# Chapter 6
# The Impact of Real-World Mathematical Modelling Problems on Students' Beliefs About the Nature of Mathematics

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**Abstract** Students' beliefs on the nature of mathematics greatly influence their interests and attitudes towards the subject. Misconceptions regarding mathematics, such as the problems always having a unique and exact answer, can become obstacles for student learning. Research has found that mathematical modelling experiences could help students see the relevance of mathematics in the real world and their lives, but more attention is needed as to whether they affect other beliefs. This study focuses on exploring high school students' views about mathematics when they work autonomously on solving real-world mathematical modelling problems during the selection process of the teams that represented Chile at the International Mathematical Modelling Challenge. The findings suggest that exposure to these modelling tasks has the potential to modify participants' beliefs, for instance, with regards to the existence of many solutions and correct procedures for mathematical problem-solving.

**Keywords** Beliefs · IMMC · Mathematical modelling · Nature of mathematics · Secondary students

## 6.1 Introduction

Students' views on the nature of mathematics shape the context in which students see and do mathematics and have a great influence on their attitudes towards learning the subject (Furinghetti & Pehkonen, 2002; Grigutsch et al., 1998; Pehkonen, 1995;

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Schoenfeld, 1989). During their time in school, they might develop several misconceptions regarding mathematics, which can include that mathematical problems always have a unique and exact answer, that there is only one correct procedure to solve them, or that it is too abstract without much relationship to their lives. These views can become obstacles for student learning since they shape, for instance, the way students approach mathematical tasks and problems. Many studies have shown that mathematical modelling experiences can help students see the relevance of mathematics in the real world and their daily life, see Stohlmann et al. (2016) for a review on this. In some sense, this is expected, given mathematical modelling has to do with situations that involve moving back and forth between the real-world and mathematics. Since different models could be constructed for the same modelling problem, which could lead to distinct solutions and results, one might ask whether mathematical modelling could also challenge or even change students' related misconceptions. In this chapter, we will present some evidence that suggests that working with real-world modelling tasks has also the potential to modify students' beliefs about the nature of mathematics and mathematical problem-solving.

Modelling has increasingly become a focus of mathematics education in Chile. The curriculum of mathematics has incorporated modelling as one of the fundamental skills to be developed in students: first for grades 1–6 (Mineduc, 2012), and then extended to grades 7–10 (Mineduc, 2013). For grades 11 and 12, the new curriculum guidelines (Mineduc, 2020) promote modelling transversely across all the advanced mathematical courses. Although modelling was introduced in the curriculum almost a decade ago, the little evidence available and researchers' personal experience suggest that teachers are not being prepared to teach it and that students have few opportunities to work on real-world mathematical modelling problems (Guerrero-Ortiz & Mena-Lorca, 2015; Huincahue et al., 2018; Tapia, 2016).

One of several initiatives that are being developed to change this situation is the participation of the country in the International Mathematical Modelling Challenge (IMMC), an annual school-level team-based mathematical modelling contest (Garfunkel et al., 2021). The interest was to encourage teachers to integrate modelling into their teaching and give students the possibility to face real-world modelling problems. In the challenge, teams have five days to solve a realistic and complex problem, which are only slightly simplified through some clues given in the statements that suggest, for example, assumptions that could be made, and certain approaches for constructing and testing the models. Therefore, IMMC could be considered part of the realistic or applied modelling perspective, as described in Kaiser (2017). Several participant countries conduct their own pre-contests to select the two representing teams that will participate in the international challenge (Garfunkel et al., 2021). In the case of Chile, a selection process by stages was adopted, in which teams have to solve problems of increasing difficulty. Teams that reach the last stage work on the IMMC problem and the two best reports are chosen to represent the country.

For IMMC 2019, two focus groups with participating teams were conducted to evaluate the selection process and explore students' motivations and opinions regarding their experience. Although it was not expected, throughout their discourse students described how the experience allowed them to change some preconceived ideas about the nature of mathematics and certain characteristics of mathematical work. In order to explore this phenomenon in greater depth, a follow-up quantitative study was conducted to address the following research questions:

1. What are the beliefs about the nature of mathematics of students who participate in the Chilean selection process for the IMMC?
2. Are there changes in students' beliefs before and after participating in the selection process?

Since IMMC 2020, a questionnaire of beliefs about mathematics has been applied as a pre- and post-test to participating students. The results suggest that the exposure to the realistic modelling tasks used in the process has a positive effect on changing students' misconceptions about mathematics such as it is a collection of rules and procedures that describe how to solve a problem or that to solve a mathematical task one needs to know the correct procedure.

## 6.2 Students' Beliefs and Mathematical Modelling

Students' beliefs about mathematics have been largely investigated in the last decades (Furinghetti & Pehkonen, 2002; Pehkonen, 1995; Schoenfeld, 1989). According to Furinghetti and Pehkonen (2002), mathematical beliefs consist of relatively long-lasting subjective knowledge about mathematics, as well as of the related attitudes and emotions, and can be conscious or unconscious. It is worth noting that there is no simple shared understanding of the concept of belief, nor a generally accepted definition. In addition, it is common to find beliefs in the literature used interchangeably with terms such as views, conceptions or attitudes (Pehkonen & Törner, 1996). For the purpose of this study, and to avoid the nuanced use of this and related terms, belief, view and conception are treated as if they share a similar meaning.

There are many reasons to be interested in the ideas and beliefs students have about mathematics. Beliefs shape the ways that the individual conceptualises and engages in mathematical behaviour (Schoenfeld, 2016), and can be seen as a filter, influencing all activity and thinking (Pehkonen, 1995). In addition, their views about mathematics, expressed in students' attitudes towards the discipline, offer a window into the mathematics education they are receiving (Grigutsch et al., 1998). In a more practical aspect, students' beliefs influence the way they approach mathematical tasks and problems.

An important contribution to the understanding of this topic was made by Grigutsch (1996), who identified four aspects to describe students' beliefs about mathematics:

- *Formalism:* Mathematics is characterised by the rigour, accuracy and precision of the concepts and language used for logical reasoning, argumentation, justification and proving statements. Its formal attributes, related to axiomatics and the strict use of deductive reasoning, are dominant.

- *Scheme:* Mathematics is viewed as a fixed set of procedures and rules (a toolbox) that specify exactly how to solve tasks. Therefore, it is only about learning, practising, remembering and applying routines and schemes.
- *Process:* Mathematics is seen as an activity of thinking about problems and acquiring knowledge. This process involves understanding facts, seeing connections, and creating or rediscovering mathematics to solve problems.
- *Application:* Mathematical knowledge is viewed as important for students' life: either mathematics helps to solve everyday tasks and problems, or it will be useful in the future work. In addition, mathematics is considered to have a general and fundamental benefit for society.

Based on empirical evidence, Grigutsch (1996) concludes that the aspects of *formalism* and *scheme* are positively correlated with each other and represent a static view of mathematics. In contrast, a dynamic view of mathematics is represented by the aspect of *process*. In addition, the *application* aspect of mathematics is only significantly related to the *process* aspect. In a study conducted by Maaß (2010) with 13-year-old students in a classroom context, evidence was found that students' beliefs about mathematics are mainly scheme-oriented. Students consider that a mathematical problem can be solved quickly and has only one solution and that teachers are supposed to explain to them how to solve it. Stohlmann et al. (2016), in a review of the literature of mathematical modelling in secondary grades, analysed twelve papers that focus on students' beliefs regarding mathematical modelling and applications. In general, students show mostly positive views towards mathematical modelling after modelling experiences. Students claimed that mathematics is useful for the real world and daily life (Kaiser et al., 2011; Yanagimoto & Yoshimura, 2013) or found the tasks realistic and interesting (Kaiser & Stender, 2013). However, this positive view might depend on the belief system students have. In Kaiser and Maaß (2007), students with a process or application-oriented belief system had a positive attitude towards modelling while those with a static view of the discipline had a tendency to reject it. On the other hand, two studies in which pre- and post-Likert surveys were used to assess students' beliefs about mathematics found little or no change in such beliefs (Dunne & Galbraith, 2003; Schukajlow et al., 2011). However, it is worth noting that the modelling tasks in these studies are simpler and of shorter duration than those usually presented in modelling contests such as IMMC. Finally, the two studies conducted outside of school settings explored whether students want modelling in their regular mathematics lessons, reporting positive answers (Kaiser & Stender, 2013; Kaiser et al., 2011).

## 6.3 IMMC and Selection Process in Chile

Chile began its participation in the IMMC in 2018. Given the lack of experience of Chilean students solving realistic modelling problems, the first author of this chapter, in charge of the selection of representative teams, decided to conduct a two-stage process. The premise was that the knowledge and experience achieved when solving the first problem could contribute to developing modelling and writing skills that would improve teams' performance for the last stage, in which they had to face the IMMC problem. Due to the large number of participating teams, an additional initial stage with a simpler problem was added in the following years. Moreover, a national committee of professional mathematicians was formed to contribute to the design of the problems of the first two stages, as well as to evaluate the reports.

In the first stage, teams have a fixed period of five days to solve a modelling problem whose solution must be presented in the form of a 5-page report. Then the solutions are reviewed by the committee to decide which teams continue. Feedback related to mathematical models, solutions and the quality of their report are given to each team. In the second stage, teams work on a more difficult problem, again in a five-day period, but chosen at their convenience, and the solution is presented in a 10-page report. Teams that continue to the next stage are invited to a short training session aimed to review the modelling process and give them feedback and recommendations for teamwork and report preparation. In this last stage, teams receive a Spanish translation of the IMMC problem at the beginning of their chosen five-day period and must send their solutions in the form of a report of nearly 20 pages. Then the committee chooses the two best reports to represent the country. These are translated to English by the national organisation. Table 6.1 presents a brief description of the three problems for the selection process for IMMC 2019, 2020 and 2021.

As it is mentioned above, for IMMC 2019, focus groups with two participating teams were conducted after the last stage. Students mostly had positive opinions about their experience, recognising that the initial stages and training helped them feel better prepared for the IMMC problem. However, an unexpected theme arose during the interviews: participation in the contest seemed to trigger a shift in students' views, not only about the applicability of mathematics, but also about its nature.

With the purpose to illustrate these findings, two quotes are presented. In the first one, a student claimed that their participation helped them question their belief that mathematics has to be exact and accurate:

> Before participating in this, I considered mathematics as something super exact, and super concrete. But now that we had to apply this to real situations, such as in the Colectiv-App [second stage problem] or in the carrying capacity of the Earth [IMMC problem], we learnt that mathematics applies much more to other fields. It is much wider and may be inaccurate.

It also seems that the experience contributes to seeing mathematics as a more dynamical and evolving field, as the following quote shows:

> All this also helped us to deconstruct mathematics a bit. Like the preconceived ideas that we had of maths of only formulas in which everything was done. That sometimes if only one mixes what is already done, or tries to invent other things, like that you can reach different things. [...] That nothing is 100% done, that one can continue innovating within the field, either mixing it with another area, or mixing the same area, but different topics, so you can continue inventing, keep innovating.

**Table 6.1** Short description of the problems for the IMMC 2019–2021 selection processes[1]

| Year | Stage 1 problem | Stage 2 problem | Stage 3 (IMMC problem) |
|---|---|---|---|
| 2019 | Design a model to calculate the final price and estimate the stock of products needed for a retail company that wants to extend the regular 3-month warranty to 1 year. Manufacturer's price, number of products sold in recent years and the failure rates are assumed to be known | Design a model to decide if the new service that a mobility app wants to launch, in which two differently located passengers share a car to travel to the same destination, is convenient for passengers and, if so, divide the fare in a fair way | Design two models that allow choosing the "best" hospital among all those that are accessible to a patient, a simple model that considers only the evitable mortality rate and another model that also includes other quality criteria, such as the facilities and experience of the doctors |
| 2020 | Describe the model for calculating the residential water bill in Chile and determine whether it allows the perverse incentive of increasing water consumption during the non-peak period to raise the overconsumption limit and thus reduce the bill during the peak period | Design a model to distribute the money from a wealth tax on the super-rich that reduces as much as possible the Gini coefficient of Chile. Also, propose a different measure that allows comparing the inequality between countries with similar Gini coefficients | Develop a model that identifies the Earth's carrying capacity for human life under current conditions and propose how this carrying capacity can be raised accounting for perceived or anticipated human conditions; see Garfunkel et al. (2021) |
| 2021 | Design two models to define the rate for different types of vehicles for a new ferry service that will link two towns in the extreme south of Chile: a simple model that only considers charging vehicles and another that also considers a charge to passengers | Design a model that allows an online platform to choose the candidate to integrate the constitutional convention that best represents the preferences of a voter based on a questionnaire with topics relevant to the new constitution | Develop a model to quantitatively predict the behaviours of the customers of a store during a flash sale event that potentially result in damage to products and propose a new store floor plan with optimal locations of departments and most popular sale items |

In general, students' answers suggested a move from a static view of mathematics, mostly associated with a traditional learning experience at school, to a more complex and accurate conception of the discipline. Bearing in mind these findings, it was clear the need for a more systematic study to explore the differences in students' beliefs about mathematics that are triggered by their participation in the contest.

[1] Full versions of each problem statement (in Spanish) can be found in https://www.immc.cl/recursos/problemas-immc/. IMMC problem statements (in English) can be downloaded from https://www.immchallenge.org/Pages/Sample.html.

## 6.4 Methodology

The research design involved the administration of a questionnaire that examines the beliefs about the nature of mathematics of students that participated in the contest in a pre- and post-test format. The questionnaire was developed as part of the Teacher Education and Development Study in Mathematics (TEDS-M). One of the purposes of the study was to collect information on pre-service teachers' beliefs about mathematics and its learning (Tatto, 2013) and it considered three categories of beliefs: nature of mathematics, mathematics teaching and learning and mathematics achievement. The first category measures conceptions about mathematics as a formal, structural, procedural or applied field of knowledge, and it is based in part on the work of Grigutsch (1996).

The questionnaire is a Likert scale that consists of 11 statements, each one with a score from 1 to 6 (1: "Strongly disagree", 2: "Disagree", 3: "Slightly disagree", 4: "Slightly agree", 5: "Agree" and 6: "Strongly agree"). These statements assess the individual's beliefs about the nature of mathematics in relation to two distinct scales: *Mathematics as a Set of Rules and Procedures* and *Mathematics as a Process of Inquiry*. The 11 statements are distributed across the two scales with 6 of them corresponding to the first scale and the remaining 5 to the second scale (Table 6.2). The statements of the first scale reflect a more static conception of mathematics, while the other ones could be associated with a more dynamical view of the field. It is worth noticing that the statements related to problems and problem-solving make no distinction between pure and applied mathematics problems, including modelling problems, and so it cannot be assumed that responders are aware of this difference when they answer. The reliability of this questionnaire was calculated in the TEDS-M study using Cronbach's alpha coefficient, which ranged between 0.78 and 0.97, and the items have been examined by expert panels (Tatto, 2013). This questionnaire has also been used in several studies to assess pre-service teachers' and mathematics educators' beliefs (e.g. Alfaro Víquez & Joutsenlahti, 2021; Tarasenkova & Akulenko, 2013). Therefore, the questionnaire was an appropriate instrument given the study's goals, which include measuring in a reliable way the changes in the students' beliefs about mathematics as a result of the IMMC modelling experiences. We do recognise that this questionnaire does not provide detailed information about what students believe about mathematics, but this was not necessarily the goal of this first study. The authors decided that this questionnaire was a simple and effective way of assessing changes in their beliefs, and feasible to be applied considering the constraints of the selection process.

The present study made use of the Spanish version offered by the TEDS-M team as a base for the development of a questionnaire pertinent for participating students. It was revised and adapted to the Chilean context since some words may have slightly different meanings for different Spanish-speaking countries. To evaluate the accuracy of the translation, it was translated back into English and compared to its original version.

**Table 6.2** Questionnaire statements about beliefs of the nature of mathematics

| Mathematics as a set of rules and procedures | Mathematics as a process of inquiry |
|---|---|
| 1. Mathematics is a collection of rules and procedures that describe how to solve a problem<br>2. Mathematics involves the remembering and application of definitions, formulas, mathematical facts and procedures<br>3. When solving mathematical tasks, you need to know the correct procedure<br>4. Logical rigour and precision are fundamental to mathematics<br>5. Doing mathematics requires considerable practice, correct application of routines and problem-solving strategies<br>6. Mathematics means learning, remembering and applying | 7. When doing mathematics, you can discover and try out many things by yourself<br>8. If you engage in mathematics tasks, you can discover new things (connections, rules, concepts)<br>9. Mathematical problems can be solved correctly in many ways<br>10. Many aspects of mathematics have practical relevance<br>11. Mathematics helps us solve everyday problems and tasks |

The questionnaire was applied voluntarily as a pre- and post-test to three cohorts of IMMC participants (2020, 2021 and 2022). The volunteers were students ranging from grades 7 to 11 and attending different secondary schools across Chile. The absence of students from grade 12 is due to students having to be enrolled in school at the time of participating in the international challenge, and the selection process starts the previous academic year. It is worth mentioning that most of these students had a high performance in mathematics classes and showed interest and a positive attitude towards mathematics, although they had almost no previous experience in mathematical modelling. A total of 281 students responded to the questionnaire during the pre-testing, which took place before the students started participating in the first stage of the selection process. This sample was composed of 166 individuals who identified themselves as male, 114 who identified themselves as female and one individual who identified themselves as Other. The majority of individuals in the sample (85%) belonged to grades 10 or 11. There were some students who had participated in more than one version of the IMMC, hence belonging to more than one cohort. To avoid having repeated individuals, only the first responses to the questionnaire were considered for these participants.

Out of the 281 students who completed the questionnaire during pre-testing, a group of 44 also completed the questionnaire during the post-testing. This subsample was composed of 28 individuals who identified themselves as male and 16 individuals who identified themselves as female. The grades ranged from grades 7 to 11 and, as before, the majority of these students (73%) belonged to grades 10 or 11. The post-testing questionnaire was administered to the students at different moments in time depending on the stage of the contest they reached in the selection process. For the purpose of the analysis, two groups of students will be considered. The first group ($N = 36$) included those students who completed the first stage of the selection process but did not continue to the subsequent stages, hence working on only one mathematical modelling task. The second group ($N = 8$) consists of those students who reached the second or last stage of the contest, hence working on two or three mathematical modelling tasks.

Several steps were taken to minimise biases during the data collection and analysis. First of all, a national committee of six professional mathematicians designed the problems for the first two stages and selected teams for the subsequent stages. Although one of the authors participated in this committee, each solution is assigned a code to ensure a blinded evaluation and the final decision depends on the entire committee. Second, the two authors of the study had different roles during the selection process: One author was in charge of collecting the data, while the other author was in charge of the training instance prior to Stage 3 of the process. Finally, the application of a questionnaire based on closed items also contributed to minimising biases, since the quantitative analysis performed relies less on the subjectivity of the researchers.

The changes in the students' beliefs about the nature of mathematics is described by the differences in the average level of agreement with a specific statement (LOA) between the pre- and post-test, as in previous studies (Alfaro Víquez & Joutsenlahti, 2021; Tatto, 2013). LOA is defined as the arithmetic mean of the values from the Likert scale that the sample of students assigned to a specific statement. Thus, the higher a LOA is for a specific statement, the more students agreed with such a statement. Because of the size of the sample, to measure the significance of the differences between means of the pre- and post-test, Wilcoxon signed-rank test for repeated measures was applied. The analysis of difference was conducted at two different levels. The first level corresponded to analysing the differences between pre- and post-test for the overall group of 44 students. Then, to assess whether longer exposure to mathematical modelling problems led to larger changes in students' beliefs, the same analysis was applied to the previously defined two groups. Finally, to assess whether the changes in students' beliefs between these two groups were significantly different, a Wilcoxon signed-rank test was also applied.

## 6.5 Results

We will begin the report of results by presenting a summary of the LOA corresponding to the overall group of students who participated in the IMMC and completed the pre-test ($N = 281$). These results offered us a general picture of how the participating students conceived mathematics. More specifically, these students demonstrated both a significantly higher LOA for those statements in the scale *Mathematics as a Process of Inquiry* than those statements in the scale *Mathematics as a Set of Rules and Procedures* (Table 6.3). This suggests that the Chilean students who participated in the IMMC between 2019 and 2021 might be inclined towards a dynamic conception of mathematics before participating in the selection process. It is worth noting this pattern is also seen in the results of previous studies that applied the same instrument as a pre- and post-test to pre-service teachers and mathematics educators (Alfaro Víquez & Joutsenlahti, 2021; Tarasenkova & Akulenko, 2013; Tatto, 2013).

**Table 6.3** Level of agreement (LOA) grouped by scale for all students completing the pre-test ($N = 281$)

| Statement | LOA |
|---|---|
| *Mathematics as a set of rules and procedures* | |
| 1. Mathematics is a collection of rules and procedures that describe how to solve a problem | 4.33 |
| 2. Mathematics involves the remembering and application of definitions, formulas, mathematical facts and procedures | 4.53 |
| 3. When solving mathematical tasks, you need to know the correct procedure | 4.02 |
| 4. Logical rigour and precision are fundamental to mathematics | 4.88 |
| 5. Doing mathematics requires considerable practice, correct application of routines and problem-solving strategies | 5.11 |
| 6. Mathematics means learning, remembering and applying | 4.93 |
| *Mathematics as a process of inquiry* | |
| 7. When doing mathematics, you can discover and try out many things by yourself | 5.37 |
| 8. If you engage in mathematics tasks, you can discover new things (connections, rules, concepts) | 5.48 |
| 9. Mathematical problems can be solved correctly in many ways | 5.30 |
| 10. Many aspects of mathematics have practical relevance | 5.24 |
| 11. Mathematics helps us solve everyday problems and tasks | 5.30 |

For the group of students who completed both the pre- and the post-test, a similar tendency was observed: they showed significantly higher LOA in both pre- and post-test for the statements in the scale *Mathematics as a Process of Inquiry* than those in the scale *Mathematics as a Set of Rules and Procedures*. Table 6.4 shows a summary of the LOA measures per statement corresponding to this group of students.

A more detailed analysis of the differences of the statements in the first scale, *Mathematics as a Set of Rules and Procedures*, revealed mixed results regarding both the LOA when comparing the pre- to the post-test. A decrease in LOA was observed for statements “1. Mathematics is a collection of rules and procedures that describe how to solve a problem” (−0.37), “3. When solving mathematical tasks, you need to know the correct procedure” (−0.25) and “6. Mathematics means learning, remembering, and applying” (−0.2). On the other hand, an increase in LOA was observed for the statements “2. Mathematics involves the remembering and application of definitions, formulas, mathematical facts, and procedures” (+0.23) and “4. Logical rigour and precision are fundamental to mathematics” (+0.16). The statement “5. Doing mathematics requires considerable practice, correct application of routines, and problem-solving strategies” (+0.02) showed almost no difference between pre- and post-test.

It should be noted that the two largest differences in LOA were observed for the statements 1 and 3, which are arguably two important potential obstacles for a productive understanding and successful implementation of mathematical modelling. Although none of these differences turned out to be statistically significant, the results

**Table 6.4** Summary of the LOA and differences between pre- and post-test per statement

| | Overall group ($N = 44$) | | | 1 modelling problem ($N = 36$) | | | >1 modelling problem ($N = 8$) | | |
|---|---|---|---|---|---|---|---|---|---|
| | Pre | Post | Diff | Pre | Post | Diff | Pre | Post | Diff |
| *Mathematics as a set of rules and procedures* | | | | | | | | | |
| 1. Mathematics is a collection of rules and procedures that describe how to solve a problem | 4.57 | 4.20 | −0.37 | 4.64 | 4.36 | −0.28 | 4.25 | 3.50 | −0.75 |
| 2. Mathematics involves the remembering and application of definitions, formulas, mathematical facts and procedures | 4.66 | 4.89 | 0.23 | 4.75 | 5.03 | 0.28 | 4.25 | 4.25 | 0.00 |
| 3. When solving mathematical tasks, you need to know the correct procedure | 4.18 | 3.93 | −0.25 | 4.11 | 4.06 | −0.06 | 4.50 | 3.38 | −1.13 |
| 4. Logical rigour and precision are fundamental to mathematics | 4.93 | 5.09 | 0.16 | 4.89 | 5.19 | 0.31 | 5.13 | 4.63 | −0.50 |
| 5. Doing mathematics requires considerable practice, correct application of routines and problem-solving strategies | 5.32 | 5.34 | 0.02 | 5.42 | 5.36 | −0.06 | 4.88 | 5.25 | 0.38 |
| 6. Mathematics means learning, remembering and applying | 5.18 | 4.98 | −0.20 | 5.19 | 5.06 | −0.14 | 5.13 | 4.63 | −0.50 |
| *Mathematics as a process of inquiry* | | | | | | | | | |
| 7. When doing mathematics, you can discover and try out many things by yourself | 5.41 | 5.61 | 0.20 | 5.50 | 5.61 | 0.11 | 5.00 | 5.63 | 0.63 |
| 8. If you engage in mathematics tasks, you can discover new things (connections, rules, concepts) | 5.59 | 5.59 | 0.00 | 5.58 | 5.61 | 0.03 | 5.63 | 5.50 | −0.13 |
| 9. Mathematical problems can be solved correctly in many ways | 5.34 | 5.57 | 0.23 | 5.44 | 5.53 | 0.08 | 4.88 | 5.75 | 0.88 |
| 10. Many aspects of mathematics have practical relevance | 5.30 | 5.50 | 0.20 | 5.22 | 5.47 | 0.25 | 5.63 | 5.63 | 0.00 |
| 11. Mathematics helps us solve everyday problems and tasks | 5.41 | 5.41 | 0.00 | 5.39 | 5.36 | −0.03 | 5.50 | 5.63 | 0.13 |

might indicate that exposure to mathematical modelling tasks during the contest could help students move away from the conceptions of mathematics represented by these two statements.

With respect to the second scale, *Mathematics as a Process of Inquiry*, the comparison between pre- and post-test showed that the LOA increased or remained unchanged for all the statements. The largest increases were in the statements "9. Mathematical problems can be solved correctly in many ways" (+0.23), "10. Many aspects of mathematics have practical relevance" (+0.2) and "7. When doing mathematics, you can discover and try out many things by yourself" (+0.2). All these represent beliefs that can help students successfully complete mathematical modelling tasks, which refer to a relevant characteristic of modelling problems, the recognition of the applicability of mathematics and the creative aspect of problem-solving. One way to interpret these results could be that working on the mathematical modelling tasks during the contest helps students develop desirable conceptions about mathematics that they are expected to acquire throughout their schooling.

To assess whether longer exposure to solving these mathematical modelling tasks led to larger changes in students' beliefs about mathematics, the differences in LOA between pre- and post-test for each statement were compared between those students who worked on only one modelling task ($N = 36$) and those who worked on more than one modelling task ($N = 8$). Although no significant differences were found with the Wilcoxon signed-rank test, it is possible to observe a contrast between the two groups. As shown in Table 6.4, the students who worked on only one problem showed the largest changes in LOA for the statements "1. Mathematics is a collection of rules and procedures that describe how to solve a problem" (−0.28), "2. Mathematics involves the remembering and application of definitions, formulas, mathematical facts, and procedures" (+0.28), "4. Logical rigour and precision are fundamental to mathematics" (+0.31) and "10. Many aspects of mathematics have practical relevance" (+0.25). In contrast, those students who worked on more than one modelling task showed, in general, more pronounced changes in the level of agreement. The largest differences were in the statements "1. Mathematics is a collection of rules and procedures that describe how to solve a problem" (−0.75), "3. When solving mathematical tasks, you need to know the correct procedure" (−1.13), "7. When doing mathematics, you can discover and try out many things by yourself" (+0.63) and "9. Mathematical problems can be solved correctly in many ways" (+0.88) (Fig. 6.1).

It is worth noticing that the students who worked on more than one task showed the largest decreases in LOA for the statements 1 and 3 of the scale *Mathematics as a Set of Rules and Procedures*, which could be considered as two important obstacles for successful mathematical modelling: seeing mathematics as a toolbox full of prescriptions on how to solve a problem, and where the student's task is just finding the right tool. Something similar can be said in relation to the scale *Mathematics as a Process of Inquiry*, where this group showed the largest increases in LOA for the statements 7 and 9, which can be considered as two characteristics of mathematical modelling; that is, that mathematical modelling involves discovering and trying out different strategies and that modelling tasks usually have many different correct solutions. As a whole, these results suggest that a longer exposure to solving modelling

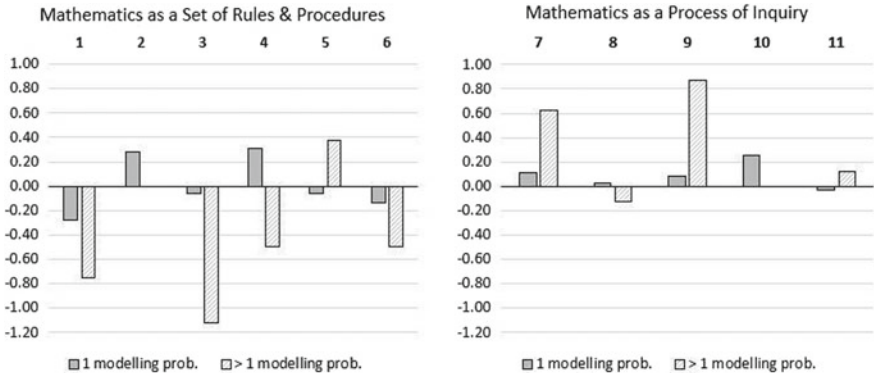


**Fig. 6.1** Comparison of pre-post-testing differences in LOA between Group 1 and Group 2 for scale 1 (left) and scale 2 (right)

tasks during the process could have a positive effect over the students' beliefs about mathematics. However, this result should be taken with caution because the sample of students who worked on more than one modelling task was relatively small, and thus, more research should be conducted to assess the validity of this claim.

## 6.6 Discussion and Conclusion

Mathematical modelling contests are running internationally and in several countries on different educational levels. A common feature is that students are challenged to work autonomously and collaboratively with realistic real-world modelling problems, such as the IMMC contest (Garfunkel et al., 2021). Many authors have pointed out the importance for students of working on these types of problems (Bracke & Geiger, 2011; Kaiser & Stender, 2013).

The results of the pre-test showed that the students participating in the selection process for IMMC demonstrated a high level of agreement with those statements in the scale *Mathematics as a Process of Inquiry*. This suggests that these students view mathematics as a dynamic discipline, where creativity, exploration and practical relevance play a central role. However, some beliefs associated with a static view of mathematics, represented by the statements of the scale *Mathematics as a Set of Rules and Procedure*s, also received a noticeable level of agreement from the students. These findings could be partially explained by the type of student who participates in the contest: in general, they have a high performance in mathematics and a positive attitude towards the discipline. In addition, it is not surprising that most of the students who show interest in participating in mathematical contests that involve working on open problems might be precisely those who hold a more dynamic view of mathematics. This is consistent with the results of Kaiser and Maaß (2007), who found that students with an application-oriented or dynamic belief system had positive attitudes towards modelling. Since these authors also concluded that students with a static or more formalism-oriented belief system tend to reject mathematical modelling, it would be interesting to study the belief system of students from the teams' schools who show high performance in mathematics but no interest in participating in IMMC.

The type of statements for which there were changes in the level of agreement are related to common aspects of the work that mathematical modelling entails. For instance, the two largest decreases in LOA were observed for two statements in the scale *Mathematics as a Set of Rules and Procedures*: "1. Mathematics is a collection of rules and procedures that describe how to solve a problem" (−0.37) and "3. When solving mathematical tasks, you need to know the correct procedure" (−0.25). These two statements endorse beliefs about mathematics that are obstacles to mathematical modelling. A similar pattern was observed in the three statements in the scale *Mathematics as a Process of Inquiry* that showed the largest increments in their levels of agreement: "7. When doing mathematics, you can discover and try out many things by yourself" (+0.20), "9. Mathematical problems can be solved correctly in many ways (+0.23)" and "10. Many aspects of mathematics have practical relevance" (+0.20). These beliefs are associated with common features of mathematical modelling: problems with many possible solutions, creative processes and applications.

The previous result suggests that the tasks students face during the IMMC selection process and contest have the potential to positively change the way they perceive mathematics. More specifically, the exposure to this type of modelling problems seems to challenge beliefs associated with a static conception of mathematics and promotes the development of others that represent a more dynamic view. Moreover, when comparing students who worked on one modelling problem with those who worked on more than one, the latter group showed a larger decrease in the level of agreement with statements related to a static view of mathematics than the former group. For example, the statement "3. When solving mathematical tasks, you need to know the correct procedure" showed a decrease of −1.13 and −0.06, respectively, and "1. Mathematics is a collection of rules and procedures that describe how to solve a problem" has a decrease of −0.75 and −0.28, respectively. In line with this, students who worked on more than one problem showed a larger increase in the level of agreement with statements promoting a dynamic view of mathematics than those who worked on only one problem. Therefore, longer exposure to realistic modelling problems appeared to have a greater effect over the students' beliefs about mathematics.

It is worth discussing the general tendency of the students in our sample, as well as those in previous studies with different populations, such as pre-service teachers and mathematics teacher educators (Alfaro Víquez & Joutsenlahti, 2021; Tarasenkova & Akulenko, 2013; Tatto, 2013), to agree more markedly with the statements in the scale *Mathematics as a Process of Inquiry* than with those in *Mathematics as a Set of Rules and Procedures*. We hypothesise that the way the statements included in each scale are presented may have played a role in this tendency. More specifically, the statements included in *Mathematics as a Process of Inquiry* appear to have a more positive sense, which may have induced the responder to agree more with them than with those in the other scale, which in general appear to be worded in a less positive sense. For instance, compare the statement "1. Mathematics is a collection of rules and procedures that describe how to solve a problem" to the statement "8. If you engage in mathematics tasks, you can discover new things (connections, rules, concepts)"; the former appears to have a less positive sense than the latter. Also consider the statements "3. When solving mathematical tasks, you need to know the correct procedure" and "7. When doing mathematics, you can discover and try out many things by yourself", where the former has a less positive sense than the latter. In both examples, the statements that could be perceived as less positive belong to the first scale, while the more positive statements are part of the second scale. We believe that a detailed revision of the way that the two scales are presented might be beneficial.

The results of this study are encouraging but have some limitations, one of which is the changes observed were not statistically significant. A reason for this could be the small sample ($N = 44$). As was previously discussed, another reason may be related to the potential limitations of the instrument utilised. Specifically, if the statements can be considered as negative or positive by the respondent, it may be difficult to assess possible changes since the respondent would show more agreement with the positive statements and more disagreement with the negative ones during both the pre- and post-testing. More empirical studies involving larger samples and perhaps other instruments could contribute to assess whether these changes actually occur and are significant.

A further research question is how students' prior beliefs about mathematics and mathematical work shape the manner in which they approach modelling tasks. Another potential future research area is related to exploring the impact of realistic modelling problems on the beliefs about mathematics held by students who usually perform poorly in mathematics classes and/or show less positive attitudes towards the learning of this discipline.

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