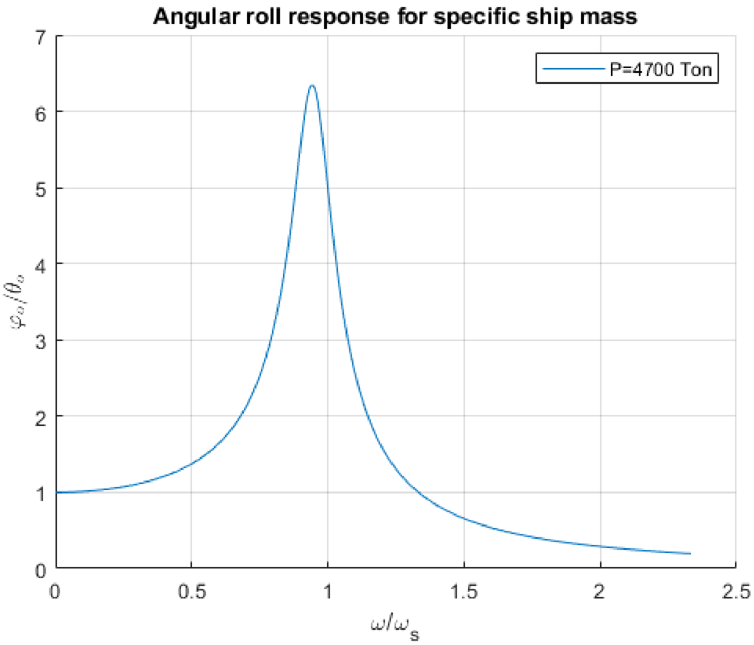


Figure 4.1: Rolling amplitude response of a ship without an ART. Own authorship.

4.2. The dynamic model of ship rolling with a U-Shaped Anti-Roll tank incorporated

It has been studied the kind of response of a ship that is not equipped with an ART stabilizer. Whereas a simple ship's dynamical response has only one DoF in the axis of heeling, a ship equipped with a U-shaped ART has two DoF in the heeling axis. The schematics of a U-shaped ART are shown in Appendix C.

In this case, the ship's response to regular waves, which are considered wave slope excitation, corresponds to the solution of the system of equations:

$$M_s\ddot{\varphi} + C_s\dot{\varphi} + K_s\varphi + M_{s,t}\ddot{\psi} + K_{s,t}\psi = m_\theta \tag{4.9}$$

$$M_{s,t}\ddot{\varphi} + K_{s,t}\varphi + M_t\ddot{\psi} + C_t\dot{\psi} + K_t\psi = 0 \tag{4.10}$$

Where the equation 4.9 represents the ship's motion due to the ship's own characteristics and the presence of a U-Shaped ART. The equation 4.10 represents the motion of the fluid in the U-shaped ART due to the characteristics of the tank itself and the influence of the ship.

The equilibrium of momenta of both equations is defined by the external excitation (wave slope) and the "ship plus ART" system characteristics (furtherly called "System"). These define the response in terms of the angle of rolling of the ship (φ) and the angle of inclination between the free surfaces of the ART (ψ). Those terms define respectively the angular acceleration ($\ddot{\varphi}$) and angular velocity ($\dot{\varphi}$) of the rolling as well as the angular acceleration ($\ddot{\psi}$) and angular velocity ($\dot{\psi}$) of the fluid moving inside the ART. The external moment exerted onto the ship (m_θ) is the same as previously defined as $m_\phi = K_s\theta$. Where, for this case of two DoF, K_s is defined in 4.12.

All the coefficients of the equations 4.9 and 4.10 depend on the ship's geometric characteristics and its implemented ART. The term C_s was previously defined in 4.4. The terms M_s and K_s represent the system characteristics. The terms M_t, C_t and K_t represent the ART characteristics. Respectively, M_t is the moment of inertia of the ART. C_t is the coefficient of the ART linear damping. K_t is the momenta related to the rigidity of the dynamical movement of the ART. Terms $M_{s,t}$ and $K_{s,t}$ represent the interaction of the ship and the ART. These terms are:

$$M_s = Pr_I^2 + I_{TG'} \tag{4.11}$$
$$K_s = (P+Q)m'g \tag{4.12}$$
$$M_t = \frac{1}{2}\rho w_2^2 w_3^2 l E_1 \tag{4.13}$$
$$C_t = \frac{1}{2}\rho w_2^2 w_3^2 l E_2 d_T \tag{4.14}$$
$$K_t = \frac{1}{2}\rho g w_2 w_3^2 l \tag{4.15}$$
$$M_{s,t} = \rho g w_2 w_3 l E_3 \tag{4.16}$$
$$K_{s,t} = \rho g w_2 w_3^2 l \tag{4.17}$$

where P and g were previously defined. And r_I', $I_{TG'}$, Q, m', ρ, l and d_T are:

- r_I' : radius of inertia (radius of gyration) of the "ship+ART" system. $\overline{M'B'}$, where is the buoyancy point of the system.
- $I_{TG'}$: moment of inertia of the tank fluid with respect to the centre of gravity of the ship-plus-tank system, (G'), see fig. C.2
- Q : mass of the fluid in the anti-roll tank
- m' : distance between the metacentre and the centre of gravity of the ship plus tank system, i.e. $m' = \overline{MG'}$
- ρ = mass density of the tank fluid
- l = the length of the tank
- d_T = the total coefficient of the linear damping of the tank

Furthermore, w_2 and w_3 are the time-dependent widths of the tank free surface and the distance between the centres of the free surfaces, respectively. See Fig. C.1 for visual references.

E_i, with $i = 1,\ 2,\ 3$, are the following Euler integrals, deduced in **Stigter**[1].

$$E_1 = \frac{1}{2}\int_{S,l}^{S,r} \frac{d_S}{n} = \frac{w+w_1}{2h} + \frac{2.31}{\alpha}\log\left(1 + \frac{y-\frac{1}{2}h}{\frac{h}{2}+\frac{w_1}{\alpha}}\right), \tag{4.18}$$

$$E_2 = \frac{1}{2}\int_{S,l}^{S,r} \frac{d_S}{n^2} = \frac{w+w_1}{2h^2} + \frac{1}{\alpha^2}\frac{y-\frac{1}{2}h}{(y+\frac{w_1}{\alpha})(\frac{h}{2}+\frac{w_1}{\alpha})}, \tag{4.19}$$

$$E_3 = -\frac{1}{2}\int_{0}^{S,r} r\cos\left(\epsilon_2 + \frac{\pi}{2}\right)d_S = (R+y+h)(\frac{w+w_1}{2}) + \frac{\alpha R}{2}(y-\frac{h}{2}), \tag{4.20}$$

Where n is the wetted tank dimension perpendicular to the tank outboard arm centreline s, while the distance r and the angle ϵ_2 describe the position vector of the fluid mass particle with reference to the centre of gravity, G' of the system. See Fig. C.1 for visual references.

The system of equations that represents the dynamic motion of rolling of the ship with ART, 4.9 and 4.10, can be formulated in matrix terms. So that both are equal to:

$$\boldsymbol{M}\ddot{\boldsymbol{x}} + \boldsymbol{C}\dot{\boldsymbol{x}} + \boldsymbol{K}\boldsymbol{x} = \boldsymbol{f} \tag{4.21}$$

where

$$\boldsymbol{x} = \begin{bmatrix}\varphi \\ \psi\end{bmatrix} \tag{4.22}$$

$$\boldsymbol{f} = \begin{bmatrix}m_\theta \\ 0\end{bmatrix} = \begin{bmatrix}K_s \\ 0\end{bmatrix}\theta \tag{4.23}$$

Respectively, $\boldsymbol{x}$ and $\boldsymbol{f}$ represent the vectors of the response and the vector of the external excitation. The matrix is formulated as:

$$\boldsymbol{M} = \begin{bmatrix}M_s & M_{st} \\ M_{st} & M_t\end{bmatrix}, \ \boldsymbol{C} = \begin{bmatrix}C_s & 0 \\ 0 & C_t\end{bmatrix}, \ \boldsymbol{K} = \begin{bmatrix}K_s & K_{st} \\ K_{st} & K_t\end{bmatrix} \tag{4.24}$$

Again, for an assumption of a simple harmonic motion, it can be assumed that the external excitation and the response have the following forms:

$$\begin{aligned}\boldsymbol{f} &= \boldsymbol{f_o}\ e^{i\omega t} \\ \boldsymbol{x} &= \boldsymbol{x_o}\ e^{i\omega t}\end{aligned} \tag{4.25}$$

Meaning that $m_\theta = \mathrm{Re}(M_\theta e^{i\omega t}))$. Thus, $\varphi = \mathrm{Re}(\Phi e^{i\omega t})$ and $\psi = \mathrm{Re}(\Psi e^{i\omega t})$. Where ω is the frequency variable, i is the complex number $i = \sqrt{-1}$ and Re stands for "Real part".

Then, applying equations 4.25 into equation 4.21, it is obtained

$$(-\omega^2\boldsymbol{M} + i\omega\boldsymbol{C} + \boldsymbol{K})\boldsymbol{x_o}\ e^{i\omega t} = \boldsymbol{f_o}\ e^{i\omega t} \tag{4.26}$$

The response $\boldsymbol{x_o}$ can be isolated as

$$\boldsymbol{x_o} = (-\omega^2\boldsymbol{M} + i\omega\boldsymbol{C} + \boldsymbol{K})^{-1}\boldsymbol{f_o} \tag{4.27}$$

obtaining the very same form of the equation B.1. Thus the transfer function is:

$$\boldsymbol{H} = (-\omega^2 \boldsymbol{M} + i\omega \boldsymbol{C} + \boldsymbol{K})^{-1} \tag{4.28}$$

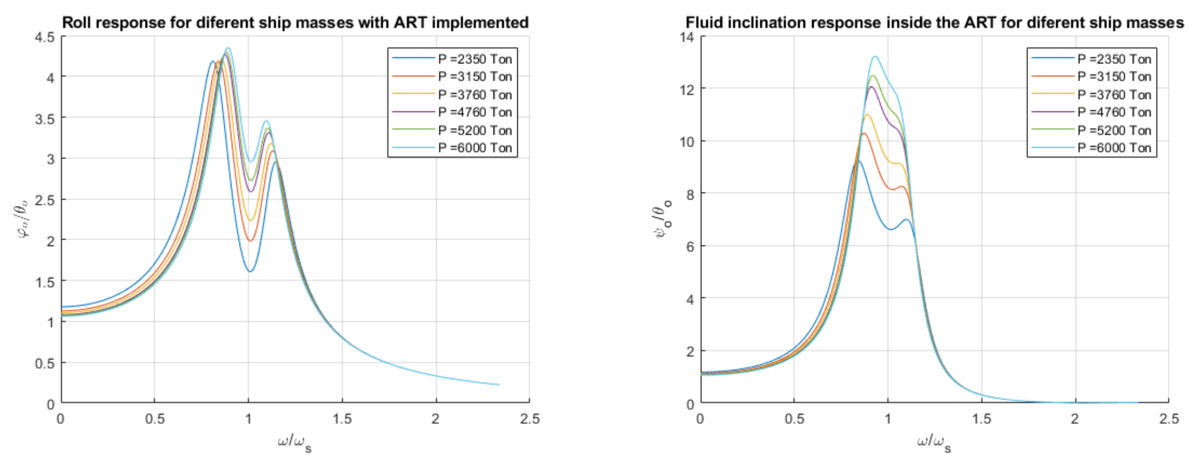
It has to be highlight that the "wave slope" is the concept that causes the momenta exerted on the ship. Thus the equation 4.1 and 4.9 has as external excitation θ, the slope of the wave. Then, as an output, it is obtained the ship roll φ. Therefore, it is obtained a frequency response function (FRF), which in this case is given by the equation 4.29:

$$\boldsymbol{x_o} = \boldsymbol{H} \boldsymbol{f_o} \tag{4.29}$$

The use of the FRF specified in the equation 4.29, allows to obtain the response of the ship roll and the inclination angle of the free surfaces inside the ART.

It is observed in the figure 2.2 how does the system responded for a given design evaluated for different ship masses. For each DoF there is one response and show how does the system behaves for a given known harmonic excitation. In this case there is at 2.2.a the response showing the amplitude of the rolling angle of the ship whereas in 2.2.b shows the response of the inclination between the lateral free surfaces of the ART.

It is highly remarkable too, that the two DoF of the system produces that in each response are two frequencies of resonance. In this case, the more mass have the ship, the more enphaissed are the peaks of the response. As well as they distance apart as the mass rises.



(a) Frequency response function of the roll amplitude of a ship with different loads (Weight or mass variation)

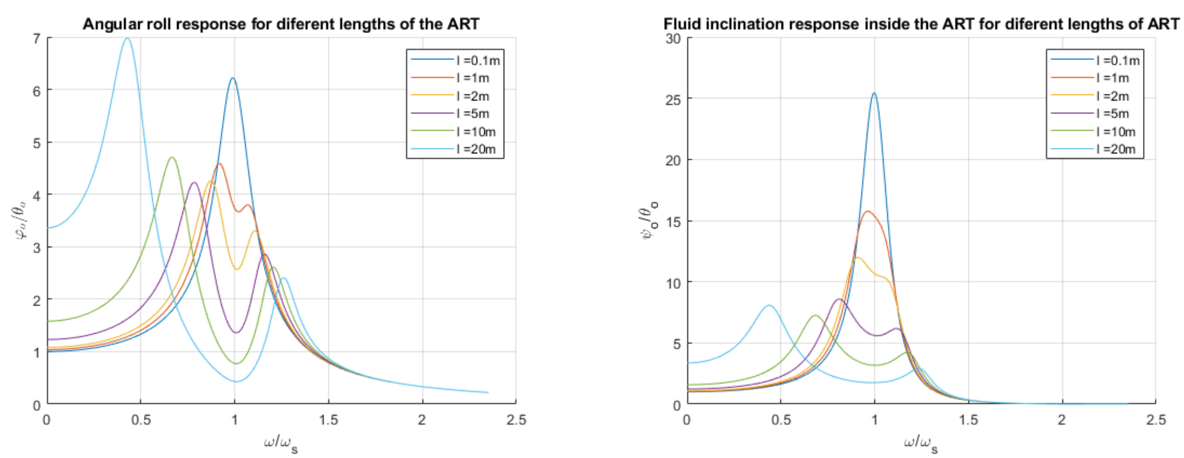
(b) Frequency response function of the inclination angle of the ART inside the ship with different loads (Weight or mass variation)

Figure 4.2: Variation of the total mass of the ship, dead weight.

Table 4.1: Example dimensions and mass of the ship and U-shaped ART

Ship	
Mass, P, (Kg)	Variable
Overall length, LOA, (m)	103.5
Waterline length, LWL, (m)	93.6
beam, B, (m)	18.8
Draught, D, (m)	3.9
Metacentric height, GM, (m)	1.673
Tank	
Water mass, Q, (kg)	$4.55 \cdot 10^4$
length, l, (m)	2
Reservoir spacing, w, (m)	12.04
Reservoir width, w_1, (m)	2.58
Conduct height, h, (m)	0.66
Vertical walls slope, alpha, (rad)	0.05
Water filling level, y, (m)	2.7

Similarly as before, when varying the length of the ART, the responses of both the Rolling angle of the ship and the Fluid inclination angle inside the ART shows the effect of the energy disipation produced by the ART (As compared with the case that there is no ART (See Fig. 4.1). The mass of the ART arises the effect on the ship rolling starts to reduce the amplitude of the roll. However, when the mass of the ART becomes to high (at $l = 20\ m$, the maximum rolling angle becomes much bigger and then it is prefferable no to install an ART insde the ship.



(a) Frequency response function of the roll amplitude of a ship with implemented with different lengths of its ART

(b) Frequency response function of the inclination angle of the ART at different lengths

Figure 4.3: Variation of the length of the ART

Table 4.2: Example dimensions and mass of the ship and U-shaped ART

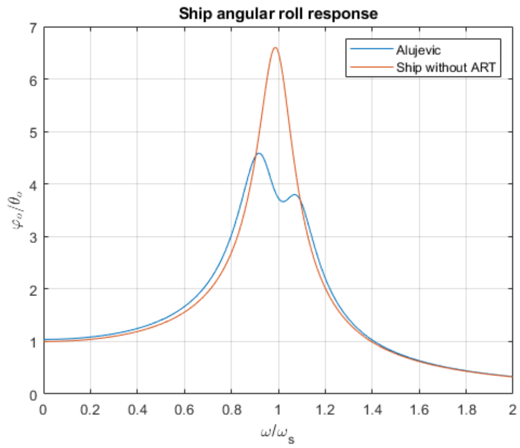
Ship	
Mass, P, (Kg)	$4700 \cdot 10^3$
Overall length, LOA, (m)	103.5
Waterline length, LWL, (m)	93.6
beam, B, (m)	18.8
Draught, D, (m)	3.9
Metacentric height, GM, (m)	1.673
Tank	
Water mass, Q, (kg)	Varies as: $f(l)$
length, l, (m)	Variable
Reservoir spacing, w, (m)	12.04
Reservoir width, w_1, (m)	2.58
Conduct height, h, (m)	0.66
Vertical walls slope, alpha, (rad)	0.05
Water filling level, y, (m)	2.7

4.3. Model validation

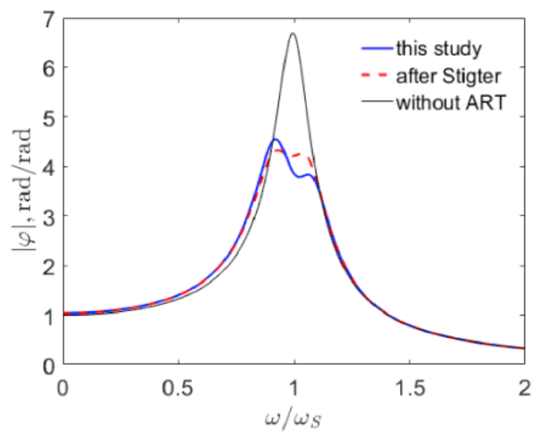
The mathematical model previously presented was developed in **Stigter**[1] and it is also used in **Alujevic**[3]. The author of this thesis implemented a code that replicates the model of **Stigter**[1] and validated it using the data presented in **Alujevic**[3].

The elaboration of the model of the dynamic response of the ship by the author of this document has been analyzed and compared with these two sources in order to corroborate that the implemented equations and the codes can be used to continue the thesis.

The developed codes gives the very same response and obtain as result a graph that matches exactly as **Alujevic** [3] proposes. As shown in Fig. 4.4, the obtained result (4.4.a) is in agreement with the original source (4.4.b).



(a) Own authorship code



(b) Original model.

Figure 4.4: Code verification by comparison with original paper figure **Alujevic**[3]

The characteristics of the ship equipped with the U-shaped ART for this nominal design are shown in table 4.3. Those are the same used in **Stigter** [1] and **Alujevic**[3] study. Furthermore, the nominal dynamic parameters obtained in this validation are exactly the same as **Alujevic**[3] wrote in his paper, shown in table 4.4. Those parameters were described in equation 4.11 to 4.17.

Table 4.3: Nominal dimensions and mass of the ship and U-shaped ART as presented by **Alujevic**[3]

Ship	
Mass, P, (Kg)	$4700 \cdot 10^3$
Overall length, LOA, (m)	103.5
Waterline length, LWL, (m)	93.6
beam, B, (m)	18.8
Draught, D, (m)	3.9
Metacentric height, GM, (m)	1.673
Tank	
Water mass, Q, (kg)	$22.8 \cdot 10^3$
length, l, (m)	1
Reservoir spacing, w, (m)	12.04
Reservoir width, w1, (m)	2.58
Conduct height, h, (m)	0.66
Vertical walls slope, alpha, (rad)	0.05
Water filling level, y, (m)	2.7

Table 4.4: Nominal dynamic parameters of the ship-plus-ART system as presented by **Alujevic**[3]

Coefficient	unit	Value
M_s	$kg \cdot m^2$	$2.67 \cdot 10^8$
C_s	$N \cdot m \cdot s$	$2.16 \cdot 10^7$
K_s	$N \cdot m$	$7.75 \cdot 10^7$
M_t	$kg \cdot m^2$	$9.84 \cdot 10^6$
C_t	$N \cdot m \cdot s$	$9.95 \cdot 10^5$
K_t	$N \cdot m$	$2.97 \cdot 10^6$
M_{st}	$kg \cdot m^2$	$2.47 \cdot 10^6$
K_{st}	$N \cdot m$	$2.97 \cdot 10^6$
w_s	$\frac{rad}{s}$	0.5385
w_t	$\frac{rad}{s}$	0.5494

From now on, it is considered that the equations had been correctly implemented into the Matlab codes. Thus the elaborated model has been validated.

Given that the model is valid, and the model says that if it is assumed that the waves are harmonic and known, then the optimal values of the dimension of the ART will depend on the frequency of the waves. If the waves change their frequency, the system changes and ceases to be optimal. Then arises the necessity to analyze the behavior of the ship's dynamical response to a stochastic sea wave excitation. Then an analysis can be performed considering those most likely and most unlikely scenarios of a ship's operational profile.

From now on can be studied the behavior of the ship response depending on the variation of parameters that can be controlled during operation, or depending on parameters that once chosen the design remain fixed.

4.4. Parametric analysis

The ship FRF that is caused by the waves depends on both the characteristics (parameters) of the ship and the U-shaped ART inside of it. For the present case, those parameters are presented in the table 4.5, which correspond to the inputs that define the problem of optimization. The other parameters that define the problem are correlated to these ones, so they are exposed in Appendix A.

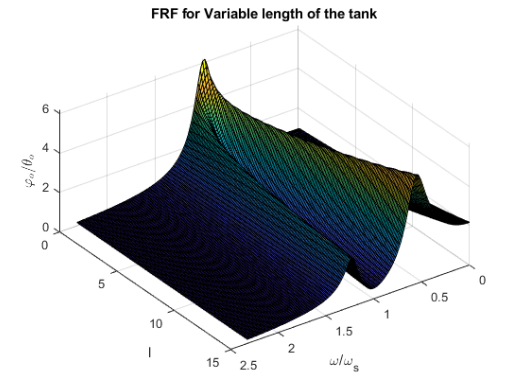
They can be classified as Design Parameters (DP), Fixed Parameters (FP), and Uncertain Parameters (UP). This classification responds to an analysis of what can be controlled and what cannot be controlled, so that the final design can be correctly optimized.

- **DP**: Those geometric parameters of the ART that varies in order to optimize the design that leads to a minimum roll during operation of the ship. Once the ship is built, these parameters will remain fixed and cannot be tuned during operation (unless specified).
- **FP**: Those parameters that, for the purpose of this study, are set to be fixed from the beginning of the design.
- **UP**: Those parameters that cannot be predicted for future operation nor when designing.

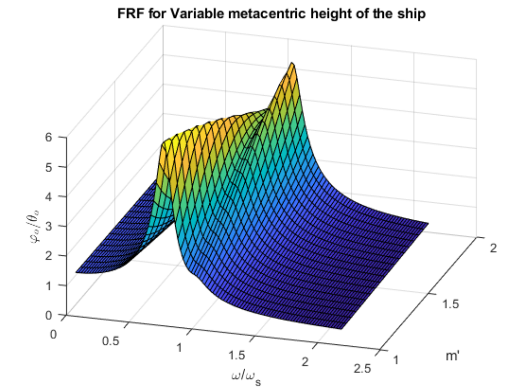
Table 4.5: Parameter classification

Parameter	Classification	
l	DP	Length of the u-shaped anti-roll tank; m
h	DP	Height of the connecting circuit between the vertical reservoirs; m
w	DP	Length of the connecting circuit; m
w_1	DP	Width of thelateral reservoirs of the ART at its base; m
y	DP	Height of water in equilibrium of both reservoir columns; m
α	DP	External wall angle of inclination of the vertical reservoir columns, rad
χ_t	UP	Dimensionless parameter of the linear damping of the tank
P	UP	Mass of the ship ; kg
m'	UP	Metacentric height of the system. Previously defined as $m' = \overline{MG'}$; m.
$\%IR$	UP	The quotient of the radius of inertia (ij) and breadth of the ship. Usually $\approx 0.4 - 0.5$
χ_s	UP	Dimensionless parameter of the linear damping of the ship roll
R'	DP	Distance between the center of gravity of the system and the bottom of the conduct of the ART; m.
B	FP	Breadth of the ship, m
x	DP	Overall height of the lateral reservoirs, m

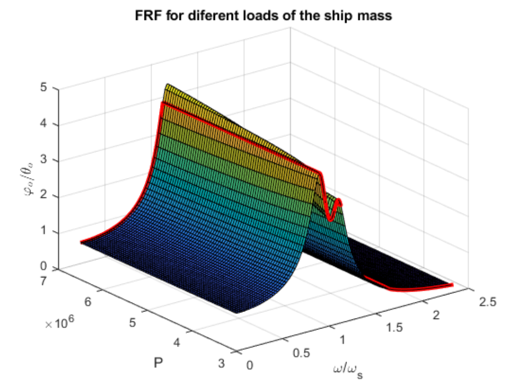
The aim of the study is to design an optimal ART for a ship so that the rolling during operation can be minimized for all possible scenarios of the sea. Therefore, it is useful to analyze how the behavior of rolling changes when some of these defined parameters vary.



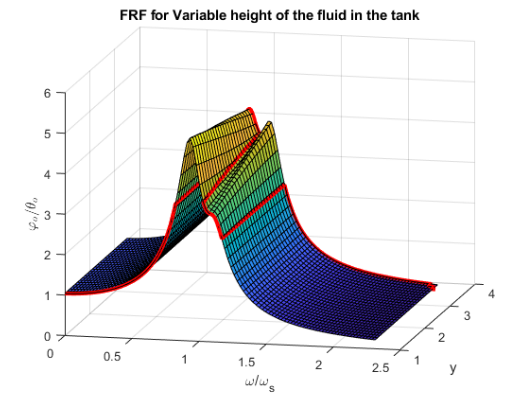
(a) Behavior of the response for different length of that ART (l).



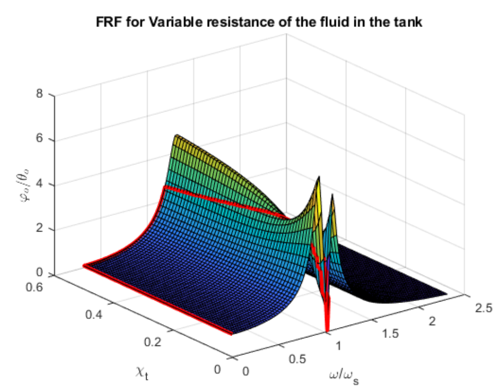
(b) Behavior of the response for different metacentric heights of the system (m').



(c) Behavior of the response for different masses of the system (P).



(d) Behavior of the response for different heights of the water level inside the ART (y).



(e) Behavior of the response for different damping parametes of the ART (χ_t).

Figure 4.5: System response for different parameters varying. The used nominal dynamic parameters are the ones specified in tables 4.1 and 4.2.

It is shown in the sample figure 4.5 that the damping of the ART (χ_t at 4.5.e), the length of the ART (l at 4.5.a) and the metacentric height (m' at 4.5.b) are those parameters that produce most significant changes in the behavior of the response of the system roll. Whereas the weight of the system (P at 4.5.c) and the height of the water level of the ART (y at 4.5.d) shows a constant behavior of the response, although the response of these two has amplitude variation. It can be seen that (P) and (y) do maintain a constant "two DoF" profile along the variation of the parameter. On the contrary, (χ_t) and (l) have variations in amplitude as well as a transition from a single DoF to two DoF. This means that the variation of those last parameters affects more in the ship roll variation of the response. Thus it is predictable that for an optimization of the design these parameters will have more decisive implications on the final design of the ART for a given ship.

In the case of χ_t, what variates is the resistance that the fluid's flow generates as it moves from side to side of the U-shaped ART. This can be controlled through the use of valves that restricts the flow or by fins that corrects the direction of the fluid movement. The variation of the quantity with which this phenomena is actuated upon is called χ_t, and is a non-dimensional number related to the dissipation of energy due to the flow restriction.

It is shown in figure 4.5.e the behavior of the FRF of the ship given different values of χ_t. The path of optimal operation, for a harmonic excitation, is marked with the red line. Which highlight the minimum rolling state for a given wave frequency.

If the frequency of excitation is well known, then is easy to choose the dimension of the parameter. It has to be chosen the value of the parameter that produces the minimum amplitude response. The issue is that the excitation is highly unpredictable. One way to approach this, is to select the value of the parameter in such a way that the integration of the amplitude of the response along a frequency domain is minimum. But, this has drawbacks, the integration of the frequency does not consider the most probable frequencies and the second is that the computation cost is too high. For this issue is proposed to generate a filter, where the distribution of the frequency is described by power spectral densities and the integration is implemented using the Lyapunov solution, which gives much more accurate results as well as a much faster computation time.

Even though this model, which is a deterministic one, allows to obtain an optimal design for a steady state of the sea for a harmonic excitation and for a well-known wave frequency, it does not give a proper solution of a realistic case,

To obtain a realistic simulation of the frequency response of the ship roll it has to be introduced a wave model that defines a proper external excitation. However, introducing such a model is complex (**Stigter**[1]), so this work proposes an alternative way to incorporate the nature of the waves.

Then arises the question, How to design a ship plus ART system incorporating random excitation of a realistic sea? This issue is approached by simulating a stochastic method of excitation and use it to perform an optimization to maximize the reduction that produces an ART design. The optimization method is proposed as follows.

4.5. The optimization problem to select the most efficient ART

The optimization performed in **Stigter**[1] and **Alujevic**[3] were made with the methods h_∞ and H_2. In both cases, the quotient $f = \frac{w_s}{w_t}$, with w_s and w_t the natural frequency of the ship system and the natural frequency of the ART respectively, were artificially set as $f = 1$. This means that for this setting, it is the most likely design condition that produces the ART will be in a 180º de-phase [1]

But this study proposes the following problem. The global optimization defined to help to decide the dimensions o the ART is:

$$\begin{aligned} \min_{DP} \quad & (-\% \ of \ Angle \ roll \ reduction) \\ \text{s.t.} \quad & Geometrical \ restrictions \\ & Fisical \ restriction \ of \ the \ ship + ART \ system \end{aligned} \tag{4.30}$$

Where the design parameters (DP) are defined in the table 4.5. The implemented code allows choosing which DP may vary for the optimization.

To implement the proposed method of optimization is needed to have a good description of the sea. This means it has to be used a description of an "irregular" sea. This is made by power spectral densities that describe the energy content of the waves. This tool has to be applied stochastically over the mathematical model. This means that the response for this stochastically excited model will be statistical values. Specifically, it will be variances. These responses are the ones used to calculate the reduction of roll amplitude that has to be inserted in the equation 4.30 to finally select the dimensions of an ART.

To generate a more realistic scenario is artificially incorporated a mathematical tool that receives as input Gaussian white noise and generates a wave description in terms of PSD. This tool is the filter. Then, given the scenario of different kind of variabilities in the parameters that define the problem, it can be proposed a design procedure at different stages that lead to an optimal solution for the mentioned description of the ocean. To understand the procedure in detail refer to Chapter 7.

The explanation of the generation of the filter via the Lyapunov solution proceeds in the following two Chapters 5 and 6.

Chapter 5

Excitation model: White Noise case

To generate a random excitation model it is implemented the Lyapunov equation, which gives as solution the variance of the response of a linear system. In this study, the variance is used to appreciate the reduction of the standard deviation value of the ship roll comparing the situation with an ART incorporated against the situation without an ART in the same ship.

The goal now is to optimize the ship design assuming that the sea is white noise (WN). Specifically, the wave slope (θ) is the WN.

However, it has to be previously ensured that the definition of the system complies with the Lyapunov equation requirements. This means, that the (1) system is linear and (2) that the excitation is defined as a Gaussian WN. For the case of the ship roll model, the first condition is complied with, and the second is assumed so that furtherly it can be used the Lyapunov solution to generate the filter that represents the behavior of the sea.

The Lyapunov solution is a method to analytically solve a transfer function of Power Spectral Densities. But the definition of the problem has to comply with both conditions mentioned before. As a transfer function, the Lyapunov solution may be represented as the block diagram of 5.1.

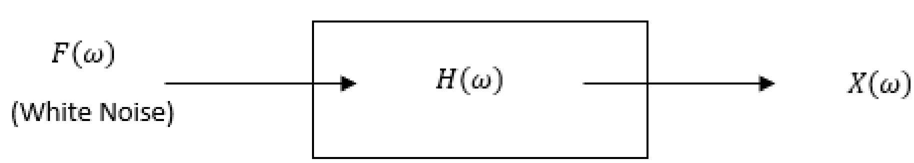


Figure 5.1: Block diagram of the Lyapunov solution process

The block diagram expressed in 5.1 is founded in the following deduction:

For linear system under stationary excitations, the relationship between f and x for the system $M\ddot{x} + C\dot{x} + Kx = f(t)$ in frequency domain is:

$$X(\omega) = H(\omega)F(\omega) \tag{5.1}$$

with $H(\omega) = (-\omega^2 M + i\omega C + K)^{-1}$ de transfer function. It can be computed then, the first and second statistical moments, obtaining:

$$\mu_x(\omega) = H(\omega)\mu_f(\omega) \tag{5.2}$$

$$\mathcal{S}_{xx}(\omega) = H(\omega)\mathcal{S}_{ff}(\omega)H(\omega)^* = |H(\omega)|^2\mathcal{S}_f(\omega) \tag{5.3}$$

So the relationship between spectral densities is:

$$\mathcal{S}_{xx}(\omega) = |H(\omega)|^2\mathcal{S}_{ff}(\omega) \tag{5.4}$$

If the relationship is integrated

$$\underbrace{\frac{1}{2\pi}\int_{-\infty}^{\infty} S_{xx}(\omega)d\omega}_{\sigma_x^2} = \frac{1}{2\pi}\int_{-\infty}^{\infty} |H(\omega)|^2 S_{ff}(\omega)d\omega \tag{5.5}$$

then the variance for the response is:

$$\sigma_x^2 = \frac{1}{2\pi}\int_{-\infty}^{\infty} |H(\omega)|^2 S_{ff}(\omega)d\omega \tag{5.6}$$

This integral can be solved analytically using the Lyapunov solution solving the following equation 5.8. Where the diagonal elements of the matrix $\boldsymbol{X}$ are the variances resulting from the integration of 5.6.

5.1. Lyapunov equation

By its definition, the Lyapunov equation exists for a given vector "$\boldsymbol{y}$" that complies with the state space representation

$$\dot{\boldsymbol{y}} = \boldsymbol{A}\boldsymbol{y} + \boldsymbol{B}\boldsymbol{\eta} \tag{5.7}$$

where $\boldsymbol{A}$ and $\boldsymbol{B}$ are matrix, $\boldsymbol{y}$ a vector and $\boldsymbol{\eta}$ represents the WN external excitation.

For the equation 5.7 exists only one matrix $\boldsymbol{X}$ that complies with the equation:

$$\boldsymbol{A}\boldsymbol{X} + \boldsymbol{X}\boldsymbol{A}^\intercal + \boldsymbol{Q} = 0 \tag{5.8}$$

If $\boldsymbol{X}$ results to be a matrix defined as positive (with all Eigen values greater to zero), then $\boldsymbol{Q}$ is defined as $\boldsymbol{Q} = \boldsymbol{B}\boldsymbol{B}^\intercal$. But considering the Power Spectral Density (PSD) $\boldsymbol{Q} = \boldsymbol{B}\cdot S_o\cdot\boldsymbol{B}^\intercal$. Where S_o is the constant PSD of the WN. $\boldsymbol{X}$ is called the Lyapunov solution and is a matrix that contains variance values of the components of the $\boldsymbol{y}$ response vector. For the actual case of study, it contains the variance of the angle of rolling and the velocity of rolling produced by the sea considered as white noise wave slope. As well as the variance of the angle of inclination of the free surfaces inside the ART and the velocity of inclination of the free surfaces inside the ART.

To use the Lyapunov solution tool, the system is expressed in terms of state space. Which represents the relation of a vector with its derivative. In this specific case, the relation of the angle of rolling with the velocity of rolling (For the single DoF of the ship without an ART implemented), and, for the two DoF, adding the relation of the angle of inclination of the free surfaces inside the tank with its associated velocity (Two DoF of the ship with an ART implemented).

5.2. State space: Generalized formulation

A second-order linear system described by "n" DoF can be described as the equation 5.9,

$$\boldsymbol{M}\ddot{\boldsymbol{x}} + \boldsymbol{C}\dot{\boldsymbol{x}} + \boldsymbol{K}\boldsymbol{x} = \boldsymbol{f} \tag{5.9}$$

where for this section $\boldsymbol{M}$, $\boldsymbol{C}$, and $\boldsymbol{K}$ are the characteristic squared-matrix ($\in \mathbb{M}_{n,n}$) of the linear system of n DoF described in the equation 5.9 and $\boldsymbol{f}$ is the vector representing the external forces expressed in equation 5.10

$$\boldsymbol{f} = \boldsymbol{f_o}\eta \tag{5.10}$$

where $\boldsymbol{f_o} \in \mathbb{R}^n$ and μ is WN.

The Lyapunov solution can be applied by taking the change of variables

$$\boldsymbol{y} = \begin{bmatrix} \boldsymbol{x} \\ \dot{\boldsymbol{x}} \end{bmatrix} \tag{5.11}$$

where $\boldsymbol{x}$ is defined as the vector of the response of the system. So, $\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \end{bmatrix}$. Given that $x \in \mathbb{R}^n$, then $\boldsymbol{y}$ is a vector of dimensions $\mathbb{R}^{2n}$.

In matrix form, the derivative of the vector $\boldsymbol{y}$ of the equation 5.11 may be written as

$$\dot{\boldsymbol{y}} = \begin{bmatrix} \dot{\boldsymbol{x}} \\ \ddot{\boldsymbol{x}} \end{bmatrix} = \begin{bmatrix} \dot{\boldsymbol{x}} \\ \boldsymbol{M}^{-1}[\boldsymbol{f} - \boldsymbol{C}\dot{\boldsymbol{x}} - \boldsymbol{K}\boldsymbol{x}] \end{bmatrix} \tag{5.12}$$

The equation 5.12 can be re-ordered in such a way that imitates the equation 5.7. Obtaining,

$$\dot{\boldsymbol{y}} = \underbrace{\begin{bmatrix} \mathbf{0}_{n,n} & \mathbb{I}_{n,n} \\ -\boldsymbol{M}^{-1}\boldsymbol{K} & -\boldsymbol{M}^{-1}\boldsymbol{C} \end{bmatrix}}_{\boldsymbol{A}} \boldsymbol{y} + \underbrace{\begin{bmatrix} \mathbf{0}_{n,0} \\ \boldsymbol{M}^{-1}\boldsymbol{f} \end{bmatrix}}_{\boldsymbol{B}} \eta \tag{5.13}$$

Thus, if η is white noise, the equation 5.13 can be solved using Lyapunov. Given that it is assumed that for this Chapter the wave slope(θ) is white noise. Then it is ensured that the requirement for a Lyapunov equation shown in the Eq. 5.7 complies with the requirements to be applied over the mathematical model of ship roll.

5.3. Lyapunov solution: Generalized formulation

Given that the system can be solved via Lyapunov, the solution is obtained by finding $\boldsymbol{X}$ in equation 5.8;

$$\overbrace{\begin{bmatrix} \mathbf{0}_{n,n} & \mathbb{I}_{n,n} \\ -\boldsymbol{M}^{-1}\boldsymbol{K} & -\boldsymbol{M}^{-1}\boldsymbol{C} \end{bmatrix}}^{\boldsymbol{A}} \boldsymbol{X} + \boldsymbol{X} \overbrace{\begin{bmatrix} \mathbf{0}_{n,n} & \mathbb{I}_{n,n} \\ -\boldsymbol{M}^{-1}\boldsymbol{K} & -\boldsymbol{M}^{-1}\boldsymbol{C} \end{bmatrix}^{\intercal}}^{\boldsymbol{A}^{\intercal}} + \underbrace{\begin{bmatrix} \mathbf{0}_{n,1} \\ \boldsymbol{M}^{-1}\boldsymbol{f} \end{bmatrix}}_{\boldsymbol{B}} S_o \underbrace{\begin{bmatrix} \mathbf{0}_{n,1} \\ \boldsymbol{M}^{-1}\boldsymbol{f} \end{bmatrix}^{\intercal}}_{\boldsymbol{B}^{\intercal}} = \mathbf{0} \tag{5.14}$$

where $\boldsymbol{X} \in \mathbb{M}_{n,n}$, $\boldsymbol{A} \in \mathbb{M}_{n,n}$ and $\boldsymbol{B} \in \mathbb{R}^n$. The PSD $S_o \in \mathbb{R}$.

To obtain the Lyapunov solution related to the stabilization problem is implemented by a computational code of the mathematical model recently explained in Chapter 1. Obtaining the solution gives as result the matrix "$\boldsymbol{X}$", which is called the "Covariance matrix of the state variable". The values of interest are those from the diagonal of the matrix.

$$\boldsymbol{X} = \begin{bmatrix} \sigma_3^2 & \dots & \dots & \iddots \\ \vdots & \sigma_2^2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ \iddots & \dots & \dots & \sigma_n^2 \end{bmatrix} \tag{5.15}$$

However, this Covariance matrix of the state variable needs to be corrected to obtain those variances of the sought response from the system. For this, is used a matrix called "Covariance matrix of the output variable" ($\boldsymbol{Y}$). This matrix is made by using an auxiliary matrix called "Sensitivity matrix" ($\boldsymbol{C}$), so that both allow to extract the wanted output variances. It is made in the following way:

$$\boldsymbol{Y} = \boldsymbol{C}\boldsymbol{X} \tag{5.16}$$

Finally, the "Output response vector" is $\boldsymbol{S}$ is:

$$\boldsymbol{S} = \boldsymbol{C}\boldsymbol{y} \tag{5.17}$$

5.4. Lyapunov solution applied to the Ship case and the Ship-plus-ART case

For clarity of notation, the indicators of the ship without ART case are denoted with the subscript "$_s$" and the indicators of the ship with ART incorporated case are denoted with the subscript "$_a$".

5.4.1. Lyapunov solution for the ship without ART (single DoF)

In the case of a single DoF, the equation 5.13 is proposed as:

$$\dot{\boldsymbol{y}}_{\boldsymbol{s}} = \boldsymbol{A_s}\boldsymbol{y_s} + \boldsymbol{B_s}\theta \tag{5.18}$$

where

$$\boldsymbol{y_s} = \begin{bmatrix} \varphi \\ \dot{\varphi} \end{bmatrix},\ \dot{\boldsymbol{y}}_{\boldsymbol{s}} = \begin{bmatrix} \dot{\varphi} \\ \ddot{\varphi} \end{bmatrix},\ \boldsymbol{A_s} = \begin{bmatrix} 0 & 1 \\ -\frac{k_s}{m_s} & -\frac{c_s}{m_s} \end{bmatrix},\ \boldsymbol{B_s} = \begin{bmatrix} \boldsymbol{0} \\ \frac{f_1}{m_s} \end{bmatrix} \tag{5.19}$$

Thus the Covariance matrix of the state of variable ($\boldsymbol{X_s} \in \mathbb{M}_{2,2}$) is:

$$\boldsymbol{X_s} = \begin{bmatrix} \sigma^2_{\varphi_s} & \ddots \\ \ddots & \sigma^2_{\dot{\varphi}_s} \end{bmatrix} \tag{5.20}$$

It is obtained the matrix where the values of interest represent the following definitions:

- σ_{φ_s}: Standard deviation of the rolling motion amplitude of the ship due to the energy spectra of the sea waves considered as WN.
- $\sigma_{\dot{\varphi}_s}$: Standard deviation of the rolling angular velocity of the ship due to the energy spectra of the sea waves considered as WN.

Even though for the scope of the present study the only value of interest is the variance of the rolling amplitude, the mathematical code of optimization was made automatically to choose an optimization over the roll amplitude or the velocity of rolling. For that case, the Covariance matrix of the output variable ($\boldsymbol{Y_s}$) and the Sensitivity matrix ($\boldsymbol{C_s}$) are:

$$\boldsymbol{Y_s} = \boldsymbol{X_s} = \begin{bmatrix} \sigma^2_{\varphi_s} & \ddots \\ \ddots & \sigma^2_{\dot{\varphi}_s} \end{bmatrix}, \ \boldsymbol{C_s} = \mathbb{I}_{\mathbf{2,2}} \tag{5.21}$$

The output response for this case is:

$$\boldsymbol{S_s} = \boldsymbol{C}_s \boldsymbol{y_s} \tag{5.22}$$

5.4.2. Lyapunov solution for the ship with an ART implemented (two DoF)

In the case of a single DoF, the equation 5.13 is proposed as:

$$\dot{\boldsymbol{y}}_{\boldsymbol{a}} = \boldsymbol{A_a} \boldsymbol{y_a} + \boldsymbol{B_a} \theta \tag{5.23}$$

where

$$\boldsymbol{y_a} = \begin{bmatrix} \varphi \\ \psi \\ \dot{\varphi} \\ \dot{\psi} \end{bmatrix}, \ \dot{\boldsymbol{y}}_{\boldsymbol{a}} = \begin{bmatrix} \dot{\varphi} \\ \dot{\psi} \\ \ddot{\varphi} \\ \ddot{\psi} \end{bmatrix}, \ \boldsymbol{A_a} = \begin{bmatrix} \mathbf{0}_{2,2} & \mathbb{I}_{\mathbf{2,2}} \\ -\boldsymbol{M}^{-1}\boldsymbol{K} & -\boldsymbol{M}^{-1}\boldsymbol{C} \end{bmatrix}, \ \boldsymbol{B_a} = \begin{bmatrix} \mathbf{0}_{2,1} \\ \boldsymbol{M}^{-1}\boldsymbol{f_1} \end{bmatrix} \tag{5.24}$$

Thus the Covariance matrix of the state of variable ($\boldsymbol{X_s} \in \mathbb{M}_{4,4}$) is:

$$\boldsymbol{X_a} = \begin{bmatrix} \sigma^2_{\varphi_a} & \cdots & \cdots & \ddots \\ \vdots & \sigma^2_{\psi_a} & \ddots & \vdots \\ \vdots & \ddots & \sigma^2_{\dot{\varphi}_a} & \vdots \\ \ddots & \cdots & \cdots & \sigma^2_{\dot{\psi}_a} \end{bmatrix} \tag{5.25}$$

It is obtained the matrix where the values of interest represent the following definitions:

- σ_{φ_a}: Standard deviation of the rolling motion amplitude of the ship due to the energy spectra of the sea waves considered as WN.
- σ_{ψ_a}: Standard deviation of the inclination angle of the free surfaces inside the ART due to the energy spectra of the sea waves considered as WN.
- $\sigma_{\dot{\varphi}_a}$: Standard deviation of the rolling angular velocity of the ship due to the energy spectra of the sea waves considered as WN.
- $\sigma_{\dot{\psi}_a}$: Standard deviation of the angular velocity of the inclination angle of the free surfaces inside the ART due to the energy spectra of the sea waves considered as WN.

And finally, the Covariance matrix of the output variable ($\boldsymbol{Y_a}$) and the Sensitivity matrix ($\boldsymbol{C_a}$) are:

$$\boldsymbol{Y_a} = \boldsymbol{X_a} = \begin{bmatrix} \sigma^2_{\varphi_a} & \dots & \dots & \ddots \\ \vdots & \sigma^2_{\psi_a} & \ddots & \vdots \\ \vdots & \ddots & \sigma^2_{\dot{\varphi}_a} & \vdots \\ \ddots & \dots & \dots & \sigma^2_{\dot{\psi}_a} \end{bmatrix}, \ \boldsymbol{C_a} = \mathbb{I}_{\boldsymbol{4,4}} \tag{5.26}$$

The output response for this case is:

$$\boldsymbol{S_a} = \boldsymbol{C_a y_a} \tag{5.27}$$

5.5. Optimization for white noise excitation input

The Lyapunov solution can be used to analyze the RMS (which is equal to de standard deviation) of the roll angle and the roll velocity. This is made by the comparison of the ship plus ART system response against the ship without ART response. It allows quantifying how effective is the reduction of roll caused by the presence of the ART inside the ship.

Thus, the reduction in percentage which can be applied to the optimization problem 4.30 is given by:

$$\% \ of \ \ Angle \ roll \ reduction \ \ = (1 - \frac{\sigma_{\varphi_a}}{\sigma_{\varphi_s}}) \cdot 100 \tag{5.28}$$

and the reduction of rolling velocity is given by:

$$\% \ of \ \ Roll \ velocity \ reduction \ \ = (1 - \frac{\sigma_{\dot{\varphi}_a}}{\sigma_{\dot{\varphi}_s}}) \cdot 100 \tag{5.29}$$

Chapter 6

Excitation model: Filtered Power Spectral Density case

The aim of the study is the minimization of the ship's roll, varying the ART design dimensions. Mathematically, this is represented using the equation 5.28 in equation 4.30. But this represents the optimization for WN input excitations, and WN is not a good representation of the ocean behavior. Thus, is needed to describe the ocean correctly.

To obtain such a description of the ocean is used a filter, implemented in the Lyapunov solution explained in Chapter 5. This technique is used in "Control System Theory", and its purpose is to imitate the measured PSD of a signal so that the filter can artificially generate a similar PSD of the original signal.

A PSD is a measure of signal's power content versus frequency and is used to characterize broadband signals. The statistical average of a signal analyzed in terms of its frequency content is called "Spectrum".

For the case of this study, the filter is wanted to imitate the behavior of the ocean. So the signal is the slope of the wave and the PSD is the power contained in a defined range of frequency of the waves.

To obtain a good representation of the ocean's PSD, the filter has to be tuned. This means that the parameters of the filter are correctly chosen. This tuning process is called "Theory of Realization".

To use the filter to obtain the response of the ship's roll, it has to be coupled the state space representation of the ship system with the state space representation of the filter system, so the final state space representation of the overall filter. The input of the filter is WN, and inside of it (as a latent parameter) is obtained the PSD of the wave slope of the ocean (and its associated RMS). It is then introduced as input to the ship system to finally obtain the response of the ship roll. The representation of the filter is shown in Figure 6.1.

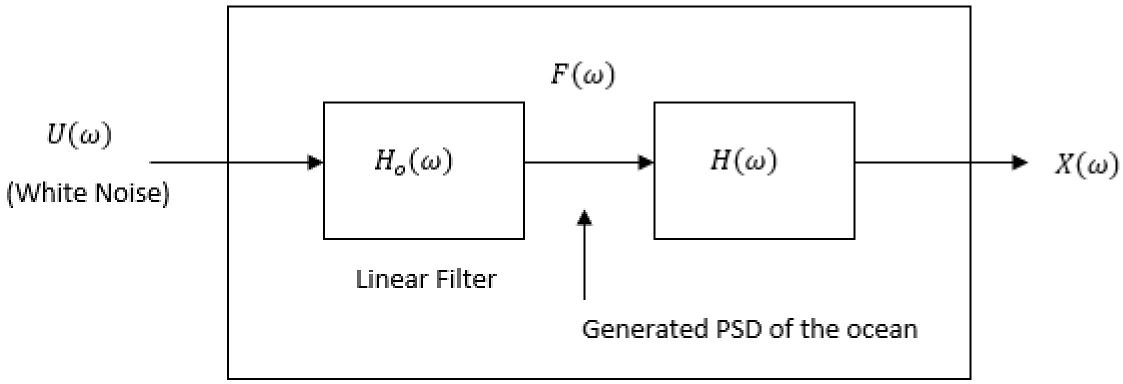


Figure 6.1: Block diagram of the filter process.

The ocean behaves in several conditions. For each of them, there is a characteristical PSD. So the optimum problem 4.30 has to be implemented considering the probability to operate in each state.

In the following sections, it is explained how to process and define the filter as well as to define the way to approach the definition of the total reduction of the roll to be optimized.

6.1. Power spectral densities of the ocean

The international towing conference [7] (ITTC) has adopted the Bretscheneider spectrum [4] as the standard wave energy spectrum to represent the conditions that occur in the open ocean. It is also called **ITTC two-parameter spectrum** and represents the spectrum of the height of the wave.

$$S_{T,H_{\frac{1}{3}}} = \frac{A}{\omega^5} e^{\frac{-B}{\omega^4}}, \qquad m^2 s \tag{6.1}$$

where the "two parameter" are:

$$A = 172.75 \frac{\overline{H}_{\frac{1}{3}}^2}{\overline{T}^4}, \qquad \frac{m^2}{s^4} \tag{6.2}$$

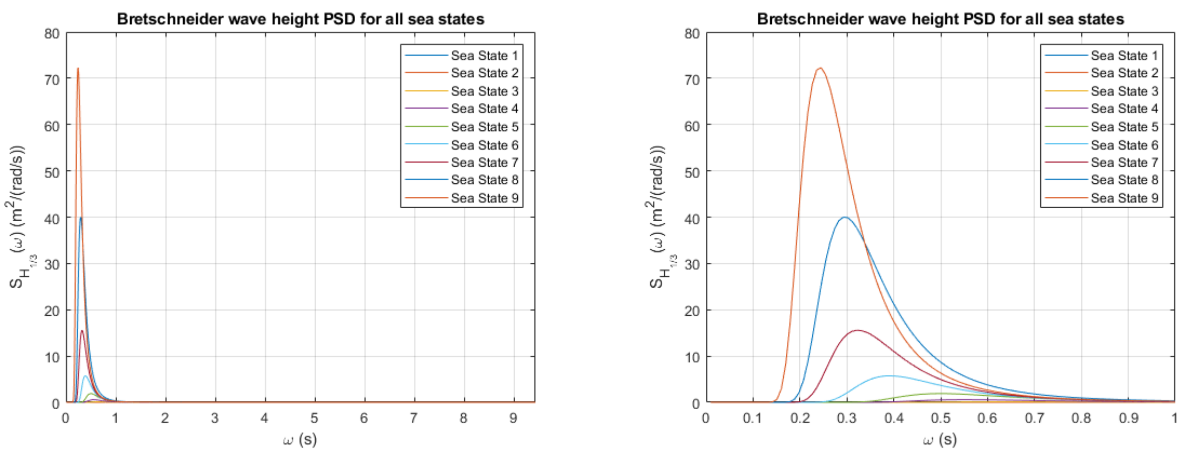
$$B = \frac{691}{\overline{T}^4}, \qquad s^{-4} \tag{6.3}$$

depending on the "Mean value of the significant wave height" and the "Most probable modal wave period; respectively $\overline{H}_{\frac{1}{3}}$ and $\overline{T}$.

The PSD of the height of the wave is plotted for all sea states 6.1 in figure 6.2 and the characteristics of each state of the ocean are given in table 6.1.

Table 6.1: Sea State code and properties of the wave spectrum. Coupled data from [3] and [8]. Sustained in data from [9]

Sea State	Description of sea	Significant wave height range [m]	Mean value of the significant wave height $\overline{H}_{\frac{1}{3}}$ [m]	Most probable modal wave period $\overline{T}$ [s]
1	Calm	0-0.1	0.06	0
2	Smooth	0.1-0.5	0.3	5.3
3	Slight	0.5-1.25	0.88	7.5
4	Moderate	1.25-2.5	1.88	8.8
5	Rough	2.5-4	3.25	9.7
6	Very Rough	4-6	5	12.4
7	High	6-9	7.5	15
8	Very High	9-14	11.5	16.4
9	Phenomenal	Over 14	14	20



(a) Wave height PSD for frequency domain from 0 to 9 rad/s

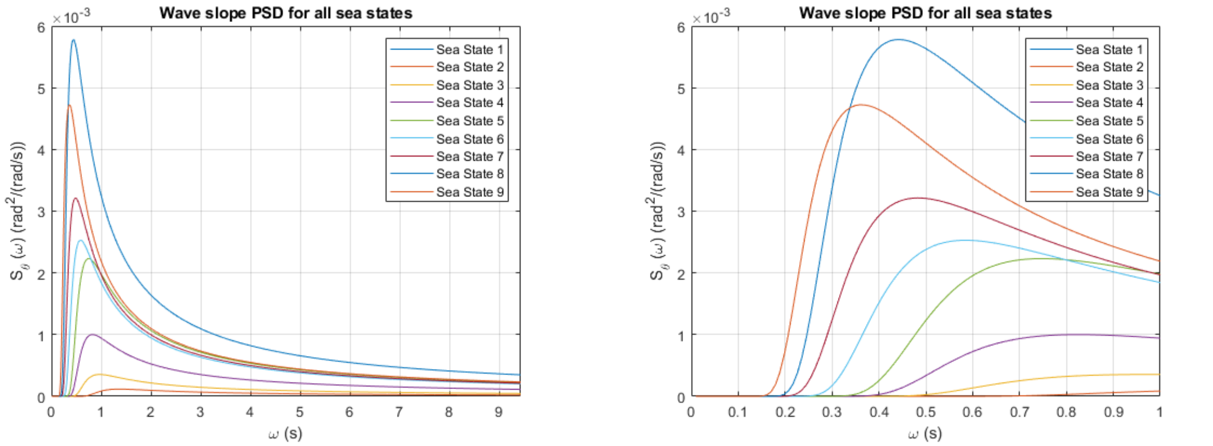
(b) Wave height PSD for frequency domain from 0 to 1 rad/s

Figure 6.2: Bretschneider Wave height Power Spectral Densities of all Sea State of the ocean

From the equation of Bretscheneider 6.1, it is possible to obtain the PSD of the slope of the wave. This last PSD is the needed one to be generated by the filter, given that the mathematical model used in this study operates with a wave slope input.

The generation of the PSD of the slope of the wave is obtained by multiplying the equation 6.1 by $\frac{\omega^4}{g^2}$. Where g is the constant of gravity and ω is the frequency sample. Then,

$$S_{T,\theta} = \frac{A}{\omega g^2} e^{\frac{-B}{\omega^4}}, \qquad \frac{rad^2}{\frac{rad}{s}} \tag{6.4}$$



(a) Wave slope PSD for frequency domain from 0 to 9 rad/s

(b) Wave slope PSD for frequency domain from 0 to 1 rad/s

Figure 6.3: Wave slope Power Spectral Densities of all Sea State of the ocean

The PSD of the slope of the wave plotted in figure 6.3 contains statistical information on the properties of the ocean. It is then used a filter to generate an artificial PSD that contains similar information about the waves of the ocean.

6.2. Linear filter of minimum squares

It has been seen that a characteristic function of power spectral density complies with the equation

$$S_{xx}(\omega = |H(\omega)|^2 S_{ff}(\omega) \tag{6.5}$$

To define the filter is the same as generating a combination of H_ω and S_{ff} such as it generates an "S_{xx}" as close as possible to S_θ, the real (measured) PSD of the slope of the wave of the ocean.

It is selected in this study the simplest of filters, which is the linear filter of minimum squares represented by the "spring-mass" system of the figure 6.4.

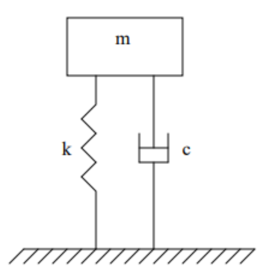


Figure 6.4: Rolling amplitude response of a ship without an ART. Own authorship.

This filter is descripted by the equation of motion 6.6.

$$\ddot{x} + 2\zeta_o\omega_o\dot{x} + \omega_o^2 x = -a \tag{6.6}$$

and the PSD "$S_{ff}(\omega)$" is set to be constant for any "ω". Then, $S_{ff}(\omega) = S_o$. The transfer function is $H_o(\omega) = (-1 + 2i\zeta_o\omega_o + w_o^2)^{-1}$. This combination generates the "Tuned PSD"

$$S_{tun}(x) = |H_o|^2 S_o \tag{6.7}$$

which is the PSD used in the filter.

6.3. Theory of realization: The filter tuning

The tuning of the filter consists on selecting the values ω_o, ζ_o, and S_o in such a way that the tuned ("spring-mass system") PSD is as close as possible to the PSD of the slope of the wave.

The optimum values for these parameters are mathematically obtained by performing the following optimization:

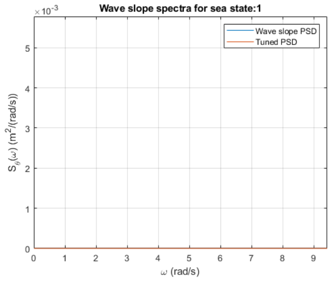
$$\begin{aligned} \min_{w_o,\zeta_o,S_o} \quad & \sqrt{\sum(S_\theta(\omega_i) - S_{tun}(\omega_i))^2} \qquad \textit{for all } \omega_i \in \omega \textit{ range} \\ \text{s.t.} \quad & 0 \leq w_o, \zeta_o, S_o \end{aligned} \tag{6.8}$$

The optimum values are called with the subscript "$_f$". So, the optimum values are defined as:

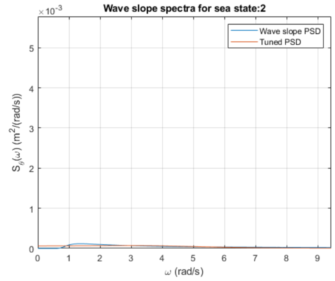
$$\omega_o = \omega_f \tag{6.9}$$
$$\zeta_o = \zeta_f \tag{6.10}$$
$$\mathcal{S}_o = \mathcal{S}_f \tag{6.11}$$

and those are the ones that, once tuned, are applied to the Lyapunov filter.

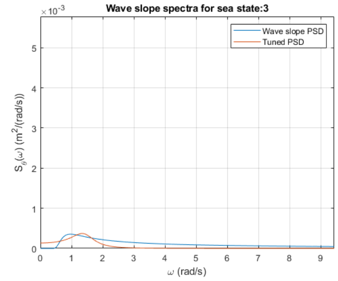
It is shown in figure 6.5 the tuning of the filters of each sea state PSD for a frequency range $\omega \in [0, 2.5 \cdot \pi]$



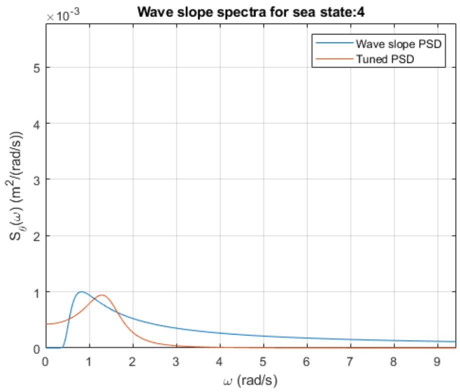
(a) Filter tuning for sea state 1



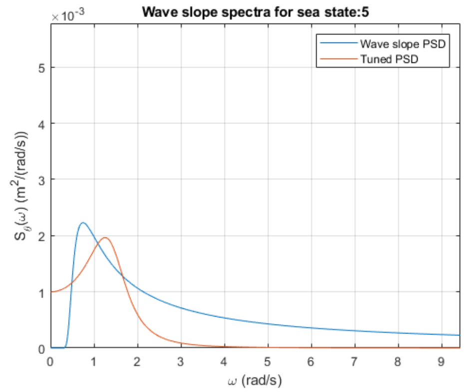
(b) Filter tuning for sea state 2



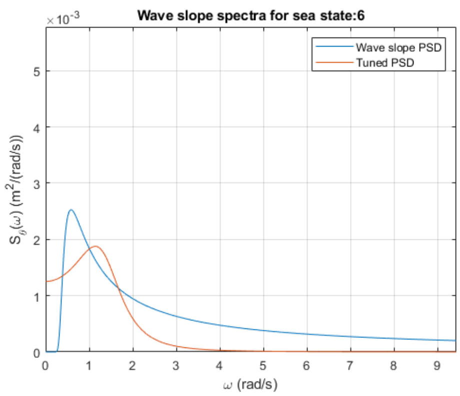
(c) Filter tuning for sea state 3



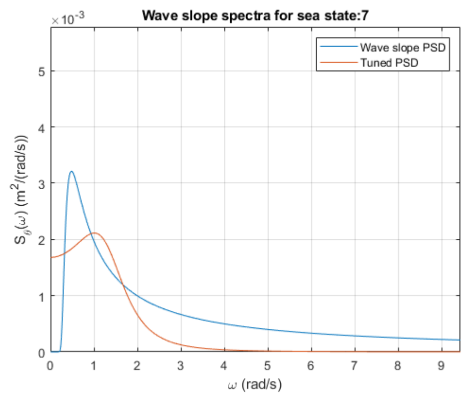
(d) Filter tuning for sea state 4



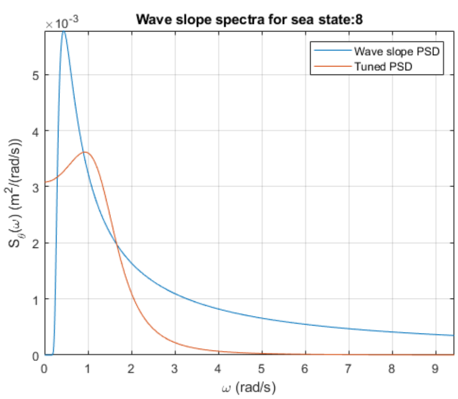
(e) Filter tuning for sea state 5



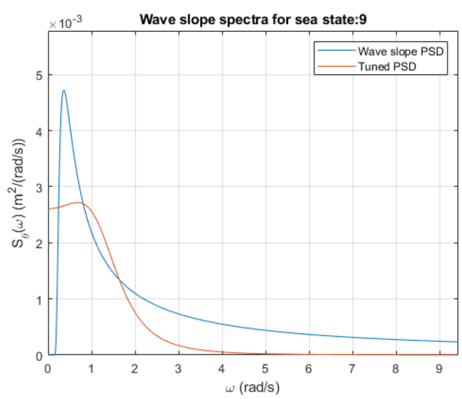
(f) Filter tuning for sea state 6



(g) Filter tuning for sea state 7



(h) Filter tuning for sea state 8



(i) Filter tuning for sea state 9

Figure 6.5: Filters tuning for all sea states for a frequency domain $\omega \in [0, 2.5 \cdot \pi]$

The values of the optimum tuning parameters for each sea state are:

Table 6.2: Optimum values of the tuning parameters over a frequency domain $\omega \in [0, 2.5 \cdot \pi]$

Sea State	ω_f	ζ_f	S_f
1	1.22752	0.65233	0.00000
2	4.98171	0.57734	0.03817
3	1.49965	0.31784	0.00068
4	1.49986	0.35958	0.00215
5	1.50000	0.38654	0.00506
6	1.49990	0.45998	0.00634
7	1.49987	0.52309	0.00851
8	1.49992	0.55490	0.01561
9	1.49983	0.63157	0.01319

6.4. State space and Lyapunov formulation of the filter

The dynamic system of the filter is represented by equation 6.6. This system represented in state space is as follows:

$$\dot{\boldsymbol{y}}_{\boldsymbol{f}} = \boldsymbol{A}_{\boldsymbol{f}} \boldsymbol{y}_{\boldsymbol{f}} + \boldsymbol{B}_{\boldsymbol{f}} a \tag{6.12}$$

where a is WN and

$$\boldsymbol{y}_{\boldsymbol{f}} = \begin{bmatrix} x_f \\ \dot{x}_f \end{bmatrix}, \ \dot{\boldsymbol{y}}_{\boldsymbol{f}} = \begin{bmatrix} \dot{x}_f \\ \ddot{x}_f \end{bmatrix}, \ \boldsymbol{A}_{\boldsymbol{f}} = \begin{bmatrix} 0 & 1 \\ -\omega_f^2 & -2\zeta_f \omega_f \end{bmatrix}, \ \boldsymbol{B}_{\boldsymbol{f}} = \begin{bmatrix} 0 \\ -1 \end{bmatrix} \tag{6.13}$$

Thus the Covariance matrix of the state of variable ($\boldsymbol{X}_{\boldsymbol{f}} \in \mathbb{M}_{2,2}$) is:

$$\boldsymbol{X}_{\boldsymbol{f}} = \begin{bmatrix} \sigma_{x_f}^2 & \ddots \\ \ddots & \sigma_{\dot{x}_f}^2 \end{bmatrix} \tag{6.14}$$

It is obtained the matrix where the values of interest represent the following definitions:

- σ_{x_f}: Standard deviation of the output motion of the filter considering an external excitation of WN. Once the filter is tuned, this value represents the standard deviation of the slope of the wave.
- $\sigma_{\dot{x}_f}$: Standard deviation of the output velocity of motion of the filter considering an external excitation of WN. Once the filter is tuned, this value represents the standard

deviation of the changing severity of the slope of the wave.

The Covariance matrix of the output variable $\boldsymbol{Y_f}$ is

$$\boldsymbol{Y_f} = \begin{bmatrix} \sigma_{x_f}^2 & \ddots \\ \ddots & \sigma_{\dot{\varphi}_s}^2 \end{bmatrix} \tag{6.15}$$

For this case, the filter output response is set as:

$$S_{ff} = \boldsymbol{C_S y_s} = x_f \tag{6.16}$$

being $\boldsymbol{C_S} = \begin{bmatrix} 1 & 0 \end{bmatrix}$, for the purpose to extract only the information on the slope of the wave. This means, when the filter is tuned, it complies

$$S_{ff} = \theta \tag{6.17}$$

(not to be confused with S_f, which is one of the tuning factors of the theory of realization.

6.5. Determination of the filter: Couplement of Lyapunov systems

For clarity of notation, the indicators of the ship without ART case are denoted with the subscript "$_t$" and the indicators of the ship with ART incorporated case are denoted with the subscript "$_T$". The filter case is expressed with the subscript "$_f$".

6.5.1. Filter coupling for the ship without an ART (single DoF)

To couple the Filter to the Lyapunov representation of the ship without the ART dynamic system it has to be selected the proper output state variable to re-enter it inside the filter and generate the wanted excitation (the wave slope). This is made in the equation 6.17.

$$\boldsymbol{y_t} = \begin{bmatrix} \boldsymbol{y_s} \\ \boldsymbol{y_f} \end{bmatrix} = \begin{bmatrix} \varphi \\ \dot{\varphi} \\ \theta \\ \dot{\theta} \end{bmatrix} \tag{6.18}$$

were $\boldsymbol{y_s}$ is the variable of state of the ship system defined in equation 5.19 and $\boldsymbol{y_f}$ is defined in equation 6.13. Assuming that the filter has been already tuned for $\boldsymbol{y_f}$.

The derivative of $\boldsymbol{y_t}$ is

$$\boldsymbol{\dot{y}_t} = \begin{bmatrix} \boldsymbol{\dot{y}_s} \\ \boldsymbol{\dot{y}_f} \end{bmatrix} = \begin{bmatrix} \dot{\varphi} \\ \ddot{\varphi} \\ \dot{\theta} \\ \ddot{\theta} \end{bmatrix} \tag{6.19}$$

Knowing that the equation of state 5.18 represents the system of the ship, where θ is set to be the output of the filter (as equation 6.17). Then, applying equation 6.17 into equation 5.18 it is obtained

$$\boldsymbol{\dot{y}_s} = \boldsymbol{A}_s \boldsymbol{y_s} + \boldsymbol{B}_s S_{ff} \tag{6.20}$$

but S_f complies wth equation 6.16, too. Then the equation 6.20 becomes

$$\boldsymbol{\dot{y}_s} = \boldsymbol{A}_s \boldsymbol{y_s} + \boldsymbol{B}_s \boldsymbol{C_f} \boldsymbol{y_f} \tag{6.21}$$

Finally, the couplement of the State equation of the filter, is needed to representate both equations 6.21 and 6.12. Which, when inserted into the equation 6.19, can be re-ordered as:

$$\boldsymbol{\dot{y}_t} = \begin{bmatrix} \boldsymbol{A}_s & \boldsymbol{B}_s \boldsymbol{C_f} \\ \boldsymbol{0_{2,2}} & A_f \end{bmatrix} \begin{bmatrix} \boldsymbol{y_s} \\ \boldsymbol{y_f} \end{bmatrix} + \begin{bmatrix} \boldsymbol{0_{2,1}} \\ \boldsymbol{B_f} \end{bmatrix} a \tag{6.22}$$

Obtaining the characteristic matrix of the Lyapunov Filter for the ship without ART case:

$$A_t = \begin{bmatrix} \boldsymbol{A}_s & \boldsymbol{B}_s \boldsymbol{C_f} \\ \boldsymbol{0_{2,2}} & A_f \end{bmatrix} \tag{6.23}$$

$$B_t = \begin{bmatrix} \boldsymbol{0_{2,1}} \\ \boldsymbol{B_f} \end{bmatrix} \tag{6.24}$$

Matrix that allows to obtain the solutions of the filter for this case (It is computationally used the equation 5.8).

$$\boldsymbol{X_t} = \begin{bmatrix} \sigma^2_{\varphi_t} & \dots & \dots & \ddots \\ \vdots & \sigma^2_{\dot{\varphi}_t} & \ddots & \vdots \\ \vdots & \ddots & \sigma^2_{\theta_t} & \vdots \\ \ddots & \dots & \dots & \sigma^2_{\dot{\theta}_t} \end{bmatrix} \tag{6.25}$$

with the Covariance matrix of the output variable $\boldsymbol{Y_t}$ as

$$\boldsymbol{Y_t} = \boldsymbol{C_t} \boldsymbol{X_t} = \begin{bmatrix} \sigma^2_{\varphi_t} & \dots & \dots & \ddots \\ \vdots & \sigma^2_{\dot{\varphi}_t} & \ddots & \vdots \\ \vdots & \ddots & \sigma^2_{\theta_t} & \ddots \end{bmatrix} \tag{6.26}$$

where

$$\boldsymbol{C_t} = \begin{bmatrix} \mathbb{I}_{\boldsymbol{3,3}} & \boldsymbol{0_{3,1}} \end{bmatrix} \tag{6.27}$$

to generate an output state variable $\boldsymbol{S_t}$ equal to

$$\boldsymbol{S_t} = \boldsymbol{C}_t \boldsymbol{y_t} = \begin{bmatrix} \varphi \\ \dot{\varphi} \\ \theta \end{bmatrix} \tag{6.28}$$

6.5.2. Filter couplement for the ship with an ART implemented (two DoF)

To couple the Filter to the Lyapunov representation of the ship without the ART dynamic system it has to be selected the proper output state variable to re-enter it inside the filter and generate the wanted excitation (the wave slope). This is made in the equation 6.17.

It is defined

$$\boldsymbol{y_T} = \begin{bmatrix} \boldsymbol{y_a} \\ \boldsymbol{y_f} \end{bmatrix} = \begin{bmatrix} \varphi \\ \psi \\ \dot{\varphi} \\ \dot{\psi} \\ \theta \\ \dot{\theta} \end{bmatrix} \tag{6.29}$$

were $\boldsymbol{y_a}$ is the variable of state of the ship-plus-ART system defined in equation 5.24 and $\boldsymbol{y_f}$ is defined in equation 6.13. Assuming that the filter has been already tuned for $\boldsymbol{y_f}$.

Therefore, the derivative of $\boldsymbol{y_T}$ is

$$\dot{\boldsymbol{y}}_{\boldsymbol{T}} = \begin{bmatrix} \dot{\boldsymbol{y}}_a \\ \dot{\boldsymbol{y}}_f \end{bmatrix} = \begin{bmatrix} \dot{\varphi} \\ \dot{\psi} \\ \ddot{\varphi} \\ \ddot{\psi} \\ \dot{\theta} \\ \ddot{\theta} \end{bmatrix} \tag{6.30}$$

Knowing that the equation of state 5.23 represents the system of the ship-plus-ART, where θ is set to be the output of the filter (as equation 6.17). Then, applying equation 6.17 into equation 5.23 it is obtained

$$\dot{\boldsymbol{y}}_{\boldsymbol{a}} = \boldsymbol{A}_a \boldsymbol{y_a} + \boldsymbol{B}_a S_{ff} \tag{6.31}$$

but S_{ff} complies with equation 6.16, too. Then the equation 6.31 becomes

$$\dot{\boldsymbol{y}}_{\boldsymbol{a}} = \boldsymbol{A}_a \boldsymbol{y_a} + \boldsymbol{B}_a \boldsymbol{C_f} \boldsymbol{y_f} \tag{6.32}$$

Finally, the couplement of the State equation of the filter is needed to represent both equations 6.32 and 6.12. Which, when inserted into the equation 6.30, can be re-ordered as:

$$\dot{\boldsymbol{y}}_{\boldsymbol{T}} = \begin{bmatrix} \boldsymbol{A_a} & \boldsymbol{B_a C_f} \\ \boldsymbol{0_{2,4}} & A_f \end{bmatrix} \begin{bmatrix} \boldsymbol{y_a} \\ \boldsymbol{y_f} \end{bmatrix} + \begin{bmatrix} \boldsymbol{0_{4,1}} \\ \boldsymbol{B_f} \end{bmatrix} a \tag{6.33}$$

Obtaining the characteristic matrix of the Lyapunov Filter for the ship-plus-ART case:

$$A_T = \begin{bmatrix} \boldsymbol{A_a} & \boldsymbol{B_a C_f} \\ \boldsymbol{0_{2,4}} & A_f \end{bmatrix} \tag{6.34}$$

$$B_T = \begin{bmatrix} \boldsymbol{0_{4,1}} \\ \boldsymbol{B_f} \end{bmatrix} \tag{6.35}$$

Matrix that allows obtaining the solutions of the filter for this case. It is computationally used the equation 5.8)

$$\boldsymbol{X_T} = \begin{bmatrix} \sigma^2_{\varphi_T} & \cdots & \cdots & \cdots & \cdots & \ddots \\ \vdots & \sigma^2_{\psi_T} & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \sigma^2_{\dot{\varphi}_T} & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \sigma^2_{\dot{\psi}_T} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \sigma^2_{\theta_T} & \vdots \\ \ddots & \cdots & \cdots & \cdots & \cdots & \sigma^2_{\dot{\theta}_T} \end{bmatrix} \tag{6.36}$$

with the Covariance matrix of the output variable $\boldsymbol{Y_T}$ as

$$\boldsymbol{Y_T} = \boldsymbol{C_T X_T} = \begin{bmatrix} \sigma^2_{\varphi_T} & \cdots & \cdots & \cdots & \cdots & \ddots \\ \vdots & \sigma^2_{\psi_T} & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \sigma^2_{\dot{\varphi}_T} & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \sigma^2_{\dot{\psi}_T} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \sigma^2_{\theta_T} & \ddots \end{bmatrix} \tag{6.37}$$

where

$$\boldsymbol{C_T} = \begin{bmatrix} \mathbb{I}_{\boldsymbol{5,5}} & \boldsymbol{0_{5,1}} \end{bmatrix} \tag{6.38}$$

to generate an output state variable $\boldsymbol{S_T}$ equal to

$$\boldsymbol{S_T} = \boldsymbol{C_T y_T} = \begin{bmatrix} \varphi \\ \psi \\ \dot{\varphi} \\ \dot{\psi} \\ \theta \end{bmatrix} \tag{6.39}$$

6.6. Optimization approach for realistic wave slope excitation

The Filter of Lyapunov applied to the mathematical model of ship roll is used to analyze the RMS (which is equal to de standard deviation) of the roll angle and the roll velocity. This is made by the comparison of the ship plus ART system response against the ship without ART response. It allows quantifying how effective is the reduction of rolling caused by the presence of the ART inside the ship.

Thus, the reduction in percentage which can be applied to the optimization problem 4.30 is given by:

$$\%\ of\ Angle\ roll\ reduction\ = (1 - \frac{\sigma_{\varphi_T}}{\sigma_{\varphi_t}}) \cdot 100 \tag{6.40}$$

and the reduction of rolling velocity is given by:

$$\%\ of\ Roll\ velocity\ reduction\ = (1 - \frac{\sigma_{\dot{\varphi}_T}}{\sigma_{\dot{\varphi}_t}}) \cdot 100 \tag{6.41}$$

The standard deviation used in equation 6.40 are found in the terms $\boldsymbol{Y_{t_{1,1}}}$ and $\boldsymbol{Y_{T_{1,1}}}$; and the standard deviation used in equation 6.41 are found in $\boldsymbol{Y_{t_{2,2}}}$ and $\boldsymbol{Y_{T_{2,2}}}$

Using equation 6.40 in the proposed system of optimization 4.30 it es possible to perform an optimization for a single state of the ocean that leads to a feasible solution given that the Filter does represent the ocean with some degree of accuracy.

Considering that the final design of an ART is fixed for all sea states it is proposed to optimize the roll reduction for all sea states considering the probabilities of occurrences. Then, the global problem of optimization is defined as:

$$\begin{aligned} \min_{DP} \quad & \sum_{SS}((-\%\ of\ Angle\ roll\ reduction)p(SS)) \\ \text{s.t.} \quad & Geometrical\ restrictions \\ & Fisical\ restriction\ of\ the\ ship + ART\ system \end{aligned} \tag{6.42}$$

where SS are the Sea States and $p(SS)$ the probability of occurrence of that particular Sea State.

Chapter 7

Methodology: Proposed framework for the optimization of the design of U-shaped Anti-Roll tanks under stochastics seas

The problem to be solved for a naval architect designer is to dimension an ART in such a way that the reduction of the ship roll motion is maximum, as well as minimize the required space to install the ART. For this purpose, is used a mathematical model that describes the roll motion [1]. But this model is only valid under harmonic excitation, and it does not represent reliably the behavior of the sea. Thus, this model is processed to generate results for stochastically defined non-harmonic seas. Refer to chapter 4 to understand the model.

Then, it is needed to find a way to describe the sea where the input excitation enters to the model and the response is a reliable value of rolling measurement. For this aim, is used a "filter" that receives as input Gaussian white noise, and gives as output the RMS of the roll of the ship given the input characteristics of the ocean.

Stochastic descriptions of irregular seas are made via PSDs. So, the filter is set to be a transfer function that receives Gaussian white noise and gives a function of Power Spectral Density (PSD). Such a filter is implemented using the state space solution of Lyapunov, which is a way to create filters as proposed in Control System Theory.

The usage of the Lyapunov filter tool is due to its benefits.It offers an analytic solution to a complex integral (the PSD). So, when correctly implemented, is reliable and the computation time is much faster than numerical integration.

The most common PSD to describe the ocean is the Bretscheneider PSD (6.1). It measures the power of the ocean from the height of the waves. In this study is used the PSD of the Slope of the Wave (not the height of the wave), because the input to the model of dynamic roll motion is the slope of the wave. However, both PSDs are related; in fact, the second one is obtained from the first.

To describe the system with a PSD, the filter has to be tuned, so the output of the filter describes a reliable approximation to the PSD of the ocean in terms of RMS of the wave slope. To tune the filter it is performed the "Theory of realization", which is explained in chapter 6 section 6.3. The filter is calibrated in such a way that has to be ensured that it describes approximately the sea.

The filter is a linear system that solve a function of minimum squared values. Usually, it describes a damped mass and spring system. Once the spectral density is obtained, the filter has to enter the system (ship plus ART) model, to obtain an RMS value of the rolling angle of the ship during operation at a specific sea.

Having the response (in RMS) of the ship, it can be obtained the performance of the ART stabilizer in terms of roll reduction. Which is obtained comparing the RMS of the roll of a ship implemented with an ART and the same ship without the ART. Then, it can be performed an optimization that allows the variation of all design parameters in order to obtain the optimum dimensions of the ART that produces maximum roll reduction.

There are 9 Sea states, and for each of them, there is a different optimum ART. But the final design of the ART is only one and cannot variate during operation. So the optimization has to be performed over the variation of the design parameters in consideration of all sea states.

To perform the optimization it is defined where the ship will operate and the characteristics of that ocean zone. So that is known the distribution of the height of the waves (Rayleigh). To know the distribution is the same to know the probability of occurrence of each sea state. Thus the optimization, assuming only one final geometry, is over the reduction of the rolling of the ship at each sea state ponderated by the probability that this sea state occurs.

Finally, the optimization is performed and is selected the optimum geometry that leads to the best performance considering all possible scenarios under stochastically described seas. Then, it is possible to study how the final geometry will behave for all different operation cases, consideryng both the velocity of the ship and the direction of the ship with respect to the wave incidence.

In the following list, it is resumed the steps of the methodology.

1. Definition of the problem.
2. Validation of mathematical model.
3. Create a model to describe a reliable behavior of the sea: the Filter using the Lyapunov solution.
4. Find a description of the ocean in terms of PSD.
5. Implement Lyapunov solution of the problem using WN as input excitation.
6. Adapt the filter to create, from WN, a similar PSD of the ocean wave slope: Theory of realization and tuning of the filter.
7. Obtain the RMS value of the roll of the ship.
8. Create the model of optimization of the problem: Defining objective function and variable parameters.
9. Perform optimization considering the probabilities of operation at each sea state.
10. Obtain the optimum design.
11. Analysis of the projected performance of the selected design at idle operation (standing still at 0 knots at open seas) and from 0 to 20 knots of velocity:
    a) Reduction of all sea states against maneuvering direction.
    b) Comparison of all sea states against maneuvering direction.

A simplified view of the methodology of the study is presented in the block diagram of figure 7.1

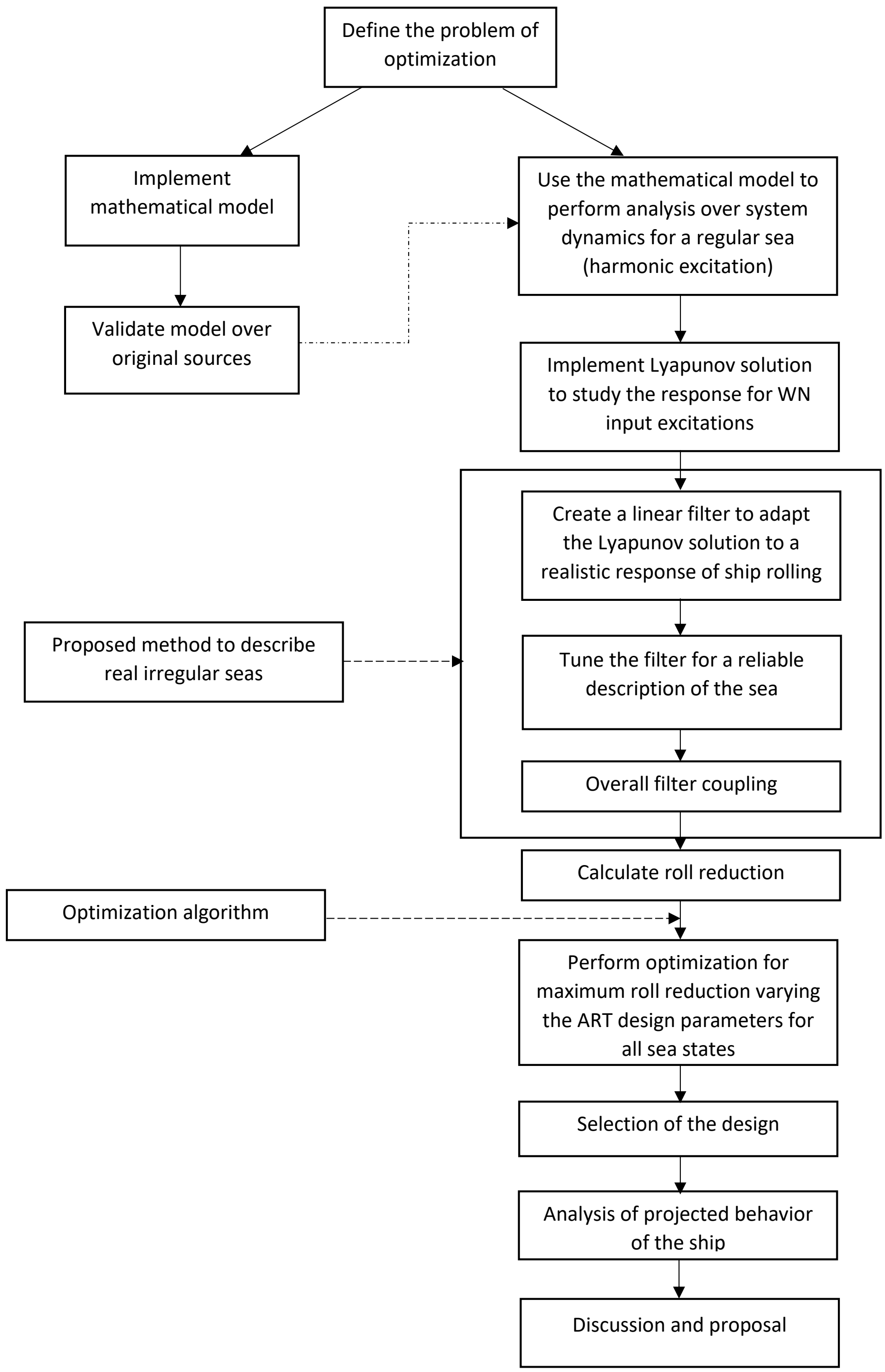


Figure 7.1: Diagram of the complete process of the study.

On the other hand, a simplified view of the study case is presented in the block diagram of figure 7.2.

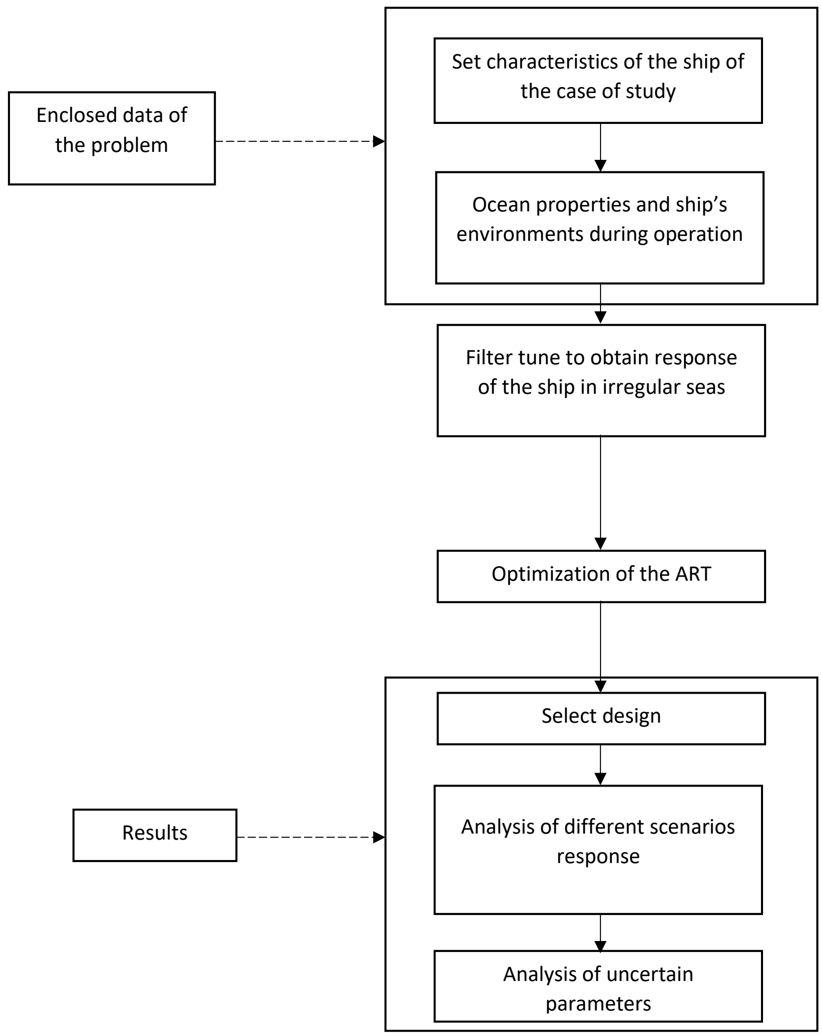


Figure 7.2: Diagram of the case of study.

Chapter 8

Case of study: Optimization of a U-Shaped ART for a specific ship

8.1. Characteristics of the ship chosen for the case of study: Offshore Patrol Vessel (OPV)

The case of study is the design of an ART for an Offshore Patrol Vessel, whose main purpose is to patrol the Chilean coast and Chilean maritime exclusive economic zone.

The characteristics of the ship system that needs to enter the algorithm of optimization are:

Table 8.1: Parameter input of the case of study.

Parameter	Value	unit	Classification	Description
P	1828	Ton	UP	Mass of the ship ; kg.
m'	1.5	m	UP	Metacentric height of the system.
$\%_{IR}$	0.5	-	UP	The quotient of the radius of inertia (ij) and breadth of the ship.
χ_s	0.075	-	UP	Dimensionless parameter of the linear damping of the ship roll.
χ_t	0.092	-	UP	Dimensionless parameter of the linear damping of the ART internal fluid (water).
B	13	m	FP	Breadth of the ship, m.
D	3.8	m	FP	Draught of the ship, m.

8.2. Ocean properties and OPV's operation predicted environment

The OPV will operate in the Extratropical South Pacific (ETSP) [10]. See figure 8.1 for location reference highlighted in red color.

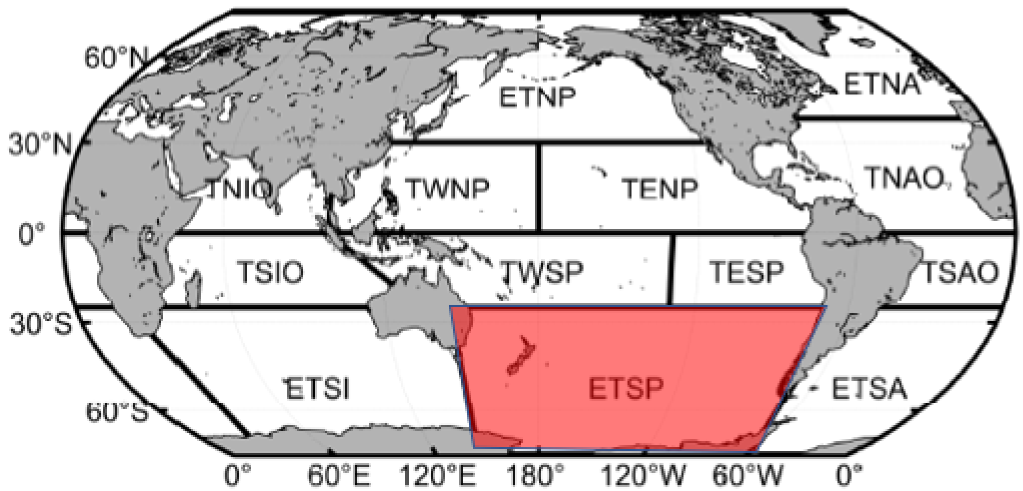


Figure 8.1: Selected zone of the ocean: ETSP.

For that location, the characteristics of the waves are shown in figure 8.2.

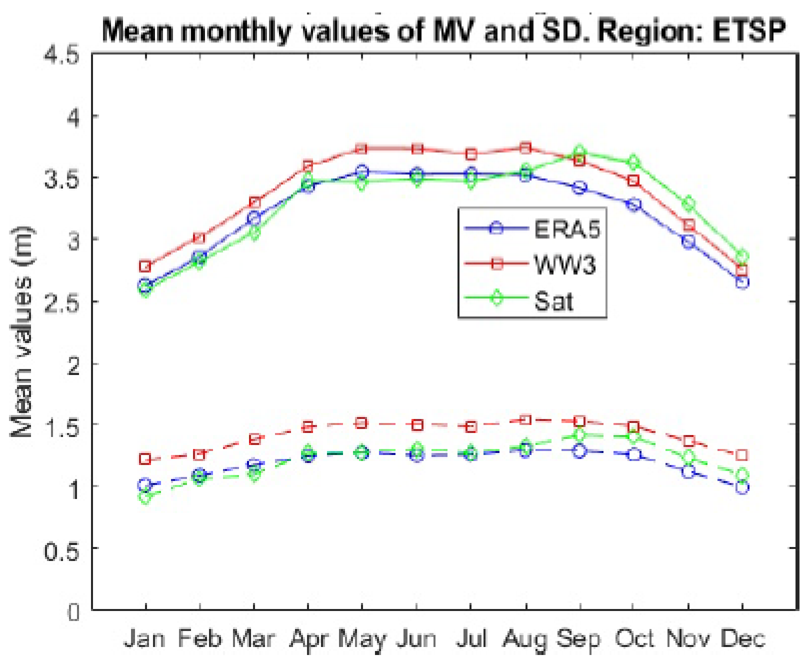


Figure 8.2: Monthly wave data of ETSP location. Mean values and standard deviation values (in dashed lines) [10].

Where the approximated values of the graph, used in this study case, are shown in the table 8.2

Table 8.2: Monthly and annual wave data of ETSP location.

	Jan'	Feb'	'Mar'	'Apr'	'May'	'Jun'	Jul'	Aug'	Sep'	Oct'	Nov'	'Dec'	Annual mean
Mean height [m]	2.9	3.1	3.55	3.75	3.85	3.8	3.8	3.85	3.7	3.5	3.2	2.95	3.50
Stand dev [m]	1.25	1.3	1.4	1.45	1.45	1.4	1.4	1.48	1.5	1.3	1.15	1.1	1.35

The distribution that represents the overall behavior of the height of the waves of the sea is the Rayleigh distribution [8]

$$p(x) = \frac{x}{a} e^{\frac{-x^2}{2a}} \tag{8.1}$$

where for this case 'x' represents the height of the waves and 'a' is the "mode", that complies with $2a = mean\ value\ of\ x^2$. The use of the Rayleigh cumulative distribution function allows calculating the probability of occurrence of each sea state, using the intervals of integration specified in table 6.1.

For the Rayghlieh distribution the mode is calculated as $a = mean\ \cdot \sqrt{\frac{2}{\pi}}$. Then for this case is taken:

$$a = annual\ mean\ height\ \cdot \sqrt{\frac{2}{\pi}} \tag{8.2}$$

Then it is obtained the characteristic curve of the probability of density function 8.3:

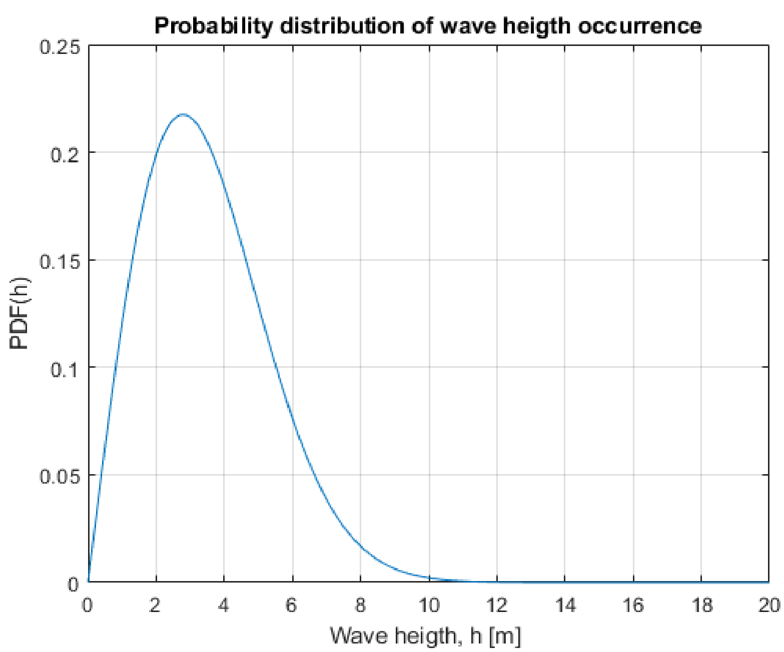


Figure 8.3: Adjusted Rayleigh probability density function for data of figure 8.2 and table 8.2.

It is presented the characteristics of the ETSP ocean where the OPV will operate and the probabilities of occurrences calculated from the Rayleigh distribution.

Table 8.3: Probabilities of occurrences of sea states in the ETSP.

Sea State	Probability of occurrence	Mean Significant Wave Height	Unit	Most Probable Modal Wave Period	Unit
1	0.0642%	0.06	m	0.0001	s
2	1.5296%	0.3	m	5.3	s
3	7.9602%	0.88	m	7.5	s
4	23.5258%	1.88	m	8.8	s
5	31.1578%	3.25	m	9.7	s
6	25.8721%	5	m	12.4	s
7	9.3418%	7.5	m	15	s
8	0.5482%	11.5	m	16.4	s
9	0.0003%	14	m	20	s

8.3. Fitting tune parameters of the Filter

The tune parameters are the same as in table 6.2. To perform the fitting of the parameters it was executed the optimization explained in equation 6.8.

8.4. Optimization of the ART for the OPV characteristics

To perform the optimization that obtains the best design for a maximum reduction of roll it was executed the problem defined in equation 6.42 using the characteristics of the ship defined in 8.1.

The algorithm used to find the solution is $fmincon$ of *Matlab*, which is a function to solve non-linear problems with non-linear and linear constraints. To run the code, it is needed seeds for the Design Parameters. The objective function is defined in 6.42, the constraints and the seeds were the following ones:

- Constraint 1: The mass of the ART cannot be more than 6% the mass of the ship.

$$Q = (2 \cdot w_1 \cdot y + y^2 \cdot \tan(\alpha) + h \cdot w) \cdot \rho \cdot l \leq 0.06 \cdot P \tag{8.3}$$

  where $\rho = 1025 \frac{Kg}{m^3}$ value is in appendix A.

- Constraint 2: The maximum length of the ART, 'l', is 6 m (see table 8.5).

- Constraint 3: The maximum height of the water in the reservoirs of the ART, 'y', is 2.3 m (see table 8.5).

- Constraint 4: The height of the conduit is lower that the level of the water.

$$h \leq y \tag{8.4}$$

- Constraint 5: The maximum width of the ART is no more than 95% of the OPV's breadth.

$$w + 2w_1 + 2 \cdot x \cdot \tan(\alpha) \leq 0.95 \cdot B \tag{8.5}$$

- Constraint 6: The water of the ART cannot slosh with the ceiling of the lateral reservoirs [1].

$$2y - h \leq x \tag{8.6}$$

- Constraint 7: The highest part of the ART cannot be upper that the main deck. Defining the main deck at 'D' meters upper from the water level.

$$x + |m' + R' - \%_{IR}| \cdot B \leq 1.4 \cdot D \tag{8.7}$$

The seeds are

Table 8.4: Seed for the algorithm of optimization.

Parameter	Classification	Description
l	DP	Length of the u-shaped anti-roll tank; m
h	DP	Height of the connecting circuit between the vertical reservoirs; m
w	DP	Length of the connecting circuit; m
w_1	DP	Width of the lateral reservoirs of the ART at its base; m
y	DP	Height of water in equilibrium of both reservoir columns; m
α	DP	External wall angle of inclination of the vertical reservoir columns, rad
R'	DP	Distance between the center of gravity of the system and the bottom of the ART, m.
x	DP	Overall height of the lateral reservoirs, m.

Where DP means Design Parameters and represents all geometrical characteristics of the ART.

The seed values and the associated limits were:

Table 8.5: Inputs to the optimization algorithm.

	l	h	w	w_1	y	α	R'	x
Lower bound	0.5	0.3	5	0.5	1.2	0.0001	0	2.5
Seed	5	0.5	7.5	2	2	0.05	5	3.5
Upper bound	6	1.2	12	2.8	4	0.17	8	5

8.5. Results

It is presented as results:

- The selected design.
- The analysis of response at operation for each sea state (maneuvering).

8.5.1. ART Optimum design

The obtained shape of the ART is presented in figure 8.4. Its dimensions are specified in table 8.6.

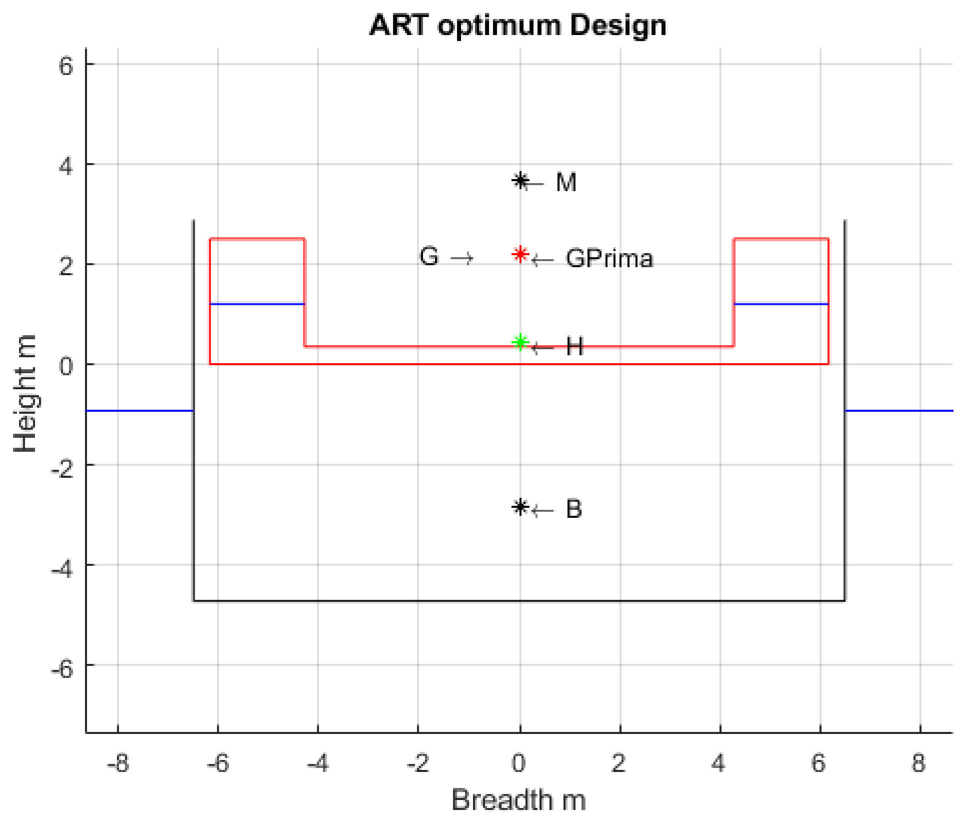


Figure 8.4: Optimum ART design for OPV operating in ETSP location

Table 8.6: Parameter values of the optimum design

Property	Lower bound	Optimum	Upper bound
l	0.5	3.62	6
h	0.3	0.35	1.2
w	5	8.55	12
w_1	0.5	1.9	2.8
y	1.2	1.200000241	4
α	0.0001	0.000104076	0.17
R'	0	2.18	8
x	2.5	2.500000271	6

With a total reduction of **20.2%** for a ponderation of all sea states by its probability of occurrence.

8.5.2. Responses of the system for different scenarios and manouvering

In figures 8.6 to 8.9 it is shown how the ART reduction of rolling behaves as function of the direction, α, of the ship with respect to the waves and velocity, V_S (See figure 8.5 to understand the reference system).

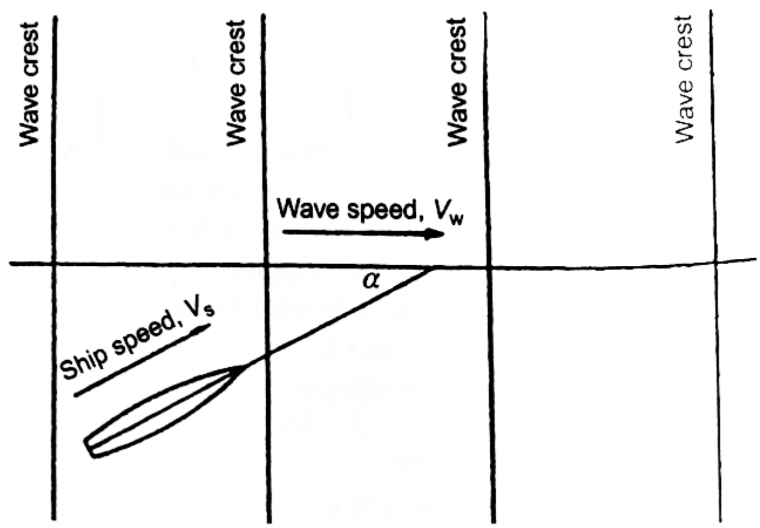


Figure 8.5: Angle of direction of a ship (**Tupper**[8]).

This difference between an idle ship and a ship in operation at a given velocity is that, when moving, the frequency of excitations of the ship is no more equal to the frequency of the waves of the ocean. Instead, the frequency of excitation of the ship depends on the frequency of the waves, the ship's velocity, and the direction (heading). The resulting frequency of excitation of the ship is called the "frequency of encounter". It is described by the equation

$$\omega_e = \omega \left(1 - \frac{\omega V_s \cos \alpha}{g} \right) \tag{8.8}$$

where V_S is the ship velocity in $\frac{m}{s}$, α the angle of direction . As the frequency changes, so do the modal period of the wave, as the frequency is the inverse of the period. Then, for each modal period can be calculated a PSD in the equation 6.1, changing the RMS of the response for every direction of the wave and the velocity of the ship. As it is shown in figure D.1 and 8.5.

It is explained through, how to perform this directional analysis with the proposed procedure.

- Calculate the modal period related to each angle of direction at a particular velocity and a particular mean wave height.

$$T_e = \frac{2\pi}{\omega_e} = \frac{T_w}{1 - \frac{\omega V_s \cos\alpha}{g}} \tag{8.9}$$

$$H_e = 0.5H_{\frac{1}{3}} + 0.5H_{\frac{1}{3}} \cos\left(\cos(\pi \cdot \alpha)\right) \tag{8.10}$$

- Calculate the PSD of the wave slope with that modal period.

$$S_e = S_{ITTC}(T_e, H_e) \cdot \left(1 - \frac{2\omega V_s \cos\alpha}{g}\right)^{-1} \tag{8.11}$$

$$S_{slope} = \frac{\omega_e^4}{g^2} S_e \tag{8.12}$$

- Calculate the response in RMS for that particular angle using the tuning factor of $S_{slope}(\alpha, V_S, SeaState)$ into the filter. Where $H_{\frac{1}{3}}$ is defined by the sea state.

- Plot responses for each angle.

So, there will have to be fitted as many filters as samples in the directional angle at one velocity at a certain mean sea state.

The most common velocities of operation for an OPV are low speeds or idle (0 to 6 knots).

From the figures 8.6 to 8.9 it is observed twofold phenomena. First, the better performance of the reduction occurs during operation at the most probable sea states. This is seen in figures 8.7, 8.8 and E.6; respectively representing operation at sea states 5,6 and 4. The second phenomenon is that for every case, from sea state 1 to 9, the better performance of the reduction is when the wave incidence comes diagonally onto the ship, as well as when the incidence is at beam seas (from the side of the ship). On the other hand, the reduction decreases when the ship is heading directly onto the waves or when it is following the waves.

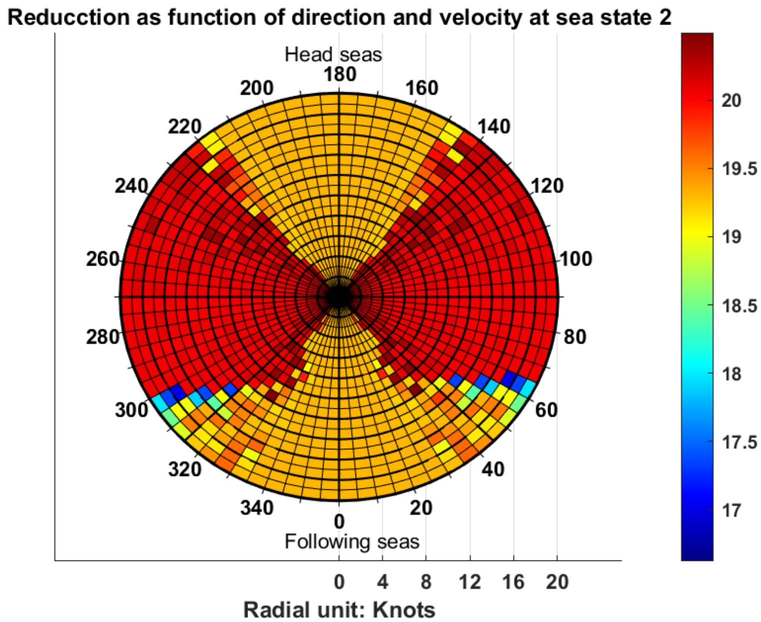


Figure 8.6: Reduction during operation at sea state 2.

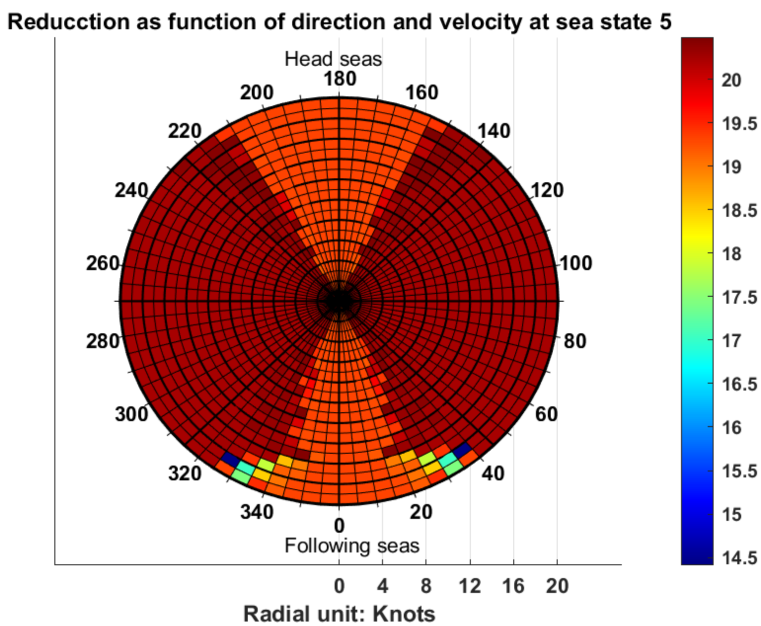


Figure 8.7: Reduction during operation at sea state 5.

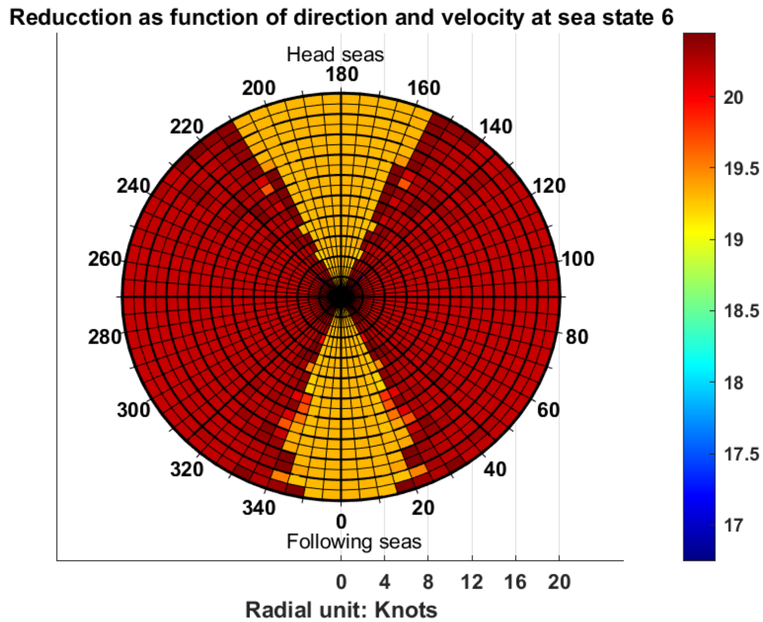


Figure 8.8: Reduction during operation at sea state 6.

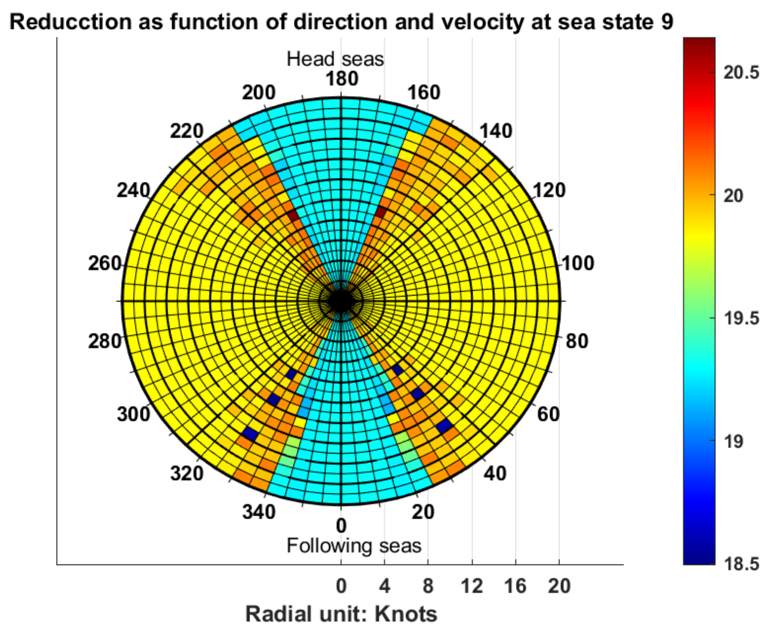
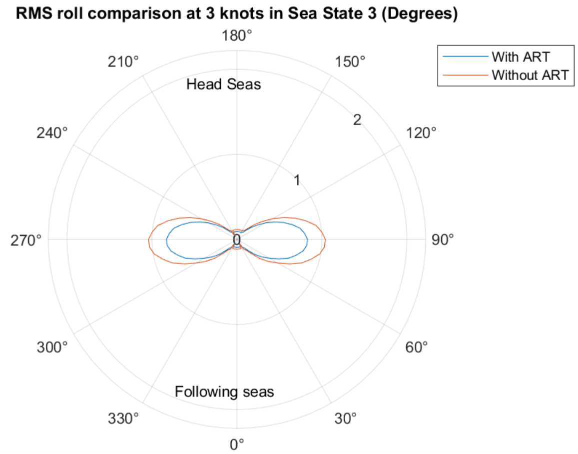
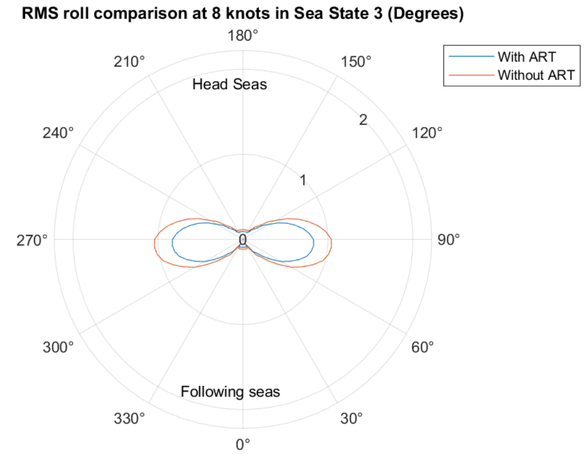


Figure 8.9: Reduction during operation at sea state 9.

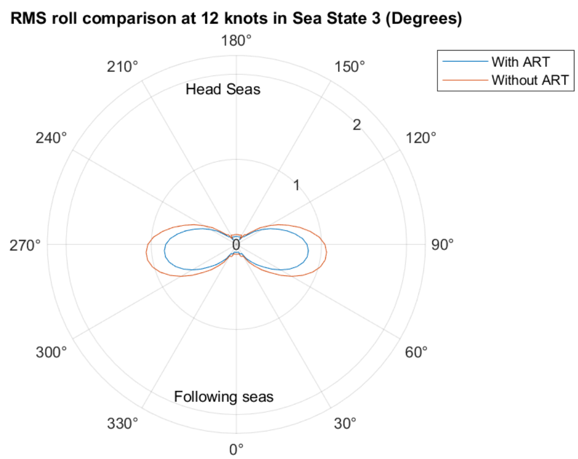
From figures 8.10 to 8.12 it can be seen twofold phenomena as well. First, for every sea state, the maximum RMS of the roll of the ship at low velocities is when the incidence of the waves comes at beam seas. As the velocity increases, the maximum RMS of the roll of the ship shifts to the stern, where the wave incidence comes diagonally from the back. The second phenomenon is that in some cases of operation, the RMS of the ship roll increases rapidly and then falls sharply. This phenomenon occurs only when the ship is following the waves where $\alpha \in (0, 90)$ and $\alpha \in (270, 360)$. In the discussion, those phenomena of figures 8.6 to 8.12 are interpreted.



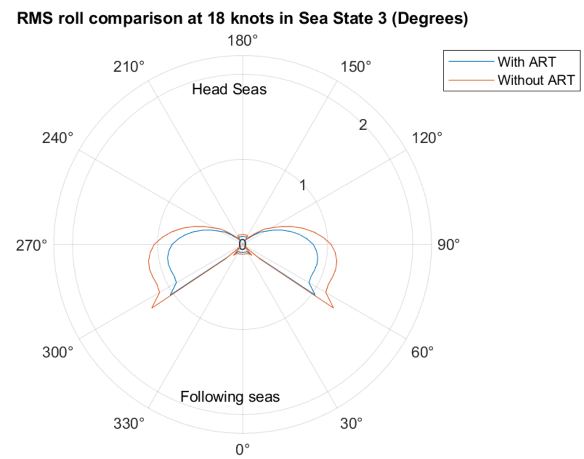
(a) Velocity: 3 Knots.



(b) Velocity: 8 Knots.

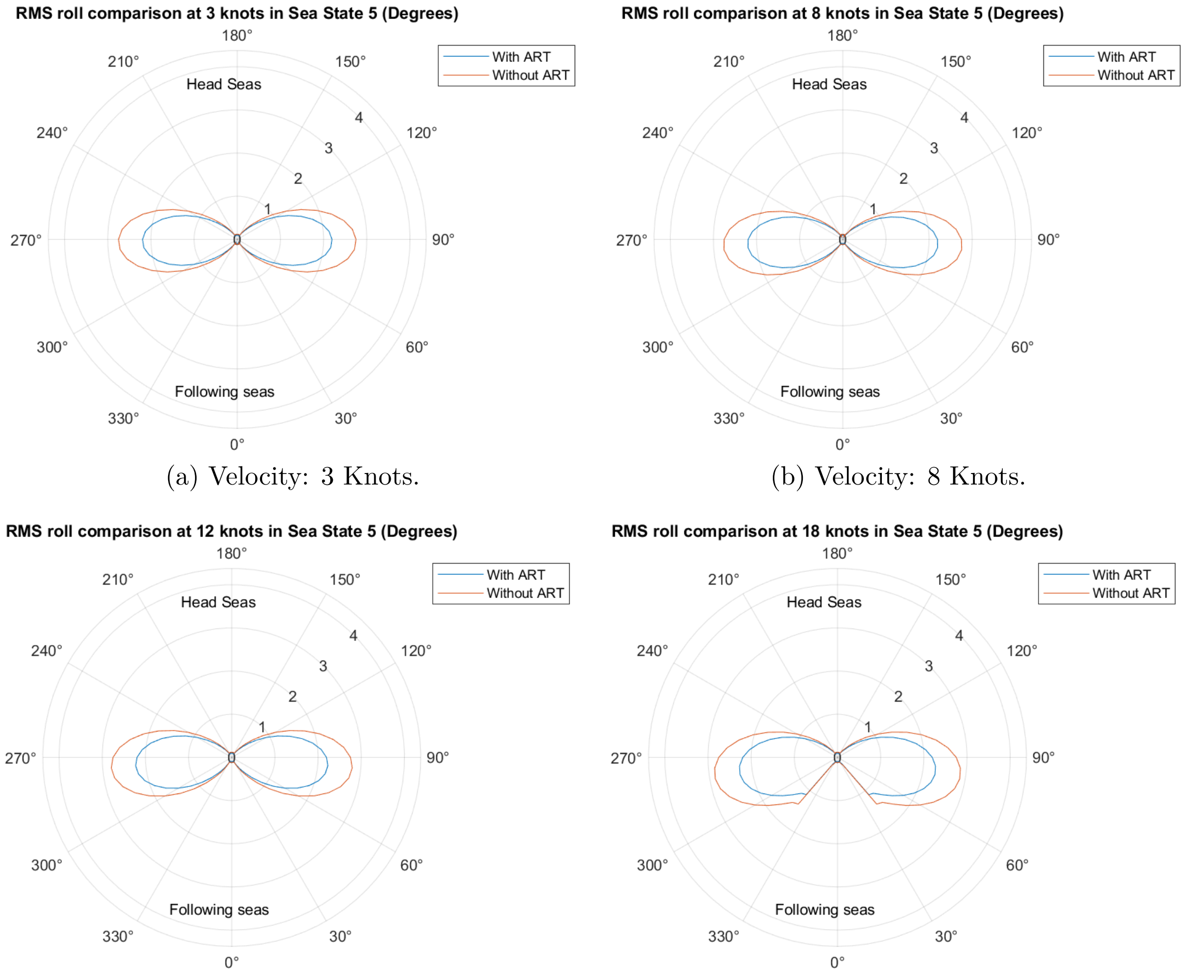


(c) Velocity: 12 Knots.



(d) Velocity: 18 Knots.

Figure 8.10: Comparison of performance of the OPV with the designed ART and without it operating at sea state 3.



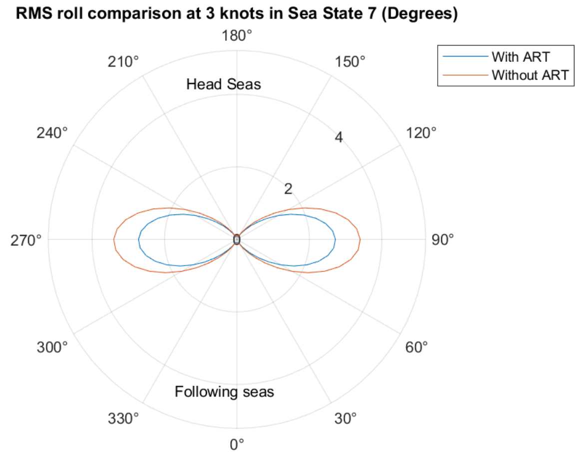
(a) Velocity: 3 Knots.

(b) Velocity: 8 Knots.

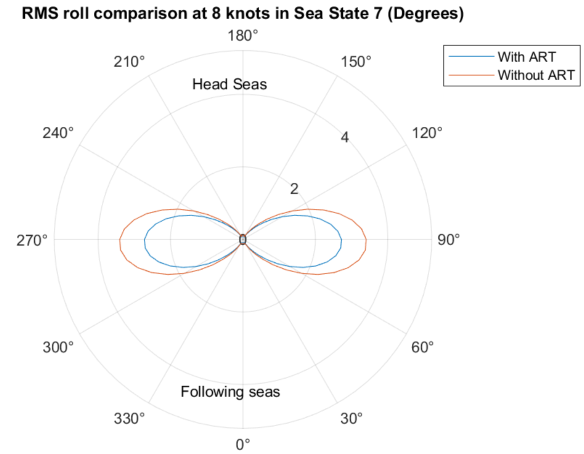
(c) Velocity: 12 Knots.

(d) Velocity: 18 Knots.

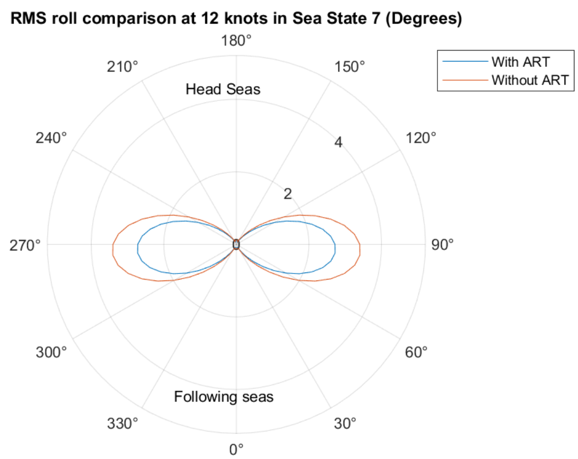
Figure 8.11: Comparison of performance of the OPV with the designed ART and without it operating at sea state 5.



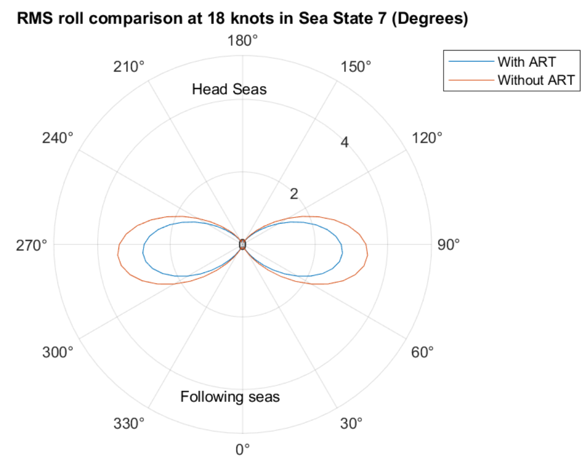
(a) Velocity: 3 Knots.



(b) Velocity: 8 Knots.



(c) Velocity: 12 Knots.



(d) Velocity: 18 Knots.

Figure 8.12: Comparison of performance of the OPV with the designed ART and without it operating at sea state 7.

Chapter 9

Analysis and Discussion

The natural frequency of the obtained optimum design of the ART is almost the same natural frequency of the ship system. Complying $f = \frac{\omega_s}{\omega_t} = 0.97$ which is close to 1. This means that for any excitation, the ART tends to be in 180 de-phase with the OPV's roll. This condition, as mentioned in chapter 4, is artificially imposed in both optimization methods of [1] and [3]. But in this particular optimization, it was not imposed. Instead, the optimization obtains that the best design of ART complies with the mathematic condition that dictates that the optimum stabilization is obtained when the maximum momenta is exerted from the tank to stabilize the ship. This occurs when the ship and the ART are in "180 phase"; and this is most likely to happen when $f \approx 1$ [1].

According to constraint 7 (8.7), R' tends to be the minimum as possible. This means that the upper the ART is placed, the better the performance of roll reduction. This phenomenon is sustained in the results of [1], allowing to infer that the optimization of this study is well implemented.

According to constraint 5 (8.5), B tends to be the maximum permissible width occupying 95% of the ship breadth. This means that the wider the ART the more righting lever moment is exerted by the ART to stabilize the ship. This phenomenon is also sustained in the results of [1], allowing newly to infer that the optimization of this study is well implemented.

The optimum design does not occupy the maximum permissible mass. It uses $\approx 2\%$ of the mass of the ship, instead of 6%. Meaning that the process not only optimizes the reduction of rolling but minimizes the occupied space of the ART, allowing the designer to use space for other substantial purposes in the seakeeping of the ship. This phenomenon implies that the length of the ART (l) does not necessarily has to be the highest possible value, as is the present case. Furthermore, the energy that produces the rolling absorption comes from the energy of the waves[3], so having the ART produces advantages, 20% of reduction, with the only drawback of the occupied space of the ART. Then the ART allows the stabilization of the OPV for freely obtained energy.

Both the height of the reservoir of the ART (x) and the height of the column of water (y) tend to be as close to the minimum permissible value (See lower bounds of x and y in table 8.6). This means that those parameters are not in a local optimum but in the better possibility allowed by the constraint set. In particular, x is limited by the vertical position of the ART, constraint 7 (8.7). As R' tends to minimize to find an optimum reduction, it forces x to be lower. For constraint 6 (8.6) of prevention of sloshing of the water with the ceiling of the reservoirs, y is forced to be lower. Then both values reach almost the lowest bounds. This means that R' has much more influence than x and y on the maximization of the reduction of roll.

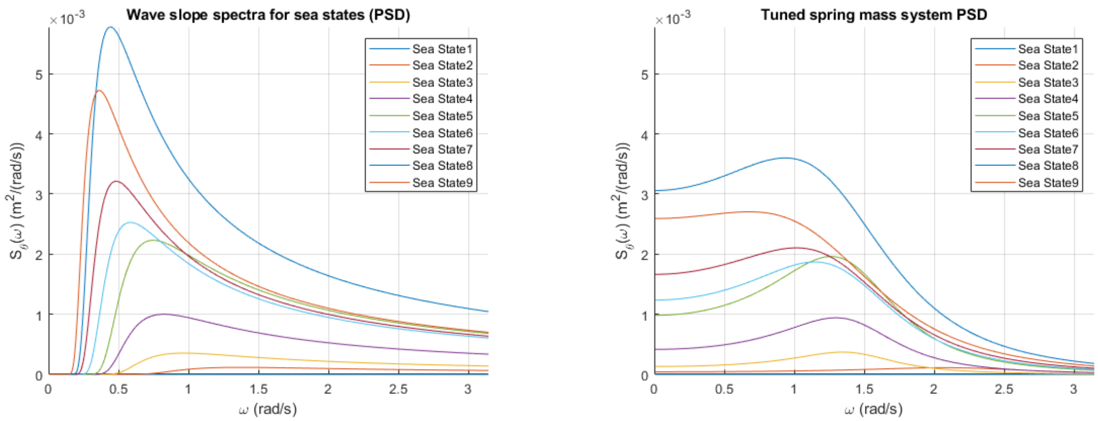
It was mentioned in the results that from figures, E.6, 8.7 and 8.8 can be seen two principal phenomena, among others. The first one is that the better performance of the reduction occurs during operation at sea states 4, 5, and 6. This is because the optimization is set to ponderate the roll reduction by the probability of occurrence of the sea state of operation. Thus, the better performance of the ART has to be during operation in those sea states most likely to occur, which are sea states 4,5, and 6. In the opposite case, the sea state less likely to occur are the ones with the lower performance, as can be seen for sea states 2 and 9. Refer to appendix E to appreciate how variates the performance of reduction for each sea state.

The second phenomenon that can be seen from figures 8.10 to 8.12 is that the maximum RMS of the roll of the ship at low velocities is when the incidence of the waves comes at beam seas. As the velocity increases, the maximum RMS of the roll of the ship shifts to the stern, where the wave incidence comes diagonally from the back. This occurs given that the change of velocities and direction affects directly the period of encounter and the mean significant wave height. These are the two parameters that define the power spectral density of Bretscheneider 6.1. Thus for each direction, for velocity, and for each sea state the power spectral density changes causing the shift of the roll RMS to the back of the ship.

The process that adds the biggest bias to the complete procedure, is the fitting of the tune parameters ω_f, ζ_f, and S_f. In this, the fitting was made for the most probable modal wave periods of each sea state. It can be seen that figure 9.1.a differs in shape and amplitude compared to figure 9.1.b. This difference is special for the cases of sea states 7, 8, and 9; see figure 6.5.g, 6.5.h, and 6.5.i.

The more accurate the filter is, the more accurate will be the final quantification of the roll. To address the issue of representation of the ocean, it should be executed another kind of filter, like the "Pierson-Moskowits" and the "Kanai-Tajimi". However, in the obtained representation of the ocean for cases 1 to 6, the tunning of the filter represents the ocean with more degrees of reliability. Knowing that the most probable sea states, as said by the Rayleigh distribution in table 8.3, are the 3, 4, 5, and 6 sea states; and that the reduction of rolling is ponderated for the probabilities of occurrences of the sea states Then it can be said that the overall process does not propagate severe biases.

It can be seen in figure 9.1 that this bias diminishes the reliability of the representation of the ocean for sea states lower than 7.



(a) Wave slope PSD for most probable modal wave period and mean heights of the waves

(b) Tuned wave slope PSD of the original Wave slope PSD

Figure 9.1: Comparison between original and tuned PSD made for the filter input.

Once the filter is done, and assuming that the model has been correctly implemented (which was proved in the validation), then the final response of quantification of rolling is reliable due to the versatility of the Lyapunov solution. it gives an analytic solution to a complex integral, fastly, and highly accurately.

If some kind of low-energy-cost control is wanted to be implemented there are two possibilities. The first one is to control the damping parameter, which is the same as controlling the energy absorption of the ART. The second one is to control the rigidity of the system, which is the same as controlling the natural frequency of the ART. Viewed in the equation 4.10 each kind of control varies C_t and K_t respectively.

$$M_s\ddot{\varphi} + C_s\dot{\varphi} + K_s\varphi + M_{s,t}\ddot{\psi} + K_{s,t}\psi = m_\theta \tag{9.1}$$

$$M_{s,t}\ddot{\varphi} + K_{s,t}\varphi + M_t\ddot{\psi} + C_t\dot{\psi} + K_t\psi = 0 \tag{9.2}$$

This kind of control is called semi-passive because it changes the characteristics of the system, it does not exert an external force. To understand how these kinds of control are physically applied refer to appendix F

Chapter 10

Conclusion

The analysis of the optimization method led to conclude that the natural frequency of the ART has to be almost the same as the natural frequency of the Ship; which is the same conclusion that came up with **Stigter**[1] study.

The optimization process leads to conclude that the maximization of roll reduction is not necessarily related to the increment of the total mass of the Antiroll Tank. In this case, the optimum occupies 2% of a maximum permissible 6% of the ship.

The proposed method of optimization leads to infer that an optimum ART has to be in the highest possible position; as well as to occupy the maximum permissible width of the total breadth of the ship. Both results are inferred in **Stigter**[1] too. Furthermore, the level of the water, the height of the reservoirs, and the length of the ART have less influence on the selection of the optimum ART dimensions than the vertical position of the ART (with respect to the ship) and the width of the ART.

The roll of a ship depends not only on the characteristics of the Ship+ART system but in the mode of operation of the ship. Depends on the state of the sea, the velocity of the ship, and the direction of maneuvering. This study leads to infer that, if the optimization of the ART considers operation at all sea states, and if those sea states are ponderated by their probabilities of occurrence, then the best performance of the ART will be at the most probable sea state.

Given that the energy that produces the rolling absorption comes from the energy of the waves [3], then is convenient to place the ART in the OPV. It ensures a 20% reduction without using a considerable amount of space. If for some reason, it is required to ensure an amount of reduction greater than 20%, then it would have to be applied control system over the ART, or simply use another different kind of stabilizer.

Points of further formulation:

It is left for further work to analyze the influence of the other angular DoF (yaw and pitch) on the Ship rolling response.

It is left for further work to perform an uncertainty and bias quantification in the process of "Filter Tuning", and analyze how the biases propagate to the final result of roll reduction.

It is left to further work to perform an analysis of uncertainty quantification over the complete process for unceratin mass, metacentric height, and radius of gyration. Performing quantification of errors and biases of the reduction of roll motion. Thus it would be possible to quantify the reliability of the ART design and predict in which scenarios it could possibly fail to accomplish the goal of stabilizing the ship.

It is left to further work on the possibility to control the parameters of the ART: damping (χ_t) and stiffness (μ_t) coefficients. Controlling the highlighted parameters in equation 9.1. This kind of passive control regulates the ART characteristics as a function of the circumstances such as shipload and sea states of operation. Which are uncertain or difficult to predict.

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ANNEXES

Annex A: Parameters and keywords of this study

A.1. Parameters

A.1.1. Design parameters for the ship + U-shaped ART system

1. l: length of the u-shaped anti-roll tank (along the longitudinal axis); m.
2. h: height of the connecting circuit between the vertical reservoirs; m.
3. y: height of water in equilibrium of both reservoir columns; m.
4. w: length of the connecting circuit; m.
5. w_1: width of the tanks at its base; m.
6. α: angle of inclination of the vertical reservoir columns, rad.
7. P: mass of the ship, kg.
8. m': distance between the metacenter and the center of gravity of the ship plus tank system, m.
9. B: breadth of the ship, m.
10. $\%_{IR}$: Percentage: ij/B: the quotient of the radius of inertia (ij) and breadth of the ship
11. χ_s: Dimensionless parameter of the linear damping of the ship roll, is related to the energy dissipation.
12. χ_t: Dimensionless parameter of the linear damping of the tank, is related to the energy dissipation at the ART.
13. R': Distance between the centre of gravity of the system and the bottom conduct that connects the lateral reservoirs of the ART.

Annex B: Frequency Response Function and Response Amplitude Operators fundamentals

Both FRF and RAOs represent the same phenomenon, it describes the response of a dynamical system to different kind of external excitations.

Formulating a dynamic vibrations problem can be done by the usage of what is called a transfer function. Such tool allows seeing an engineering problem as a "cause" followed by a "consequence" so that the relation between both of them is described by the "transfer function". A common representation of a transfer function is shown in figure B.1.

Figure B.1: Transfer function representation

where an external excitation (input) affects a systems that respond with an output. That output is called the Frequency Response Function (FRF) and can describe different kinds of phenomena.

Mathematically the FRF is represented by

$$\boldsymbol{X} = \boldsymbol{H}\ \boldsymbol{F} \tag{B.1}$$

Where F represents the input "F" to the system, "X" represents the response (output) and "H" represents the transfer function.

Having as an excitation any force exerted upon a system as a simple harmonic motion, H can be easily obtained.

The specific case of the ship's rolling response is represented in figure B.2:

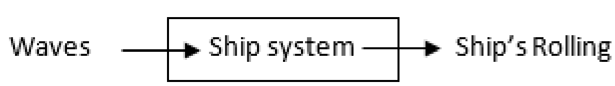

Figure B.2: Transfer function representation for the case of rolling of a ship

Annex C: U-shaped ART basic schematics

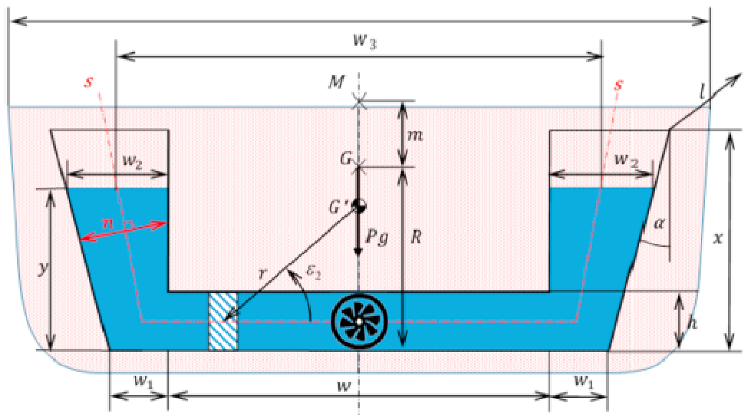

Figure C.1: Basic dimensions of ship and U-tank. Intellectual property of **Alujevic**[3].

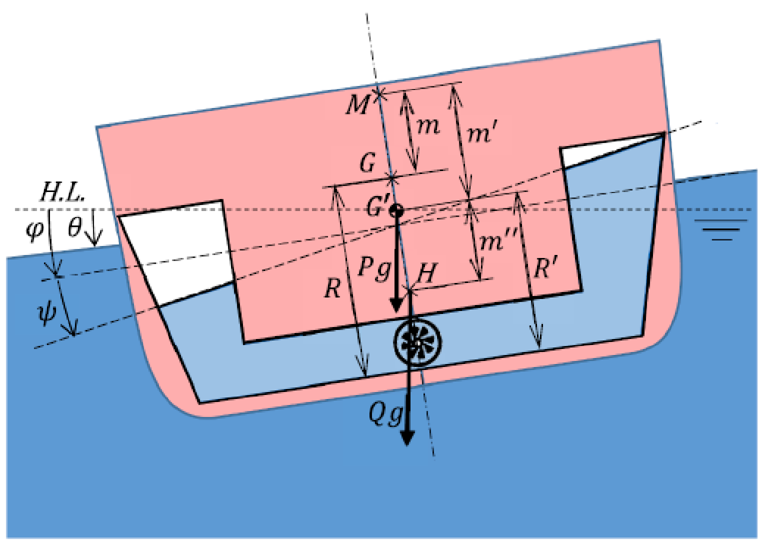

Figure C.2: The scheme of a ship equipped with an anti-roll tank. Intellectual property of **Alujevic**[3].

Annex D: Convention of ship maneuvering reference system

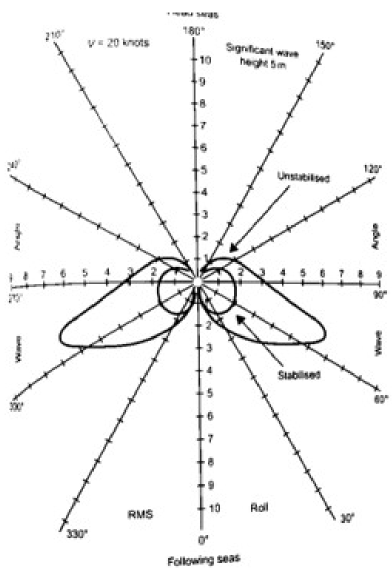

Figure D.1: Directional behavior at defined velocity and defined significant wave height [8].

Annex E: Roll reduction of the OPV for all sea states

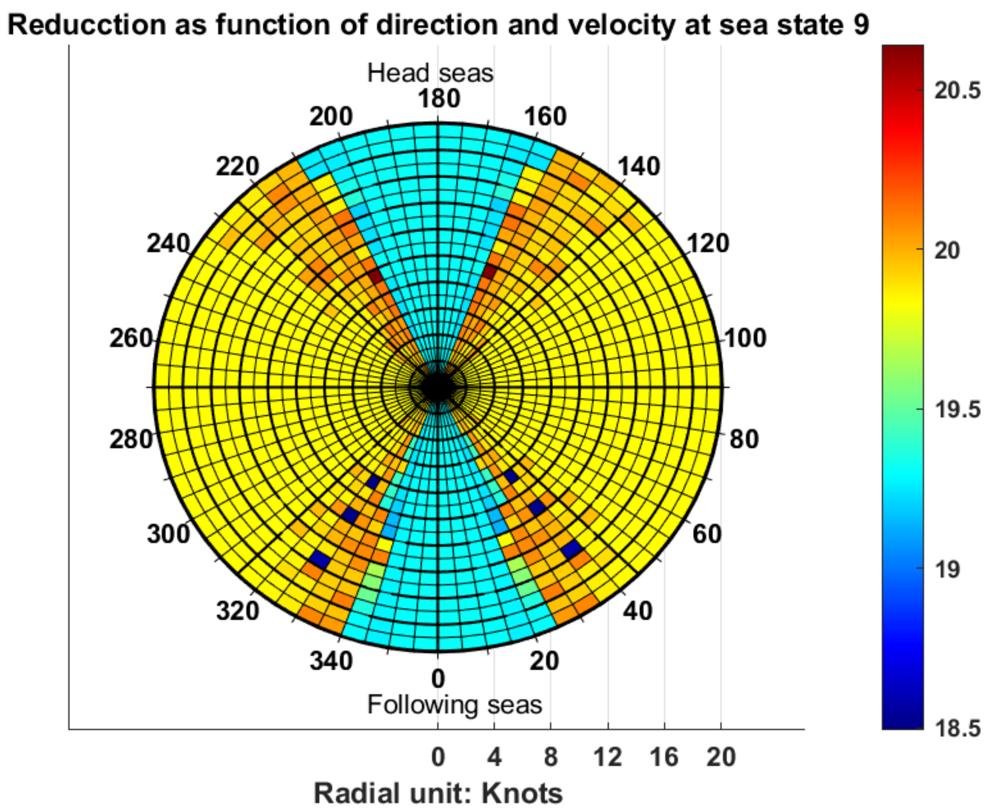

Figure E.1: Reduction during operation at sea state 9.

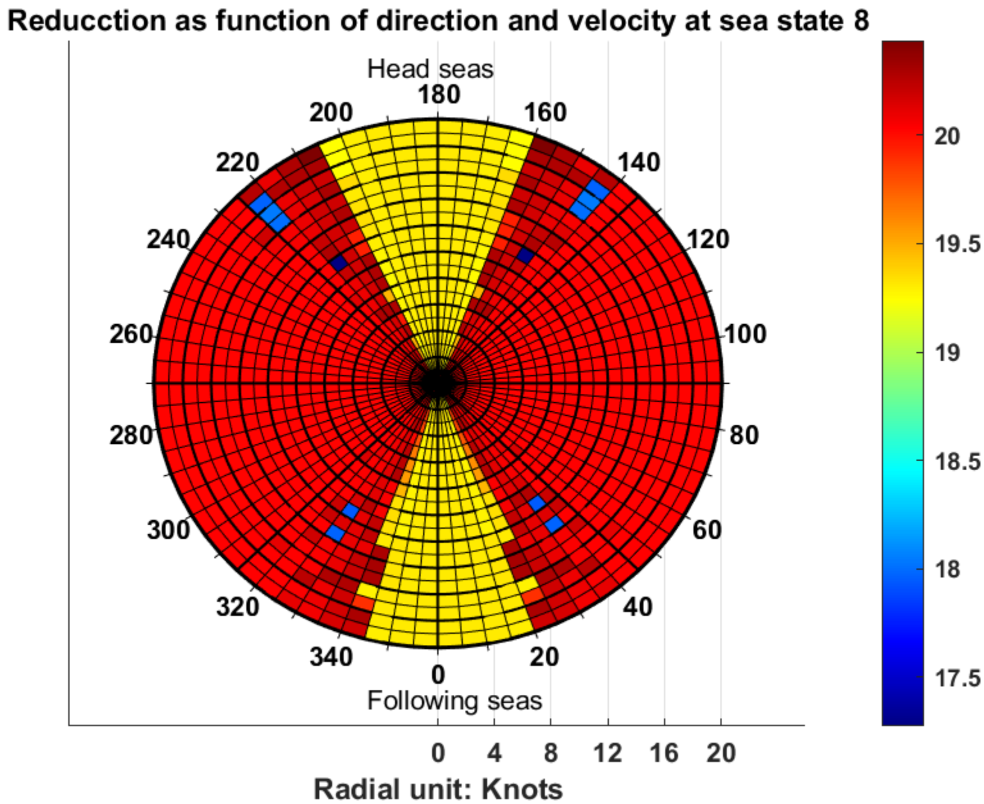

Figure E.2: Reduction during operation at sea state 8.

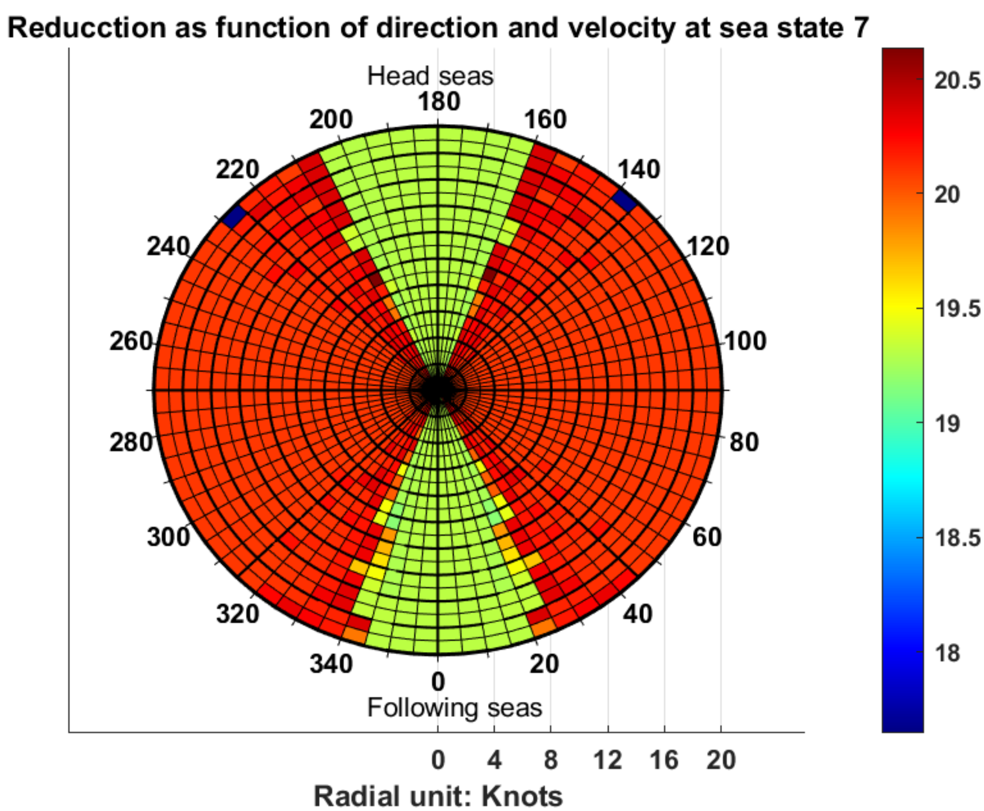

Figure E.3: Reduction during operation at sea state 7.

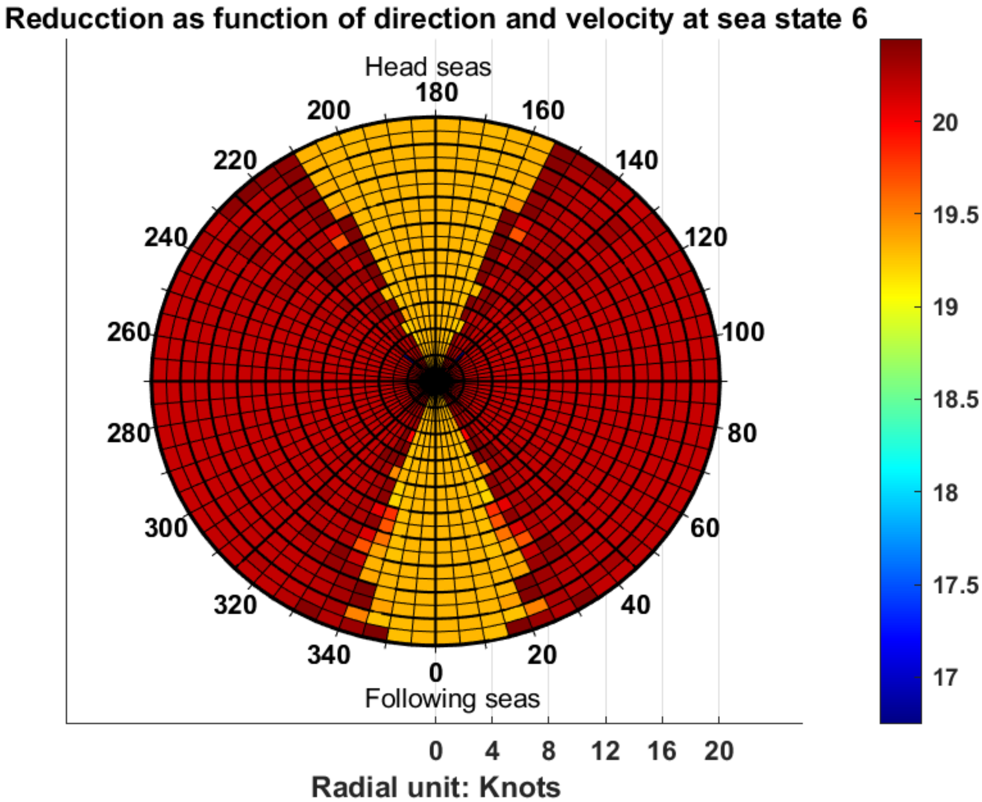

Figure E.4: Reduction during operation at sea state 6.

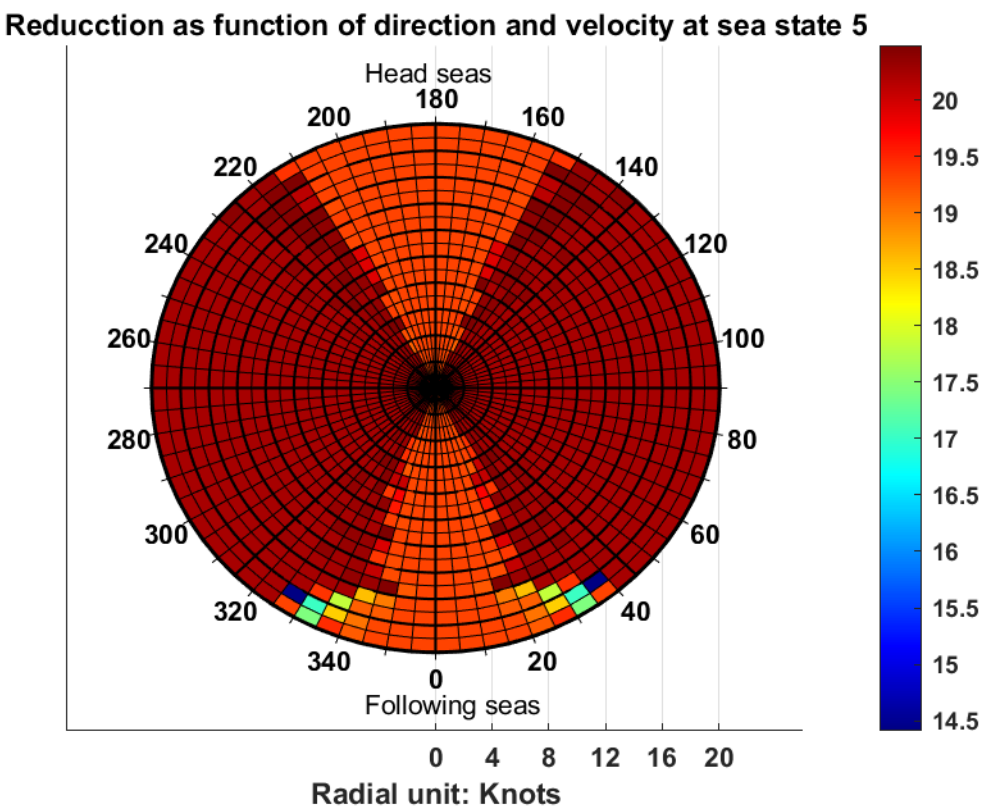

Figure E.5: Reduction during operation at sea state 5.

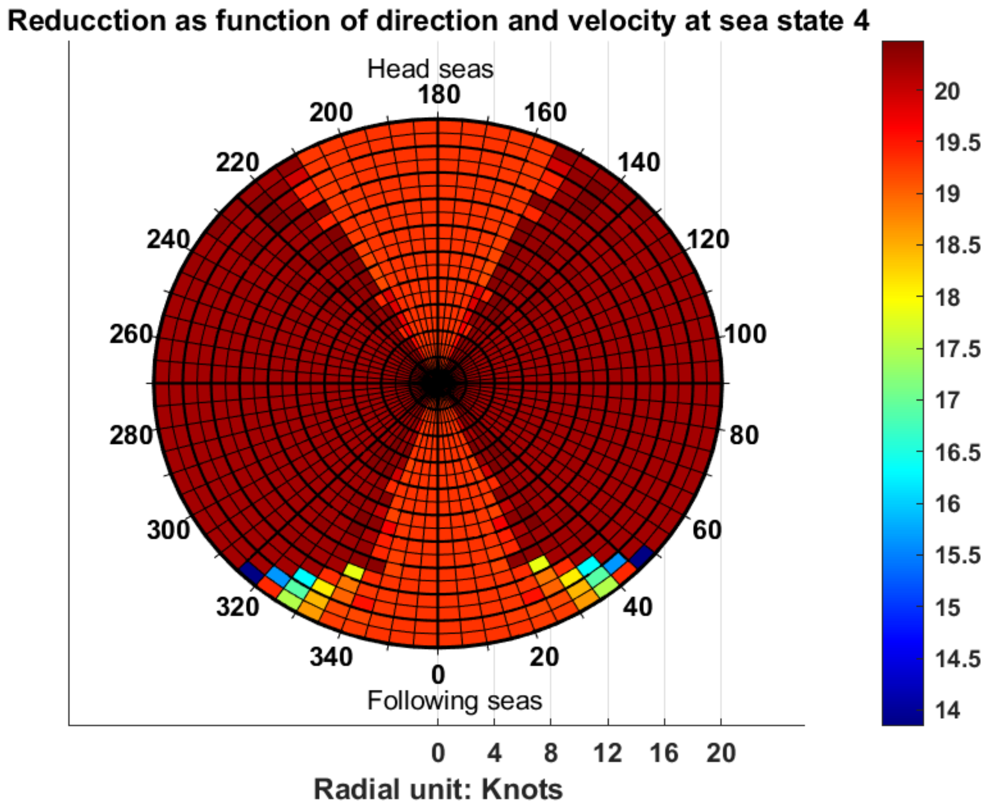

Figure E.6: Reduction during operation at sea state 4.

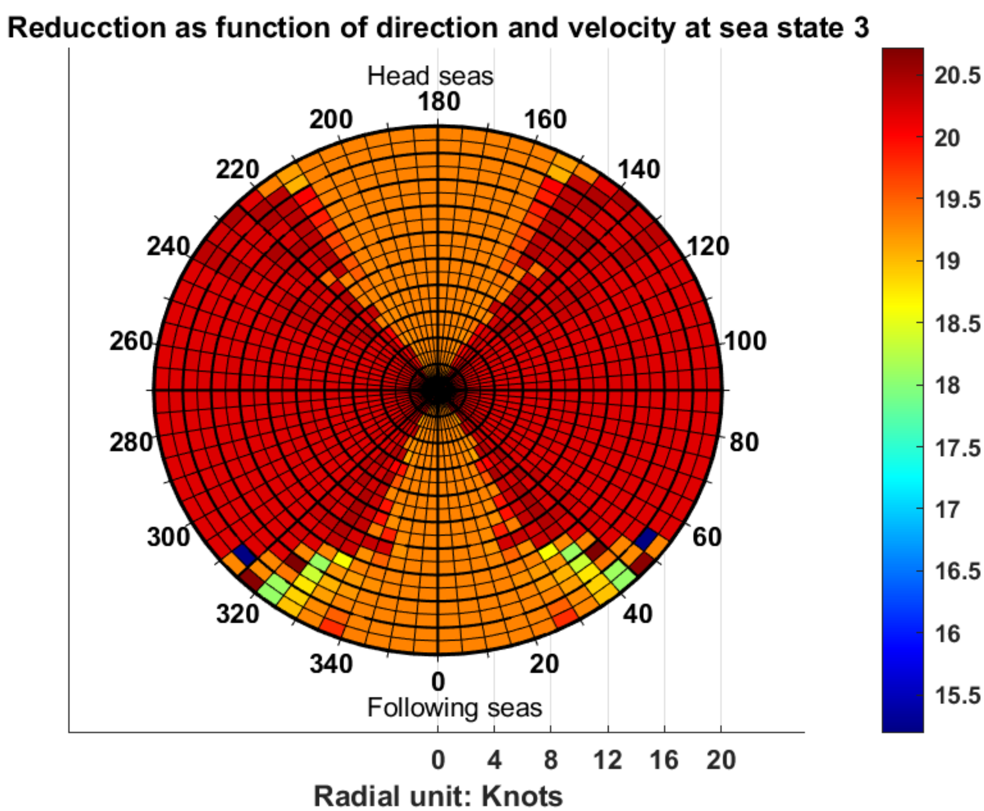

Figure E.7: Reduction during operation at sea state 3.

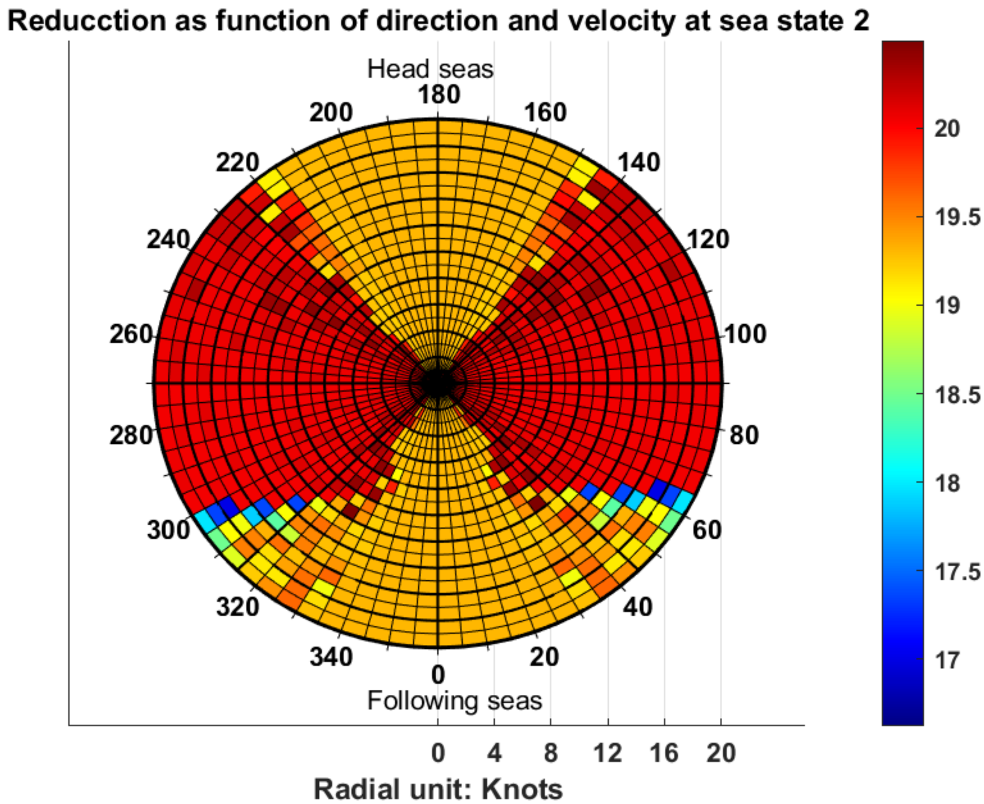

Figure E.8: Reduction during operation at sea state 2.

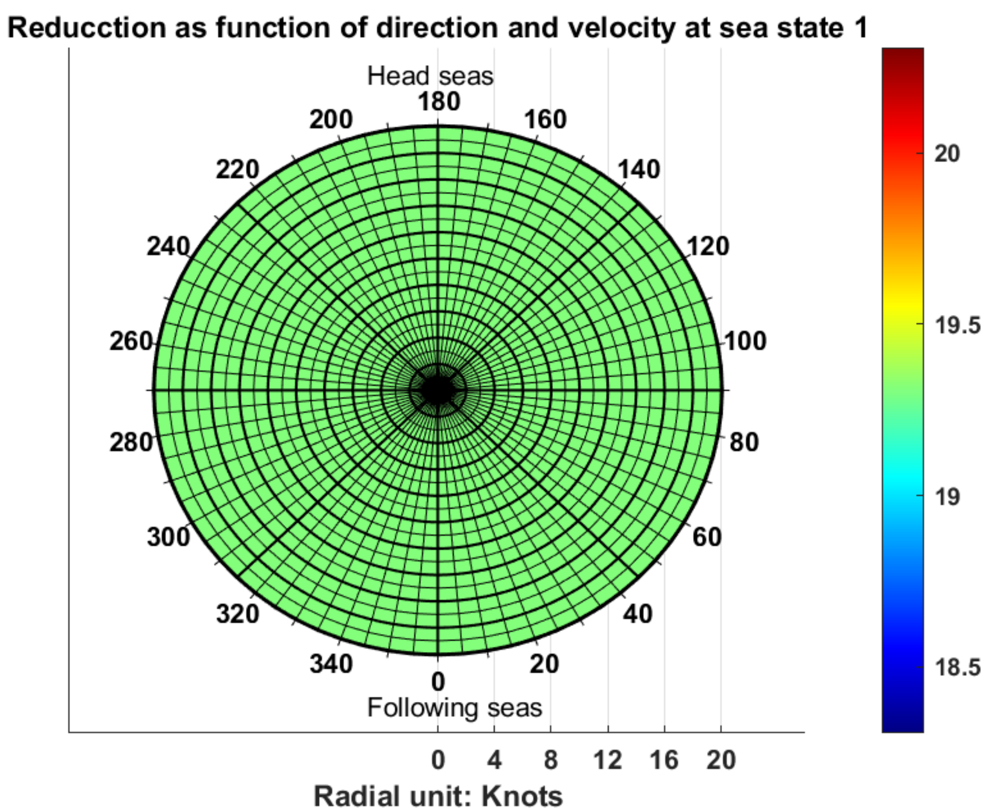

Figure E.9: Reduction during operation at sea state 1.

Annex F: The U-shaped Anti-roll tank stabilisation phenomena

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F.1. Operation principle of a U-shaped Anti-roll tank

U-shaped tanks are anti-rolling systems that consume minimum energy to diminish the ship rolling. The sea, causing the vessel to roll, delivers the necessary energy to reduce roll.

The vessel's roll is used to cause an oscillatory counter athwart movement of the water in the U-shaped tank system. The aim to effectively cause an athwart movement is to always keep the system tuned to counteract and reduce roll.

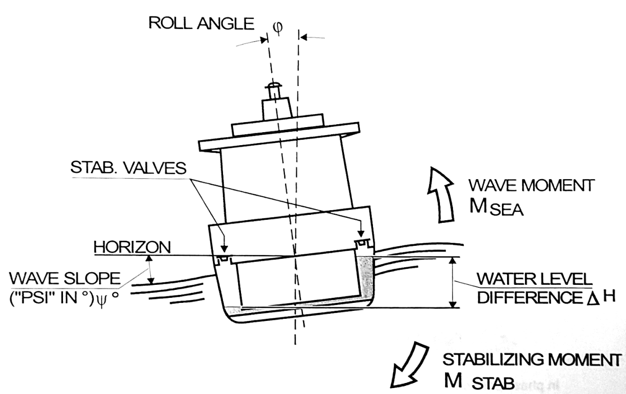


Figure F.1: Simple schematics of U-shaped ART operation principle.
Courtesy of Armada de Chile, Rolls Royce manuals

The way the movement is measured is given by the equations 4.9 and 4.10 shown in the third chapter, and it is the result of the sum of the moment exerted by the ocean onto the ship with the moment exerted by the U-shaped ART onto the ship. The are different ways to control the tuning of the U-shaped ART in order to assure that those moments will interact always against each other to reduce roll.

The most intuitive design and empirically control way used are:

- Barriers or valves in the inferior connecting duct.
- Air flow control in an upper connecting duct.

The first option is what the Stigter model represents and describes.It can control the mass of the fluid inside the U-shaped ART and the dissipation of energy through the fluid flow obstruction caused by the valves.

The second option control the mass of the fluid inside the U-shaped ART and the rigidity of the system, thus it concentrates mainly in shifting the natural frequency of the ship.

While those are two different methods to control the reduction of rolling, both works in the same way.

In a passive operational range, the principle of suppression between both ocean and ART exerted moments onto the ship works by the following sequence:

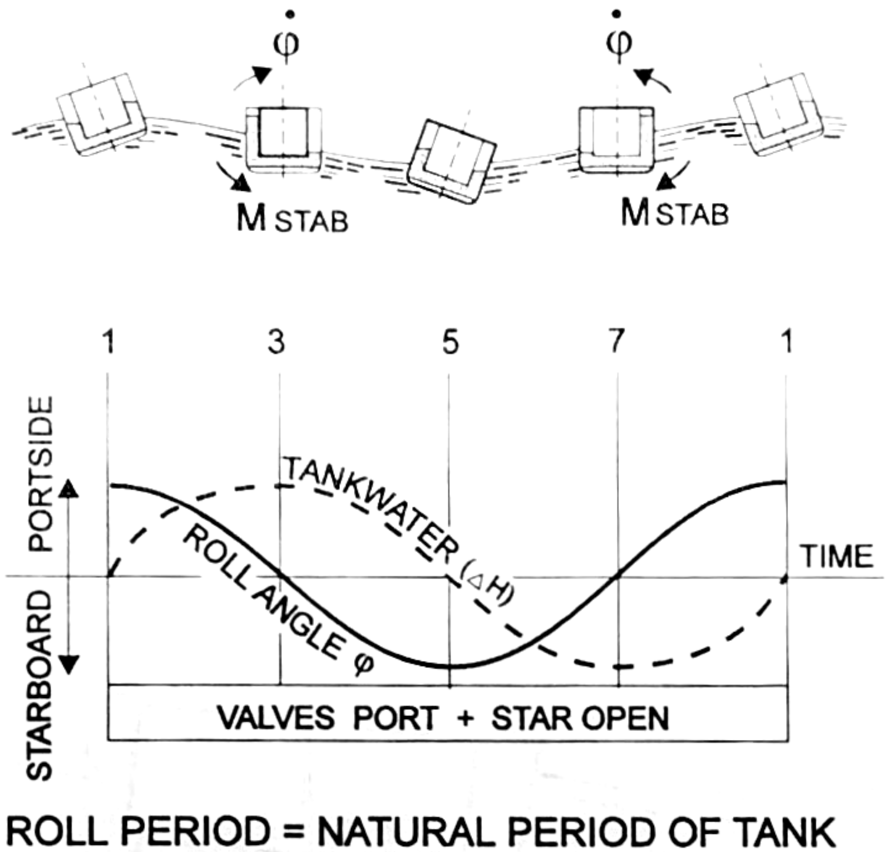


Figure F.2: Phase Cycle - Ship and tank in Anti-Resonance, non controlled.
Image from: Rolls Royce operation manual of OPV Armada de Chile.

1. In phase position 1, the ship has reached its maximum roll angle to port and the tank fluid is flowing with maximum velocity from the starboard side of the tank into the portside of the tank.The roll angle velocity $\dot{\varphi}$ at this moment is zero, the tank fluid has the same level in both side tanks and the ship to right to starboard.

2. In phase position 3, the ship is moving with maximum roll angle velocity from port to starboard, and the tank fluid has reached the maximum level on portside, and thus

acting downwards with maximum stabilising moment(M_{satb}) against the roll and wave moment (M_{sea}).

3. Between phase position 3 and 5, the ship continues to roll to starboard, and the tank fluid start to flow into the starboard tank.

4. At the phase position 5, the tank fluid is flowing with maximum velocity from portaside tank into the starboard tank.

5. The ship has reached its maximum roll angle to starboard and start to right in order to roll at 7 phase position with maximum roll angle velocity from starboard to portside. The tank fluid has reached its maximum position in the starboard tank side and is again acting with maximum stabilising moment M_{stab} against the roll and the wave moment M_{sea} which are now acting upwards on starboard.

As soon as the ship rolls periods is slightly longer than the natural period of the tank system, due to reduced GM values or the effect of the waves, is suggested that he tank fluid should be controlled. A reference of how this could be done is shown in the image XX, in which is illustrated every step of the phenomena.

This system has to be tuned for given frequencies of the waves so that the ship rolling does not enter in resonance. For this, the schematics would be the one described in the following items:

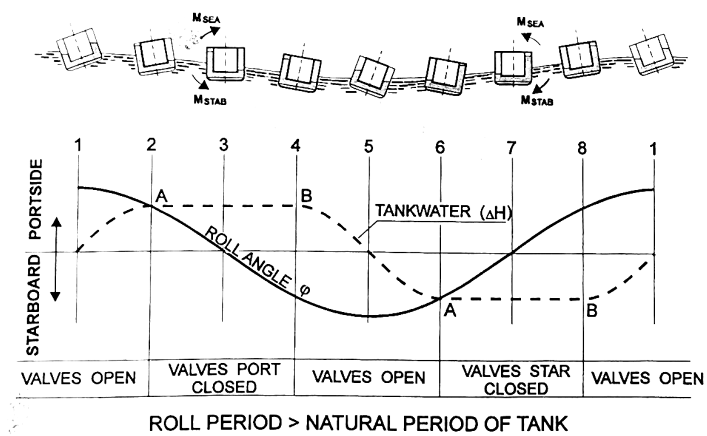


Figure F.3: Phase cycle for roll periods longer than natural period of the tank.
Image from: Rolls Royce operation manual of OPV Armada de Chile.

1. Phase position 1 correspond to the same as shown in figure XX (the last one). The ship has reached the maximum angle to port and starts to right to starbord. At this point the tank fluid is flowing with maximum velocity from starboard to portside due to the effect of gravity.

2. When at 2, the tank fluid has obtained the maximum level in the portside tank, the valves are closed by the automatic control (point A in the dashed curve).

3. At 3, the ship continues to roll to starboard and -due to closed valves- the tank water is prevented from flowing into the low side of the tank, thus creating the stabilizing

moment M_{stab}, acting against the roll motion.
The water is kept blocked in the portside tank, due to the low pressure created in the upper part of the tank, from position "2" to "4" where the automatic control gives a signal fro opening of the valves (point B on the dashed line). The air which now flows to through these opened valves into the portside tank enables the water to flow from portside (point B) into the starbord tank.

4. In phase position 5, when the ship has obtained its maximum roll angle to stardboard, the tank water is flowing with maximum velocity into the starboard side tank.

5. After phase position 5, the ship starts to right while the tank fluid continues to flow to starbord in order to reach its maximum level at "6".

6. The valves on the starboard are now closed in order to block the water in this position. The tank fluid is once more prevented from flowing back due to the closed valves and thus acts, between positions "6" and "8", against the roll motion. During this phase, the vessel is forced - as before on portside between phase position "2" and "4"- to lift up the fluid on the upwards moving ship's side, thus producing the stabilising effect.

7. At "8" the control once more -as at "4"- determines the moment to open the valves, this time on starbord, the tank fluid flows from starboard to portside and the cycle starts once more.

F.2. Variation Of dynamic response for different design parameters

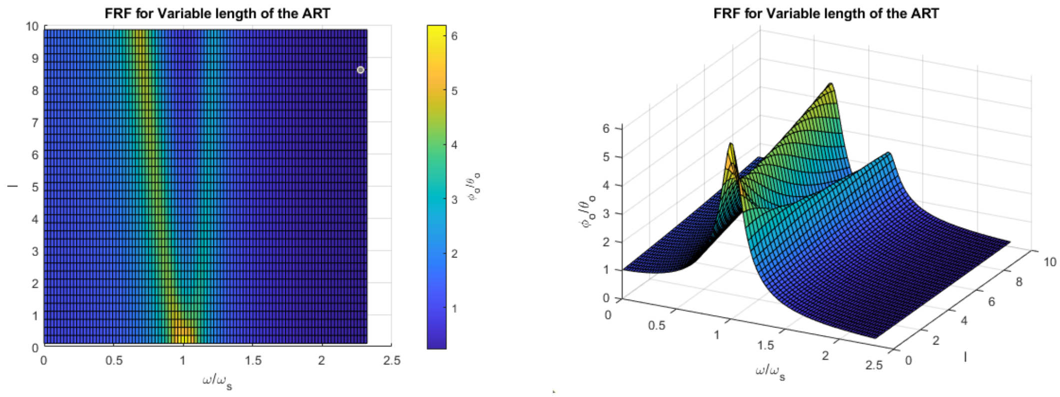
F.2.1. Design variability

The variation of the design parameters allows to compare and chose the optimum side of the ART given the ship's characteristics. The difference between these variables with the operation control variables is that the design variables will remain fixed once the design of the ship is finished.

F.2.1.1. Variability of the length of the U-shaped ART

Th most relevant analysis for the variability of a design parameter is the variation of the length of the U-Shaped ART. This defines the weight capacity of the tank to store the fluid. Thus, it defines the overall energy that can be dissipated and the power absorption that the ART performs to reduce the rolling amplitude of the ship.

It can be appreciated in the figure F.4.b that the FRF of the ship varies considerably for different lengths of the ART. In fact, what happens is that the larger the ART is, the more separated are the resonance of the rolling amplitude.



(a) Colormap indentation for variation in tank's length

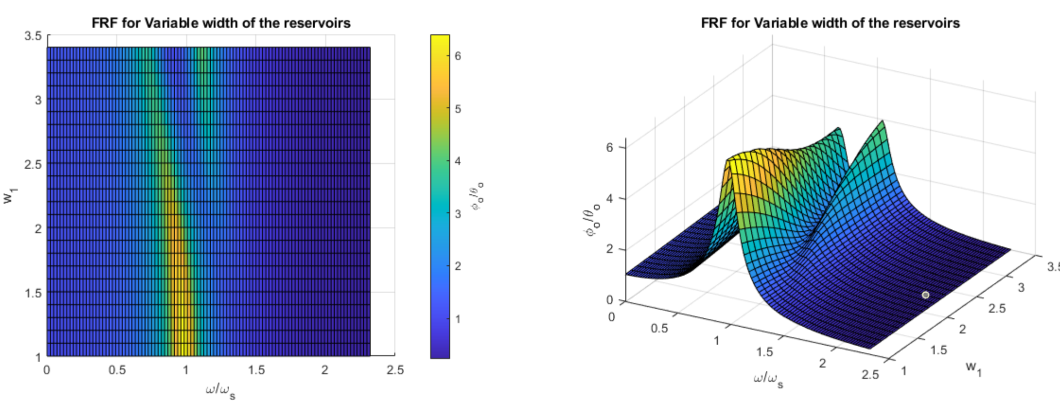
(b) Surface map of FRF behaviour for different range of length of the tank

Figure F.4: Variable length of the U-shaped ART

F.2.1.2. Variability of the width of the lateral reservoirs

Another relevant parameter that influences considerably in the design of an ART is the width of the base of the lateral reservoirs. It impacts on the quantity of flow that passes from once side to the other; thus it is strongly related to the definition of the resistance of the fluid inside the tank χ_t.

In the figure F.5 can be appreciated the behaviour of this parameter "w_1". It is observed that impacts considerably in the variation of the FRF.



(a) Colormap indentation for variation lateral reservoir width

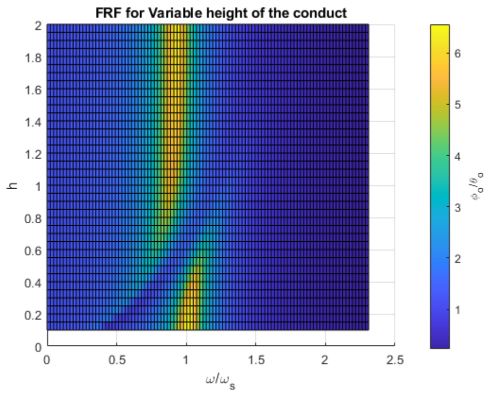
(b) Surface map of FRF behaviour for different lateral tank width

Figure F.5: Variable width of the lateral reservoirs

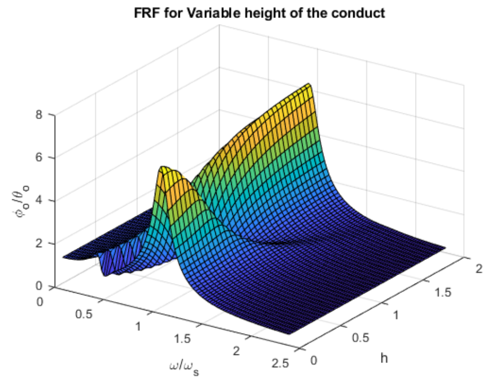
F.2.1.3. Variability of the height of the conduct between reservoirs

Similarly to the latest case, the height of the conduct that connects the lateral reservoirs impacts over the quantity of flow that passes from once side to the other. Thus, is strongly related too, to the definition of the resistance of the fluid inside the tank χ_t.

It can be observed that the change in the "h parameter influences considerably in the shape of the change of the FRF of the ship. However, its influence over the FRF behaves exactly on the contrary to the behaviour of the "w_1 parameter. This relation forces to find a pair of values that enable a proper range of the "χ_t" variable, so that the resistance that the fluid of the tank exerts on the dissipation of energy is the optimum one.



(a) Colormap indentation for variation in height of the conduct



(b) Surface map of FRF behaviour for different height of the connecting circuit

Figure F.6: Variable height of the conduct between reservoirs

F.2.2. Other operational variabilities

During operation of a ship there are intrinsic changes related to manoeuvring, propulsion and ship usage. The most important is the variation in the ship displacement given the fuel consumption. Although the ships weight may vary considerably, the change in the response does not change in such a difference. It can be appreciated that the FRF suffers a minimal change.

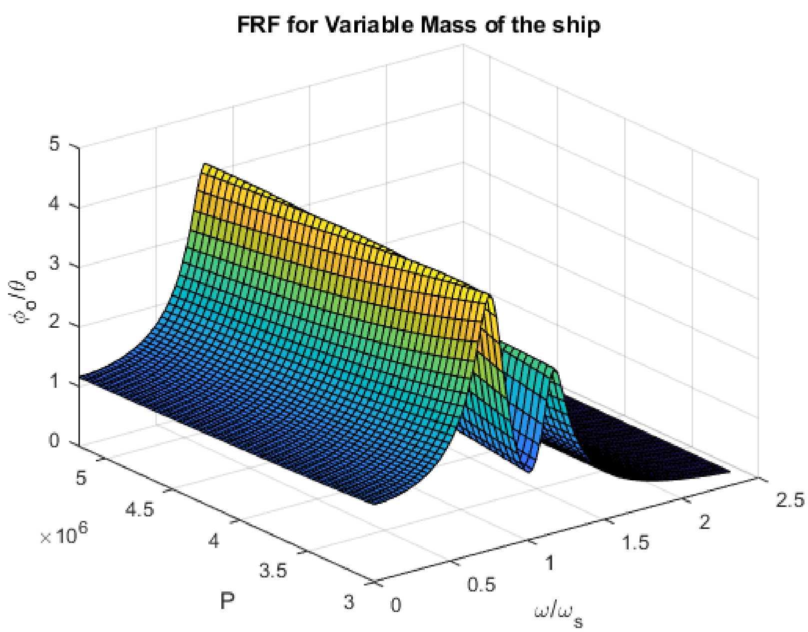


Figure F.7: Minimal variation of FRF due to ship displacement changes