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WELFARE PERFORMANCE OF THE URBAN EQUILIBRIUM AND THE ROLE OF TRANSPORT

TESIS PARA OPTAR AL GRADO DE DOCTOR EN SISTEMAS DE INGENIERÍA

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ANÁLISIS DE BIENESTAR EN EL EQUILIBRIO URBANO Y EL ROL DEL TRANSPORTE

Muchas ciudades en el mundo comparten problemas como la congestión, largos tiempos de viaje, contaminación y el aumento de los precios de las viviendas. Sin embargo, difieren en otras características, como la forma en que las personas se desplazan, reflejada en la proporción de viajes en transporte público, y en el patrón espacial de riqueza y pobreza. La literatura reporta que el transporte desempeña un papel clave en el equilibrio urbano. El objetivo principal de esta tesis es investigar cómo diferentes políticas de transporte se relacionan con la forma y las características de las ciudades, tanto a corto como a largo plazo.

En el primer capítulo, estudiamos la relación entre dos enfoques utilizados para analizar el bienestar en el modelo monocéntrico: la maximización de la utilidad y la minimización de los recursos. La literatura asume que ambos enfoques son equivalentes, concluyendo que un planificador Rawlsiano elegiría el equilibrio de mercado ya que minimiza el uso de recursos. Mostramos que si bien el equilibrio de mercado minimiza el uso de recursos en el caso de propietarios ausentes, no maximiza la utilidad de los residentes ni la suma del excedente de los residentes y las rentas de la tierra. Mostramos que el efecto es considerable, lo que afecta en gran medida las conclusiones de las políticas de transporte.

En el segundo capítulo, estudiamos impuestos que mejoran el bienestar en el equilibrio urbano. Primero, utilizando el modelo monocéntrico, mostramos que el equilibrio que maximiza la utilidad difiere del equilibrio de mercado en ausencia de externalidades y con residentes homogéneos. Mostramos que este efecto previo no se limita a los modelos monocéntricos: utilizando un modelo cuantitativo, realizamos un análisis numérico basado en una estimación cuantitativa de Berlín, mostrando que, incluso en ausencia de externalidades, un impuesto a la propiedad puede aumentar la utilidad esperada de todos los residentes. Concluimos que la suposición típica de que las rentas de la tierra se escapan de la economía tiene implicaciones significativas en los análisis de bienestar en modelos cuantitativos de ciudades.

En el tercer capítulo, estudiamos la eficiencia de tres políticas de transporte (tarificación de congestión, subsidios al transporte público y pistas exclusivos para buses) en un modelo no monocéntrico que permite la reubicación y la reurbanización. Encontramos que cuando se implementa una primera política, el aumento en el bienestar debido a la introducción de una segunda política es moderado. Además, utilizando un escenario estilizado de reurbanización cíclica, analizamos los impactos de estas políticas de transporte no solo a largo plazo, sino también a mediano plazo, cuando la reurbanización es solo parcialmente posible y cuando la mayoría del cambio urbano se debe a la reubicación. Utilizando este escenario, mostramos que cuando se introducen más de una política de transporte en momentos diferentes, las ganancias en bienestar y la estructura urbana resultante pueden diferir significativamente según el orden de introducción (es decir, existe dependencia de la trayectoria).

ABSTRACT
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WELFARE PERFORMANCE OF THE URBAN EQUILIBRIUM AND THE ROLE OF TRANSPORT

Many large cities share common problems such as congestion, long commuting times, pollution, and rising housing prices. However, they may differ on other features, such as the way that people commute –reflected in the share of public transport commuting trips– and in the spatial pattern of wealth and poverty. It has been long recognized and documented that transportation and commuting play a key role in the urban equilibrium. The main goal of this thesis is to investigate how different transport policies affect the structure and interact with the shape and features of cities, both in the short run and long run.

In the first chapter, we study the relationship between two approaches used to study welfare in the monocentric city model: maximization of equilibrium utility and minimization of resources. The literature assumes that both approaches are equivalent, concluding that a Rawlsian planner would choose the market equilibrium since it minimizes the use of resources. We show that while the market equilibrium minimizes resource usage in (and only in) the absentee landlord case, it does not maximize residents' utility nor the sum of residents' surplus and land rents. The same result holds for almost any land ownership structure, and we show that the effect is sizable, strongly impacting transport policy conclusions.

In the second chapter, we study welfare-improving taxes in the urban equilibrium. First, using the monocentric city model, we show that the utility-maximizing city differs from the market outcome in the absence of externalities and with homogeneous residents. The only exception is the extreme case of public land ownership in which all the differential rent is transferred lump-sum to residents. We show that the previous effect is not restricted to monocentric models: using a quantitative model, we show that the result holds. We conduct a numerical analysis using a quantitative estimation of Berlin, showing that, even in the absence of externalities, a property tax can increase the expected utility of all residents. We conclude that the typical assumption that land rents accrue to absentee landlords has significant implications in welfare analyses in quantitative models of cities.

In the third chapter, we study the efficiency of three transport policies (congestion pricing, public transport subsidies, and dedicated bus lanes) in a non-monocentric model that allows for relocation and redevelopment. We find that when any first policy is implemented, the welfare increase due to the introduction of a second policy is only moderate. Additionally, by using a stylized setting of cyclical redevelopment, we analyze these transport policies' impacts not only in the long run but also in the medium run, where redevelopment is only partially possible and when the core of the urban change is due to relocation. Using this setting, we show that when more than one transport policy is introduced at different time frames, the welfare gains and the final resulting urban structure may significantly differ depending on the introduction order (i.e., there is path dependence).

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Chapter 1

Introduction

Many large cities share common problems such as congestion, long commuting times, pollution, and rising housing prices. However, they may differ on other features, such as the way that people commute –reflected in the share of public transport commuting trips– and in the spatial pattern of wealth and poverty. Cities are complex systems whose structure is shaped by the interaction between people, developers and landlords, regulators and the Government, and service and manufacturing firms, together with economic forces such as economies of scale, agglomeration, and market failures such as externalities; still, it has been long recognized and documented that transportation and commuting play a key role in the urban equilibrium. The main goal of this thesis is to investigate how different transport policies affect the structure and interact with the shape and features of cities, both in the short run and long run. However, to study the impact of transport policies, it is necessary to understand the welfare properties of the urban equilibrium in the absence of externalities. In other words, if the urban equilibrium does not maximize welfare when no frictions are present, then when transportation generates externalities, any instrument aimed to improve welfare should consider not only these externalities.

This thesis encompasses five chapters, including both the introduction and conclusions. Chapters 2, 3, and 4 are self-contained entities, each making distinct contributions to the overall body of work. Below, we provide a brief overview of the contribution of each of these chapters.

In Chapter 2, we study the relationship between two approaches used to study welfare in the monocentric city model: maximization of equilibrium utility and minimization of resources understood as the sum of total non-land consumption, total commuting cost, and the opportunity cost of the urban land. In particular, the literature assumes that both approaches are equivalent, concluding that a Rawlsian planner would choose the market equilibrium since it minimizes the use of resources (Duranton and Puga, 2015). In a nutshell, we show that, in general, the minimization of resource usage is not equivalent to the maximization of equilibrium utility nor the maximization of a welfare function defined as the monetary value of households' utility and absentee landlord rents. Furthermore, while the market equilibrium minimizes resource usage in (and only in) the absentee landlord case, it does not maximize residents' utility nor the sum of residents' surplus and land rents. The same result holds for almost any land ownership structure, and we show that the effect is sizable, strongly impacting transport policy conclusions.

In Chapter 3, we study welfare-improving taxes in the urban equilibrium. First, using the monocentric city model, we show that the utility-maximizing city differs from the market outcome in the absence of externalities and with homogeneous residents. The only exception is the extreme case of public land ownership in which all the differential rent is transferred lump-sum to residents. In any other case, a judiciously designed revenue neutral tax-subsidy schedule achieves a higher equilibrium utility by increasing the city size and indirectly transferring land rents from the absentee landlords to the city residents. Note that even those that are taxed are better off. Then, we study transport policies in this setting. For public transport, we show that the fare that decentralizes the first-best scenario is below marginal cost, and thus the system should be subsidized, even in the absence of externalities. In the case of car congestion pricing, we show that the welfare-maximizing toll may be non-monotonic, yielding a city that is more extended and with more aggregated mileage than the unpriced city. We show that the previous effect is not restricted to monocentric models: using a quantitative model, we show theoretically that the result holds. Then, to shed more light on the size and relevance of the finding, we conduct a numerical analysis using the quantitative estimation of Berlin presented in Ahlfeldt et al. (2015), showing that, even in the absence of externalities, a property tax can increase the expected utility of all residents by 2.4%. Additionally, we show that a labor tax and a corporate tax can increase the expected utility by 1.0% and 1.9%, respectively. We conclude that the typical assumption that land rents accrue to absentee landlords has significant implications in welfare analyses in quantitative models of cities. The literature usually presents welfare estimations that combine general equilibrium channels with the indirect redistribution analyzed in this paper.

In Chapter 4 we study the the efficiency and substitutability of three transport policies (congestion pricing, public transport subsidies, and dedicated bus lanes). We argue that although the literature has extensively studied the impacts and the desirability of policies aimed to correct transport externalities in cities, most of the previous work does not consider the interaction of these policies with the underlying urban form. Moreover, within those that consider the urban form, two main assumptions have been used: (i) urban form is fixed (i.e., no redevelopment occurs as a response to policies), or (ii) the underlying city is malleable (i.e., full redevelopment occurs as a response to policies). To fill this gap, we propose a non-monocentric model that allows for relocation and redevelopment, featuring endogenous location decisions for firms and households of two skill groups and two transport modes (private car and transit), allowing for cross-congestion effects between cars and buses. We use this model to study the interaction of three transport policies with the underlying urban form: congestion pricing, transit subsidies, and dedicated public transport infrastructure. Then, we propose a stylized setting of cyclical redevelopment that allows us to study the impact of transport policies not only in the long run but also in the medium run, where redevelopment is only partially possible and when the core of the urban change is due to relocation. We find that (i) dedicated lanes are able to attract users to the transit system much more effectively than the other two policies, and (ii) the substitutability between policies is large. In other words, when any first policy is implemented, the welfare increase due to the introduction of a second policy is only moderate. Then, when redevelopment is only partially possible and when the core of the urban change is due to relocation, we show that (i) short-term and long-term approaches might greatly underestimate and overestimate, respectively, the welfare gains of any set of policies, and (ii) when more than one transport policy is introduced at different time frames, the welfare gains and the final resulting urban structure may significantly differ depending on the introduction order.

Chapter 2

On the relationship between maximization of equilibrium utility and minimization of resources in the urban equilibrium

Abstract: This paper studies welfare in the building block of the urban economics field, the monocentric city model. We analyze the relationship between the two most common approaches used to evaluate policies, regulation and welfare: maximizing the equilibrium utility and minimizing total resources. In this context, resources refer to the combined total of non-land consumption, commuting costs, and the opportunity cost of urban land. Previous literature assumes that these approaches are equivalent, suggesting, in consequence, that when all individuals are ex-ante equal, a Rawlsian planner would select the market equilibrium due to its resource-minimizing nature. We show that this result only holds if the excess land rents are fully captured and redistributed to the population in equal shares. In particular, then, for the very common assumption of an absentee landlord, or even if a fraction of the land rents profits leaks from the economy, both approaches no longer coincide.

2.1. Introduction

The monocentric city model has been a fundamental building block in urban economics. Due to its simplicity and elegance, this framework has been widely used to analyze the welfare effects of interventions such as zoning regulations and transportation infrastructure on urban dynamics. For this purpose, two main approaches have been employed in the literature. Table 1 presents a summary of references for both approaches. The first approach is based on the analysis of the equilibrium utility reached by households. These papers assume that all land rents are captured and redistributed lump-sum in equal shares among households. Under this framework, the literature shows that the market equilibrium maximizes equilibrium utility (Kanemoto, 1980). The second approach is based on the concept of resource usage, minimizing resources used to achieve a given equilibrium utility, assuming that all land rents vanish from the city. Under this framework, the literature shows that the market equilibrium minimizes resource usage (Fujita and Thisse, 2013).

Table 2.1: Welfare functions used in the literature.

Reference	Welfare Function	Objective
Oron et al. (1973)	Equilibrium Utility	Optimal congestion tolls
Robson (1976)	Equilibrium Utility	Optimal road land allocation in the presence of congestion
Kanemoto (1977)	Equilibrium Utility	Optimal road land allocation in the presence of congestion
Arnott (1979b)	Resources	Optimal city size
Arnott (1979a)	Resources	Optimal valuation of land in the presence of congestion
Kanemoto (1980)	Equilibrium Utility	Optimal congestion tolls
Pines and Sadka (1985)	Equilibrium Utility	Optimal congestion tolls and lot size zoning
Straszhem (1987)	Equilibrium Utility	Equilibrium structure of cities
Papageorgiou and Pines (1999)	Resources	Equilibrium structure of cities
Brueckner (2005)	Resources	Optimal transport subsidies
Verhoef (2005)	Equilibrium Utility	Second-best congestion pricing schemes
Brueckner (2007)	Equilibrium Utility	Urban growth boundaries in the presence of congestion
Kono et al. (2012)	Equilibrium Utility	Optimal building size in the presence of congestion
Pines and Kono (2012)	Equilibrium Utility	Floor area ratio regulations in the presence of congestion
De Lara et al. (2013)	Resources	Congestion pricing schemes
Fujita and Thisse (2013)	Resources	Equilibrium structure of cities
Tikoudis et al. (2015)	Equilibrium Utility	Congestion pricing schemes
Duranton and Puga (2015)	Resources	Equilibrium structure of cities
Kono and Kawaguchi (2017)	Equilibrium Utility	Congestion pricing and land use regulations
Tikoudis et al. (2018)	Equilibrium Utility	Congestion pricing schemes
Kono et al. (2019)	Equilibrium Utility	Property taxes in the presence of congestion

Note the gap here: one approach considers that land rents are fully captured, while the other assumes that all land rents accrue to absentee landlords, and in both cases it is shown that the market equilibrium maximizes the welfare function considered. Thus, two questions remain open: Does the market equilibrium maximize utility (the first approach's welfare function) when all land rents vanish from the city (the second approach's assumption)? Does the market equilibrium minimize resource usage (second approach's welfare function) when all land rents are captured and redistributed in equal shares (first approach's assumption)? If the answer to these questions were affirmative, both approaches would be equivalent irrespective of the assumption regarding land ownership. However, to the best of our knowledge, the literature does not provide a conclusive answer to this. Moreover, some authors seem to assume that the answer to these questions is affirmative since, with an absentee landlord, they assert that the market equilibrium maximizes utility because it minimizes resource usage (e.g., Duranton and Puga, 2015).

In this paper, we fill the gap between the two approaches, presenting a unifying framework of welfare analysis and land ownership. In this framework, the government is able to capture a fraction $\mu \in [0, 1]$ of the excess land rents, which are then distributed lump-sum to all residents. Using this framework, we show that both approaches used to study welfare in the monocentric city model are never equivalent. In other words, a planner maximizing equilibrium utility would choose a different equilibrium compared to a planner minimizing resource usage for all level of land rent capture and redistribution. This result has profound implications when analyzing the effect of policies on the urban equilibrium, since the choice of the approach used to evaluate welfare (utility versus resource usage) is momentous. The decision about the welfare function to use is not innocuous and might have substantial implications over policy results (e.g., first-best policies or welfare gains from interventions).

Our paper contributes to the vast literature that analyzes welfare, externalities, and regulation using the monocentric city model. Contrary to the typical belief, we show that when there is an absentee landlord, a planner maximizing utility would never choose the market equilibrium. Consequently, many of the policies and interventions that have been usually studied with an absentee landlord are not optimal in this sense.

The rest of the paper is organized as follows. Section 2 revisits the textbook monocentric model, and then presents (i) the two most common approaches used to study welfare in the monocentric city model, and the efficiency results that the literature reports (ii) the unifying framework of land ownership. Then, Section 3 presents our main results, showing that the utility maximization approach is never equivalent to the minimization of resources approach. Finally, Section 4 concludes.

2.2. The monocentric city model

2.2.1. The textbook model

Early works from Alonso (1964), Mills (1967), and Muth (1969) developed the starting point of the modern urban economics literature in what is called the monocentric city model. In that model, there is a central business district (CBD) in which production takes place and that accommodates all jobs. The core of the model is that people consume a numeraire good, housing, and must commute to the CBD, where they work to obtain a fixed income. While extremely simple, the monocentric model provides the basis of our understanding of several aspects of urban economics, and is a building block to teach urban economics. To simplify the discussion, in what follows, we first formally describe the model and notation, and show what the market equilibrium entails. Modern references for an excellent and intuitive presentation include Brueckner (1987) and Duranton and Puga (2015).

Consider L identical households that locate along a closed linear city parametrized by $x \in [0, \overline{x}]$, where the CBD and the endogenous city boundary are located at $x = 0$ and $x = \overline{x}$ respectively. No one else from abroad may come to the city (what is known as a *closed-city*). The land is, initially, assumed to be owned by an absentee landlord that does not demand housing. All city residents work at the CBD, earning a fixed income y, and commute from their residential location using a single mode (e.g., private car). Commuting costs increase linearly with the distance to the CBD and include both the time cost of the trip and the operating cost of the vehicle. The total commuting cost per unit of distance is denoted t. Every household maximizes its utility, represented by a quasi-concave function $u(c, q)$ that depends on the consumption of a composite good c and housing q (measured by the floor size), both depending on the location. Thus, in this classical monocentric city model, the trade-off is between accessibility and dwelling size. The price of c is normalized to one irrespective of location, while the price of q is denoted by p and varies with location. The urban market equilibrium has several components. First, each household must make their choices –c, q and x– maximizing their utility:

$$\begin{aligned} \max \quad & u(c(x), q(x)) \\ \text{s.t.} \quad & y = tx + c(x) + p(x)q(x) \end{aligned} \tag{2.1}$$

Solving for c from the budget constraint, problem (2.1) can be rewritten as:

$$\max \quad v\left(y - p(x)q(x), q(x)\right) \tag{2.2}$$

where v denotes the indirect utility function. Since city residents choose q optimally conditional on prices, the first-order condition is then:

$$\frac{\partial v\left(y - p(x)q(x), q(x)\right)}{\partial q} \Bigg/ \frac{\partial v\left(y - p(x)q(x), q(x)\right)}{\partial c} = p \tag{2.3}$$

The second component of the equilibrium is spatial (and Nash): no one can improve its situation by unilaterally deviating to a different location. Consequently, the utility of all households must be the same everywhere along the city:

$$u(c(x), q(x)) = \overline{u} \quad 0 \leq x \leq \overline{x} \tag{2.4}$$

Conditions (2.3) and (2.4) allow to find solutions for $p(x)$ and $q(x)$, which we denote $\hat{p}(x, y, \overline{u})$ and $\hat{q}(x, y, \overline{u})$. The market equilibrium is complete with two additional conditions. First, the urban land rent in the city boundary equals the agricultural land rent r_a (i.e., residents outbid agricultural users in the city):

$$\hat{p}(\overline{x}, y, \overline{u}) = r_a \tag{2.5}$$

Second, the urban population L must fit inside the city:

$$\int_0^{\overline{x}} \frac{1}{\hat{q}(x, y, \overline{u})} dx = L \tag{2.6}$$

Equation (2.5) allows for obtaining the value of $\overline{x} = \overline{x}(y, \overline{u}, r_a)$, while equation (2.6) is used to obtain the value of the equilibrium utility $\overline{u} = \overline{u}(\overline{x}, y, L)$.

The spatial equilibrium is given by $p(x)$, $q(x)$, $\overline{x}$ and $\overline{u}$, and they are characterized by four equations: (2.3), (2.4), (2.5) and (2.6).

In regard to the textbook model presented above, the literature shows that the equilibrium is unique. Moreover, this equilibrium is Pareto efficient, in that, by construction, in equilibrium, no city resident has incentives to relocate (Fujita, 1989; Papageorgiou and Pines, 1999).

2.2.2. Approaches used to study welfare in the monocentric city model

From the textbook model described in Subsection 2.2.1, the literature deviates in different directions depending on the objective of the paper. First, there is a stream of papers that study and analyze the equilibrium structure of cities without referring to any welfare analysis. Since land ownership does not change the equilibrium's qualitative results, these papers maintain the assumption that an absentee landlord owns all the land so that rents vanish from the city. This stream of papers includes the seminal works of Alonso (1964), Mills (1967), and Muth (1969), but also many others, such as Brueckner (1987) or Cooke (1988). Note that this is the textbook approach: For example, it is the approach used in Fujita (1989, Chapter 2), Brueckner (2011, Chapter 2), Fujita and Thisse (2013, Section 3) or Duranton and Puga (2015, Section 8).

A second stream of literature focuses on welfare analysis within a monocentric city model, as opposed to only describing the market equilibrium. This is the most relevant line of research to this paper. Within this stream, two main approaches have been usually employed.

The most common approach is based only on the analysis of the equilibrium utility reached by households: the higher the equilibrium utility reached after an intervention is, the more desirable that intervention, net of its costs. These papers, though, differently than the previously presented textbook approach, assume that all land rents are captured and redistributed lump-sum in equal shares among households. Formally, households' income now includes a second component, R/L, where R denotes the excess land rents:

$$R = \int_0^{\overline{x}} p(x) - r_a \, dx \tag{2.7}$$

This framework has been used to analyze policies that deal with traffic congestion (Kanemoto, 1980; Oron et al., 1973; Tikoudis et al., 2015, 2018; Verhoef, 2005), and to study second-best policies in congested cities, such as land use regulations (Brueckner, 2007; Kono and Kawaguchi, 2017; Pines and Kono, 2012; Pines and Sadka, 1985) and property taxes (Kono et al., 2019), among others.

The main efficiency result of this framework (shown by Kanemoto, 1980, among others) is as follows:

Proposition 1. *When the land rents are fully captured and redistributed lump-sum in equal shares among the city residents, the market equilibrium maximizes equilibrium utility. In other words, the market equilibrium is the solution to the following planning problem:*

$$(P_{\max}) \qquad \max \quad \overline{u}$$

$$s.t. \quad u(c(x), q(x)) = \overline{u}, \qquad 0 \leq x \leq \overline{x} \tag{2.8a}$$

$$\int_0^{\overline{x}} \frac{1}{q(x)} dx = L \tag{2.8b}$$

$$\frac{R}{L} + y - tx - c(x) - p(x)q(x) = 0 \qquad 0 \leq x \leq \overline{x} \tag{2.8c}$$

$$R = \int_0^{\overline{x}} p(x) - r_a \, dx \tag{2.8d}$$

$$p(\overline{x}) = r_a \tag{2.8e}$$

The other main approach that studies welfare is based on the concept of resource usage, defined as the sum of total non-land consumption, total commuting cost, and the opportunity cost of the urban land. This approach considers the opposite end of the spectrum regarding land ownership than those that maximize equilibrium utility: an absentee landlord, with rents vanishing from the city. This approach has been used, for instance, to study policies in the presence of traffic congestion (De Lara et al., 2013), and the desirability of transit subsidies (Brueckner, 2005).

The main efficiency result of this framework (presented in Fujita and Thisse, 2013, among others) is as follows:

Proposition 2. *When the land rents vanish from the economy, the market equilibrium minimizes the resource usage, defined as the sum of total non-land consumption, total commuting cost, and the opportunity cost of the urban land. In other words, the market equilibrium* *coincides with the solution of the following minimization problem:*

$$
\begin{aligned}
(P_{\min}) \qquad \min \quad & \left(\int_0^{\overline{x}} \left[\frac{tx + c(x)}{q(x)} + r_a \right] \, dx \right) \\
s.t. \quad & u(c(x), q(x)) = \overline{u}, \qquad\qquad 0 \leq x \leq \overline{x} && (2.9a) \\
& \int_0^{\overline{x}} \frac{1}{q(x)} dx = L && (2.9b) \\
& R = \int_0^{\overline{x}} p(x) - r_a \, dx && (2.9c) \\
& p(\overline{x}) = r_a && (2.9d)
\end{aligned}
$$

Based on the previous result, Brueckner (2005) concludes that the market outcome is efficient, while Duranton and Puga (2015) states that a Rawlsian planner –seeking to maximize the equilibrium utility– would choose the market equilibrium. Note that these conclusions are not straightforward, since the result of Proposition 2 deals with minimization of resources and not maximization of equilibrium utility. Moreover, as made explicit from Propositions 1 and 2, these two approaches consider different assumptions regarding the land ownership. To fill the gap between the two approaches, and to study whether they are equivalent or not, we now present a unifying framework of land ownership.

2.3. Relationship between maximization of equilibrium utility and minimization of resources

2.3.1. A general monocentric model for land rents distribution

We propose a unifying framework, in line with (Papageorgiou and Pines, 1999), where the government is only able to capture a fraction $\mu \in [0, 1]$ of the excess land rents, which are then distributed lump-sum in equal shares to all residents. Considering $\mu = 1$ leads to the most common setting used to study welfare in a monocentric city model, where all the rents are redistributed equally among residents, while considering $\mu = 0$ leads to the case where landlords are absentee and all land rents vanishes (the textbook case). With this framework, the households' budget constraint is:

$$\frac{\mu R}{L} + y = tx + c(x) + p(x)q(x) \tag{2.10}$$

A natural interpretation of our framework is to consider an agent that owns the land, but that does not affect the urban equilibrium. Still, the city government is able to impose a tax rate on the land value from rents equal to μ, which is distributed equally among residents. In view of this interpretation, we believe the more sensible approach to model cities is to consider $0 < \mu \ll 1$. For instance, in the US, rental income is subject to ordinary tax rates, ranging from 10% to 37%. Furthermore, since the US tax system treats landlords as business entities, expenses (repairs, maintenance, mortgage interest, etc.) can be used to offset the taxable rental income (Sommer and Sullivan, 2018), lowering the effective tax rate. Note that for this range of values, neither the assumptions of Proposition 1 nor those of Proposition 2 hold.

We generalize the Proposition 2 to allow for all possible land ownership structures ($\mu \in [0,1]$) using the framework presented in Subsection 2.3:

Proposition 3. *The market equilibrium, subject to a fixed utility, minimizes a resource function, defined as the sum of total non-land consumption, total commuting cost, and the opportunity cost of the urban land, minus the share of rents received by city residents. In other words, the market equilibrium coincides with the solution of the following minimization problem:*

$$(P_{\min_\mu}) \qquad \min \quad \left(\int_0^{\overline{x}} \left[\frac{tx + c(x)}{q(x)} + r_a \right] dx \right) - \mu R$$

$$s.t. \quad u(c(x), q(x)) = \overline{u}, \qquad 0 \leq x \leq \overline{x} \tag{2.11a}$$

$$\int_0^{\overline{x}} \frac{1}{q(x)} dx = L \tag{2.11b}$$

$$R = \int_0^{\overline{x}} p(x) - r_a \; dx \tag{2.11c}$$

$$p(\overline{x}) = r_a \tag{2.11d}$$

Proof. See Appendix A.1. □

Note that when $\mu = 0$ (i.e., all land rents vanish from the city), we recover the result presented in Proposition 2: the market equilibrium minimizes resource usage.. Nonetheless, the resource function that is minimized by the market equilibrium is different than the one in Proposition 2 for any other land ownership structure. In particular, the result of Proposition 2 no longer holds when the rents are fully captured ($\mu = 1$), making the connection to Proposition 1 far from obvious. To shed light on this relationship, we now present our main result, that connects the resource minimization and the utility maximization approaches:

Proposition 4. *Consider the following Rawlsian optimization problem* ($P_{\max_\mu}$)*:*

$$(P_{\max_\mu}) \qquad \max \quad \overline{u}$$

$$s.t. \quad u(c(x), q(x)) = \overline{u}, \qquad 0 \leq x \leq \overline{x} \tag{2.12a}$$

$$\int_0^{\overline{x}} \frac{1}{q(x)} dx = L \tag{2.12b}$$

$$\frac{\mu R}{L} + y - tx - c(x) - p(x)q(x) = 0 \qquad 0 \leq x \leq \overline{x} \tag{2.12c}$$

$$R = \int_0^{\overline{x}} p(x) - r_a \; dx \tag{2.12d}$$

$$p(\overline{x}) = r_a \tag{2.12e}$$

The problem of utility maximization ($P_{\max_\mu}$) *is equivalent to the minimization of a modified resource function, defined as the sum of total non-land consumption, total commuting cost, the opportunity cost of the urban land, and the share of rents received by the absentee landlord.*

In other words, problem ($P_{\max_\mu}$) is equivalent to:

$$(\tilde{P}_{\min_\mu}) \qquad \min \quad \left(\int_0^{\overline{x}} \left[\frac{tx + c(x)}{q(x)} + r_a \right] \, dx \right) + (1 - \mu) R$$

$$s.t. \quad u(c(x), q(x)) = \overline{u}, \qquad 0 \leq x \leq \overline{x} \tag{2.13a}$$

$$\int_0^{\overline{x}} \frac{1}{q(x)} dx = L \tag{2.13b}$$

$$R = \int_0^{\overline{x}} p(x) - r_a \, dx \tag{2.13c}$$

$$p(\overline{x}) = r_a \tag{2.13d}$$

PROOF. Appendix A.2. □

Corollary 1. *The sum of total non-land consumption, total commuting cost, and the opportunity cost of the urban land are minimized by the market equilibrium only when $\mu = 0$, while the minimization of those resources is equivalent to the maximization of equilibrium utility if and only if $\mu = 1$.*

Thus, as Propositions 3 and 4 show, the market equilibrium is the solution to a modified resource minimization problem, but this problem is never equivalent to the maximization of equilibrium utility. Furthermore, both problems never present the same objective function (for a given μ). Note that the objective function of the problems presented in Propositions 3 and 4 are the same when $\mu = 0$ in the first problem, while $\mu = 1$ in the second one. In this case, both problems aim to minimize the sum of total non-land consumption, total commuting cost, the opportunity cost of the urban land, as considered by Fujita (1989) and Fujita and Thisse (2013). Nonetheless, in this case both problems refer to different land ownership structures, with different underlying conditions (e.g., different households' income). Thus, even in this case they are not equivalent. Importantly, this implies that it is incorrect to argue that, because in the case of the absentee landlord case resources are minimized in the market equilibrium, a planner would choose that same allocation. Moreover, in general, the welfare analysis (for instance, the impact of transport or housing policies) using either of these approaches leads to different results.

2.3.2. Numerical Analysis

In order to provide intuition about our results, we simulate the urban equilibrium structure of a city. We base our numerical analysis on US values previously used in the literature, with parameters summarized in Table 2.2. We use, for practical purposes, a Cobb-Douglas utility function $u(c, q) = c^{1-\alpha} q^{\alpha}$, setting $\alpha = 0.35$. This implies that every household spends 35% of their income net of transportation costs on housing, which is consistent with the average expenditure reported by the US Department of Labor (2018)'s Consumer Expenditure Survey.[1] The hourly wage is set at US$25.27 which, following Bertaud and Brueckner (2005), is obtained using the median income per household of the 2018 US census (US$63,179) and assuming 2000 work hours/year. For the agricultural rent value, we also follow Bertaud and

[1] The US Department of Labor (2018)'s Consumer Expenditure Survey reports an average income net of taxes and transportation costs of US$57,480, of which US$20,091 is spent on housing.

Brueckner (2005): we consider the average US agricultural land value for the year 2019 of US$3,160/acre. Assuming a 5% discount rate, we arrive to r_a = US$101,120/sq. mi. We set $L = 120,000$ households, which would be equivalent to roughly $800,000$ households in a circular (rather than linear as in our model) city.

Table 2.2: Main parameter values.

Parameter	Value
α	0.35
y [$]	63,179
r_a [$/sq. mi]	101,120
L [households]	120,000
μ	0

Considering the extreme case of $\mu = 0$ (absentee landlords), Figure 2.1 shows the income net of transportation cost and land prices of the market equilibrium –that minimizes resources– compared to the Rawlsian first best, which seeks to maximize the equilibrium utility. As this figure depicts, a Rawlsian planner would not choose the market equilibrium. Moreover, this planner would choose an equilibrium that involves a more extended city, where people near the city center end up with a lower income net of transportation cost. This, in turn, reduces the competition for space near the CBD, with land prices being lower in the maximum utility equilibrium.

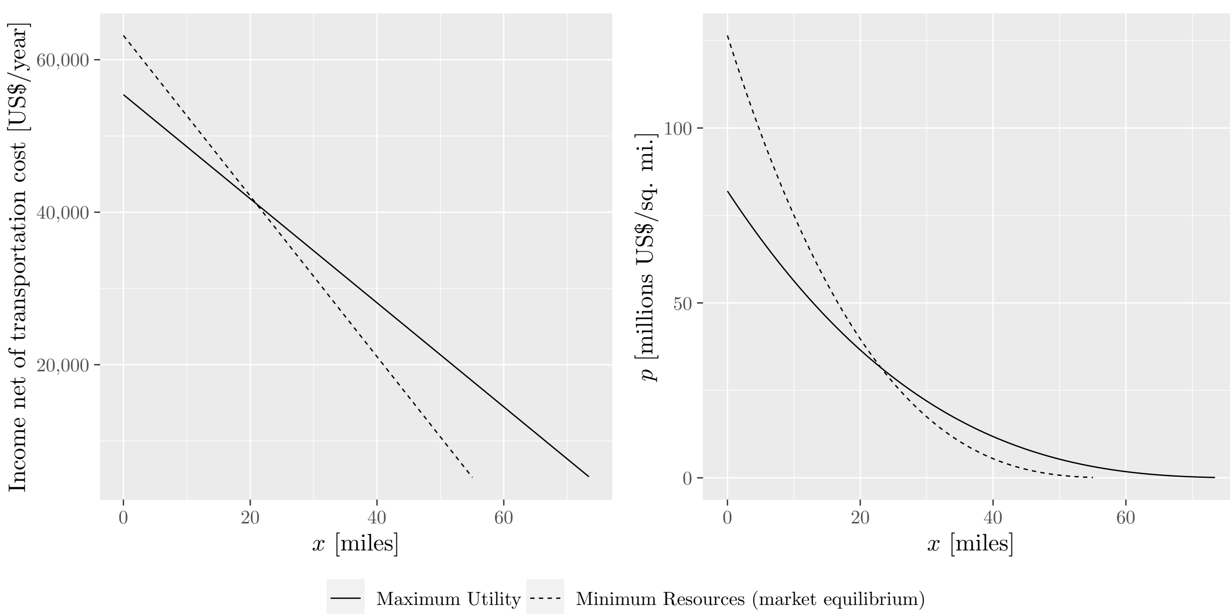


Figure 2.1: Income net of transportation cost and housing prices, for $\mu = 0$.

2.4. Concluding remarks

In this paper, we study the building block in urban economics: the monocentric city model, and we shed light on the different approaches used to evaluate welfare. In particular, we discuss the two main approaches considered in the literature: maximization of equilibrium utility and minimization of resources. Previous literature seems to assume that both approaches are equivalent, and consequently, may be used interchangeably. However, there has been a key underlying difference in the way the two approaches have been used, and that has obscured a clean comparison: in one approach, all the land rents are captured and distributed to city residents, while in the other, all the land rents accrue to absentee landlords, and thus, they leak from the economy.

To fill the previous gap, we propose a unifying framework of land ownership where a fraction $\mu \in [0, 1]$ of the land rents are captured and returned to city residents. Using this framework, we show that the assumption that maximizing equilibrium utility and minimizing resource usage are equivalent never holds. In other words, a planner maximizing equilibrium utility would always choose a different equilibrium compared to a planner minimizing resource usage.

Note that our result is not only a technical one. Indeed, the monocentric city model, due to its tractabilty and ease of intuition, has been widely used to evaluate the impact of different urban and transportation policies. In this regard, our result implies that most of the welfare results found using the monocentric city model might change if a different welfare function and assumption about the land rents is used. This is far from desirable, especially when considering that the literature is highly fragmented, with a high number of papers using either welfare function discussed in this paper. To address this issue and provide more robust evaluations of policies in the monocentric city model, we strongly recommend considering the results presented in this paper. Policymakers and researchers should conduct sensitivity analyses across different welfare functions, enabling them to gain a clearer understanding of the robustness of their conclusions.

Chapter 3

Welfare-improving taxes in the urban equilibrium

Abstract: This paper studies welfare-improving taxes in the urban equilibrium. First, using the monocentric city model, we show that the utility-maximizing city differs from the market outcome in the absence of externalities and with homogeneous residents. The only exception is the extreme case of public land ownership in which all the differential rent is transferred lump-sum to residents. In any other case, a judiciously designed tax schedule achieves a higher equilibrium utility by increasing the city size and indirectly transferring land rents from the absentee landlords to the city residents. Then, we show that this effect is not restricted to monocentric models: using a quantitative model, we conduct a numerical analysis using the quantitative estimation of Berlin presented in Ahlfeldt et al. (2015), showing that, even in the absence of externalities, a property tax can increase the expected utility of all residents by 2.4%. Additionally, we show that a labor tax and a corporate tax can increase the expected utility by 1.0% and 1.9%, respectively. We conclude that the typical assumption that land rents accrue to absentee landlords has significant implications in welfare analyses in quantitative models of cities. The literature usually presents welfare estimations that combine general equilibrium channels with the indirect redistribution analyzed in this paper.

3.1. Introduction

Cities are a fundamental part of economic life. According to The Economist, they occupy just 2% of the earth's land surface but are home to more than half of the world's population and generate 80% of all economic output.[2] In cities, many agents interact: people, who choose residence and workplace locations, consumption and transport modes; developers and landlords, who affect the size and number of dwellings as well as the rental and land prices; the government, which, via regulations, taxes, subsidies, and transport policies, shapes the playground; and service and manufacturing firms, who demand land and labor to produce. It is the interaction of all these agents and their decisions, together with economic forces such as congestion, scale and agglomeration economies, that shape the cities' structure.

It is not surprising thus that multiple taxes and policies have been studied or proposed to increase welfare in cities. Examples include income taxes (e.g., Tscharaktschiew and Hirte, 2010), property taxes (e.g., Kono et al., 2019; Song and Zenou, 2006), city-specific minimum wage policies (e.g., Pérez, 2022) congestion pricing (e.g., Brinkman, 2016; Mun et al., 2005; Verhoef, 2005; Zhang and Kockelman, 2016a), land use regulations (e.g., Anas and Rhee, 2007; Brueckner, 2007; Turner et al., 2014), among others. All of these taxes and policies, however, are aimed at correcting an externality (e.g., congestion or agglomeration), a distortion (e.g., preexisting land regulations or labor taxes) or taking advantage of residents' heterogeneity (e.g., different skill groups or preferences).

In this paper, we deal with a fundamental question about welfare in cities: when is the urban market equilibrium welfare maximizing? We show how the assumption of land ownership makes the equilibrium differ from welfare maximization, even without market failures such as externalities. As Redding and Rossi-Hansberg (2017) highlight, "the urban economics literature has a long tradition of abstracting from land rents by postulating the existence of absentee landlords who receive all the rents but are not explicitly modeled." We find that when landlords are absentee, the market equilibrium utility can be increased by *indirectly* transferring surplus from the landlords to residents through taxes. Because benefits capitalized in land rents accrue to landowners, a judiciously designed tax schedule can indirectly redistribute the benefits to residents by softening the competition for land and decreasing land rents.

In the first part of the paper, we demonstrate analytically using the monocentric city model that, unless land rents are distributed in equal shares to residents, there is an outcome with equal utility for all city residents that brings higher welfare than the market outcome. The effect is most prominent when landlords are absentee but also exists when a share of the excess land rents accrues to residents. Importantly, our result holds even when the welfare function is the sum of the absentee landlords' rents and the surplus of citizens (measured by the compensating variation). This welfare-maximizing equilibrium can be decentralized by a location-specific tax that is an increasing function of the households' expenditure on housing and the corresponding lump-sum redistribution of total revenue. The intuition of the result is as follows. The maximum possible welfare that can be achieved in equilibrium is when all land rents accrue to residents. If this is not the case because landlords are absentee, the best alternative is to decrease overall land rents to some extent through the tax scheme.

We further show that when traffic congestion externalities are considered, equilibrium util-

[2] https://innovationmatters.economist.com/the-future-of-cities. Accessed on August 2020.

ity maximization calls for a location-specific tax that combines the Pigouvian tax and the tax that works as an alternative to complete land rent redistribution. As we show with numerical examples, the size of the second component may be significant and, for example, make the toll schedule non-monotonic and even decrease with distance in the suburbs. Also, a naive implementation of a Pigouvian toll equal to the marginal external cost could be welfare decreasing. In other words, implementing the Pigouvian toll can decrease welfare when the redistributive tax is ignored.

In the second part of the paper, we study welfare-maximizing taxes in a quantitative spatial model that features homogeneous residents and a set of discrete locations differing in productivity, amenities, and transport costs, among others. Using the parameterization of Ahlfeldt et al. (2015) without externalities for Berlin, we show that at least three different types of taxes (property, labor, and corporate) may induce a substantial welfare increase. These taxes are indistinguishable from each other in the monocentric model but have different effects under this more general framework. Using numerical analysis, we show that an ad-valorem property tax can increase the expected utility of all residents by 2.4%, while labor and a corporate tax can increase the expected utility by 1.0% and 1.9%, respectively. These results show that the effect is sizable, and its potential interaction with other policies is relevant.

The relevance of this finding is underlined in view of two phenomena in today's cities. First, there has been a surge in out-of-town home buyers in cities around the world, which detracts from the validity of the fully public land assumption. For example, Cvijanović and Spaenjers (2021), using a sample of the 1992 to 2016 period, shows that around 16.5% of the property transactions in Paris were due to non-resident buyers. Similarly, Favilukis and Van Nieuwerburgh (2021), using a sample of the 2004 to 2016 period, obtains a 10% share of out-of-town buyers in Manhattan and a 5% share for the entire New York City metro. Second, considering the tax rates for land rents, the full capture is the exception rather than the norm: for instance, in the US, rental income is subject to ordinary tax rates, ranging from 10% to 37%. Furthermore, since the US tax system treats landlords as business entities, expenses (repairs, maintenance, mortgage interest, etc.) can be used to offset the taxable rental income, lowering the effective tax rate (Sommer and Sullivan, 2018).

Our paper contributes to the literature that studies the efficient spatial distribution of economic activity. Fajgelbaum and Gaubert (2020) develop a framework that combines elements of quantitative trade and location choice models with heterogeneous workers to study the first-best (spatial) allocations and the transfers and subsidies needed to implement them. They show that there exists scope for welfare-enhancing spatial policies even when spillovers are common across locations. In this paper we show that there exists scope for welfare-improving spatial policies even in the absence of spillovers, provided that land rent (or returns to fixed factors) is not fully redistributed.

We also contribute to the literature that analyzes place-based policies. The effect presented in this paper will have significant interactions with other largely discussed policy options in cities when externalities are present. For example, in light of our results, it is straightforward to conclude that, in monocentric cities without full capture of the excess land rents, imposing an urban growth boundary is welfare-reducing since the city is already too small. Additionally, the properties of second-best congestion pricing alternatives, such as cordon charging, flat per-kilometer charges, or urban growth boundaries, may be quite different from what has been previously argued (Anas and Rhee, 2007; Brueckner, 2007; Mun et al., 2003, 2005;

Verhoef, 2005), depending on the land rent tax rate.

Our work also contributes to the recent and fast-growing literature that uses quantitative spatial models of cities to estimate the welfare effects of transportation infrastructure, among others. The treatment of land ownership in these types of models is diverse. For example, in Ahlfeldt et al. (2015), there is an absentee landlord, and the land rents are not spent within the city. Heblich et al. (2020) assumes that floor space is owned by landlords who fully spend the rents on consumption goods. In Fajgelbaum and Gaubert (2020), workers collectively own a national portfolio of the returns to fixed factors, such as land. As we show in this paper, the assumption about ownership affects the welfare analysis because the computed value will include the benefit of the policy and the benefits that come from the indirect transfer of land rents. For example, Tsivanidis (2022) estimates that 58% of welfare benefits of the Bus Rapid Transit system implemented in Bogota, TransMilenio, come from travel time savings. Production externalities and the size of the shock explain the rest. We argue that the latter general equilibrium channels are not adequately identified as they are also reflecting indirect redistribution of land rents.

The rest of the paper is structured as follows: Section 2 presents our main results in the basic monocentric city model. Section 3 shows how our main results translate to the quantitative model of Berlin. Finally, Section 4 concludes.

3.2. Welfare improving taxes in a monocentric city

We begin our analysis with the monocentric city model with homogeneous individuals. We consider absentee landlords and restrict attention to the equilibrium outcomes reached after any policy intervention. This is important because, in this model, the utilitarian first-best allocation requires that utility varies over space, and this is not possible to decentralize. Therefore, the relevant welfare measure is the (unique) resident equilibrium utility, which can be interpreted as a Rawlsian benevolent planner. It can also be thought of as the problem of a utilitarian planner that explicitly considers urban equilibrium as a constraint. The simple model allows us to isolate the main force behind our results and show that the urban market equilibrium does not always maximize equilibrium utility even in this overly –yet broadly used– simplified setting.

We closely follow the model in Brueckner (1987) and consider H identical households that locate along a closed linear city parametrized by $0 \leq x \leq \overline{x}$, where the CBD and the endogenous city boundary are located at $x = 0$ and $x = \overline{x}$, respectively.[3] All city residents work at the CBD, earn y as income, and commute from their residential location using a single mode (e.g., private car). Commuting costs increase linearly with the distance to the CBD and include both the trip's time cost and the vehicle's operating cost. Each household maximizes its utility, represented by a quasi-concave function $u(c, l)$ that depends on the consumption of a composite good c and housing l, measured as floor space, both depending on the location. The trade-off between transportation costs and dwelling size, thus, is central to the classical monocentric city model. The price of c is normalized to one irrespective of location, while the price of l is denoted by Q and varies with location. At the city boundary,

[3] Considering a circular city with a dense radial road network does not change any of the results of this Section. The only difference would be that the density function must be multiplied by $\theta(x)$, where $\theta(x)$ is the number of radians of land available at each x.

the residential rent $Q(\overline{x})$ must equal the exogenous agricultural rent Q_a.

Then, we consider that the city government can impose a tax rate on the land value to the absentee landlords equal to $\mu \in [0,1]$. The revenue from this tax is distributed equally among residents. As in Papageorgiou and Pines (1999), by varying μ, we allow for various scenarios that range from a complete redistribution of land rents ($\mu = 1$) to the traditional approach in urban economics of abstracting from these rents ($\mu = 0$). In this framework, we study whether the market equilibrium maximizes welfare, that is, it reaches the maximum equilibrium utility possible, or whether a policy intervention that requires no additional resources can induce an urban equilibrium with higher utility. The policy intervention we consider is a location-specific tax that we denote $\tau(x)$, which generates revenues that will be redistributed lump-sum to all residents. If the optimum is achieved with $\tau \equiv 0$, then a Rawlsian planner would choose the market outcome, implying that the city size, the allocation of people to dwellings, the size of those dwellings, and consumption patterns are the best that can be achieved in equilibrium. If $\tau \not\equiv 0$, then the market equilibrium is not utility-maximizing.

We first briefly characterize the equilibrium in the absence of taxes to clearly show the role of the excess land rents in the welfare analysis. The first equilibrium component is the individual maximization problem:

$$\begin{aligned} \text{max} \quad & u(c(x), l(x)) \\ \text{s.t.} \quad & y + \frac{\mu R}{H} = tx + c(x) + Q(x)l(x) \\ \text{with} \quad & R = \int_0^{\overline{x}} Q(x) - Q_a \, dx \end{aligned} \tag{3.1}$$

where Eq. (3.1) is the budget constraint, t is the commuting cost per kilometer, R is aggregate land rents, and $\mu R/H$ is the lump-sum transfer by the planner of the revenue from taxation on land rents.

The remaining components, summarized below, are as follows. No one can improve its situation by unilaterally deviating to a different location (3.2a); the residential rent equals the agricultural rent at the city boundary (3.2b); and the urban population H must fit inside the city (3.2c).

$$u(c(x), l(x)) = \overline{U}, \quad 0 \leq x \leq \overline{x} \tag{3.2a}$$

$$Q(\overline{x}) = Q_a \tag{3.2b}$$

$$\int_0^{\overline{x}} \frac{1}{l(x)} dx = H \tag{3.2c}$$

The model leads to the usual properties of urban equilibrium gradients along the city. When moving away from the CBD, housing prices decline, housing consumption increases, and population density decreases. The following proposition establishes an essential result for building intuition.

Proposition 5. *Increasing the tax rate $\mu \in [0,1]$ on the land value to the absentee landlords increases equilibrium utility. Its impact on the urban equilibrium gradients is the same as those that result from an income increase. In particular, the rent gradient rotates counter-clockwise, reducing the price of dwellings closer to the CBD. The city expands, making it less* *dense overall, and commuting costs are overall increased.*

PROOF. Appendix. □

We now move to the problem of designing taxes that increase the equilibrium utility. To study this, we formulate the following welfare maximization problem:

$$
\begin{aligned}
(P) \qquad & \max \quad \overline{U} && && (3.3a)\\
F \text{ s.t.} \quad & u(c(x), l(x)) = \overline{U}, && 0 \le x \le \overline{x} && (3.3b)\\
& \frac{\mu R}{H} + \frac{G}{H} + y = tx + \tau(x) + c(x) + Q(x)l(x) && 0 \le x \le \overline{x} && (3.3c)\\
& R = \int_0^{\overline{x}} Q(x) - Q_a \, dx && && (3.3d)\\
& \int_0^{\overline{x}} \frac{1}{l(x)} dx = H && && (3.3e)\\
& G = \int_0^{\overline{x}} \frac{\tau(x)}{l(x)} dx && && (3.3f)\\
& Q(\overline{x}) = Q_a && && (3.3g)
\end{aligned}
$$

In the problem (P), (3.3b) restricts the outcome to be an urban equilibrium in that no one can reach higher utility by reallocating. (3.3c) is the income constraint, where t denotes the commuting cost per kilometer, R denotes the excess land rents, and G denotes the tax revenue. (3.3d) defines the excess land rents, (3.3f) defines the tax revenue. (3.3e) establishes that the total urban population H fits inside the city, and (3.3g) indicates that the residential rent equals the agricultural rent at the city boundary.

By solving this problem, we obtain the result that $\tau(x)$ is different from zero unless $\mu = 1$ and that, under mild conditions, it is decreasing in x:

Proposition 6. *If the planner does not fully capture land rents, i.e., $\mu < 1$, there exists a spatially dependent tax schedule $\tau(x)$ that increases welfare if revenue is redistributed lump-sum among residents:*

$$\tau(x) = (1 - \mu)\frac{Q(x)l(x)}{|\sigma|}$$

where σ is the income-compensated price elasticity of demand for housing, $\sigma(x) = \dfrac{\partial Q(x)}{\partial x} \Big/ \dfrac{\partial l(x)}{\partial x} \cdot \dfrac{l(x)}{Q(x)}$.

PROOF. Appendix B.1. □

Proposition 7. *A sufficient condition for $\tau'(x) < 0 \; \forall x \in [0, \overline{x}]$ is that the income-compensated demand for housing σ is a non-increasing function of x and is inelastic everywhere, i.e., $\sigma > -1$. This holds, for example, for Cobb-Douglas utility functions $u(c, l) = c^{1-\beta} l^{\beta}$, where*

$\sigma(x) = \beta - 1$, and for CES utility functions $u(c,l) = ((1-\beta)c^\rho + \beta l^\rho)^{\frac{1}{\rho}}$ with $\rho < 0$. Implementing τ when these conditions hold leads to a more extended city than the market equilibrium city.

Proof. Appendix B.2 □

Proposition 6 shows that, except for the extreme case where all the excess land rents are captured, i.e., $\mu = 1$, there is a location-specific set of taxes that can achieve an equilibrium with higher utility. The tax is proportional to the expenditure on housing at each location, and, following Proposition 7, the tax decreases with the distance to the CBD under very mild conditions. For instance, when the utility is represented by a Cobb-Douglas function of the form $u(c,l) = c^{1-\beta} l^\beta$, the tax is $\tau(x) = (1-\mu)\frac{\beta}{1-\beta}\overline{y}(x)$, with $\overline{y}(x)$ being the income net of transportation costs at x.[4] Consequently, in this case, the instrument τ can be understood as an income tax with a flat rate of $(1-\mu)\frac{\beta}{1-\beta}$.

The intuition behind this result is as follows. As Proposition 5 shows, the maximum equilibrium utility is achieved when all excess land rents are redistributed to residents ($\mu = 1$). Proposition 6 shows that there is no room for welfare improvement in this case as the toll effectively becomes null. However, it also shows that as long as a fraction of land rents are not captured, welfare can be improved. Intuitively, and following from Proposition 5, as μ decreases, the equilibrium utility decreases, and the room for improvement is larger. Indeed, as land rents cannot be fully captured, a judiciously designed tax works as an alternative. The tax schedule mimics the consequences of an income increase: it makes the city center more expensive and the outskirts more attractive. As a result, it reduces the competition for space close to the city center, increases the city length, and reduces the excess land rents. The outcome is an indirect transfer of land rents from absentee landlords to the residents, yielding higher welfare.

The tax decreases the absentee landlords' rents and indirectly transfers them to the residents to achieve a higher equilibrium utility. However, its direct net effect at each location, $\tau(x) - G/H$, is not always positive. The places near the CBD face a tax that exceeds the lump sum compensation, yet –most importantly– the utility achieved in equilibrium is higher as floor space is cheaper. In a way, taxation softens the competition between individuals for central floor space. In places near the city boundary, the redistribution minus the tax, $G/H - \tau(x)$, translates into a subsidy. The floor space price increases, but it is more than compensated by the increase in income.[5]

The monocentric city model is useful for its simplicity but not sophisticated enough for quantifying tax policies. In this framework, $\tau(x)$, can be indistinguishably interpreted as a location tax, an ad-valorem property tax (see Proposition 6), and even as an income (net of commuting costs) tax. This is because location, income, and thus expenditure on housing are mechanically related. Section 3 studies welfare-improving taxes in a quantitative spatial model of a city where taxes generate different distortions.

Before moving to the quantitative analysis, it is important to highlight two key features of our analysis above. First, the results presented up to this point are rooted in the spatial setting of the urban equilibrium and are not due to the absentee landlord's exclusion from

[4] $\overline{y}(x) = \frac{\mu R}{H} + \frac{G}{H} + y - tx - \tau(x)$

[5] Formally, $G/H - \tau(x) < 0 \ \forall x \in [0, x^*)$ and $G/H - \tau(x) > 0 \ \forall x \in (x^*, \overline{x}]$, for some $x^* \in (0, \overline{x})$.

the welfare function. Second, policy instruments such as urban growth boundaries and the taxes needed to correct negative externalities interact with the tax in Proposition 6. This interaction may change the intuition developed from earlier models. We formalize these two features below.

Consider the following social welfare function that integrates both the city's residents and the absentee landlord:

$$SW = H \cdot CV(\mu) + (1-\mu)R \tag{3.4}$$

where $CV(\mu)$ corresponds to the compensating variation of residents with respect to the market equilibrium when they receive a share μ of the land rents. As Proposition 8 shows, there is a large class of utility functions where the gain in utility measured as the compensating variation is greater than the loss in the absentee landlord's rents.

Proposition 8. *Suppose $u(c, l)$ is strictly quasi-concave and $\sigma > -1$, where σ is the income-compensated price elasticity of demand for housing. Then, the market equilibrium does not maximize the social welfare function in Eq.* (3.4) *for any $\mu < 1$.*

Proof. See Appendix B.3. □

As for the policy implications of our analysis, note that Propositions 6 and 7 establish that the tax not only improves welfare but increases the city's extension. Therefore, from a welfare standpoint, the equilibrium city is too compact. Consequently, an urban growth boundary (UGB) works in the opposite direction of welfare maximization.

A more interesting interaction occurs when road congestion externalities exist. That is, an individual's commuting cost depends on the number of other people using the road. In particular, we follow the traditional approach of Arnott (1979b) and assume that the commuting costs incurred by a resident living at a distance x from the CBD are

$$t(x) = \int_0^x g(T(z))dz$$

where $T(x)$ is the total traffic flow at x, and g is such that $\frac{dg}{dT} > 0$.

In this case, by following the derivation in Appendix B.1 but with $t(x)$ as the commuting costs, we can show that the welfare maximizing toll is:

$$\tau(x) = MEC(x) + (1-\mu)\frac{p(x)q(x)}{|\sigma|} \tag{3.5}$$

where $MEC(x)$ is the marginal external cost of a trip originated at x, and the second term is the tax obtained in Proposition 6.

This result essentially shows that equilibrium utility maximization calls for a tax that combines both effects: the marginal external cost due to traffic congestion and the tax that works as an alternative to full land rent redistribution. As we show below with numerical examples, the size of the second component may be significant and, for example, make a naive implementation of a Pigouvian toll equal to the marginal external cost to be welfare decreasing. This is, implementing the Pigouvian toll decreases welfare when the redistributive tax is ignored.

3.2.1. Numerical Analysis

To build intuition about the results presented above, we simulate the urban equilibrium structure of a city and show, first, the tax schedule for different values of μ and then the interaction with traffic congestion. We base our numerical analysis on US values, with parameters summarized in Table 3.1. We use, for practical purposes, a Cobb-Douglas utility function $u(c,l) = c^{1-\beta}l^{\beta}$, setting $\beta = 0.35$. This implies that every household spends 35% of their income net of transportation costs on housing, which is consistent with the average expenditure reported by the US Department of Labor (2018)'s Consumer Expenditure Survey.[6] The hourly wage is set at US$25.27, which, following Bertaud and Brueckner (2005), is obtained using the median income per household of the 2018 US census (US$63,179) and assuming 2000 work hours/year. For the agricultural rent value, we also follow Bertaud and Brueckner (2005): we consider the average US agricultural land value for the year 2019 of US$3,160/acre. Assuming a 5% discount rate, we obtain Q_a = US$ 101,120/sq. mi. We set $H = 120,000$ households, which would be equivalent to roughly $800,000$ households in a circular (rather than linear as in our model) city.

Table 3.1: Main parameter values.

Parameter	Value
$u(c,l)$	$c^{1-\beta}l^{\beta}$
β	0.35
y [$]	63,179
Q_a [$/sq. mi]	101,120
H [households]	120,000
μ	{0.1,0.5,0.9}

In Figure 3.1, we show, for different values of μ, the resulting combination of taxes and subsidies for every location. The lower the value of μ, the more aggressive the optimal tax's effect is. For example, for $\mu = 0.1$, the tax $\tau(x)$ is 48% of the residents' net income.[7] On the other hand, for $\mu = 0.9$, the optimal tax represents only 5% of the residents' net income. This taxation has sizable impacts on the extension of the resulting city. Compared to the market equilibrium city, the utility-maximizing city is up to 16.8 miles more extended, representing a 29.8% increase (for $\mu = 0.1$). This result shows that the effects of the land ownership structure over the utility-maximizing urban form are far from negligible. The difference between the market and the utility-maximizing city becomes "small" only when the percentage of land capture is high. Even with 50% of land rent capture, the market outcome delivers a city that should expand more than 15% in extension.

[6] The US Department of Labor (2018)'s Consumer Expenditure Survey reports an average income net of taxes and transportation costs of US$57,480, of which US$20,091 is spent on housing.

[7] As noted before, when the utility is Cobb-Douglas, the optimal tax rate is constant throughout the city.

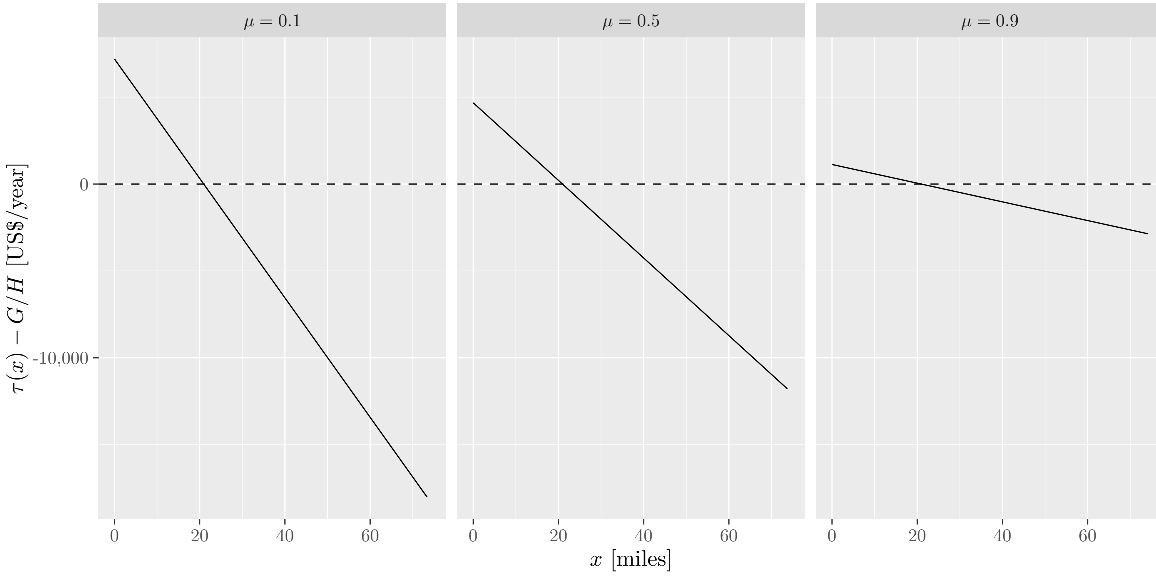


Figure 3.1: Optimal $\tau(x) - G/H$ for different values of μ.

Table 3.2: Comparison of market equilibrium against Rawlsian optimum.

Equilibrium	Market	Optimal	Market	Optimal	Market	Optimal
μ	0.1		0.5		0.9	
$\overline{U}$	49.56	50.41	55.45	55.80	62.93	62.95
x [miles]	56.51	73.35	63.23	73.73	71.76	74.15
R [millions US$]	$2,013.55$	$1,782.36$	$2,252.88$	$2,084.78$	$2,556.78$	$2,510.83$
Tax rate [%]	0	48	0	27	0	5

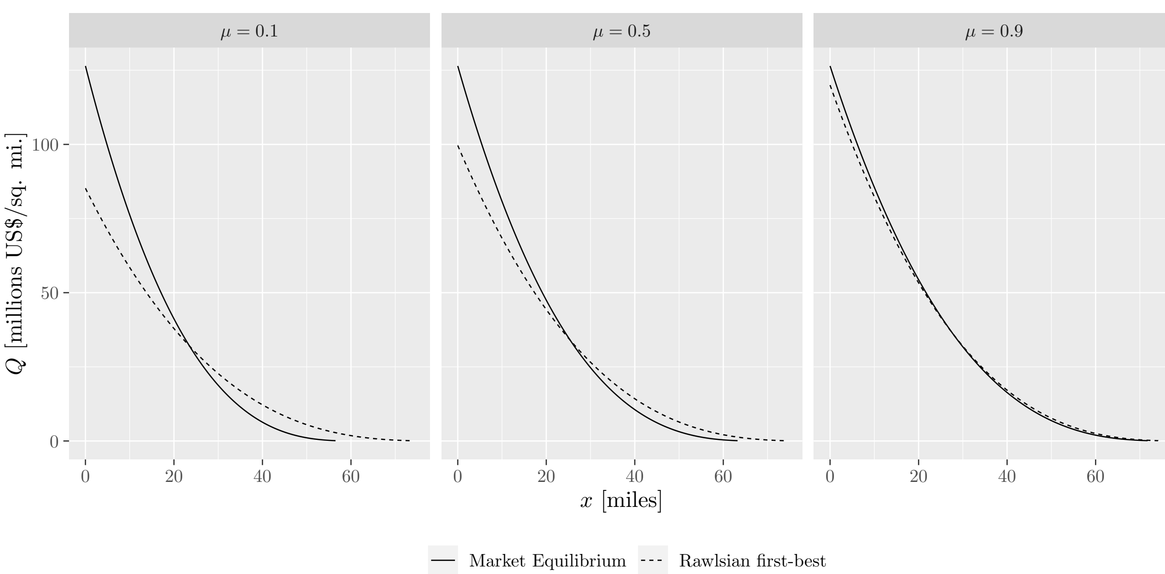


Figure 3.2: Rental prices - Market equilibrium versus Rawlsian first-best.

As a side effect of this policy, rental prices in the city center decrease (Figure 3.2). For instance, in the CBD, rental prices reduce by 32.61%, 21.20% and 5.11% for $\mu = 0.1$, $\mu = 0.5$ and $\mu = 0.9$, respectively. This implies, in return, a lower amount of excess land rents, as Table 3.2 shows. Finally, although the utility gains when using the optimal redistribution policy seem modest (ranging from 0.03% to 1.71% in our example, as shown in Table 3.2), this is expected and in line with estimations of surplus gains when using, for example, optimal road pricing policies (see, e.g., Verhoef, 2005).

3.2.1.1. Traffic congestion

To include traffic congestion, we consider a commuting cost per mile given by the Bureau of Public Roads (BPR) function, in line with recent papers studying congestion pricing in the monocentric city model (e.g. De Lara et al., 2013; Li et al., 2012; Tikoudis et al., 2018). In particular, we consider the following BPR function:

$$g(T(x)) = t_0 + c_t t_1 \left[1 + \rho_1 \left(\frac{T(x)}{K} \right)^{\rho_2} \right] \tag{3.6}$$

where t_0 is the per-mile monetary cost of the trip, c_t is the value of travel time, t_1 is the free-flow travel time per mile, $T(x)$ is the total traffic at x, K is the capacity of the road, and ρ_1 and ρ_2 are positive constants.

For the monetary cost, t_0, we use the current US federal allowance for business mileage of US$ 0.575/mile, which translates to $t_0 =$ US\$359.38/mile when considering 1.25 workers/-household, 250 working days/year, and two daily trips. To obtain the time cost component, we assume that the commuting time is valued at the hourly wage rate, i.e., $c_t =$ US\$25.27. Then, we consider a free-flow speed of 50 miles/h, leading to an annualized free-flow travel time of $t_1 = 12.5$ h/mile.[8] Finally, we assume a road capacity of $K = 54,000$ veh/h, and we consider widely used parameter values for the BPR function, $\rho_1 = 0.15$, $\rho_2 = 4$ (Small and Verhoef, 2007). These values produce a city that presents an average commuting speed of 22 miles/h and an average commuting time between 47 and 59 minutes.[9] The average travel speed and commuting time are roughly consistent with those reported by the 2017 National Household Travel Survey (McGuckin and Fucci, 2018).

Table 3.3 provides a summary of the results, comparing the unpriced equilibrium against two congestion pricing policies: charging the marginal external cost of every trip (the Pigouvian toll) and charging the optimal toll from Equation (3.5). The utility gains when charging the optimal congestion pricing toll are up to 1.1% and 1.7% with respect to the city with no pricing and to the city under marginal cost pricing, respectively. As discussed in Verhoef (2005), this is expected and in line with estimations of surplus gains from optimal road pricing in urban areas. Nevertheless, charging the optimal congestion toll has a sizable effect on the urban form. In Figure 3.3, we show that for the parameters considered, the component of the optimal pricing scheme that comes from land rents offsets the marginal external cost, resulting in a non-monotonic toll that decreases with distance in the outer city. This result

[8] We get this free-flow speed assuming that, on average, 80% of every trip is made using highways at a free-flow speed of 60 miles/h, while the other 20% uses main roads at a free-flow speed of 20 miles/h. This gives a free-flow speed of 52 miles/h, which we round to 50 miles/h.

[9] To obtain the travel speed in our model, we consider the time component of $g(T(x))$ to obtain the equivalent travel time, and with this, the travel speed.

has profound implications. First, the optimal city ends up being more extended than the unpriced city for a wide range of values of μ. For $\mu = 0.1$, the city under the optimal pricing is 17.3 miles longer than the unpriced city, representing a 22.5% increase. For $\mu = 0.5$, the optimal city is 11.2% longer than the unpriced city. Only when μ reaches 0.9 is the optimal city marginally smaller than the unpriced city; note that the optimal toll is still decreasing with distance in the final part of the city.

At the same time, the optimal toll reduces the excess land rents for low values of μ by means of decreasing rental prices in the inner city. This reduction is once again sizable (4.4% for $\mu = 0.1$). A straightforward implication follows: for most values of μ, charging only the marginal external cost operates precisely in the opposite way as the optimal toll, reducing the extension of the city and the aggregate mileage while increasing the excess land rents. Thus, welfare might be decreased in these situations compared to the no-pricing situation. It is only for high values of μ that the optimal toll is closer to the Pigouvian toll, producing similar effects on the urban form: for $\mu = 0.9$, the city under the optimal pricing compared to the unpriced city is smaller (0.2 miles or a 0.25% reduction), with a decreased aggregate mileage (-8.3%), and leads to an increase in excess land rents (8.2%).

Table 3.3: Equilibrium results - No congestion pricing vs. marginal cost pricing vs. optimal pricing.

Congestion Policy	Unpriced	Pigouvian	Optimal	Unpriced	Pigouvian	Optimal	Unpriced	Pigouvian	Optimal
μ		0.1			0.5			0.9	
Equilibrium utility $\overline{u}$	52.12	51.66	52.55	57.80	57.79	58.16	64.88	65.58	65.60
City extension [miles]	76.80	72.72	94.07	85.17	81.36	94.68	95.60	92.32	95.36
Aggregate land rents [millions US$]	1,862.97	2,012.29	1,780.94	2,066.04	2,251.31	2,083.11	2,318.79	2,554.77	2,508.82
Aggregate mileage [thousand miles]	2,551.57	2,216.74	2,911.85	2,829.70	2,480.04	2,912.14	3,175.91	2,814.31	2,912.30

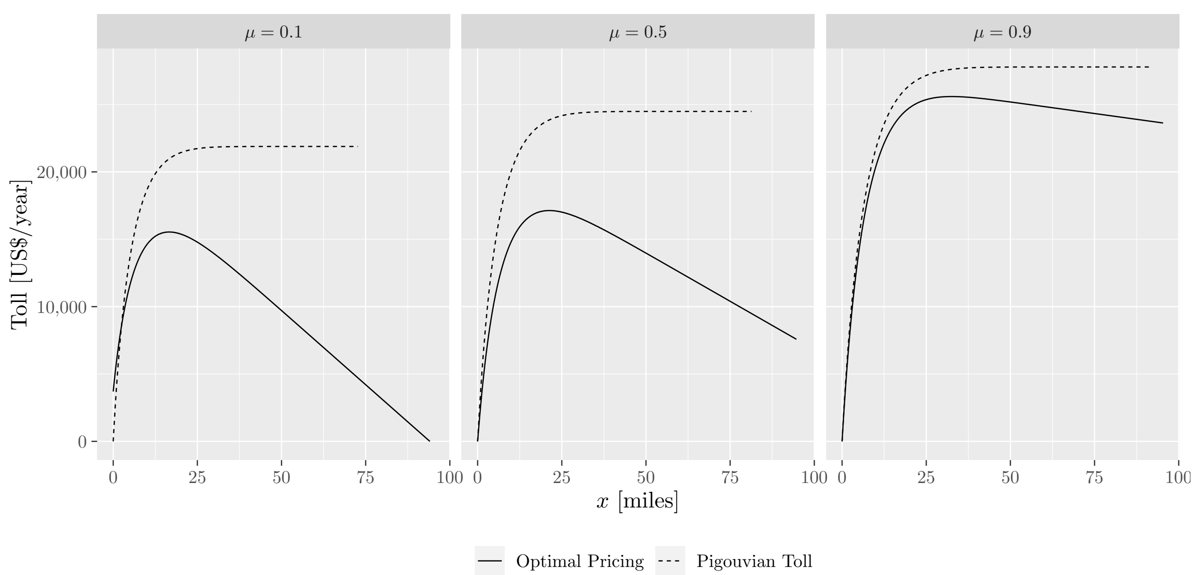


Figure 3.3: Pigouvian versus optimal toll.

3.3. Welfare improving taxes in a quantitative model

We now study welfare-improving taxes in a richer model that allows for quantifying the general equilibrium effects of implementing taxes. Unlike the monocentric model, the quantitative model of a city includes the firm's location choice, endogenous wages, residential and commercial floorspace developers, and commuting costs that depend on the transportation network. We first generalize the main result of Section 2 by showing that when not all land rents are redistributed and in the absence of externalities, the equilibrium utility can be increased by implementing judiciously designed taxes. We then numerically study the impact of three instruments (a property tax, an income tax, and a corporate income tax) using the structural estimation of Berlin provided in Ahlfeldt et al. (2015).

3.3.1. The Model

The exposition of the model and notation follows closely that of Ahlfeldt et al. (2015). However, in the theoretical analysis, we simplify some elements that are not key to our study. Consider a closed city consisting of S discrete blocks, each one with a land area denoted by K_i, $i = 1, \ldots, S$. Land space is divided among households and firms. In particular, in every block i, an exogenous amount K_{Li} is allocated to develop commercial floorspace, while the remaining amount, K_{Mi}, is destined for developing dwellings. Floor space is provided by developers that use the land as their only input. Therefore, even though the land is exogenously allocated between land uses, the square meters built for commercial and residential use at every location are endogenous. We assume that an exogenous number of workers H populate the city, and they attain an endogenous expected utility $\overline{U}$.

The treatment of land ownership in these types of models is diverse. For example, in Ahlfeldt et al. (2015), there is an absentee landlord, and the land rents are not spent within the city. Heblich et al. (2020) assumes that floor space is owned by landlords who fully spend the rents on consumption goods. In Fajgelbaum and Gaubert (2020), workers collectively own a national portfolio of the returns to fixed factors, such as land. To simplify the analysis, we consider that floor space developers own the land they use. However, the city government can impose a tax rate on the land rents equal to $\mu \in [0, 1]$, and then distribute its revenue equally among residents. Thus, μ is the share of the returns to land that accrues to workers in equal parts and $1 - \mu$ is the share that is not spent within the city. When $\mu = 0$, we recover the absentee landlord assumption and when $\mu = 1$, the collective ownership.

3.3.1.1. Workers

Every worker chooses a residential location and a working location. The utility U_{ij} that a worker obtains when living in block i and working in block j is assumed to take a Cobb-Douglas form with parameter $0 < \beta < 1$. Utility is derived from the consumption of a numeraire good c_{ij} and housing l_{ij}:

$$U_{ij} = \frac{B_i z_{ij}}{d_{ij}} \left(\frac{c_{ij}}{\beta}\right)^{\beta} \left(\frac{l_{ij}}{1-\beta}\right)^{1-\beta} \tag{3.7}$$

In Equation 3.7, B_i represents residential amenities in block i. d_{ij} is an iceberg commuting cost, $d_{ij} = \exp\{\kappa t_{ij}\}$, where t_{ij} is the commuting time between blocks i and j. z_{ij} is an idiosyncratic utility shock capturing the idea that workers can have idiosyncratic reasons for living and working in different blocks in the city. We assume that the shock z_{ij} is drawn from a Frechet distribution with scale parameters $T_i > 0$ and $E_j > 0$, and shape parameter $\varepsilon > 1$:

$$F(z_{ij}) = e^{-T_i E_j z_{ij}^{-\varepsilon}} \tag{3.8}$$

After observing her realizations for idiosyncratic shocks, workers choose a residential and working block to maximize her utility, subject to an income constraint:

$$c_{ij} + l_{ij} Q_i + \tau_{ij} = w_j + I \tag{3.9}$$

In Equation 3.9, Q_i is the residential floor space price in block i, τ_{ij} is a location-specific tax, w_j is the wage paid at block j, and I is the lump-sum transfer from the government of the tax revenue. With this, utility maximization allows us to obtain the indirect utility from living in block i and working in block j, u_{ij}:

$$u_{ij} = \frac{z_{ij} B_i \left[w_j - \tau_{ij} + I\right] \left[Q_i\right]^{\beta-1}}{d_{ij}} \tag{3.10}$$

Using well-known properties from the Frechet distribution, the probability that a worker chooses to live in block i and work in block j, π_{ij}, is given by:

$$\pi_{ij} = \frac{T_i E_j \left(d_{ij} \left[Q_i\right]^{1-\beta}\right)^{-\varepsilon} \left(B_i \left[w_j - \tau_{ij} + I\right]\right)^{\varepsilon}}{\sum_{r=1}^{S} \sum_{s=1}^{S} T_r E_s \left(d_{rs} \left[Q_r\right]^{1-\beta}\right)^{-\varepsilon} \left(B_r \left[w_s - \tau_{rs} + I\right]\right)^{\varepsilon}} \equiv \frac{\Phi_{ij}}{\Phi} \tag{3.11}$$

Using Equation (3.11), and letting R_i denote the developers' profit in block i, the lump sum transfer is given by:

$$I = \frac{G}{H} + \frac{\mu \sum_i R_i}{H} \tag{3.12}$$

where $G = H \sum_{i=1}^{S} \sum_{j=1}^{S} \pi_{ij} \tau_{ij}$ is the revenue from the location specific tax and

The number of workers living in block i, H_{Ri}, is given by:

$$H_{Ri} = H \sum_{j=1}^{S} \pi_{ij} \tag{3.13}$$

Finally, similarly to Ahlfeldt et al. (2015), the expected utility that workers obtain, $\overline{U}$, is given by:

$$\overline{U} = \gamma \Phi^{1/\varepsilon} \tag{3.14}$$

where $\gamma = \Gamma\left(\frac{\varepsilon-1}{\varepsilon}\right)$, with $\Gamma(\cdot)$ the Gamma function.

3.3.1.2. Firms

Firms operate under constant returns to scale, using floor space L_M and labor H_M as inputs to produce a single numeraire good. The production technology is assumed to take a Cobb-

Douglas form with parameter $0 < \alpha < 1$, so that the output in block j, y_j, is given by:

$$y_j = A_j H_{Mj}^{\alpha} L_{Mj}^{1-\alpha} \tag{3.15}$$

In Equation (3.15), A_j is the productivity parameter of firms located in block j. Firms choose labor and floor space to maximize its profit Π_j, as shown in Equation (3.17), where q_j is the commercial floor price. We assume that the numeraire good is freely traded, and consequently:

$$\sum_j y_j = H \sum_{ij} \pi_{ij} c_{ij} \tag{3.16}$$

Profit maximization and the zero profit condition allow us to characterize the relative demand for each input (Equation 3.18) and the equilibrium commercial floor price, q_j (Equation 3.19) for each block j where firms operate.

$$\Pi_j = y_j - w_j H_{Mj} - q_j L_{Mj} \tag{3.17}$$

$$H_{Mj} = \left(\frac{\alpha A_j}{w_j}\right)^{1/(1-\alpha)} L_{Mj} \tag{3.18}$$

$$q_j = (1-\alpha)\left(\frac{\alpha}{w_j}\right)^{\alpha/(1-\alpha)} A_j^{1/(1-\alpha)} \tag{3.19}$$

Finally, the job market-clearing condition equates the number of workers employed in a block j with the number of workers choosing to work in that block:

$$H_{Mj} = \sum_{s=1}^{S} \frac{\pi_{is}}{\sum_{j=1}^{S} \pi_{ij}} H_{Ri} = \sum_{s=1}^{S} \frac{E_s\left(\left[w_s - \tau_{is} + G/H + \mu \sum_l R_l/H\right]/d_{is}\right)^{\varepsilon}}{\sum_{j=1}^{S} E_j\left(\left[w_j - \tau_{ij} + G/H + \mu \sum_l R_l/H\right]/d_{ij}\right)^{\varepsilon}} H_{Ri} \tag{3.20}$$

3.3.1.3. Land market

We assume that floor space L is the output of a sector that uses land K as input. Additionally, we consider that land developers own the land they use. However, the city government can impose a tax rate on the land rents equal to $\mu \in [0,1]$. Consequently, developers only face a fraction μ of the land price. Then, the block i construction sector's profit maximization problem is given by:

$$\begin{aligned} \Pi_i^C =& (Q_i L_{Ri}(K_{Ri}) + q_i L_{Mi}(K_{Mi}) - \mu \mathbb{R}_{Ri} K_{Ri} - \mu \mathbb{R}_{Mi} K_{Mi}) \\ \text{s.t.} \quad & K_{Li} \leq \overline{K}_{Li} \\ & K_{Mi} \leq \overline{K}_{Mi} \end{aligned}$$

where $\mathbb{R}_i$ and $\mathbb{R}_{Mi}$ are the residential and commercial land prices, respectively. With this, we define R_i as:

$$R_i = \mu \mathbb{R}_{Ri} K_{Ri} + \mu \mathbb{R}_{Mi} K_{Mi} \tag{3.21}$$

Residential land clearing follows from workers' utility maximization, combined with the distribution of the idiosyncratic utility:

$$(1-\beta)\frac{\mathbb{E}\left(w_s - \tau_{is} + G/H + \mu\sum_l R_l/H\right| i)\, H_{Ri}}{Q_i} = L_{Ri} \tag{3.22}$$

In Equation 3.22, $\mathbb{E}\left(w_s - \tau_{is} + G/H + \mu\sum_l R_l/H\right| i)$ is the expected net income obtained by workers living in block i (Equation 3.23).

$$\mathbb{E}\left(w_s - \tau_{is} + G/H + \mu\sum_l R_l/H\middle| i\right) = \sum_{s=1}^{S}\pi_{is|i}\left(w_s - \tau_{is} + G/H + \mu\sum_l R_l/H\right) \tag{3.23}$$

In Equation (3.23), $\pi_{is|i}$ is the probability that, conditional on living in block i, a worker chooses to work in block s:

$$\pi_{is|i} = \frac{\pi_{is}}{\sum_j \pi_{ij}} = \frac{E_s((w_s - \tau_{is} + G/H + \mu\sum_l R_l/H)/d_{is})^{\varepsilon}}{\sum_{j=1}^{S} E_j((w_j - \tau_{ij} + G/H + \mu\sum_l R_l/H)/d_{ij})^{\varepsilon}} \tag{3.24}$$

On the other hand, commercial land clearing follows from the profit maximization and zero-profit conditions:

$$\left(\frac{(1-\alpha)A_j}{q_j}\right)^{1/\alpha} H_{Mj} = L_{Mj} \tag{3.25}$$

3.3.2. The taxes

In this Section, we numerically study the impact of three instruments that try to proxy the optimal redistribution presented in the previous section: (i) a property tax τ^V, such that residents and firms face floorspace prices $(1+\tau^V)Q_i$ and $(1+\tau^V)q_i$ respectively, (ii) an income tax, such that residents receive wages $(1-\tau^I)w_j$, (iii) a corporate income tax, such that firms revenue is $(1-\tau^F)y_j$. In any case, we consider that the taxes revenue is returned equally among residents. For this numerical analysis, we use the structural estimation of Berlin provided in Ahlfeldt et al. (2015).

Before moving on to the numerical analysis, we deem it essential to discuss how each of the taxes affects the agents' decisions and the mechanism by which land rents may change. The income tax is linear with a rate of τ_I, and it works similarly as in the monocentric city model. Relative to the unregulated equilibrium, a flat-rate income tax makes the locations where firms offer higher wages less attractive. Therefore, it directly impacts those firms and their willingness to pay for commercial floor space. In equilibrium, the first-order effect of the tax should be to lower floor space prices in otherwise more expensive places and increase the floor space prices in otherwise cheaper places. Consequently, the linear income taxation should soften the competition for floor space and reduce the aggregate land rents.

The second instrument is an ad valorem property tax rate τ_V applied equally to residential and commercial properties. Its effects are straightforward, as it directly impacts the willingness to pay for floor space of households and firms, and thus shifts the demand downwards. As a result, it decreases aggregate land rents.

Finally, we study the implementation of a corporate income tax τ_F. As the equilibrium is perfectly competitive and the production technology exhibits constant returns to scale, it only directly affects the aggregate output and, therefore, the demand for labor and floor space. In this case, the taxation works as a proportional reduction in the location-specific productivity (A_j). Thus, its impacts are relatively stronger for the more productive workplaces, which are the ones with a higher willingness to pay for floor space. An analogous mechanism explains the effect on wages. In summary, both effects work in the direction of reducing land rents.

3.3.3. Numerical Analysis

In this Section, we numerically study the existence of welfare-improving taxes for the parametrization of Berlin, provided in Ahlfeldt et al. (2015), which in the model presented in Section 3.3.1 is equivalent to set $\mu = 0$ and $\tau^I = \tau^V = \tau^F = 0$. The region is characterized by 12,309 statistical blocks ("Blöcke"), each with an area of around 50,000 square meters. Figure 3.4 depicts the distribution of these blocks in the city of Berlin.

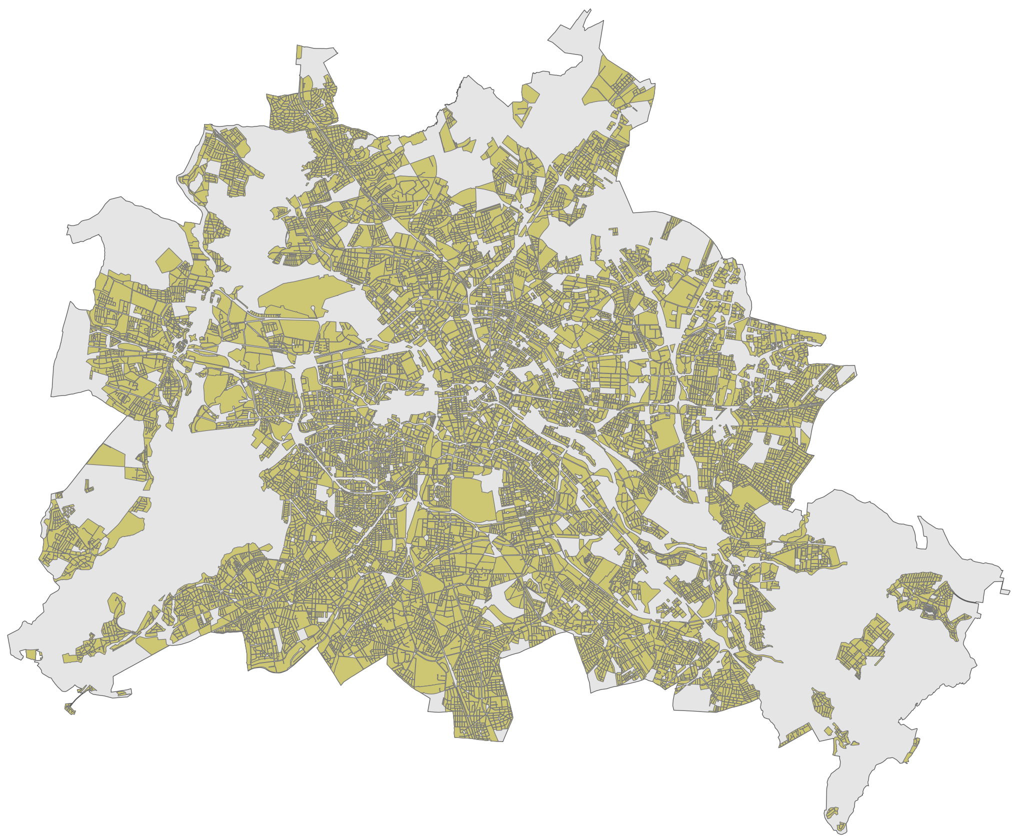


Figure 3.4: Statistical blocks - Berlin.

Using data for 2006, the authors provide the structural estimation of unobserved location characteristics using the observed variables $\{\mathbb{Q}, \boldsymbol{H}_M, \boldsymbol{H}_R, \boldsymbol{t}, \boldsymbol{K}\}$, and the model's parameters $\{\alpha, \beta, \varepsilon, \kappa, \lambda\}$.[10] Since most of the unobserved parameters enter the model isomorphically,

[10] $\{\alpha, \beta, \varepsilon, \kappa, \lambda\} = \{0.2, 0.25, 6.83, 0.01, 0.75\}$. In addition, we consider $R_a = 0$ and $\mathbb{P} = 1$.

the authors define the following composites:

$$\tilde{A}_i = A_i E_i^{\alpha/\varepsilon} \tag{3.26}$$

$$\tilde{B}_i = B_i T_i^{1/\varepsilon} \zeta_{Ri}^{1-\beta} \tag{3.27}$$

$$\tilde{w}_i = w_i E^{1/\varepsilon} \tag{3.28}$$

where ζ_{Ri} takes value 1 for completely specialized residential blocks and value ξ_i for blocks with some commercial use. Then, there exist unique vectors of unobserved location characteristics $\{\tilde{\boldsymbol{A}}, \tilde{\boldsymbol{B}}, \tilde{\boldsymbol{w}}\}$ that are consistent with the observed data. Our numerical analysis starts from the structural estimation of these unobserved composites obtained in Ahlfeldt et al. (2015). Nevertheless, note that the scale parameters $\boldsymbol{E}$ cannot be uniquely identified from the observed variables. Consequently, to evaluate the equilibrium under different scenarios numerically, we draw values for $\boldsymbol{E}$ from a uniform distribution $U[0,1]$.[11] With these values, we can obtain values for the productivity parameters $\boldsymbol{A}$ using Equation (3.26). Finally, note that the unobserved parameters B_i and T_i still enter the model isomorphically. Thus, the values for the composites $\tilde{B}_i$ are enough to evaluate the model numerically.

3.3.3.1. Ad-valorem property tax

Then, we consider taxes in this city when $\mu = 0$. First, we start by studying the effect of the ad valorem property tax. In Figure 3.5, we show the relative efficiency of this tax for different values of τ^V, noting that τ^V represents the market city. From this figure, departing from the market city by imposing an ad valorem property tax, and then redistributing it lump sum increases the expected utility of residents. In particular, the expected utility can be increased by up to 2.4% by using a tax rate of 36%.

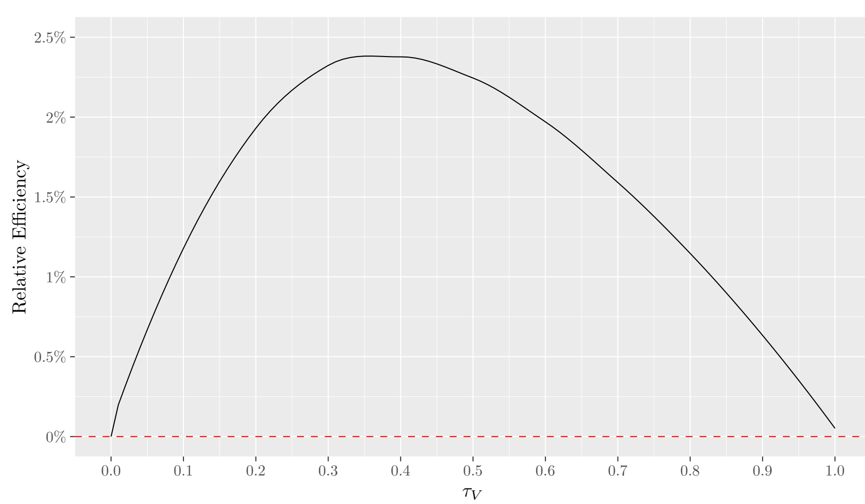


Figure 3.5: Relative efficiency of the ad valorem property tax, for different values of τ_V.

The main force behind the expected utility's gain is an indirect transfer from landowners

[11] Computational experiments show that drawing values for $\boldsymbol{E}$ from a different distribution does not change the qualitative results presented in this Section, and does not greatly change the effect size. Consequently, we choose a uniform distribution for simplicity.

to city residents by reducing the excess land rents, similar to the effect presented in the monocentric city model. Indeed, one of the effects of the property tax is a change in the marginal rate of substitution between the numeraire good and residential floor space in the case of individuals, and between labor and commercial floor space in the case of firms. Consequently, firms and residents reduce their floor space consumption (Figure 3.6). Thus, the output of developers decreases, and since the land input is fixed, the equilibrium land prices decrease (Figure 3.7). In turn, this translates into a decrease of 30.6% in the excess land rents.

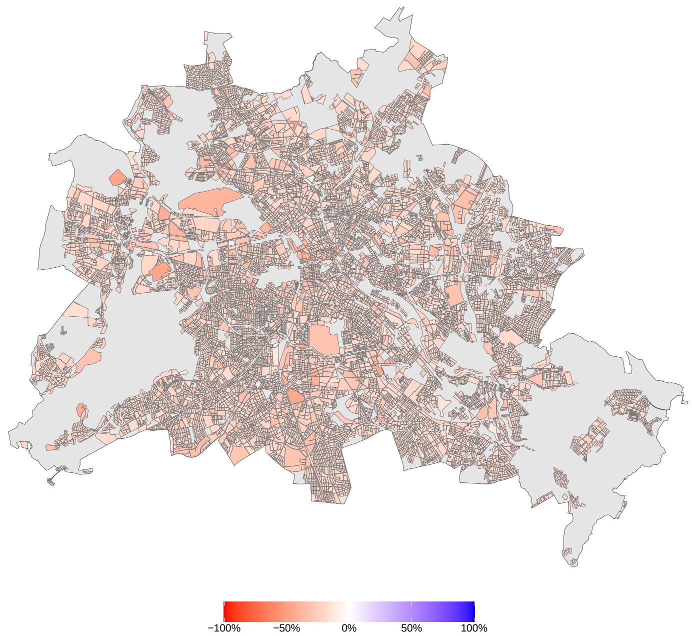


Figure 3.6: Percentage change in floor space **L** - Optimal τ^V versus market city.

3.3.3.2. Corporate income tax

We show in Figure 3.8 the relative efficiency of the corporate income. Like the property tax, departing from the market city by setting a corporate tax increases the expected utility of residents. In particular, the expected utility can be increased by up to 1.9% by using a corporate income tax of 21%. It is only for very high values of τ_F that the tax produces an outcome with a lower expected utility than the benchmark.

The intuition behind this result is similar to the one for the ad-valorem tax: taxing the firms' revenue is equivalent to reducing the price of the output produced by firms, which leads to lower production. Then, since we are considering a closed city, where total labor is fixed, firms respond by reducing their floor space usage in general (Figure 3.9). As mentioned above, this ultimately translates into lower excess land rents, with a decrease of 16.9% compared to the market city. Note, however, that since this tax only affects firms, the magnitude of the effect is smaller than with a property tax.

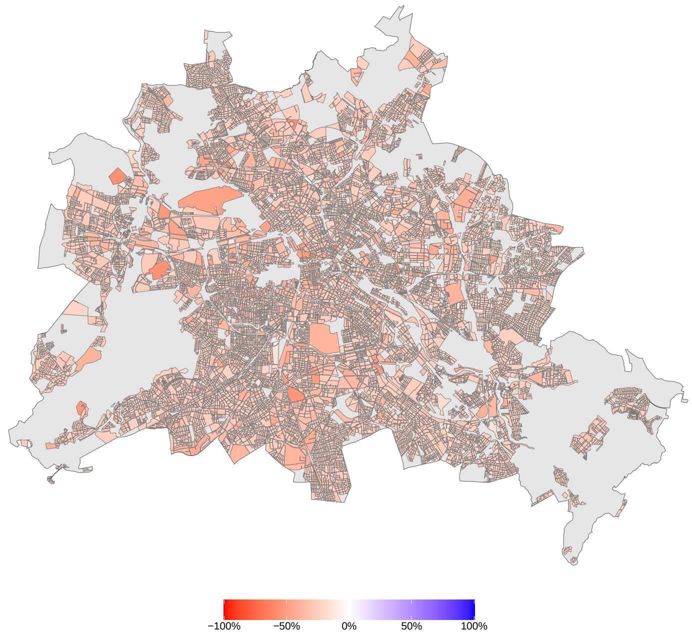


Figure 3.7: Percentage change in land price $\mathbb{R}$ - Optimal τ^V versus market city.

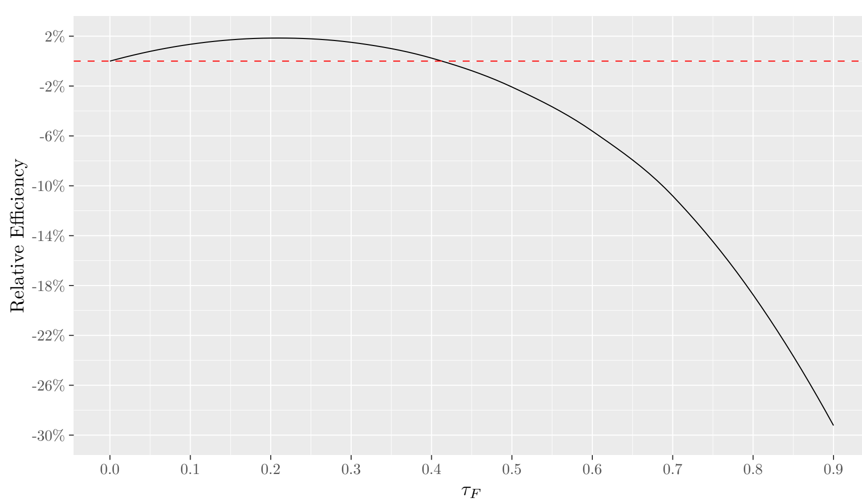


Figure 3.8: Relative efficiency of the corporate income tax, for different values of τ_F.

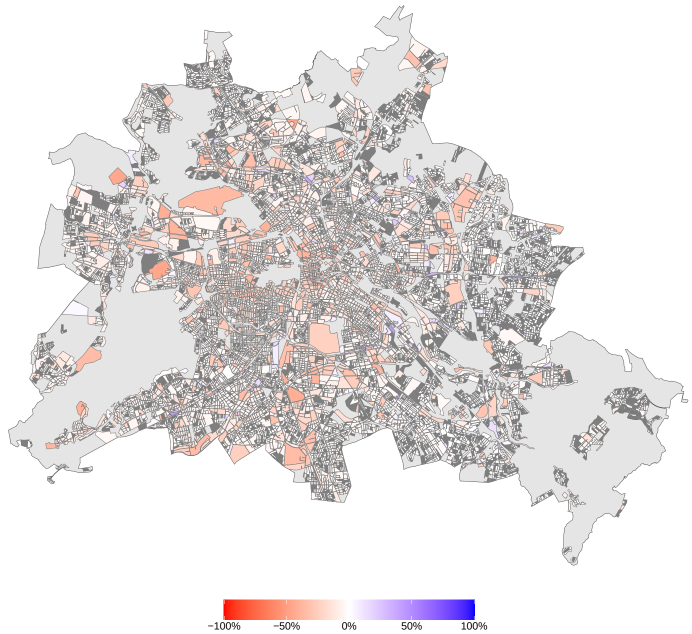


Figure 3.9: Percentage change in commercial floor space $\theta\mathbf{L}$ - Optimal τ^F versus market city.

3.3.3.3. Income tax

Finally, we show in Figure 3.10 the relative efficiency of the income tax for different values of τ_I. Our results show that the expected utility can be increased by up to 1.0% by using an income tax of 23%. To see the mechanism behind this increase in the expected utility, note that the income tax can be interpreted as a labor tax imposed on employers. If we denote $\overline{w}_j \equiv w_j(1-\tau^I)$, then city residents' get paid $\overline{w}_j$ when working on block j, while the cost of labor for firms is given by $w_j = \overline{w}_j/(1-\tau^I)$. Thus, any $\tau^I > 0$ increases firms' labor cost. Then, when considering that total labor is fixed, firms react by substituting towards floor space but also by decreasing total output. In particular, the aggregate production when considering the optimal τ^I is 26.7% lower than in the market city.

This decrease in the output translates, in general, to a lower use of floor space for production, as Figure 3.11 reveals. As a consequence, excess land rents decrease by 30.9%, compared to the market city. Even though the size of the reduction in the land rents is higher than with, for example, the corporate income tax, note that the welfare-increasing effect of this income tax is partially offset by residents facing lower wages –a consequence of the labor tax–. Indeed, the average wage in the city decreases by 31.0%.

3.3.3.4. Comparison

Finally, similarly to the monocentric city model, the welfare-improving effect of the different taxes decreases when the government can capture a fraction of the excess land rents. Table 3.4 shows that the relative efficiency of the property, income, and corporate tax is only 0.24%, 0.24% and 0.07%, respectively, when $\mu = 0.9$: since city residents receive a 90% share of the

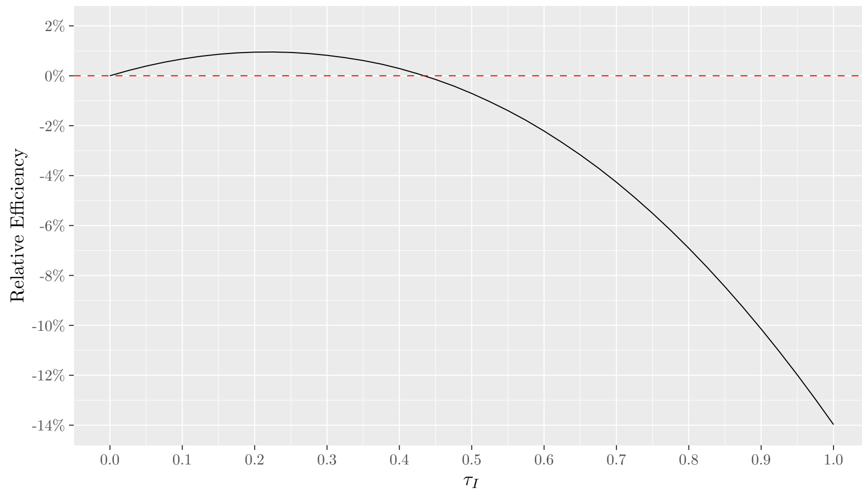


Figure 3.10: Relative efficiency of the income tax, for different values of τ^I.

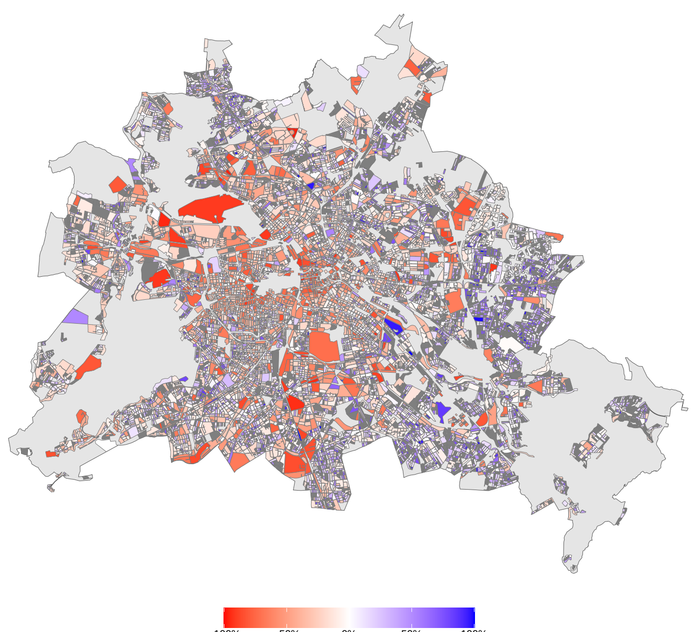


Figure 3.11: Percentage change in commercial floor space $\theta\mathbf{L}$ - Optimal τ^I versus market city.

excess land rents, it is not desirable to highly reduce them. In fact, for example, the optimal property tax reduces the excess land rents by 26.7% when $\mu = 0.1$, but only by 17.4% and 7.4% when $\mu = 0.5$ and $\mu = 0.9$, respectively.

Table 3.4: Comparison of market equilibrium against the equilibrium when using the optimal taxes.

Equilibrium	Market	τ_V	τ_F	τ_I	Market	τ_V	τ_F	τ_I	Market	τ_V	τ_F	τ_I
μ	0.1				0.5				0.9			
$\overline{U}$	5.27	5.37	5.35	5.31	5.54	5.57	5.56	5.56	5.81	5.83	5.82	5.82
Relative Efficiency	-	1.81%	1.42%	0.68%	-	0.68%	0.50%	0.43%	-	0.24%	0.19%	0.07%
R	195,791	143,013	164,409	185,606	199,113	164,395	175,230	191,224	202,599	187,698	190,465	195,811
Tax rate [%]	0	34	19	21	0	20	15	17	0	15	8	8

3.4. Concluding remarks

In this paper, we study welfare improving taxes in cities of equals in the absence of externalities. For this, first, we present a monocentric city model with mixed ownership, where city residents in equal shares jointly own a fraction μ of the land, and an absentee landlord owns the rest $(1 - \mu)$. We show that for any $\mu \neq 1$, a Rawlsian planner would employ a revenue-neutral combination of location-specific taxes and a lump-sum redistribution to transfer income from the city center to the outskirts. The intuition behind this result is that this instrument serves as a second-best alternative to full land rent capture, increasing residents' utility at the expense of the absentee landlord by means of lowering excess land rents. Previous studies have asserted that a Rawlsian planner would choose the market outcome, but since this efficiency result only holds for the full land rent capture scenario, we believe that this is, most likely, not consistent with ownership structures observed in reality. For example, μ may be interpreted as either the property tax rate or the tax rate on rental income; therefore, low values of μ should be considered. For instance, in the US, the tax rate on rental income ranges from 10% to 37%. On the other hand, realistic values for property tax rates in the US range from approximately 0.5% to 2.4% of the property value (Song and Zenou, 2006). However, since in our model the tax is levied on rents, following Brueckner and Kim (2003) and using a 5% discount rate, realistic property tax rates translate to values of μ, ranging from 0.1 to 0.5. Importantly, the results we present are rooted in the spatial setting of the urban equilibrium and are not due to the absentee landlord's exclusion from the welfare function. Indeed, we show that, under some mild conditions, even when absentee landlords are included in the welfare function, a redistribution of income increases welfare.

The issue of which model of land ownership is better suited goes beyond the monocentric city model. For example, Tsivanidis (2019) studies the welfare effects of transit infrastructure using a quantitative urban model, while considering, first, a public ownership scenario, and then, absentee landlords. Zhang and Kockelman (2016b) extend the model of Lucas and Rossi-Hansberg (2002) to incorporate congestion externalities, analyzing welfare under different policies for the case where rents are uniformly redistributed to residents. Diamond and McQuade (2019) estimate spillovers of affordable housing developments in a non-parametric setting, studying the welfare impacts on homeowners, renters, and absentee landlords. Thus, to show that the effect we present is not restricted to the monocentric city model, we study welfare improving taxes in the quantitative model of Berlin presented in Ahlfeldt et al. (2015), which features an absentee landlord. Using the parametrization provided by the authors, we find that, even in the absence of externalities, a property tax can increase the expected utility of all residents by 2.4%. Additionally, we show that a labor tax and a corporate tax can increase the expected utility by 1.0% and 1.9%, respectively. The main force behind these welfare gains is the same as in the monocentric city model: the redistributing instrument reduces competition for space, thus decreasing the excess land rents.

Although part of these results has been previously suggested (see the Appendix in Kanemoto, 1977, for a short discussion about the absentee landlord case), to the best of our knowledge, this is the first paper that formalizes these facts and shows that, in general, the market outcome in the absence of externalities does not maximize equilibrium utility for any case other than public ownership. We believe this formalization is fundamental to the literature since it emphasizes that policy results might be widely different depending on the nature of land ownership. Indeed, the effect presented in this paper will have significant interactions with urban policies in cities without full land rent capture when externalities are present, since any pricing instrument should include a redistributing component and depart from the marginal cost. In addition, in the case of welfare analysis in a monocentric city without full land rent capture, it is straightforward to conclude that policies aimed to restrict the city size are undesirable, since the city is already too small.

Naturally, some caveats to the previous results need to be considered, although we do not believe the qualitative results will be different when they are considered. For example, note that even though we show that our results hold if we include the absentee landlord in the welfare function, for a fully closed analysis of welfare, one might want to consider that landlords locate within the city. In this case, although the extent of previous implications might change, the main force behind our results would still be present: since landlords would end up with a higher income than renters, the model would become one with household heterogeneity. Regardless of the equilibrium location of different income groups, our result would hold within each group, as all residents of the same income group must achieve the same utility level and would, therefore, have different marginal utilities of income. Our result could also hold between groups if low-income groups locate in the suburbs. As they have lower marginal utilities of income, there would be gains by reallocating income between groups. Specifically, when the income elasticity of commuting costs is greater than the income elasticity of the demand for housing, landlords will occupy the inner city, while renters would locate in the outer city (Duranton and Puga, 2015). Since a Rawlsian planner would only care about the renters, and provided that the fraction μ is low enough, a redistribution from the city center to the outskirts should be welfare increasing, since marginal utilities of income would still increase with distance (within each income class).

Chapter 4

Efficiency of transport policies: The role of redevelopment

Abstract: This paper studies the efficiency and substitutability of three transport policies (congestion pricing, public transport subsidies, and dedicated bus lanes) in a non-monocentric model that allows for relocation and redevelopment. We find that (i) dedicated lanes are able to attract users to the transit system much more effectively than the other two policies, and (ii) the substitutability between policies is large. In other words, when any first policy is implemented, the welfare increase due to the introduction of a second policy is only moderate. Additionally, by using a stylized setting of cyclical redevelopment, we analyze these transport policies' impacts not only in the long run but also in the medium run, where redevelopment is only partially possible and when the core of the urban change is due to relocation. Using this setting, we show that (i) short-term and long-term approaches might greatly underestimate and overestimate, respectively, the welfare gains of any set of policies, and (ii) when more than one transport policy is introduced at different time frames, the welfare gains and the final resulting urban structure may significantly differ depending on the introduction order.

4.1. Introduction

Although the literature has extensively studied the impacts and the desirability of policies aimed to correct transport externalities (see Santos et al., 2010a,b, for an extensive review of transport policies aimed to internalize the negative externalities caused by the use of the private car, and that try to incentive the use of public transportation, respectively), less attention has been given to the interaction of such policies with the underlying urban form. Indeed, most of the previous work does not consider the underlying urban form when studying transport policies (e.g., Basso and Silva, 2014; Parry and Small, 2009; Proost and Van Dender, 2008), or consider that it is fixed beforehand, i.e., there is a spatial component in the modeling, but neither redevelopment nor relocation occurs (e.g., Ho et al., 2013; Li et al., 2012; Mun et al., 2003, 2005). Thus, the impact of the urban form is mostly ignored.

On the other hand, the works that consider the possibility of redevelopment or relocation implicitly consider a strong assumption: the city is malleable. In other words, the city can be rebuilt every time any of the conditions change, ignoring the fact that buildings are not quickly replaceable (e.g., Anas and Rhee, 2006; Brinkman, 2016; Li and Wang, 2018; Tikoudis et al., 2018; Verhoef, 2005; Zhang and Kockelman, 2016b). This framework is useful when comparing intercity differences at a given point in time or, in general, to describe the long-run effect of a change in the underlying conditions over the urban form (Brueckner, 1987). Still, it fails to capture the short-term or medium-term effects, where space for redevelopment in cities may be very limited.

In this paper, we study the efficiency and substitutability of three transport policies (congestion pricing, public transport subsidies, and dedicated bus lanes) in a model that allows for relocation and redevelopment, based on the model of Anas and Rhee (2006). We find that the substitutability between policies is large. In other words, when any first policy is implemented, the welfare increase due to the introduction of a second policy is only moderate. Additionally, by using a stylized setting of cyclical redevelopment based on Brueckner and Rosenthal (2009), we analyze these transport policies' impacts not only in the long run but also in the medium run, where redevelopment is only partially possible and when the core of the urban change is due to relocation. Using this setting, we show that when a single policy is used, the welfare-maximizing policy depends on the timeframe considered. Moreover, when more than one transport policy is introduced at different time frames, the welfare gains and the final resulting urban structure may significantly differ depending on the introduction order (i.e., there is path dependence).

The contributions of this paper are twofold. First, we present an extension to the non-monocentric model by Anas and Rhee (2006). This extension considers endogenous location decisions for firms and households of two skill groups. Additionally, our extension features two transport modes (private car and transit), allowing for cross-congestion effects between cars and buses. We use this model to evaluate the impacts of three urban transport policies: congestion pricing, transit subsidies, and dedicated public transport infrastructure. Second, we propose a simplified setting of durable housing, based on the work by Brueckner and Rosenthal (2009). Using this setting, we show that neither a short-term model (i.e., fixed urban form) nor a long-term model (i.e., a malleable city) might correctly estimate the welfare gains of any set of transport policies.

The rest of this paper is structured as follows. In Section 2, we describe the basics of the long-run model. In Section 3, we develop a numerical analysis of the long-run welfare effects of transport policies. In Section , we present a stylized setting of durable housing, and we provide a numerical welfare analysis. Finally, in Section 5, we provide some concluding remarks.

4.2. The model

The interaction between transportation and the structure of cities is at the core of the urban theory (Duranton and Puga, 2015). Consequently, the literature provides different approaches to study the configuration of cities and the welfare implications of urban policies, starting with the monocentric model proposed by Alonso (1964), Mills (1967), and Muth (1969). As workers living further from the city center face higher commuting costs, this must be compensated in equilibrium by a lower land rent further from the city center. This model quickly became the workhorse in the urban and transportation economics literature due to its transparency and tractability, and it has been used to analyze the impact of traffic congestion policies (e.g., Kanemoto, 1980; Oron et al., 1973; Tikoudis et al., 2015, 2018; Verhoef, 2005), land use regulations (e.g., Brueckner, 2007; Kono and Kawaguchi, 2017; Pines and Kono, 2012; Pines and Sadka, 1985) and property taxes (Kono et al., 2019), among others.

Although the monocentric city model is able to explain a number of phenomena observed in cities, as years went by, there was an increasing sense this model is relevant for a diminishing number of cities (Glaeser and Kahn, 2004). Indeed, in many cities, people live and work outside of the central district (Glaeser et al., 2008). Moreover, in the monocentric city model, firms do not use any land and are located at a single central point, and thus, this model, by assumption, designates some areas for business use and others for residential use. In contrast, firms –as residents– relocate in the long-term and use land as an input, thus producing complex patterns of residential and commercial land use (Duranton and Puga, 2015). Motivated by the previous limitations, the literature later provided models that try to endogenize the location of both firms and workers throughout the city. Two famous examples of this are Ogawa and Fujita (1980) and Fujita and Ogawa (1982). These papers propose a non-monocentric framework of a linear city, where firms and residents compete for land and where firms benefit from proximity to each other due to agglomeration externalities. This framework was later extended to a circular city by Lucas and Rossi-Hansberg (2002), allowing for a production technology that permits substitution between land and labor, and consumers that can choose any quantities of both land and goods. This model has been used to evaluate congestion policies Zhang and Kockelman (2016b). As another famous example of a polycentric city model, Anas and Xu (1999) proposes a tractable framework where firms and residents locate into differentiated and discrete locations. As opposed to the Lucas and Rossi-Hansberg (2002)'s framework, in Anas and Xu (1999)'s model, the structure of the city is not explained by agglomeration externalities. This model has also been used to evaluate congestion policies Anas and Rhee (2006).

Finally, more recently, quantitative models have been used to study the interaction between transportation and the structure of cities. Since the seminal contribution by Ahlfeldt et al. (2015), quantitative spatial models of cities have become the frontier framework to study urban policies. These models feature commercial and residential location choice, endogenous wages, residential and commercial floorspace developers, and commuting costs that depend on the transportation network. Moreover, they connect directly to the observed data, allowing to predict the impact of realistic public policy interventions (Redding, 2023). This framework has been used to evaluate the welfare effects of transportation infrastructure (Allen and Arkolakis, 2022; Tsivanidis, 2019; Warnes, 2020).

The model we use in this paper draws from the polycentric models of Anas and Xu (1999) and Anas and Rhee (2006). The choice of this framework is motivated by several reasons. First, we deviate from the first framework (monocentric city model), since, as we previously discussed, many cities depart from the assumption of monocentricity. In particular, our computational experiments are motivated by non-monocentric cities. In this regard, the literature shows that the welfare impacts of urban policies may be greatly different in the monocentric city model compared to non-monocentric models. For instance, Brueckner (2000), based on a monocentric intuition, discusses that a not-too-stringent urban boundary can increase, while Anas and Rhee (2006), based on a non-monocentric model, that an urban boundary of any stringency could be welfare decreasing. Moreover, as we are interested in both medium and long-term effects of transportation policies, we find it reasonable to assume that not only residents but also firms relocate. Second, we also deviate from the third framework (quantitative urban models). Although this framework provides a realistic representation of cities, they lack the ability to simulate the effects of transportation policies over transportation costs. Indeed, to the best of our knowledge, there is no contribution in this line of research that considers a travel network with endogenous travel times. This prevents us from introducing transportation externalities. This leaves with the second framework, which, we believe, provides a good compromise between a realistic representation of cities and a tractable model. In this framework, to focus solely on transportation externalities –rather than agglomeration externalities–, we choose to use the Anas and Xu (1999) approach. Additionally, since our results are mostly based on numerical simulations –due to the complexity of analytical first-best policies–, we believe that the discrete nature of this approach –as opposed to the continuous nature of the Lucas and Rossi-Hansberg (2002)'s approach– provides a numerical advantage. Moreover, as we discuss in Section 4.4, this discrete nature provides a straightforward approach to cyclical redevelopment.

We now describe the basics of our model.

4.2.1. City

We study a closed city of (exogenous) radius S [km]. We divide this region into I concentric zones, each one with an area of A_i [km^2] available to residences, workplaces and roads. Additionally, we assume that travel only occurs along rays. Figure 4.1 depicts an example of the city distribution, with $I = 4$. Each zone i includes the points –in polar coordinates– (x, ϕ), $x \in [x_{i-1}, x_i)$, $\phi \in [0, 2\pi]$, where $x_0 = 0$, $x_I = S$.

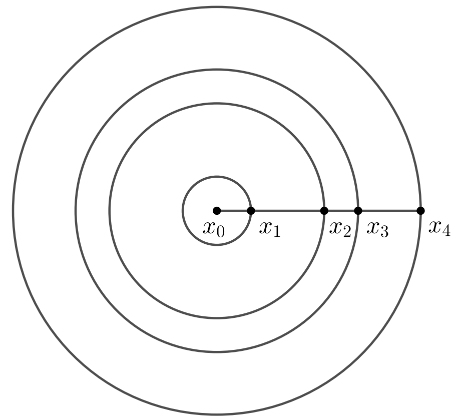


Figure 4.1: City distribution, with $I = 4$.

Zones are assumed to group locations with the same characteristics (e.g., amenities), and consequently, we consider that locations within a zone are perfect substitutes.

4.2.2. Households

We consider that the city is populated by residents from two skills groups, indexed by $s \in \{L, H\}$. Each group s consists of N_s residents. Every resident chooses a residential location i, and a working location j. Then, conditional on the residential and working location, they choose a commuting mode $k \in \{\text{Car}, \text{Bus}\}$, based on the utility they get from each alternative. To model the locations and mode choices, we consider a nested Logit model (Ben-Akiva, 1973). We assume that residents follow a two-step process: they jointly choose residential and workplace location[12], and then, conditional on those decisions, they choose a travel mode (similar to the process presented in Dröes and Rietveld, 2015; Schindler et al., 2021; Tsivanidis, 2019, among others).

The utility $\hat{U}_{ijks}$ that a resident of type s obtains if they locate in i, work in j and commute using mode k is composed of two terms: a deterministic component U and an idiosyncratic shock ε_{ijks} that varies with the worker's blocks of employment, block of residence and transport mode :

$$\hat{U}_{ijks} = U(c_{ijks}, q_{ijks}, T^f_{ijks}) + \varepsilon_{ijks} \tag{4.1}$$

We consider that the systematic utility U is represented by a quasi-concave function that depends on the consumption of a composite good c_{ijks}, housing q_{ijks} (measured by the floor size) and leisure time T^f_{ijks}. The price of c is normalized to 1, while the price of q depends on the zone i, and it is denoted by r_i. Thus, city's residents, conditional on i, j, k, maximize their systematic utility, subject to an income constraint:

$$c_{ijks} + r_i q_{ijks} + \tau_{ijk} M_{ijks} = w_{js} T^w_{ijks} + \frac{L(\boldsymbol{r})}{N} + \frac{E}{N} \tag{4.2}$$

[12] Previous research has shown that joint residential-workplace choice models fit empirical data better than sequential residential-workplace choice models (Jiao et al., 2015; Waddell, 1993).

In Equation (4.2), τ_{ijk} and M_{ijks} are the transport monetary cost per trip and the number of commuting trips made by residents of group s commuting from i to j using mode k, respectively.[13] w_{js} is the wage per unit of time paid by firms located in zone j to workers of the skill group s. T^w_{ijks} is the time allocated to work by residents of group s commuting from i to j using mode k. $L(\boldsymbol{r})$[14] corresponds to the excess land rents in the city:

$$L(\boldsymbol{r}) = \sum_{i=1,11} (r_i - r_A)A_i + \sum_{i=2}^{10} (r_i - r_A)A_i \tag{4.3}$$

where r_A is the agricultural rent and A_i is the total area of zone k. In others words, each resident receives an equal share of the differential land rent. Finally, E corresponds to a redistribution of two sources of income/spending in the city: congestion toll revenues (if a toll is in place) and a percentage of operating cost of the bus system (if a subsidy is in place). In particular, we assume that both the congestion toll revenue and the bus system' subsidized cost are beared by city's residents in equal shares. Consequently:

$$E = TR - OC \cdot X \tag{4.4}$$

In Equation (4.4), TR is the congestion toll revenue, OC is the operating cost of the bus system and X is the bus system' subsidy percentage.[15]

We consider that each resident has a time budget T, which is allocated over work (T^w), leisure (T^f) and commuting (T^c). Similar to Verhoef (2005), we assume that the number of commuting trips is equal to the amount of working time supplied. [16] Consequently, we can think T^w as the number of days worked, each of a fixed duration in hours. Thus, the time spent commuting is:

$$T^c_{ijks} = T^w_{ijks} \cdot T^t_{ijk} \tag{4.5}$$

where T^t_{ijk} is the commuting time per round trip from zone i to zone j using mode k (described in Subsection 4.2.4.2).

Then, the time budget constraint is written as:

$$T = T^f_{ijks} + T^w_{ijks} + T^c_{ijks} \tag{4.6}$$

$$= T^f_{ijks} + T^w_{ijks}\left(1 + T^t_{ijk}\right) \tag{4.7}$$

[13] Note that the monetary cost per trip, τ_{ijk}, does not depend on the group. Indeed, we assume that the transport system cannot discriminate between groups. This cost is described in Subsection 4.2.4. Nonetheless, the number of commuting trips made by each group, M_{ijks}, may differ. Consequently, the total monetary cost faced by each group will also be different.

[14] When necessary, we use bold letters to denote a vector. For example, $\boldsymbol{r} = (r_1, \ldots, r_{11})$.

[15] Note that this approach can also be interpreted as using the congestion toll revenue to finance (at least partially) the bus system subsidy. Indeed, when considering a target subsidy percentage X, the toll revenue could be used to finance a percentage $Y = \min\left\{\frac{TR}{OC}, X\right\}$. Then, E would be characterized by $E = (TR - OC \cdot Y) - OC \cdot (X - Y) = TR - OC \cdot X$, the same expression of Equation (4.4).

[16] Thus, $M_{ijks} = T^w_{ijks}$.

From (4.7), we can obtain the working time:

$$T^w_{ijks} = \frac{T - T^f_{ijks}}{1 + T^t_{ijk}} \tag{4.8}$$

Thus, the income constraint (2.10) can be rewritten as:

$$\frac{L(\boldsymbol{r})}{N} + \frac{E}{N} + \frac{w_{js} - \tau_{ijk}}{1 + T^t_{ijk}}(T - T^f_{ijks}) - c_{ijks} - r_i q_{ijks} = 0 \tag{4.9}$$

Then, solving the maximization problem for a given (i, j, k, s), we can obtain the Marshallian demands for the composite good and land, $c_{ijks}(r_i, w_{js}, \tau_{ijk}, T^t_{ijk}, \Omega_{ijks})$, $q_{ijks}(r_i, w_{js}, \tau_{ijk}, T^t_{ijk}, \Omega_{ijks})$, and the optimal leisure time allocation, $T^f_{ijks}(r_i, w_{js}, \tau_{ijk}, T^t_{ijk}, \Omega_{ijks})$, with Ω_{ijks} the gross budget (or income net of transportation costs):

$$\Omega_{ijks}(w_{js}, \boldsymbol{r}, \tau_{ijk}, T^t_{ijk}) = \frac{L(\boldsymbol{r})}{N} + \frac{E}{N} + \frac{w_{js} - \tau_{ijk}}{1 + T^t_{ijk}} \cdot T \tag{4.10}$$

The proportion of commuters of group s that choose mode k, conditional on the choice of residential location i and workplace location j is given by (see, e.g., Koppelman and Wen, 1998):

$$\mathcal{P}_s(k|ij) = \frac{\exp(U_{ijks}/\lambda_{ijs})}{\sum\limits_{k' \in \{\text{Car,Bus}\}} \exp(U_{ijk's}/\lambda_{ijs})} \tag{4.11}$$

while the proportion of commuters of group s that choose the pair (i, j) is given by:

$$\mathcal{P}_s(ij) = \frac{\exp(\lambda_{ijs} \cdot \Gamma_{ijs})}{\sum\limits_{(i',j') \in (1,\ldots,I) \times (1,\ldots,I)} \exp(\lambda_{ijs} \cdot \Gamma_{i'j's})} \tag{4.12}$$

where Γ_{ijs} is the s group's expected utility of nest (i, j), often referred as the *log-sum term* (Ben-Akiva, 1973):

$$\Gamma_{ijs} = \ln\left(\sum_{k' \in \{\text{Car,Bus}\}} \exp(U_{ijk's}/\lambda_{ijs})\right) \tag{4.13}$$

In eqs. (4.11) to (4.13), λ_{ijs} measures the degree of independence between the random part of utility of each alternative in nest i, j for the skill group s. Indeed, as McFadden (1977) points out, the value of $(1 - \lambda_{ijs})$ can be used as a proxy of correlation among alternatives of the same nest. In particular, when $\lambda_{ijs} = 1 \; \forall i, j$, the nested logit model for skill group s collapses to the multinomial logit. With the previous results, the number of residents of group s that commute from residential location i to workplace location j using the mode k, N_{ijks}, is given by:

$$N_{ijks}(\boldsymbol{r}, \boldsymbol{w}_s, \boldsymbol{\tau}, \boldsymbol{T^t}, \boldsymbol{\Omega}) = N_s \cdot \mathcal{P}_s(ij) \cdot \mathcal{P}_s(k|ij) \tag{4.14}$$

Finally, the labor (total hours) supplied by residents of group s working in zone j, W_{js}, is given by:

$$W_{js}(\boldsymbol{r}, \boldsymbol{w}_s, \boldsymbol{\tau}, \boldsymbol{T^t}, \boldsymbol{\Omega}) = \sum_{\substack{i=1,\dots,I \\ k\in\{\text{Car},\text{Bus}\}}} N_{ijks} \cdot T^w_{ijks} \tag{4.15}$$

4.2.3. Workplaces

In each zone i a firm is located. Firms operates under constant returns to scale, using land Q_i and labor M_i to produce goods. Following Card (2009), we consider that the labor input M_i is a CES aggregate over the effective labor input (total hours) M_{is} of each skill group s:

$$M_i(M_{iH}, M_{iL}) = \left(M_{iL}^{\frac{\sigma-1}{\sigma}} + M_{iH}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \tag{4.16}$$

where σ is the elasticity of substitution between skill groups. Then, firm's total production, F_i, is given by:

$$F_i(M_{iH}, M_{iL}, Q_i) = \delta M_i^k Q_i^{1-k}, \quad 0 < k < 1 \tag{4.17}$$

where δ is the total factor productivity (TFP). Firms then maximize its profit Π_i:

$$\max \Pi_i = F_i - r_i Q_i - w_{iL} M_{iL} - w_{iH} M_{iH} \tag{4.18}$$

From the first order conditions of Problem (4.18), we arrive to the demand for labor, as a function of the demand for land:

$$M_{iL}(Q_i, w_{iH}, w_{iL}, r_i) = Q_i \frac{k}{1-k} \left(\frac{1}{w_{iL}}\right)^{\sigma} \frac{r_i}{w_{iL}^{1-\sigma} + w_{iH}^{1-\sigma}} \tag{4.19}$$

$$M_{iH}(Q_i, w_{iH}, w_{iL}, r_i) = Q_i \frac{k}{1-k} \left(\frac{1}{w_{iH}}\right)^{\sigma} \frac{r_i}{w_{iL}^{1-\sigma} + w_{iH}^{1-\sigma}} \tag{4.20}$$

Additionally, considering perfectly competitive input and output markets, lands rents rise to the value that ensure that firms make zero profit:

$$r_i = (1-k)\left[\delta k^k (w_{iL}^{1-\sigma} + w_{iH}^{1-\sigma})^{k/(\sigma-1)}\right]^{1/(1-k)} \tag{4.21}$$

Finally, note that from conditions (4.19), (4.20) and (4.21), it follows that:

$$w_{iL} = \Lambda \left(\frac{Q_i}{M_{iL}} r_i^{(1-\sigma(1-k))/k}\right)^{1/\sigma} \tag{4.22}$$

$$w_{iH} = \Lambda \left(\frac{Q_i}{M_{iH}} r_i^{(1-\sigma(1-k))/k}\right)^{1/\sigma} \tag{4.23}$$

where Λ is a constant depending on k, σ, δ. Thus, as expected, wages decrease with the amount of labor required, and they increase with the amount of land used.

4.2.4. Transportation system

4.2.4.1. Monetary Costs

We consider that two modes are available to commuters: car and buses. In the case of buses, the only financial outlay is the fare. Consequently:

$$\tau_{ij,\text{bus}} = \begin{cases} \phi_i \left(\dfrac{x_i - x_{i-1}}{3} \right) & \text{if } i = j \\ \phi_i \left(\dfrac{x_i - x_{i-1}}{2} \right) + \phi_j \left(\dfrac{x_j - x_{j-1}}{2} \right) + \sum_{m=i+1}^{j-1} \phi_m (x_m - x_{m-1}) & \text{if } i < j \\ \phi_i \left(\dfrac{x_i - x_{i-1}}{2} \right) + \phi_j \left(\dfrac{x_j - x_{j-1}}{2} \right) + \sum_{m=j+1}^{i-1} \phi_m (x_m - x_{m-1}) & \text{if } i > j \end{cases} \tag{4.24}$$

In Equation (4.24), ϕ_i is fare per mile charged to users travelling through zone i. Since we are not modeling intrazonal location decisions, we assume that residents living and working in the same zone travel, on average, a third of the zone length.[17] Additionally, for residents living and working in different zones, we assume they travel, in average, half the length of the origin and destination zones and the full length of every intermediate zone.

In the case of cars, we follow previous contributions (e.g., Li et al., 2012; Li and Wang, 2018), and we assume that, in addition to a possible toll, the monetary cost of the trip has a component that increases linearly with distance. In particular, the monetary cost faced by residents commuting by car from zone i to zone j is given by:

$$\tau_{ij,\text{car}} = \begin{cases} c_0 + (c_1 + \xi_i) \left(\dfrac{x_i - x_{i-1}}{3} \right) & \text{if } i = j \\ c_0 + (c_1 + \xi_i) \left(\dfrac{x_i - x_{i-1}}{2} \right) + (c_1 + \xi_j) \left(\dfrac{x_j - x_{j-1}}{2} \right) + \sum_{m=i+1}^{j-1} (c_1 + \xi_m)(x_m - x_{m-1}) & \text{if } i < j \\ c_0 + (c_1 + \xi_i) \left(\dfrac{x_i - x_{i-1}}{2} \right) + (c_1 + \xi_j) \left(\dfrac{x_j - x_{j-1}}{2} \right) + \sum_{m=j+1}^{i-1} (c_1 + \xi_m)(x_m - x_{m-1}) & \text{if } i > j \end{cases} \tag{4.25}$$

In Equation (4.25) c_0 is the fixed cost of the trip (e.g., parking charges in the workplace), while c_1 is the variable cost per mile (e.g., fuel cost). Finally, ξ_i is (if in place) the toll per mile charged to car users when traversing through zone i. Then, we can define the toll revenue TR as:

$$\begin{aligned} TR = \sum_{i=1}^{I} \sum_{s \in \{L,H\}} \Bigg\{ & N_{ii,\text{car},s} \cdot \xi_i \left(\frac{x_i - x_{i-1}}{3} \right) \\ & + \sum_{\substack{j=1 \\ j \neq i}}^{I} N_{ij,\text{car},s} \cdot \xi_i \left(\frac{x_i - x_{i-1}}{2} \right) + \sum_{\substack{j=1 \\ j \neq i}}^{I} N_{ji,\text{car},s} \cdot \xi_i \left(\frac{x_i - x_{i-1}}{2} \right) \\ & + \sum_{l=1}^{i-1} \sum_{m=i+1}^{I} N_{lm,\text{car},s} \cdot \xi_i (x_i - x_{i-1}) + \sum_{l=1}^{i-1} \sum_{m=i+1}^{I} N_{ml,\text{car},s} \cdot \xi_i (x_i - x_{i-1}) \Bigg\} \end{aligned} \tag{4.26}$$

[17] Assuming that both residential and workplaces locations are uniformly distributed within each zone, then, the expected travel distance for residents living and working in the same zone is one third of the zone length.

In Equation (4.26), the first term is the revenue due to intrazonal trips. The second and third terms represent the revenue associated with the starting and ending zone of every interzonal trip. Finally, the fourth and fifth terms represent the revenue due to flow passing through each zone.

4.2.4.2. Commuting times

The commuting time from i to j using mode k is given by:

$$T^t_{ijk} = \begin{cases} \frac{1}{2}\left(\zeta^{\text{inward}}_{ik} + \zeta^{\text{outward}}_{ik}\right) + \frac{t_{ik}}{3} & \text{if } i = j \\ \zeta^{\text{outward}}_{ik} + \frac{t_{ik}}{2} + \frac{t_{jk}}{2} + \sum_{n=i+1}^{j-1} t_{nk} & \text{if } i < j \\ \zeta^{\text{inward}}_{ik} + \frac{t_{ik}}{2} + \frac{t_{jk}}{2} + \sum_{n=j+1}^{i-1} t_{nk} & \text{if } i > j \end{cases} \tag{4.27}$$

In Equation (4.27), ζ^d_{ik} is the d-direction ($d \in$ {inward, outward}) waiting time in zone i when using mode k, while t_{ik} is the travel time of traversing zone i using mode k. The first case in (4.27) corresponds to intrazonal commuting: since we are not modeling intrazonal locations decisions, we assume that half of the residents living and working in the same zone travel in each direction. Furthermore, we assume that these residents travel a third of the zone length. The second and third case in (4.27) correspond to interzonal commuting. In this case, we assume that residents living and working in different zones travel half the length of the origin and destination zones and the full length of every intermediate zone. Finally, for simplicity, we follow Anas and Rhee (2006) and we assume that travel time t_{ik} is the same regardless of travel direction.

To define both the waiting and travel times, we require the flows traversing each zone. We compute the flow traversing zone i using mode k in direction d ($d \in$ {inward, outward}), F^d_{ik}, as:

$$F^d_{ik} = \begin{cases} \sum_{s\in\{L,H\}}\left(\frac{1}{2}\cdot\frac{N_{iiks}}{3} + \frac{1}{2}\cdot\sum_{\forall j<i} N_{ijks} + \frac{1}{2}\cdot\sum_{\forall j>i} N_{jiks} + \sum_{l=1}^{i-1}\sum_{m=i+1}^{I} N_{mlks}\right) & \text{if } d = \text{inward} \\ \sum_{s\in\{L,H\}}\left(\frac{1}{2}\cdot\frac{N_{iiks}}{3} + \frac{1}{2}\cdot\sum_{\forall j>i} N_{ijks} + \frac{1}{2}\cdot\sum_{\forall j<i} N_{jiks} + \sum_{l=1}^{i-1}\sum_{m=i+1}^{I} N_{lmks}\right) & \text{if } d = \text{outward} \end{cases} \tag{4.28}$$

In Equation (4.28), the first term in the right-hand side represents the intrazonal traffic. We factor this term by 1/3 since we assume that this flow travels, in average, only a third of the zone length. Additionally, we factor this term by 1/2 since we assume that half of this flow moves in each direction. The second and third term are the traffic originating and ending in zone i, respectively. We factor these two terms by 1/2 since we assume that these residents travel, in average, only half the length of zone i. Finally, the fourth term represents the flow passing through zone i.

Then, we follow De Cea and Fernández (1993), and we model the waiting time as follows:

$$\zeta^d_{ik} = \begin{cases} 0 & \text{if } k = \text{car} \\ \dfrac{\sigma_1}{f^d} + \sigma_2\cdot\left(\dfrac{F^d_{i,\text{bus}}}{f^d\cdot g}\right)^{\sigma_3} & \text{if } k = \text{bus} \end{cases} \tag{4.29}$$

In Equation (4.29), f^d is the bus line frequency in direction d. The first term for the second case represents the waiting time due to the interval between buses. The typical value of σ_1 used in the literature is 0.5, which assumes fixed headways and commuters that arrive with an uniform distribution. The second term for the second case represents the crowding externality: the waiting time increases as the demand exceeds the capacity of the system. In this term, σ_2, σ_3 are positive parameters, g is the bus capacity, and consequently, the denominator is the total capacity of the bus line.

Finally, to define the travel time of each mode, we assume that in each zone the land allocated to roads is converted into road capacity using a factor ρ, leading to an endogenous road capacity K_i. Thus, note that our model involves deciding the amount of land allocated to roads. To compute t_{ik}, we follow the approach of Basso and Silva (2014). In particular, t_{ik} depend on whether car and buses share road capacity or not. We study both cases.

1. *Travel time by bus:* When bus lanes are in place in zone i, a percentage n_i is allocated to them. Then, we model congestion using a BPR function, and consequently, the bus travel time –when using bus lanes– is given by:

$$t_{i,\text{bus}} = (x_i - x_{i-1}) \left[a_{\text{bus}} \left(1 + b \left(\frac{(f^{\text{inward}} + f^{\text{outward}}) \cdot e}{n_i \cdot K_i} \right)^c \right) \right] \tag{4.30}$$

   In Equation (4.30), the first term is the distance traveled when traversing zone i, while the second term denotes the travel time per mile: a_{bus} is the bus free flow travel time per mile, b and c are positive coefficients to model congestion, and e is a factor to convert a bus into the equivalent number of passenger cars.

   When bus lanes are not in place, buses and cars share the road. Thus, the travel time changes, and is given by:

$$t_{i,\text{bus}} = (x_i - x_{i-1}) \left[a_{\text{bus}} \left(1 + b \left(\frac{(f^{\text{inward}} + f^{\text{outward}}) \cdot e + F_{i,\text{car}}^{\text{inward}} + F_{i,\text{car}}^{\text{outward}}}{K_i} \right)^c \right) \right] \tag{4.31}$$

2. *Travel time by car:* Similar to buses, the travel time by car depends on whether there are bus lane in placer or not. When bus lanes are in place in zone i, a percentage n_i is allocated to them. Consequently, only a capacity $(1 - n_i) \cdot K_i$ is available to car users. The travel time is then given by:

$$t_{i,\text{car}} = (x_i - x_{i-1}) \left[a_{\text{car}} \left(1 + b \left(\frac{F_{i,\text{car}}^{\text{inward}} + F_{i,\text{car}}^{\text{outward}}}{(1 - n_i) \cdot K_i} \right)^c \right) \right] \tag{4.32}$$

   In Equation (4.32), a_{car} is the car free flow travel time per mile.

   When bus lanes are not in place, buses and cars share the road, and the travel time is given by:

$$t_{i,\text{car}} = (x_i - x_{i-1}) \left[a_{\text{car}} \left(1 + b \left(\frac{(f^{\text{inward}} + f^{\text{outward}}) \cdot e + F_{i,\text{car}}^{\text{inward}} + F_{i,\text{car}}^{\text{outward}}}{K_i} \right)^c \right) \right] \tag{4.33}$$

4.2.4.3. Bus Operating Costs

The operating costs OC of the bus system depend on two factors: the bus fleet B and the number of vehicle-km V traveled:

$$OC = OC_b \cdot B + OC_v \cdot V \tag{4.34}$$

where OC_b is the cost per bus, and OC_v is the cost per vehicle-mile. The required fleet to operate the bus system is defined by the direction with the highest frequency:

$$B = \max_d \left\{ f^d \cdot \left(\sum_{i=1}^{I} t_{i,\text{bus}} \right) \right\} \tag{4.35}$$

On the other hand, the number of vehicle-km is proportional to the sum of the vehicle-km in each direction:

$$V = S \cdot (f^{\text{inward}} + f^{\text{outward}}) \tag{4.36}$$

Finally, we impose that the fares collected must cover the percentage of the cost that is not subsidized:

$$\begin{aligned}(1 - X) \cdot G = &\sum_{i=1}^{I} \sum_{s \in \{L,H\}} \left\{ N_{ii,\text{bus},s} \cdot \phi_i \left(\frac{x_i - x_{i-1}}{3} \right) \right. \\ &+ \sum_{\substack{j=1 \\ j \neq i}}^{I} N_{ij,\text{bus},s} \cdot \phi_i \left(\frac{x_i - x_{i-1}}{2} \right) + \sum_{\substack{j=1 \\ j \neq i}}^{I} N_{ji,\text{bus},s} \cdot \phi_i \left(\frac{x_i - x_{i-1}}{2} \right) \\ &\left. + \sum_{l=1}^{i-1} \sum_{m=i+1}^{I} N_{lm,\text{bus},s} \cdot \phi_i (x_i - x_{i-1}) + \sum_{m=1}^{i-1} \sum_{l=i+1}^{I} N_{ml,\text{bus},s} \cdot \phi_i (x_i - x_{i-1}) \right\}\end{aligned} \tag{4.37}$$

4.2.5. Urban Equilibrium

We compute the general equilibrium for a completely closed city. In this context, suppose that the land allocated to roads is fixed by the planner at some level $\{\mathbb{Q}_i\}_i$. Then, the equilibrium of our model involves residents of each group s maximizing their utility conditional on a given (i, j, k). As Subsection 4.2.2 stated, this leads to Marshallian demands for the composite good and land, $c_{ijks}(r_i, w_{js}, \tau_{ijk}, T^t_{ijk}, \Omega_{ijks})$, $q_{ijks}(r_i, w_{js}, \tau_{ijk}, T^t_{ijk}, \Omega_{ijks})$, and the optimal leisure time allocation, $T^f_{ijks}(r_i, w_{js}, \tau_{ijk}, T^t_{ijk}, \Omega_{ijks})$. Then, the transport sector equilibrium allows us to obtain transport costs $\tau_{ijk}(\boldsymbol{N})$ and times $T^t_{ijk}(\boldsymbol{N})$, and with this, we can compute full income functions $\Omega_{ijks}(w_{js}, \boldsymbol{r}, \tau_{ijk}, T^t_{ijk})$. Then, the distribution of residents in the city is obtained $N_{ijks}(\boldsymbol{r}, \boldsymbol{w}, \boldsymbol{\tau}, \boldsymbol{T^t}, \boldsymbol{\Omega})$, and with this, the supply of labor of type s, $W_{js}(\boldsymbol{r}, \boldsymbol{w}, \boldsymbol{\tau}, \boldsymbol{T^t}, \boldsymbol{\Omega})$, can be computed. In the firms side, maximization of profit leads to conditional demands for labor $M_{iL}(Q_i, w_{iH}, w_{iL}, r_i)$ and $M_{iH}(Q_i, w_{iH}, w_{iL}, r_i)$. These conditions, together with the following define the equilibrium:

$$W_{js}(\boldsymbol{r},\boldsymbol{w}_s,\boldsymbol{\tau},\boldsymbol{T^t},\boldsymbol{\Omega}) - M_{js}(Q_j,w_{jH},w_{jL},r_j) = 0 \qquad \forall j=1,\dots,11,\ s=\{L,H\} \tag{4.38}$$

$$A_i - \sum_{\substack{j=1,\dots,I\\ k\in\{\text{Car,Bus}\}\\ s=\{L,H\}}} N_{ijks}(\boldsymbol{r},\boldsymbol{w}_s,\boldsymbol{\tau},\boldsymbol{T^t},\boldsymbol{\Omega})\cdot q_{ijks}(r_i,w_{js},\tau_{ijk},T^t_{ijk},\Omega_{ijks}) - Q_i - \mathbb{Q}_i = 0 \quad \forall i=1,\dots,11 \tag{4.39}$$

$$r_i - (1-k)\left[\delta k^k(\alpha_L^\sigma w_{iL}^{1-\sigma} + \alpha_H^\sigma w_{iH}^{1-\sigma})^{k/(\sigma-1)}\right]^{1/(1-k)} = 0 \qquad \forall i=1,\dots,11 \tag{4.40}$$

Equation (4.38) equals labor (man-hours) supply and demand for each skill group s. Equation (4.39) represents the land market clearing condition. Equation (4.40) is the zero profit conditions for firms. Note that the equilibrium then depends on just four vectors of endogenous variables: $\boldsymbol{Q}$, $\boldsymbol{w}_H$, $\boldsymbol{w}_L$, $\boldsymbol{r}$. This give us $4I$ unknowns, with conditions (4.38), (4.39) and (4.40) providing the same number of equations.[18]

4.2.6. Welfare Analysis

To measure the welfare for each group s, $\mathcal{W}_s$, we use the expected utility, obtained through the logsum formula:

$$\mathcal{W}_s = \ln\left(\sum_{(i,j)\in(1,\dots,I)\times(1,\dots,I)} \exp(\mu_s\cdot\Gamma_{ijs})\right) \tag{4.41}$$

Then, the social welfare $\mathcal{W}$ is defined using a utilitarian approach:

$$\mathcal{W} = N_L\mathcal{W}_L + N_H\mathcal{W}_H \tag{4.42}$$

Then, to compare the benefits and implications the different transport policies, we build different scenarios defined as the maximization of social welfare subject to different constraints. For simplicity, we denote each scenario by a short name in capital letters. Note that, in each scenario, the land allocated to roads is also chosen.

1. Reference Scenario (REF): This describes the market equilibrium for the case where the bus system is self-financed ($X=0$), there is no congestion toll in place ($\xi_i = 0,\ \forall i$), and the road is shared by buses and cars ($n_i = 0,\ \forall i$).

2. Car congestion pricing (CON): A congestion toll is in place ($\xi_i > 0,\ \forall i$), while the bus system is self-financed ($X=0$) and the road is shared by buses and cars ($n_i = 0,\ \forall i$).

3. Dedicated bus lanes (DL): The road is no longer shared by buses and cars ($n_i > 0,\ \forall i$), and the bus system is self-financed ($X=0$), and there is no congestion toll in place ($\xi_i = 0,\ \forall i$).

4. Transit subsidization (SUBY): A $Y\%$ of the cost of the transit system is subsidized,

[18] Just as in Anas and Rhee (2006), we do not prove uniqueness of the equilibrium analytically, but we explore it numerically. We find the same equilibrium starting from different points. We tried this for a broad range of parameters.

while there is no congestion toll in place ($\xi_i = 0,\ \forall i$), and the road is shared by buses and cars ($n_i = 0,\ \forall i$).

4.3. Numerical Analysis

In this Section, we explore the welfare impacts of different policies in the model proposed. For this, we consider that the residents' utility is represented by a Cobb-Douglas function:

$$U_{ijks} = (c_{ijks})^{\beta_c} \cdot (q_{ijks})^{\beta_q} \cdot \left(T^f_{ijks}\right)^{\beta_f} \tag{4.43}$$

Following Anas and Rhee (2006), we consider a city of 15,000 residents. We then consider $N_H = 3,000$ and $N_L = 12,000$, and $I = 10$ zones, each one length two kilometers. We then follow Verhoef (2005): the agricultural land rent is assumed to be given, and units of space are chosen such that also the agricultural rent $r_A = 1$, while the units of time are chosen such that the total time endowment $T = 1$. The rest of the parameters are summarized in Table 4.1.

Table 4.1: Parameters considered.

Parameter	Definition	Value	Source
β_G, β_c, β_q, β_f	Resident's utility function parameters	0.05, 0.36, 0.15, 0.49	Anas and Rhee (2006)
λ_{ijL}	Nested logit parameter for skill group L	0.1, $\forall i, j$	Anas and Rhee (2006)
λ_{ijH}	Nested logit parameter for skill group H	0.1, $\forall i, j$	Anas and Rhee (2006)
σ	Elasticity of substitution between skill groups	1.43	Card (2009)
δ	Production scale parameter	1	Anas and Rhee (2006)
k	Production function parameter	0.86	Anas and Rhee (2006)
c_0	Fixed component of monetary travel cost by auto [$]	1.45	Li and Wang (2018)
c_1	Variable component of monetary travel cost by auto [$/km]	0.357	Basso and Silva (2014)
σ_1, σ_2, σ_3	Parameters in bus waiting time function	0.5, 0.1, 2.0	Li and Wang (2018)
a_{bus}	Bus free flow travel time [hr/km]	0.027	Basso and Silva (2014)
a_{car}	Car free flow travel time [hr/km]	0.027	Basso and Silva (2014)
b, c	Parameters in congestion function	0.15, 4	Basso and Silva (2014)
e	Factor to convert a bus into passenger cars	2.06	Basso and Silva (2014)
OC_b	Cost per bus [$/vehicle-day]	859	Basso and Silva (2014)
OC_v	Cost per vehicle-mile [$/vehicle-km]	1.78	Basso and Silva (2014)

The main results of the numerical analysis are summarized in Table 4.2. As this Table shows, the reference scenario results in a city where residents commute mainly by public transport. However, the high-skill group uses the car in a much higher proportion than the low-skill group due to their higher wages, which mimics the behavior observed in cities. Additionally, the land is allocated mainly to residential use, while, as Figure 4.2 depicts, the percentage of land allocated to roads decreases as we move further away from the geometric center. Then, Figure 4.3 depicts the rental prices and wage profiles in the city. First, it can be seen that rental prices sharply decrease with distance to the geometric center (Zone 1). This is an expected result of a model based on Anas and Xu (1999), since the supply of land increases with distance from the geometric center. Then, note that wages increase with distance to the geometric center: since rental prices decrease with distance, firms substitute land for labor. As more land is substituted for labor, the marginal product of labor increases enough to cause competitive firms to pay higher wages. Finally, note that rental prices decrease much more sharply than wages increase: wages in Zone 10 are about 60% higher than in Zone 1, while rental prices in Zone 1 are 17 times higher than in Zone 10. This is explained due to the fact that the supply of land in each zone is perfectly inelastic, whereas workers can supply more labor in the zones with greater demand. Finally, Figure 4.4 shows the residential and employment densities, both decreasing around 16 times from the city center to the city edge. However, since land available increases with distance to the geometric center, the output, labor hours, and land used in production all increase with distance.

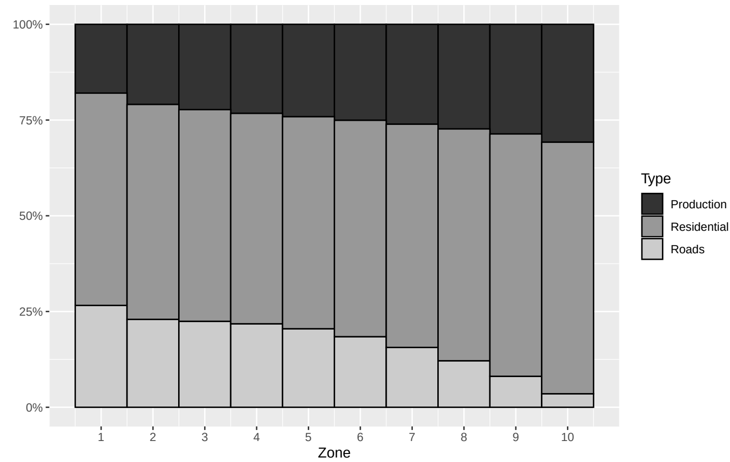


Figure 4.2: Land use (REF).

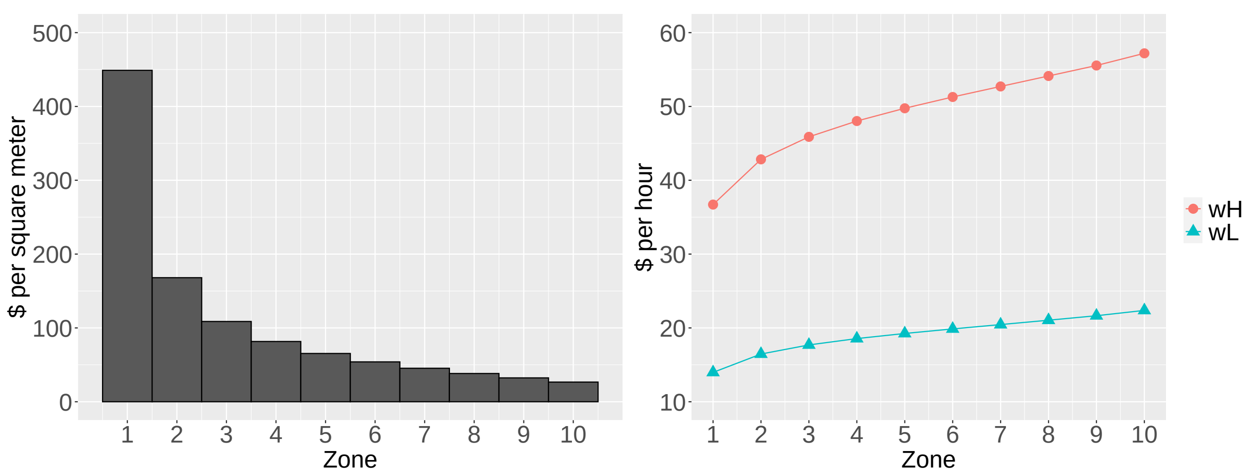


Figure 4.3: Rent and wages profile.

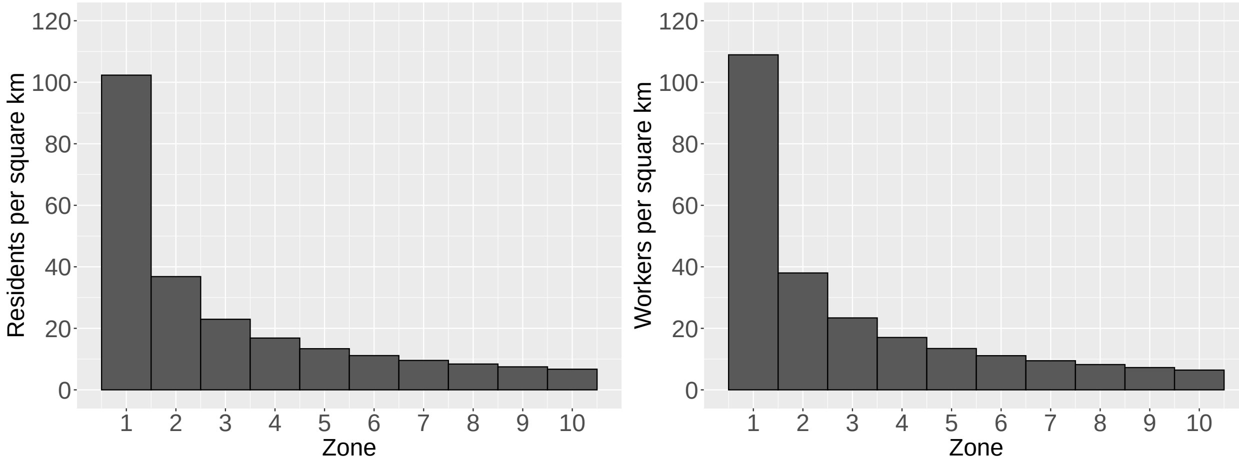


Figure 4.4: Residential and employment density.

Table 4.2: Main results.

Scenario	REF	SUB100	CON	DL	CON+DL
Welfare $\mathcal{W}$	61,181.44	60,558.73	61,390.24	61,458.26	61,552.80
Welfare change [%]	0	-1.02	0.34	0.45	0.61
Car modal share [%]	33.37	30.94	31.88	19.04	22.24
Bus modal share [%]	66.63	69.06	68.12	80.96	77.76
Car modal share - High Skill [%]	52.49	54.83	51.98	26.69	34.83
Bus modal share - High Skill [%]	47.51	45.17	48.02	73.31	65.17
Car modal share - Low Skill [%]	28.59	24.96	26.86	17.13	19.10
Bus modal share - Low Skill [%]	71.41	75.04	73.14	82.87	80.90
Residential land share [%]	59.94	58.39	61.90	66.71	65.93
Production land share [%]	26.69	27.51	26.62	28.34	27.67
Roads land share [%]	13.37	14.10	11.48	4.95	6.39
Mean travel distance [km]	6.73	6.81	6.70	6.51	6.57

Then, when we introduce a subsidy that covers 100% of the public transport fares (SUB100),[19] the welfare sharply decreases. To shed more light into this result, note that, in general, the desirability of transit subsidies is explained by the fact that increasing transit ridership reduces a negative externality, traffic congestion (since buses uses infrastructure more efficiently than cars), while increasing a positive externality, the so called Mohring effect (increased frequencies translate to shorter waiting times). However, within our setting, two opposing effects counterbalance these typically welfare-improving outcomes. First, we take into account a crowding externality, wherein increased ridership may lead to longer waiting times. This phenomenon may introduce diseconomies of scale in the public transport system, a previously reported effect (see e.g., Tirachini et al., 2010). Second, our model factors in the possibility of individuals relocating in response to transportation policies. Then, since the transport fare increases with distance, when we introduce a subsidy, the only residents that are better off are those that (i) use public transport and (ii) travel large distances. Indeed,

[19] The same comments apply to any level of subsidy, as Figures 4.7 and 4.8 show.

not only car users are worse off when a subsidy is introduced, but also public transit users travelling short distances. This, in turn, produces an incentive to choose a combination of residential and working locations that have a greater distance than in the REF scenario, thus increasing the mean travel distance in the city. Moreover, this has a second effect on the high-skill users: since a fraction of them now choose to commute a longer distance than in the REF scenario, they now find the car a better option than the public transport system –due to their higher wages, and thus their higher value of time–. Finally, this last effect produces that, in equilibrium, the planner chooses to assign more land to roads to combat the increasing congestion. In a nutshell, our results show that in the setting we propose, public transport subsidization is not a desirable policy since it (i) increases waiting time due to crowding externalities, (ii) increases travel distances, (iii) decreases the share of transit users in the high-skill group, (iv) increases the land needed for roads, thus decreasing the land available for other uses. To further explore this issue, we conduct two numerical experiments aimed at isolating the individual impact of each of these welfare-reducing effects. First, we consider a setting where there is no crowding in the public transportation ($\sigma_2 = 0$), and we compare the welfare of the reference scenario (which we call $\overline{\text{REF}}$, to differentiate it from our original reference scenario) to the welfare under different levels of subsidy (which we call $\overline{\text{SUB}}X$). Then, we consider a setting with a single zone ($I = 1$). In this way, we remove the possibility of relocation from our model. Once again, we compare the welfare of the reference scenario (which we call $\widetilde{\text{REF}}$) to the welfare under different levels of subsidy (which we call $\widetilde{\text{SUB}}X$). Figure 4.5 summarizes the results. From this figure it follows that, when there is no crowding externality, any level of subsidy decreases welfare ($\overline{\text{SUB}}X$), although the reduction is not as drastic as in our initial case. In other words, both positive externalities linked to a transit subsidies are still completely offset by the relocation of residents in the city, and the corresponding increasing in travel distances. However, when the crowding externality is in play and only a single zone exists, any level of transit subsidy ($\widetilde{\text{SUB}}X$) actually increases welfare. Consequently, it follows that our initial result is mainly due to the possibility of relocation in the city.

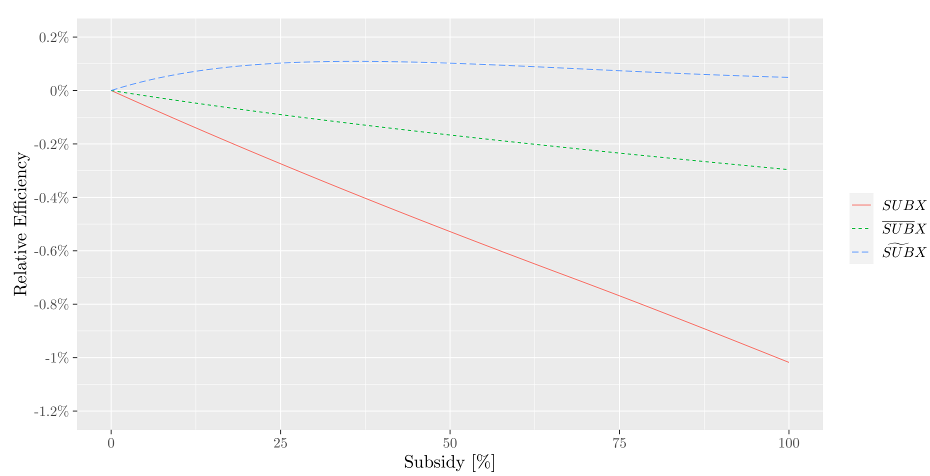


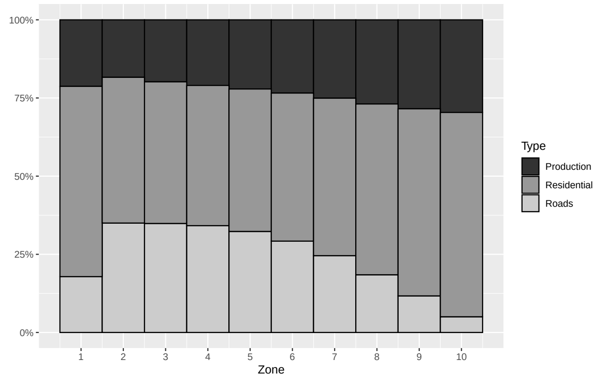
Figure 4.5: Relative Efficiency - SUBX vs $\overline{\text{SUB}}X$ vs $\widetilde{\text{SUB}}X$.

With respect to the other two policies considered (CON/DL), Table 4.2 shows that both are effective in increasing welfare. However, the underlying mechanisms of the two policies are very different. On the one hand, the congestion toll mainly favors the users that, in the REF scenario, are already using the public transport system but is not able to attract many new users. Indeed, in our setting, for larger distances, the car presents as a much more attractive alternative than the public transport, and the congestion toll is not able to change that, especially for the high-skill group. Then, although increasing the toll seems reasonable, note that the low-skill group is much more affected by any increase in the toll (which is reflected by the fact that the car modal share decrease is larger in this group). Nevertheless, the modal and location changes produced by the toll, albeit modest, are able to reduce the travel distances and the land needed for roads by a small amount, which explains the welfare gains the instrument provides. However, as Figures 4.7 and 4.8 show, its impact is much more pronounced in the low-skill group, due to the fact that this group uses the car in a much lower proportion. On the other hand, the dedicated lanes policy produces higher welfare gains by means of a sharp modal share change. Note that allocating road space exclusively to buses forces residents to move out of cars, but at the same time, if enough people start using public transport, it allows the planner to reduce the space allocated to roads. Indeed, since buses move people much more efficiently, in the DL scenario, only 4.95% of the land is allocated to roads, as opposed to the 13.37% of the REF scenario. Note that this effect is only obtainable when considering a setting where the land is able to redevelop. In other words, if using a setting that considers, for example, a fixed urban form, this welfare-improving effect of the dedicated lanes policy would not be present, and thus, its welfare gains would be much lower. We confirm this hypothesis through two experiments: first, we compute the welfare gains of policies when the land allocated to roads is fixed to the optimal level when considering the REF scenario. The results are shown in Table 4.3, where it follows that, in this scenario, the DL is no longer the prefered policy, since the CON policy achieves higher welfare gains. Second, we study the welfare gains of policies in the short and medium run. This analysis is presented in Section 4.4, but the results are similar: when the redevelopment is limited, the welfare gains of the dedicated lanes policy is only moderate, and lower than the one achieved by a congestion toll. Finally, note that, as Figures 4.7 and 4.8 show, this policy impacts both groups similarly. In other words, by replacing car travel with public transport, both groups can reap the benefits of a lower road land share.

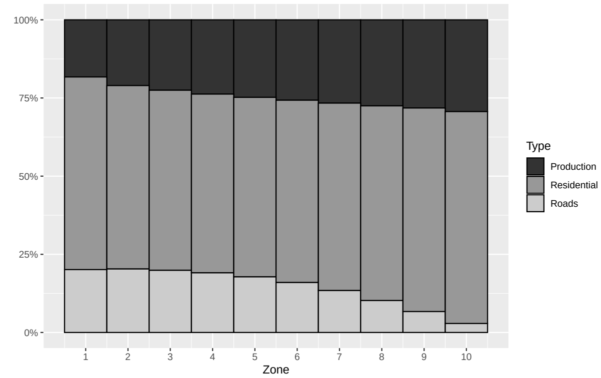
Table 4.3: Welfare impact of policies - fixed road land allocation.

Scenario	REF	SUB100	CON	DL	CON+DL
Welfare $\mathcal{W}$	61,181.44	60,531.89	61,350.29	61,265.73	61,375.56
Welfare change [%]	0	-1.06	0.28	0.14	0.32
Car modal share [%]	33.37	32.75	30.77	29.52	28.95
Bus modal share [%]	66.63	67.25	69.23	70.48	71.05

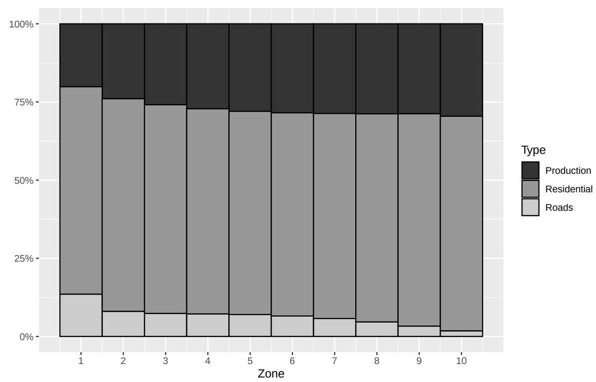
Finally, when a combination of the welfare improving policies is implemented (CON+DL), the effects in the resulting urban form are similar to the ones discussed above. However, since both policies produce a similar effect (decrease in the use of car/reduction of travel distances/reduction of land allocated to roads), the marginal contribution of the second policy, although significant, is strongly diminished. Note that in the setting we propose, the order of introduction of any of the two policies in the CON+DL is irrelevant. In other words, the resulting city would be the same regardless the congestion toll or the dedicated lanes policy is introduced first. In the next Section we explore this issue.



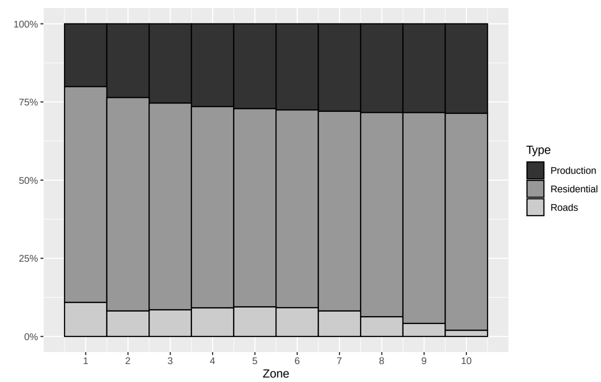
(a) SUB100.



(b) CON.



(c) DL.



(d) CON+DL.

Figure 4.6: Land use for the different policies.

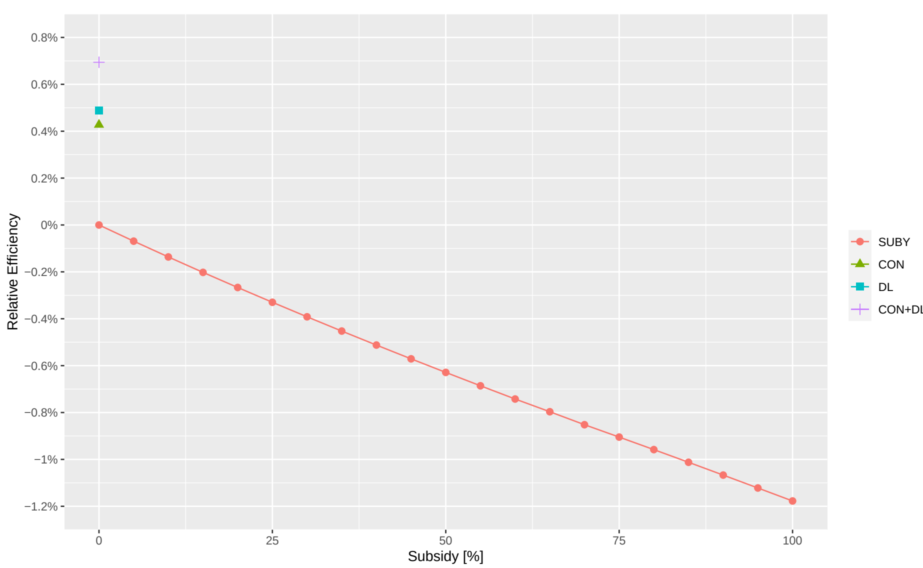


Figure 4.7: Relative Efficiency - Low skill group.

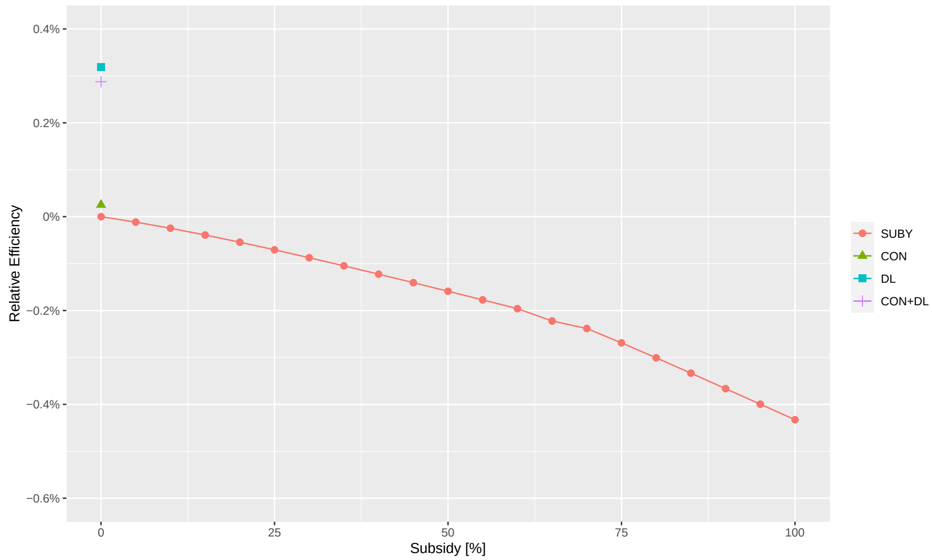


Figure 4.8: Relative Efficiency - High skill group.

4.4. Welfare analysis under cyclical redevelopment

The analysis of Section 4.3 implicitly consider a strong assumption: the city is malleable. In other words, the city can be rebuilt every time as any of the conditions change, ignoring the fact that buildings are not quickly replaceable. This framework is useful when comparing intercity differences at a given point in time or, in general, to describe the long-run effect of a change in the underlying conditions over the urban form (Brueckner, 1987). Yet, it fails to capture the short-term or medium-term effects, where space for redevelopment in cities may be very limited.

To shed light on the importance of this assumption, in this section we study the model proposed in Section 4.2 under a stylized setting of durable housing, similar to the one presented in Brueckner and Rosenthal (2009): assume that, when the city is first built, every ring is built in consecutive time periods. In other words, the i-th ring is built at $t = i$. Additionally, consider that buildings life span is given an (exogenous) number of periods L: in the time period $t = i + k \cdot L$, the ring i is redeveloped, for every integer k. In particular, when the ring i redevelops, every other ring does not. This implies that, for these rings, the land allocation is fixed (although rental prices may change).

Under this setting, we study the welfare gains of the policies we presented before. In particular, we first assume that a single policy is introduced just after the city is fully built, i.e. at $t = I + 1$. Then, we evaluate the welfare gains of that policy for every period $t = L + 1, \ldots, L + I, 2 \cdot L + 1, \ldots, 2 \cdot L + I, 3 \cdot L + 1, \ldots, 3 \cdot L + I$. In other words, we study three periods of full redevelopment of the city: from periods $t = L + 1$ to $t = L + I$, every ring is rebuilt, starting from the innermost one. The same occurs from periods $t = 2 \cdot L + 1$ to $t = 2 \cdot L + I$ and from periods $t = 3 \cdot L + 1$ to $t = 3 \cdot L + I$. In each period, we assume that a myopic planner chooses the equilibrium that maximizes welfare.

For simplicity, in the numerical analysis we consider $L = I$, i.e., just after the outermost ring redevelops, the innermost ring redevelops again. As Figure 4.9 shows that, when a single policy is used, the welfare-maximizing policy depends on the timeframe considered. Indeed, in the short term, the congestion toll provides larger welfare gains than dedicated lanes. The intuition behind this result is the same as in Section 3: dedicated lanes' welfare gains strongly depend on the ability to redevelop the city, and consequently to reduce the space allocated to roads when users migrate to public transport. However, in the short-term this is not possible, and the relative efficiency of this instrument is much lower than in the full redevelopment case, and lower than the congestion toll's relative efficiency. Note, however, that as the time t goes by and the city is fully rebuilt, this situation reverses, and dedicated lanes provide a higher relative efficiency than the congestion toll, just as in the malleable city case. Additionally, note that even after three periods of full cyclic redevelopment, the relative efficiencies of both instruments are lower than the malleable city case: 0.38% versus 0.45% in the case of dedicated lanes, and 0.32% versus 0.34% in the case of congestion tolls. Although this result is expected –since we consider a myopic planner under a cyclic redevelopment–, we believe that the main takeaway is that papers that consider a fully malleable city may greatly overestimate even the long-term welfare gains of any instrument, since the city does not rebuild at once. On the other hand, papers that consider a fixed urban form may greatly underestimate the relative efficiency of instrument, since, even in the very short-term, relocation occurs and prices and wages change. Moreover, as we discussed, the choice of a preferred instrument under either approach (fixed urban form/malleable city) might not even be the same.

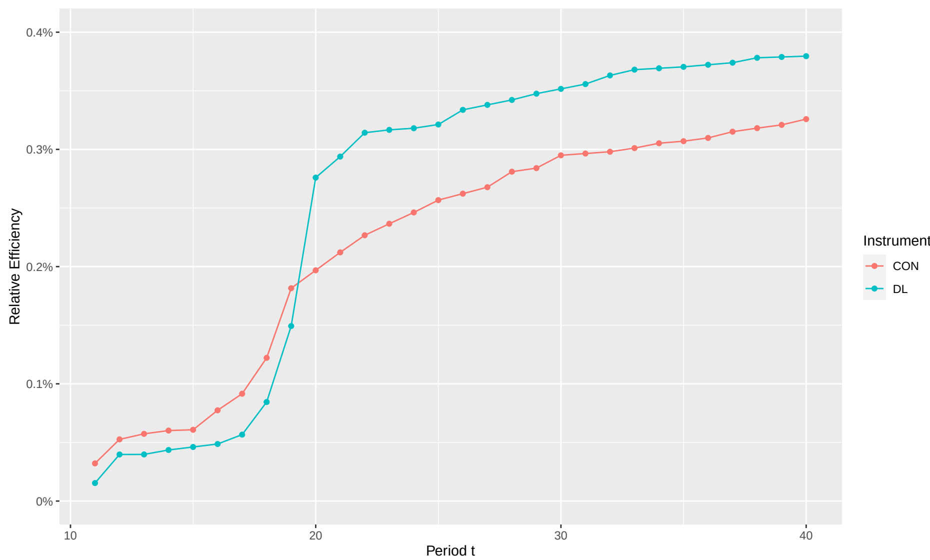


Figure 4.9: Welfare gains (DL/CON) for different values of t.

Then, we evaluate the introduction of two policies (dedicated lanes and congestion tolls) in different time periods. Figure 4.10 depicts the relative efficiency when the first policy is implemented in $t_1 = 11$, while the second is implemented in $t_2 = 16$.[20] From this Figure, it follows that, in the short and medium term, the introduction order of the policies considered is relevant: depending on which policy was first introduced, the same combination of policy may produce up to 13% less welfare gains than expected. It is only for the long-term scenarios (after two full cycles of redevelopment in our case) that the introduction order becomes irrelevant. In our case, introducing the dedicated lanes policy first produces a lower relative efficiency in the short-term ($t \leq 15$), before the second policy is introduced. The intuition is the same as in the single-policy discussion above (since for $t \leq 15$, there is indeed only one policy introduced). Then, in the medium-term ($16 \leq t \leq 31$), the DL+CON combination produces higher welfare gains than the CON+DL combination. This result is explained by two reasons: (i) The DL policy is more efficient in the medium term than the CON policy, as shown in Figure 4.9, and (ii) The DL policy requires a sharp modification of the land use, decreasing the percentage of land allocated to roads. In this regard, the DL+CON combination reduces the road land use earlier ($t = 11$) than the CON+DL combination ($t = 16$) and, thus, is able to make use of this reassignation of land. Finally, in the long term, as expected, the introduction order becomes irrelevant, and the two combinations of policies are almost equivalent, mimicking the behavior of a malleable city. However, note that just as in the single-policy case, even after three periods of full cyclic redevelopment, the malleable

[20] We evaluated the welfare gains for different values $t_2 - t_1$, and the conclusions remain largely the same. The main difference is that the smaller the value of $t_2 - t_1$, the less relevant is the introduction order of the two policies. However, for t large enough, both combinations of instruments (CON+DL/DL+CON) end up producing similar welfare gains.

city setting greatly overestimates the welfare gains of the combination of instruments: while the relative efficiency of the CON+DL policy under a malleable city case is 0.61%, the relative efficiency of the same policy at $t = 40$ is 0.53%, or 13% lower.

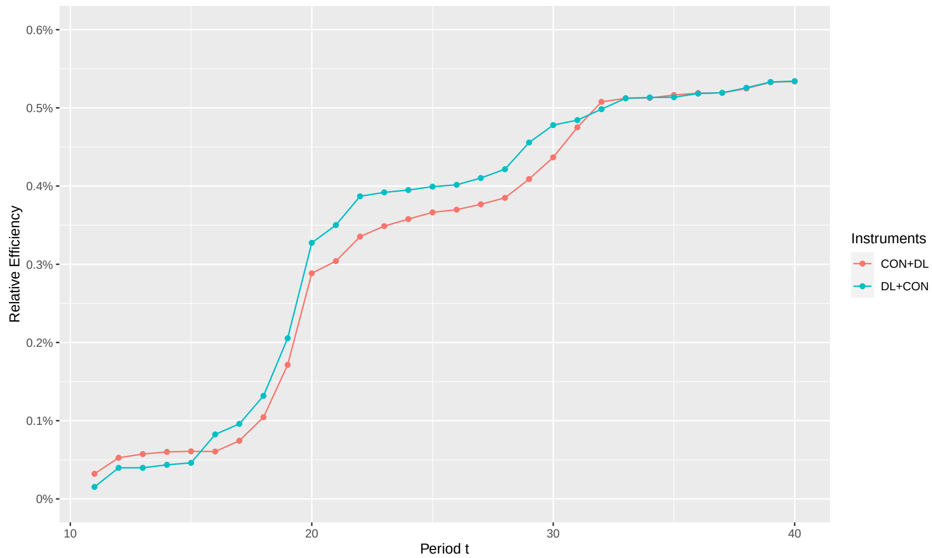


Figure 4.10: Welfare gains (DL+CON/CON+DL) for different values of t.

4.5. Concluding remarks

Although the literature has extensively studied the impacts and the desirability of policies aimed to correct transport externalities in cities, most of the previous work does not consider the interaction of these policies with the underlying urban form. Moreover, within those that consider the urban form, two main assumptions have been used: (i) urban form is fixed (i.e., no redevelopmet occurs as a response to policies), or (ii) the underlying city is malleable (i.e., full redevelopment occurs as a response to policies). To fill this gap, in this paper we propose a non-monocentric model that allows for relocation and redevelopment, featuring endogenous location decisions for firms and households of two skill groups, and two transport modes (private car and transit), allowing for cross-congestion effects between cars and buses. We use this model to study the interaction of three transport policies with the underlying urban form: congestion pricing, transit subsidies, and dedicated public transport infrastructure. Then, we propose a stylized setting of cyclical redevelopment that allow us to study the impact of transport policies not only in the long run but also in the medium run, where redevelopment is only partially possible and when the core of the urban change is due to relocation.

Our numerical experiments show that the use a transit subsidy sharply decreases the welfare in the city since it incentives longer trips, thus (i) increasing travel distances, (ii) decreasing the share of transit users in the high-skill group, (iii) increasing the land needed for roads, thus decreasing the land available for other uses. On the other hand, both congestion toll and dedicated lanes are effective in increasing welfare. However, the dedicated lanes policy produces higher welfare gains, explained by a sharp modal change to buses, and the consequent reduction of land allocated to roads, which allows users to increase their residential land consumption. Since this effect is strongly dependent on the redevelopment of land, any setting that considers a fixed urban form might underestimate the welfare gains of this instrument. Finally, by studying a combination of policies we find that the marginal contribution of the second policy, although significant, is strongly diminished.

Then, moving to the cyclical redevelopment setting, we find that, when a single policy is used, the welfare-maximizing policy depends on the timeframe considered. Indeed, since dedicated lanes' welfare gains depend on land redevelopment, in the short-term, congestion toll are a more desirable option. However, in the medium-term, where the city starts to redevelop, this conclusions reverses. We then evaluate a combination of policies, finding that in the short and medium term, the introduction order of the policies considered is relevant: depending on which policy was first introduced, the same combination of policy may produce up to 13% less welfare gains than expected. This is explained once again by the dependency of each policy on the redevelopment of the city.

Additionally, one of the main takeaways of our experiments –and a word of caution– is that papers that consider a fully malleable city may greatly overestimate even the long-term welfare gains of any instrument since the city does not rebuild at once. On the other hand, papers that consider a fixed urban form may greatly underestimate the relative efficiency of the instrument since, even in the very short term, relocation occurs and prices and wages change.

Chapter 5

Conclusions

This thesis delved into two topics: (i) the welfare properties of the urban equilibrium in the absence of externalities, (ii) how different transport policies affect the structure and interact with the shape and features of cities, both in the short run and long run. Chapters 2 and 3 are dedicated to analysis of the properties of the urban equilibrium, while Chapter 4 centers on evaluating the effectiveness of diverse transport policies within a framework that permits us to proxy the short and long-term consequences of these policies.

In Chapter 2, we examine the foundational monocentric city model in urban economics, with a focus on assessing welfare through two prevailing approaches: equilibrium utility maximization and resource usage minimization. While prior literature often treated these approaches as interchangeable, a crucial distinction has been overlooked. Specifically, one approach channels all land rents to city residents, while the other assumes that all rents accrue to absentee landlords. To bridge this gap, we introduce a unified land ownership framework, where a fraction $\mu \in [0, 1]$ of land rents is retained for city residents. Our analysis shows that the assumption of equivalence between maximizing equilibrium utility and minimizing resource usage is does not hold, and that each approach will lead to a different equilibrium in general. This result holds significant implications, as the monocentric city model is widely used to evaluate urban and transportation policies. Therefore, our findings emphasize the need for a more nuanced understanding of welfare functions and land rent assumptions, as the choice between these can substantially modify policy evaluations.

In Chapter 3, we study welfare-improving taxes in a city of equals in the absence of externalities. For this, we introduce a monocentric city model with mixed land ownership, where residents jointly own a fraction μ of the land, while absentee landlords own the rest $(1 - \mu)$. Our findings reveal that, for any $\mu \neq 1$, a Rawlsian planner would use a revenue-neutral combination of location-specific taxes and lump-sum redistribution to shift income from the city center to the outskirts. This approach increases residents' utility at the expense of absentee landlords by diminishing excess land rents. Importantly, this result holds, in general, even if absentee landlords are included in the welfare function. We extend this analysis to a quantitative urban model of Berlin with an absentee landlord and we show that property taxes, labor taxes, and corporate taxes can increase expected utility for all residents. These insights underscore that the nature of land ownership significantly influences policy outcomes, highlighting the need to consider this factor when assessing urban policies.

In Chapter 4, we address the interaction between transport policies and urban form. For this, we introduce a non-monocentric model accounting for relocation and redevelopment, featuring endogenous location choices for households and firms, two transport modes (private car and transit), and cross-congestion effects. We explore three transport policies (congestion pricing, transit subsidies, and dedicated public transport infrastructure) and introduce a cyclical redevelopment framework that allows us to proxy the short and long-term consequences of these policies. Our numerical experiments reveal that transit subsidies reduce city welfare since it incentives longer trips, thus (i) increasing travel distances, (ii) decreasing the share of transit users in the high-skill group, (iii) increasing the land needed for roads, thus decreasing the land available for other uses. In contrast, congestion tolls and dedicated lanes prove effective in increasing welfare, with dedicated lanes producing higher gains. Furthermore, we show that the choice between these policies depends on the timeframe considered, particularly influenced by land redevelopment dynamics. We also find that the introduction order of policies in combination can substantially impact welfare outcomes. Importantly, we show that models assuming either fully adaptable or fixed urban forms can lead to significant overestimations or underestimations of policy effectiveness.

Several future research lines arise from this thesis. For instance, in Chapter 3 we show that our main result holds in a quantitative model. Given the increasing prominence of quantitative spatial models in urban policy analysis since the seminal work of Ahlfeldt et al. (2015), exploring the interplay between transportation policies and the redistribution effect we present in a quantitative framework represents an interesting research direction. Notable examples, such as Tsivanidis (2019) and Warnes (2020), have utilized quantitative models for transport policy evaluation, but the interaction between transport policies and our proposed redistribution mechanism remains unexplored in empirical contexts. Additionally, Chapter 4 offers a stylized approach to redevelopment and a myopic planner to maintain tractability. However, the literature offers more intricate and realistic settings for durable housing (see, e.g., Brueckner, 1996, for a review). Exploring these complex housing dynamics could provide a deeper understanding of the magnitude and dynamics of the effects highlighted in our research.

Bibliography

Ahlfeldt, G.M., Redding, S.J., Sturm, D.M., Wolf, N., 2015. The economics of density: Evidence from the berlin wall. Econometrica 83, 2127–2189.

Allen, T., Arkolakis, C., 2022. The welfare effects of transportation infrastructure improvements. The Review of Economic Studies 89, 2911–2957.

Alonso, W., 1964. Location and land use. Toward a general theory of land rent. Cambridge, Mass.: Harvard University Press.

Anas, A., Rhee, H.J., 2006. Curbing excess sprawl with congestion tolls and urban boundaries. Regional Science and Urban Economics 36, 510–541.

Anas, A., Rhee, H.J., 2007. When are urban growth boundaries not second-best policies to congestion tolls? Journal of Urban Economics 61, 263–286.

Anas, A., Xu, R., 1999. Congestion, land use, and job dispersion: a general equilibrium model. Journal of Urban Economics 45, 451–473.

Arnott, R., 1979a. Optimal city size in a spatial economy. Journal of Urban Economics 6, 65–89.

Arnott, R.J., 1979b. Unpriced transport congestion. Journal of Economic Theory 21, 294–316.

Basso, L.J., Silva, H.E., 2014. Efficiency and substitutability of transit subsidies and other urban transport policies. American Economic Journal: Economic Policy 6, 1–33.

Ben-Akiva, M.E., 1973. Structure of passenger travel demand models. Ph.D. thesis. Massachusetts Institute of Technology.

Bertaud, A., Brueckner, J.K., 2005. Analyzing building-height restrictions: predicted impacts and welfare costs. Regional Science and Urban Economics 35, 109–125.

Brinkman, J.C., 2016. Congestion, agglomeration, and the structure of cities. Journal of Urban Economics 94, 13–31.

Brueckner, J.K., 1987. The structure of urban equilibria: A unified treatment of the muth-mills model. Handbook of regional and urban economics 2, 821–845.

Brueckner, J.K., 1996. Urban growth models with durable housing: An overview. Department of Economics, and Institute of Government and Public Affairs . . ..

Brueckner, J.K., 2000. Urban sprawl: diagnosis and remedies. International regional science review 23, 160–171.

Brueckner, J.K., 2005. Transport subsidies, system choice, and urban sprawl. Regional Science and Urban Economics 35, 715–733.

Brueckner, J.K., 2007. Urban growth boundaries: An effective second-best remedy for unpriced traffic congestion? Journal of Housing Economics 16, 263–273.

Brueckner, J.K., 2011. Lectures on urban economics. MIT press.

Brueckner, J.K., Kim, H.A., 2003. Urban sprawl and the property tax. International Tax and Public Finance 10, 5–23.

Brueckner, J.K., Rosenthal, S.S., 2009. Gentrification and neighborhood housing cycles: will america's future downtowns be rich? The Review of Economics and Statistics 91, 725–743.

Card, D., 2009. Immigration and inequality. American Economic Review 99, 1–21.

Cooke, T.W., 1988. Mortgage and rental qualification criteria, household location decisions, and urban form. Journal of Urban Economics 23, 354–369.

Cvijanović, D., Spaenjers, C., 2021. We'll always have Paris: Out-of-country buyers in the housing market. Management Science 67, 4120–4138.

De Cea, J., Fernández, E., 1993. Transit assignment for congested public transport systems: an equilibrium model. Transportation science 27, 133–147.

De Lara, M., De Palma, A., Kilani, M., Piperno, S., 2013. Congestion pricing and long term urban form: Application to paris region. Regional Science and Urban Economics 43, 282–295.

Diamond, R., McQuade, T., 2019. Who wants affordable housing in their backyard? an equilibrium analysis of low-income property development. Journal of Political Economy 127, 1063–1117.

Dröes, M.I., Rietveld, P., 2015. Rail-based public transport and urban spatial structure: The interplay between network design, congestion and urban form. Transportation Research Part B: Methodological 81, 421–439.

Duranton, G., Puga, D., 2015. Urban land use, in: Handbook of regional and urban economics. Elsevier. volume 5, pp. 467–560.

Fajgelbaum, P.D., Gaubert, C., 2020. Optimal spatial policies, geography, and sorting. The Quarterly Journal of Economics 135, 959–1036.

Favilukis, J., Van Nieuwerburgh, S., 2021. Out-of-town home buyers and city welfare. The Journal of Finance 76, 2577–2638.

Fujita, M., 1989. Urban economic theory: land use and city size. Cambridge University Press.

Fujita, M., Ogawa, H., 1982. Multiple equilibria and structural transition of non-monocentric urban configurations. Regional science and urban economics 12, 161–196.

Fujita, M., Thisse, J.F., 2013. Economics of Agglomeration: Cities, Industrial Location, and Globalization. Cambridge University Press.

Glaeser, E.L., Kahn, M.E., 2004. Sprawl and urban growth, in: Handbook of regional and urban economics. Elsevier. volume 4, pp. 2481–2527.

Glaeser, E.L., Kahn, M.E., Rappaport, J., 2008. Why do the poor live in cities? the role of public transportation. Journal of urban Economics 63, 1–24.

Heblich, S., Redding, S.J., Sturm, D.M., 2020. The making of the modern metropolis: evidence from london. The Quarterly Journal of Economics 135, 2059–2133.

Ho, H., Wong, S., Sumalee, A., 2013. A congestion-pricing problem with a polycentric region and multi-class users: a continuum modelling approach. Transportmetrica A: Transport Science 9, 514–545.

Jiao, P., Sun, T., Guo, J., Li, Y., 2015. Joint residence-workplace location choice model based on household decision behavior. Discrete Dynamics in Nature and Society 2015.

Kanemoto, Y., 1977. Cost-benefit analysis and the second best land use for transportation. Journal of Urban Economics 4, 483–503.

Kanemoto, Y., 1980. Theories of urban externalities. Amsterdam: North-Holland.

Kono, T., Joshi, K.K., Kato, T., Yokoi, T., 2012. Optimal regulation on building size and city boundary: An effective second-best remedy for traffic congestion externality. Regional Science and Urban Economics 42, 619–630.

Kono, T., Kawaguchi, H., 2017. Cordon pricing and land-use regulation. The Scandinavian Journal of Economics 119, 405–434.

Kono, T., Pines, D., Yokoi, T., 2019. Spatially-variable property tax and optimal tax composition in congested monocentric cities: George, pigou, ramsey and strotz unified. Journal of Urban Economics 112, 122–132.

Koppelman, F.S., Wen, C.H., 1998. Alternative nested logit models: structure, properties and estimation. Transportation Research Part B: Methodological 32, 289–298.

US Department of Labor, B.o.L.S., 2018. Consumer expenditure survey.

Li, Z.C., Lam, W.H., Wong, S., 2012. Modeling intermodal equilibrium for bimodal transportation system design problems in a linear monocentric city. Transportation Research Part B: Methodological 46, 30–49.

Li, Z.C., Wang, Y.D., 2018. Analysis of multimodal two-dimensional urban system equilibrium for cordon toll pricing and bus service design. Transportation Research Part B: Methodological 111, 244–265.

Lucas, R.E., Rossi-Hansberg, E., 2002. On the internal structure of cities. Econometrica 70, 1445–1476.

McFadden, D., 1977. Modelling the choice of residential location .

McGuckin, N., Fucci, A., 2018. Summary of travel trends: 2017 national household travel survey. Washington, DC: Federal Highway Administration .

Mills, E.S., 1967. An aggregative model of resource allocation in a metropolitan area. The American Economic Review 57, 197–210.

Mun, S.i., Konishi, K.j., Yoshikawa, K., 2003. Optimal cordon pricing. Journal of Urban Economics 54, 21–38.

Mun, S.i., Konishi, K.j., Yoshikawa, K., 2005. Optimal cordon pricing in a non-monocentric city. Transportation Research Part A: Policy and Practice 39, 723–736.

Muth, R., 1969. Cities and housing: The spatial patterns of urban residential land use. University of Chicago, Chicago, 4, 114-123.

Ogawa, H., Fujita, M., 1980. Equilibrium land use patterns in a nonmonocentric city. Journal of regional science 20, 455–475.

Oron, Y., Pines, D., Sheshinski, E., 1973. Optimum vs. equilibrium land use pattern and congestion toll. The Bell Journal of Economics and Management Science , 619–636.

Papageorgiou, Y.Y., Pines, D., 1999. An essay on urban economic theory. volume 1. Springer Science & Business Media.

Parry, I.W., Small, K.A., 2009. Should urban transit subsidies be reduced? American Economic Review 99, 700–724.

Pérez, J.P., 2022. City minimum wages and spatial equilibrium effects. Banco de México.

Pines, D., Kono, T., 2012. Far regulations and unpriced transport congestion. Regional Science and Urban Economics 42, 931–937.

Pines, D., Sadka, E., 1985. Zoning, first-best, second-best, and third-best criteria for allocating land for roads. Journal of Urban Economics 17, 167–183.

Proost, S., Van Dender, K., 2008. Optimal urban transport pricing in the presence of congestion, economies of density and costly public funds. Transportation Research Part A: Policy and Practice 42, 1220–1230.

Redding, S.J., 2023. Quantitative urban models. The Journal of Economic Perspectives 37, 75–98.

Redding, S.J., Rossi-Hansberg, E., 2017. Quantitative spatial economics. Annual Review of Economics 9, 21–58.

Robson, A.J., 1976. Cost-benefit analysis and the use of urban land for transportation. Journal of Urban Economics 3, 180–191.

Santos, G., Behrendt, H., Maconi, L., Shirvani, T., Teytelboym, A., 2010a. Part i: Externalities and economic policies in road transport. Research in transportation economics 28, 2–45.

Santos, G., Behrendt, H., Teytelboym, A., 2010b. Part ii: Policy instruments for sustainable road transport. Research in transportation economics 28, 46–91.

Schindler, M., Wang, J.Y., Connors, R.D., 2021. A two-stage residential location and transport mode choice model with exposure to traffic-induced air pollution. Journal of Transport Geography 93, 103044.

Small, K.A., Verhoef, E.T., 2007. The economics of urban transportation. Routledge.

Sommer, K., Sullivan, P., 2018. Implications of us tax policy for house prices, rents, and homeownership. American Economic Review 108, 241–74.

Song, Y., Zenou, Y., 2006. Property tax and urban sprawl: Theory and implications for us cities. Journal of urban economics 60, 519–534.

Straszhem, M., 1987. The theory of urban residential location, in: Handbook of regional and urban economics. Elsevier. volume 2, pp. 717–757.

Tikoudis, I., Verhoef, E.T., van Ommeren, J.N., 2015. On revenue recycling and the welfare effects of second-best congestion pricing in a monocentric city. Journal of Urban Economics 89, 32–47.

Tikoudis, I., Verhoef, E.T., van Ommeren, J.N., 2018. Second-best urban tolls in a monocentric city with housing market regulations. Transportation Research Part B: Methodological 117, 342–359.

Tirachini, A., Hensher, D.A., Jara-Díaz, S.R., 2010. Restating modal investment priority with an improved model for public transport analysis. Transportation Research Part E: Logistics and Transportation Review 46, 1148–1168.

Tscharaktschiew, S., Hirte, G., 2010. How does the household structure shape the urban economy? Regional Science and Urban Economics 40, 498–516.

Tsivanidis, N., 2019. Evaluating the impact of urban transit infrastructure: Evidence from bogotá's transmilenio. Unpublished manuscript .

Turner, M.A., Haughwout, A., Van Der Klaauw, W., 2014. Land use regulation and welfare. Econometrica 82, 1341–1403.

Verhoef, E.T., 2005. Second-best congestion pricing schemes in the monocentric city. Journal of Urban Economics 58, 367–388.

Waddell, P., 1993. Exogenous workplace choice in residential location models: Is the assumption valid? Geographical Analysis 25, 65–82.

Warnes, P.E., 2020. Transport infrastructure improvements and spatial sorting: Evidence from buenos aires. Technical Report. Working paper.

Zhang, W., Kockelman, K.M., 2016a. Congestion pricing effects on firm and household location choices in monocentric and polycentric cities. Regional Science and Urban Economics 58, 1–12.

Zhang, W., Kockelman, K.M., 2016b. Optimal policies in cities with congestion and agglomeration externalities: Congestion tolls, labor subsidies, and place-based strategies. Journal of Urban Economics 95, 64–86.

Annexes

Annex A. Proofs of Chapter 1

A.1. Proof of Proposition 3

Let $C(q(x),\overline{u})$ be the quantity of the composite good for which $u(C(q(x),\overline{u}),q(x)) = \overline{u}$. The problem stated in Proposition 3 can be restated as

$$(P_{\min}) \qquad \max \quad \left(\int_0^{\overline{x}} \frac{\overline{y} - tx - C(q(x),\overline{u})}{q(x)} - r_a \; dx \right) \tag{A.1a}$$

$$\int_0^{\overline{x}} \frac{1}{q(x)} dx = L \tag{A.1b}$$

Where $\overline{y} = y + \frac{\mu R}{L}$. From this, it follows directly from the proof of Section 3.3.3 of Fujita and Thisse (2013) that the market equilibrium solves this problem.

A.2. Proof of Proposition 4

The utility maximization problem under our setting can be stated as

$$
\begin{aligned}
(P_{\max}) \qquad & \max && \int_0^{\overline{x}} \frac{\overline{u}}{q(x)} dx && && \text{(A.2a)}\\
& \text{subject to} && u(c(x), q(x)) = \overline{u}, && 0 \leq x \leq \overline{x} && \text{(A.2b)}\\
& && \int_0^{\overline{x}} \frac{1}{q(x)} dx = L && && \text{(A.2c)}\\
& && \frac{\mu R}{L} + y - t(x) - c(x) - p(x)q(x) = 0 && 0 \leq x \leq \overline{x} && \text{(A.2d)}\\
& && R = \int_0^{\overline{x}} p(x) - r_a \; dx && && \text{(A.2e)}
\end{aligned}
$$

with $\mu \in [0, 1]$. Define M as the total budget of a household:

$$
\begin{aligned}
M &= y + \frac{\mu R}{L} && \text{(A.3)}\\
&= t(x) + c(x) + p(x)q(x) && \text{(A.4)}
\end{aligned}
$$

Using (A.4) we obtain

$$
p(x) = \frac{M - t(x) - c(x)}{q(x)} \qquad \text{(A.5)}
$$

Then, replacing (A.2e) in (A.3), we get

$$
\begin{aligned}
M &= y + \frac{\mu}{L} \int_0^{\overline{x}} p(x) - r_a \; dx\\
&= y + \frac{1}{L} \int_0^{\overline{x}} p(x) - \mu r_a - (1 - \mu)p(x) \; dx
\end{aligned}
$$

Then, using (A.5)

$$
\begin{aligned}
M &= y + \frac{1}{L} \int_0^{\overline{x}} \frac{M - t(x) - c(x)}{q(x)} - \mu r_a - (1 - \mu)p(x) \; dx\\
&= y + M + \frac{1}{L} \int_0^{\overline{x}} -\frac{t(x) + c(x)}{q(x)} - r_a - (1 - \mu)\frac{u_q}{u_c} \; dx
\end{aligned}
$$

Where we used the individual utility maximization condition $p(x) = \dfrac{u_q(c(x), q(x))}{u_c(c(x), q(x)}$. The budget constraint (A.2d) can then be replaced by

$$
\int_0^{\overline{x}} \frac{t(x) + c(x)}{q(x)} + \mu r_a + (1 - \mu)\frac{u_q}{u_c} \; dx = Ly \qquad \text{(A.6)}
$$

The Lagrangian for this problem can then be written as

$$\Lambda_1(x) = \lambda_1 \int_0^{\overline{x}} \frac{\overline{u}}{q(x)} dx + \int_0^{\overline{x}} \nu(x)(u(c(x), q(x)) - \overline{u}) dx + \delta \left[Ly - \int_0^{\overline{x}} \frac{t(x) + c(x)}{q(x)} + \mu r_a + (1-\mu) \frac{u_q}{u_c}\ dx \right] + \gamma \left[L - \int_0^{\overline{x}} \frac{1}{q(x)}\ dx \right] \tag{A.7}$$

where $\nu(x), \delta$ and γ are respectively the Lagrange multipliers associated with (A.2b), (A.6) and (A.2c). Applying the maximum principle, the first order conditions are (for the maximum principle used in the following, see Kanemoto, 1980):

$$\nu(x) u_c - \frac{\delta}{q(x)} - \delta \left(\frac{1-\mu}{u_c^2} \right) (u_{cq} u_c - u_q u_{cc}) = 0 \qquad 0 \leq x \leq \overline{x} \tag{A.8}$$

$$\nu(x) u_q - \frac{\lambda_1 \overline{u} - \delta t(x) - \delta c(x) - \gamma}{q(x)^2} - \delta \left(\frac{1-\mu}{u_c^2} \right) (u_{qq} u_c - u_q u_{cq}) = 0 \qquad 0 \leq x \leq \overline{x} \tag{A.9}$$

$$\int_0^{\overline{x}} \frac{1}{q(x)} - \int_0^{\overline{x}} \nu(x) = 0 \tag{A.10}$$

$$\frac{\lambda_1 \overline{u}}{q(\overline{x})} - \delta \left(\frac{t(\overline{x}) + c(\overline{x})}{q(\overline{x})} + r_a \right) - \frac{\gamma}{q(\overline{x})} = 0 \tag{A.11}$$

On the other hand, consider the following problem, where we seek to minimize resource usage plus a share of excess land rents:

$$(\tilde{P}_{\min}) \qquad \min \qquad \int_0^{\overline{x}} \frac{t(x) + c(x)}{q(x)} + r_a\ dx + (1-\mu) R \tag{A.12a}$$

$$\text{subject to} \qquad u(c(x), q(x)) = \overline{u}, \qquad 0 \leq x \leq \overline{x} \tag{A.12b}$$

$$\int_0^{\overline{x}} \frac{1}{q(x)} dx = L \tag{A.12c}$$

Note that the objective function can be written as

$$\int_0^{\overline{x}} \frac{t(x) + c(x)}{q(x)} + r_a\ dx + (1-\mu) R = \int_0^{\overline{x}} \frac{t(x) + c(x)}{q(x)} + \mu r_a + (1-\mu) \frac{u_q}{u_c}\ dx \tag{A.13}$$

Then, the Lagrangian is

$$\begin{aligned} \Lambda_2(x) = & -\lambda_2 \int_0^{\overline{x}} \frac{t(x) + c(x)}{q(x)} + \mu r_a + (1-\mu) \frac{u_q}{u_c}\ dx \\ & + \int_0^{\overline{x}} \kappa(x)(u(c(x), q(x)) - \overline{u}) dx + \rho \left[L - \int_0^{\overline{x}} \frac{1}{q(x)}\ dx \right] \end{aligned} \tag{A.14}$$

where $\kappa(x)$ and ρ are respectively the Lagrange multipliers associated with (A.12b) and (A.12c). The first order conditions are

$$\kappa(x)u_c - \frac{\lambda_2}{q(x)} - \lambda_2\left(\frac{1-\mu}{u_c^2}\right)(u_{cq}u_c - u_q u_{cc}) = 0 \qquad 0 \leq x \leq \overline{x} \tag{A.15}$$

$$\kappa(x)u_q - \frac{-\lambda_2 t(x) - \lambda_2 c(x) - \rho}{q(x)^2} - \lambda_2\left(\frac{1-\mu}{u_c^2}\right)(u_{qq}u_c - u_q u_{cq}) = 0 \qquad 0 \leq x \leq \overline{x} \tag{A.16}$$

$$\int_0^{\overline{x}} \frac{1}{q(x)} - \int_0^{\overline{x}} \kappa(x) = 0 \tag{A.17}$$

$$-\lambda_2\left(\frac{t(\overline{x}) + c(\overline{x})}{q(\overline{x})} + r_a\right) - \frac{\rho}{q(\overline{x})} = 0 \tag{A.18}$$

Finally, suppose that for some feasible control variables and parameters $(c^*(x), q^*(x), \overline{x}^*, \overline{u}^*)$ there exists a set of multipliers $(\nu^*(x), \lambda_1^*, \delta^*, \gamma^*)$ such that (A.8)-(A.11) hold. Then, note that there also exists a set of multipliers $(\kappa^*(x), \lambda_2^*, \rho^*)$ such that (A.15)-(A.18) also hold for $(c^*(x), q^*(x), \overline{x}^*, \overline{u}^*)$. Indeed, consider

$$(\kappa^*(x), \lambda_2^*, \rho^*) = (\nu^*(x), \delta^*, \gamma^* - \lambda_1^*\overline{u}) \tag{A.19}$$

Conversely, suppose that for some feasible control variables and parameters $(c^*(x), q^*(x), \overline{x}^*, \overline{u}^*)$ there exists a set of multipliers $(\kappa^*(x), \lambda_2^*, \rho^*)$ such that (A.15)-(A.18) hold. Then, note that there also exists a set of multipliers $(\nu^*(x), \lambda_1^*, \delta^*, \gamma^*)$ such that (A.8)-(A.11) also hold for $(c^*(x), q^*(x), \overline{x}^*, \overline{u}^*)$. Indeed, consider

$$(\nu^*(x), \lambda_1^*, \delta^*, \gamma^*) = (\kappa^*(x), m, \lambda_2^*, \rho^* + m\overline{u}) \tag{A.20}$$

for any $m \geq 0$.

Annex B. Proofs of Chapter 2

B.1. Proof of Proposition 6

For this proof, we consider a more general setting, where commuting costs may depend on the number of individuals using the same stretch of the road at the same time. To be more precise, the commuting cost per mile at x is equal to $g(F(x))$, where $F(x)$ is the total traffic at x, and g is such that $g > 0$, $g_F \geq 0$. The case without congestion is then a particular case of the proof presented in this Appendix, where $g_F = 0$. The welfare-maximization optimal control problem associated with this formulation is:

$$(P) \qquad \max \quad \int_0^{\overline{x}} \frac{\overline{U}}{l(x)} dx \tag{B.1a}$$

$$\text{s.t.} \quad u(c(x), l(x)) = \overline{U}, \qquad 0 \leq x \leq \overline{x} \tag{B.1b}$$

$$t(x) = \int_0^x g(F(z))dz, \qquad 0 \leq x \leq \overline{x} \tag{B.1c}$$

$$F(x) = \int_x^{\overline{x}} \frac{1}{l(z)} dz, \qquad 0 \leq x \leq \overline{x} \tag{B.1d}$$

$$\int_0^{\overline{x}} \frac{1}{l(x)} dx = H \tag{B.1e}$$

$$\frac{\mu R}{H} + \frac{G}{H} + y = t(x) + \tau(x) + c(x) + Q(x)l(x) \qquad 0 \leq x \leq \overline{x} \tag{B.1f}$$

$$R = \int_0^{\overline{x}} Q(x) - Q_a \ dx \tag{B.1g}$$

$$G = \int_0^{\overline{x}} \frac{\tau(x)}{l(x)} dx \tag{B.1h}$$

$$Q(\overline{x}) = Q_a \tag{B.1i}$$

where $c(x)$, $l(x)$ and $\tau(x)$ are control variables, while $\overline{U}$ and $\overline{x}$ are control parameters.

Condition (B.1d) can be replaced by the following condition:

$$F'(x) = -\frac{1}{l(x)} \qquad 0 \leq x \leq \overline{x} \tag{B.2}$$

Then, define M as the total budget of a household:

$$M = y + \frac{\mu R}{H} + \frac{G}{H} \tag{B.3}$$

$$= t(x) + \tau(x) + c(x) + Q(x)l(x) \tag{B.4}$$

Using (B.4) we obtain

$$Q(x) = \frac{M - t(x) - \tau(x) - c(x)}{l(x)} \tag{B.5}$$

Then, replacing (B.1g) and (B.1h) in (B.3), we get

$$M = y + \frac{\mu}{H}\int_0^{\overline{x}} Q(x) - Q_a \; dx + \frac{1}{H}\int_0^{\overline{x}} \frac{\tau(x)}{l(x)} dx \tag{B.6}$$

$$= y + \frac{1}{H}\int_0^{\overline{x}} Q(x) - \mu Q_a - (1-\mu)Q(x) \; dx + \frac{1}{H}\int_0^{\overline{x}} \frac{\tau(x)}{l(x)} dx \tag{B.7}$$

Then, using (B.5) in (B.7):

$$M = y + \frac{1}{H}\int_0^{\overline{x}} \frac{M - t(x) - c(x)}{l(x)} - \mu Q_a - (1-\mu)Q(x) \; dx$$

$$= y + M + \frac{1}{H}\int_0^{\overline{x}} -\frac{t(x) + c(x)}{l(x)} - Q_a - (1-\mu)\frac{u_l}{u_c} \; dx$$

Where we used the individual utility maximization condition $Q(x) = \dfrac{u_l(c(x), l(x))}{u_c(c(x), l(x)}$. Thus, the budget constraint (B.1f) can then be replaced by:

$$\int_0^{\overline{x}} \frac{t(x) + c(x)}{l(x)} + \mu Q_a + (1-\mu)\frac{u_l}{u_c} \; dx = Hy \tag{B.8}$$

Now, note that the second term of (B.8) can be integrated by parts:

$$\int_0^{\overline{x}} \frac{t(x)}{l(x)} \; dx = \int_0^{\overline{x}} -t(x)F'(x) \; dx \tag{B.9}$$

$$= - \; t(x)F'(x)|_0^{\overline{x}} + \int_0^{\overline{x}} t'(x)F(x) \; dx \tag{B.10}$$

$$= \int_0^{\overline{x}} F(x)g(F(x)) \; dx \tag{B.11}$$

where we used (B.2) for the first equality, and the boundary conditions $t(0) = 0$, $F(\overline{x}) = 0$ and the condition $t'(x) = g(F(x))$ for the last equality. Using this last in (B.8), we arrive to the budget constraint we will use:

$$\int_0^{\overline{x}} \frac{c(x)}{l(x)} + F(x)g(F(x)) + \mu Q_a + (1-\mu)\frac{u_l}{u_c} \; dx = Hy \tag{B.12}$$

Consequently, the problem (P) can be restated as:

$$(\tilde{P}) \qquad \max \int_0^{\overline{x}} \frac{\overline{U}}{l(x)} dx \tag{B.13a}$$

$$\text{s.t.} \quad u(c(x), l(x)) = \overline{U}, \qquad 0 \leq x \leq \overline{x} \tag{B.13b}$$

$$F'(x) = -\frac{1}{l(x)} \tag{B.13c}$$

$$\int_0^{\overline{x}} \frac{c(x)}{l(x)} + F(x)g(F(x)) + \mu Q_a + (1-\mu)\frac{u_l}{u_c} \; dx = Hy \tag{B.13d}$$

$$\int_0^{\overline{x}} \frac{1}{l(x)} dx = H \tag{B.13e}$$

The Lagrangian for $(\tilde{P})$ can then be written as:

$$\Lambda(x) = \int_0^{\overline{x}} \frac{\overline{U}}{l(x)} dx + \int_0^{\overline{x}} \nu(x)(u(c(x), l(x)) - \overline{U}) dx - \frac{\lambda(x)}{l(x)} \tag{B.14}$$

$$+ \delta \left[Hy - \int_0^{\overline{x}} \frac{c(x)}{l(x)} + F(x)g(F(x)) + \mu Q_a + (1-\mu)\frac{u_l}{u_c} \, dx \right] + \gamma \left[H - \int_0^{\overline{x}} \frac{1}{l(x)} \, dx \right] \tag{B.15}$$

The first order conditions are then:

$$-\frac{\delta}{l(x)} - \delta \left(\frac{1-\mu}{u_c^2} \right) (u_{cl}u_c - u_l u_{cc}) + v(x)u_c = 0 \qquad 0 \leq x \leq \overline{x} \tag{B.16}$$

$$-\frac{\overline{U} - \lambda(x) - \delta c(x) - \gamma}{l(x)^2} - \delta \left(\frac{1-\mu}{u_c^2} \right) (u_{ll}u_c - u_l u_{cl}) + v(x)u_l = 0 \qquad 0 \leq x \leq \overline{x} \tag{B.17}$$

$$-\lambda'(x) = -\delta \left(g(F(x)) + F(x)\frac{dg}{dF}(F(x)) \right) \qquad 0 \leq x \leq \overline{x} \tag{B.18}$$

$$\int_0^{\overline{x}} \frac{1}{l(x)} dx = \int_0^{\overline{x}} \nu(x) \tag{B.19}$$

$$\frac{\overline{U} - \lambda(\overline{x}) - \gamma}{l(\overline{x})} - \delta \left(\frac{c(\overline{x})}{l(\overline{x})} + F(\overline{x})g(F(\overline{x})) + \mu Q_a + (1-\mu)Q(\overline{x}) \right) = 0 \tag{B.20}$$

From (B.18), calling $MEC(x)$ the marginal external cost at x, we can get

$$\lambda(x) = \lambda(0) + \delta \left(t(x) + MEC(x) \right) \tag{B.21}$$

On the other hand, straightforward calculations using (B.16) and (B.17) lead to

$$\frac{v(x)u_l}{v(x)u_c} = \frac{u - \lambda - \delta c - \gamma + \Lambda l^2}{l(\delta + l\Omega)} \tag{B.22}$$

where

$$\Lambda = \delta \left(\frac{1-\mu}{u_c^2} \right) (u_{ll}u_c - u_l u_{cl}) \tag{B.23}$$

$$\Omega = \delta \left(\frac{1-\mu}{u_c^2} \right) (u_{cl}u_c - u_l u_{cc}) \tag{B.24}$$

Then, since $\frac{u_l}{u_c} = Q(x)$, using (B.21) and (B.22) we can obtain that, for the optimum:

$$t(x) + c(x) + Q(x)l(x) = \frac{\overline{U} - \lambda(0) - \gamma}{\delta} - MEC(x) + l^2 \left(\frac{\Lambda - Q\Omega}{\delta} \right) \tag{B.25}$$

Using the budget constraint $\frac{\mu R}{H} + \frac{G}{H} + y = \tau(x) + t(x) + c(x) + Q(x)l(x)$, we then get the optimal toll:

$$\tau(x) = \left(\frac{\mu R}{H} + \frac{G}{H} + y - \frac{\overline{U} - \lambda(0) - \gamma}{\delta}\right) + MEC(x) + l^2\left(\frac{Q\Omega - \Lambda}{\delta}\right) \tag{B.26}$$

Then, note that the optimal toll $\tau(x)$ is not uniquely identified from (B.26). Indeed, since the right-hand side of (B.26) includes the redistribution of the toll revenue, if some $\tilde{\tau}(x)$ solves (B.26), $(\tilde{\tau}(x) + C)$ also does, for any constant C. Consequently, noting that the first term in the right-hand side of (B.26) is a constant, optimal instruments' family can be written as:

$$\tau(x) = MEC(x) + l^2\left(\frac{Q\Omega - \Lambda}{\delta}\right) + C \tag{B.27}$$

for any constant C. In particular, the following $\tau(x)$ is optimal:

$$\tau(x) = MEC(x) + l^2\left(\frac{Q\Omega - \Lambda}{\delta}\right) \tag{B.28}$$

Then, note that if $g_F = 0$, the marginal external cost is zero. Thus, in that case, one of the optimal tolls is given by:

$$\tau(x) = l^2\left(\frac{Q\Omega - \Lambda}{\delta}\right) + C \tag{B.29}$$

Finally, when $\mu = 1$, $\Lambda = \Omega = 0$, so that the optimum toll in (B.29) is zero everywhere. For $\mu \neq 1$, it is easy to check that

$$\frac{Q\Omega - \Lambda}{\delta} = \left(\frac{1-\mu}{u_c^3}\right)(2u_c u_l u_{cl} - u_l^2 u_{cc} - u_c^2 u_{ll}) \tag{B.30}$$

Furthermore, $2u_c u_l u_{cl} - u_l^2 u_{cc} - u_c^2 u_{ll} > 0$ if u is strictly quasi-concave, so this last expression is non-negative for all values of μ.

Now, since $Q(x) = \frac{u_l}{u_c}\ \forall\, 0 \leq x \leq \overline{x}$, differentiating this last expression with respect to x allow us to get

$$\frac{\partial Q(x)}{\partial x} = \frac{u_{cl}\left(u_c \frac{\partial c(x)}{\partial x} - u_l \frac{\partial l(x)}{\partial x}\right) - u_l u_{cc} \frac{\partial c(x)}{\partial x} + u_c u_{ll} \frac{\partial l(x)}{\partial x}}{u_c^2} \qquad \forall\, 0 \leq x \leq \overline{x} \tag{B.31}$$

From the equilibrium condition $u(c(x), l(x)) = \overline{U}\ \forall\, 0 \leq x \leq \overline{x}$, it is easy to obtain $u_c \frac{\partial c(x)}{\partial x} = -u_l \frac{\partial l(x)}{\partial x}$. Using this last equality in (B.31) reduces to

$$\frac{\partial Q(x)}{\partial x} = \frac{\frac{\partial c(x)}{\partial x}\left(2u_c u_{cl} - u_l u_{cc} - \frac{u_c^2}{u_l} u_{ll}\right)}{u_c^2} \qquad \forall\, 0 \leq x \leq \overline{x} \tag{B.32}$$

$$= \frac{\partial c(x)}{\partial x}\frac{u_c}{u_l}\left(\frac{2u_c u_l u_{cl} - u_l^2 u_{cc} - u_c^2 u_{ll}}{u_c^3}\right) \qquad \forall\, 0 \leq x \leq \overline{x} \tag{B.33}$$

Since $\frac{\partial c(x)}{\partial x} < 0$, $\frac{u_c}{u_l} > 0$, $\frac{2u_c u_l u_{cl} - u_l^2 u_{cc} - u_c^2 u_{ll}}{u_c^3} > 0$, we conclude that $\frac{\partial Q(x)}{\partial x} < 0$. Using this in the Muth-Mills condition

$$\frac{\partial Q(x)}{\partial x} = -\frac{t + \tau'(x)}{l(x)} \qquad \forall\ 0 \leq x \leq \overline{x} \tag{B.34}$$

we arrive to $\tau'(x) > -t\ \forall\ 0 \leq x \leq \overline{x}$. Returning to (B.33):

$$\frac{\partial Q(x)}{\partial x} = \frac{\partial c(x)}{\partial x} \frac{u_c}{u_l} \left(\frac{2u_c u_l u_{cl} - u_l^2 u_{cc} - u_c^2 u_{ll}}{u_c^3} \right) \qquad \forall\ 0 \leq x \leq \overline{x} \tag{B.35}$$

this implies that

$$l^2 \left(\frac{1-\mu}{u_c^3} \right) (2u_c u_l u_{cl} - u_l^2 u_{cc} - u_c^2 u_{ll}) = l^2 (1-\mu) \frac{\partial Q(x)}{\partial x} / \left(\frac{\partial c(x)}{\partial x} \frac{u_c}{u_l} \right) \tag{B.36}$$

$$= -l^2 (1-\mu) \frac{\partial Q(x)}{\partial x} / \frac{\partial l(x)}{\partial x} \tag{B.37}$$

$$= (1-\mu) \left(-\frac{\partial Q(x)}{\partial x} / \frac{\partial l(x)}{\partial x} \cdot \frac{l(x)}{Q(x)} \right) Q(x) l(x) \tag{B.38}$$

$$= (1-\mu) \frac{Q(x) l(x)}{|\sigma|} \tag{B.39}$$

where σ is the income-compensated elasticity of demand for housing.

B.2. Proof of Proposition 7

Indeed, if the conditions stated in Proposition 7 hold, then $\frac{1}{|\sigma|}$ is a non-decreasing function of x. In addition, $\frac{\partial Q(x)l(x)}{\partial x} = (1+\sigma)l\frac{\partial Q}{\partial x}$, so that $Q(x)l(x)$ –the expenditure in housing– is a decreasing function of x if $\sigma > -1$. Then, noting that $\sigma = \beta - 1$ for a Cobb-Douglas utility function $u(c,l) = c^{1-\beta}l^{\beta}$, it is easy to check that both conditions hold. On the other hand, for a CES utility function, $u(c,l) = ((1-\beta)c^{\rho} + \beta l^{\rho})^{\frac{1}{\rho}}$, tedious yet straightforward calculations lead to

$$\sigma(x) = \sigma\left(\frac{1}{1+\left(\frac{1-\beta}{\beta}\right)^{\sigma}Q(x)^{\sigma-1}} - 1\right)$$

where $\sigma = 1/(1-\rho)$ is the elasticity of substitution. Since the first term inside the parenthesis is positive, a sufficient condition for $\sigma > -1$ is $\sigma < 1 \Leftrightarrow \rho < 0$. In addition, since $Q(x)$ is a decreasing function of x, a sufficient condition for $\sigma'(x) < 0$ is also $\sigma < 1 \Leftrightarrow \rho < 0$.

For the proof of the extension of the city, let superscripts m and τ refer to the market equilibrium and the equilibrium when using τ, respectively. We first show that $Q^{\tau}(0) < Q^{m}(0)$. Indeed, the Muth-Mills conditions are

$$\frac{\partial Q^m}{\partial x} = -\frac{t}{l^m} \tag{B.40}$$

$$\frac{\partial Q^{\tau}}{\partial x} = -\frac{t+\tau'}{l^{\tau}} \tag{B.41}$$

Integrating (B.40) from 0 to $\overline{x}^m$ leads to

$$Q^m(0) = Q_a + Ht \tag{B.42}$$

while integrating (B.41) from 0 to $\overline{x}^{\tau}$ leads to

$$Q^{\tau}(0) = Q_a + Ht + \int_0^{\overline{x}^{\tau}} \frac{\tau'(x)}{l^{\tau}(x)}dx \tag{B.43}$$

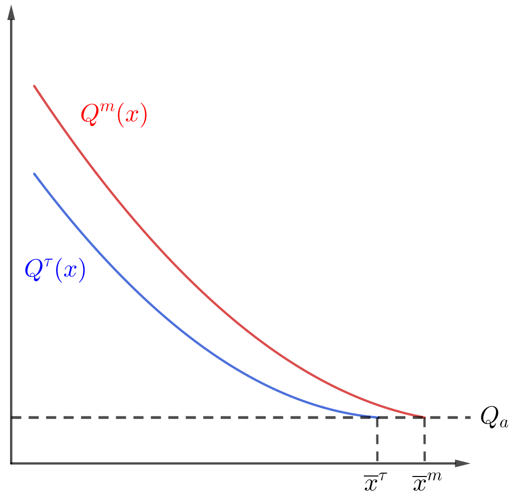
Using (B.42), (B.43) and the fact that $\tau' < 0$, we obtain $Q^{\tau}(0) < Q^m(0)$.

The second step of the proof is to show that the price curves have a single crossing point, which implies that the city has to be more extended (Figure B.1.b). There are three possible cases regarding the price curves:

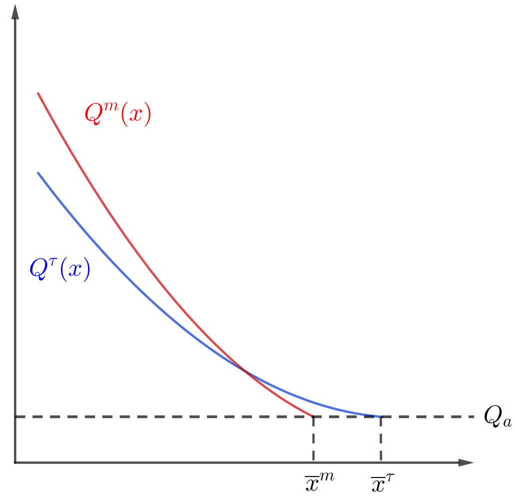
Case 1. $Q^{\tau}(x) \leq Q^m(x)\ \forall x \in [0, \overline{x}^{\tau}]$. In this case, it follows that $\overline{x}^{\tau} \leq \overline{x}^m$ (Figure B.1.a).

Case 2. $Q^{\tau}(x) \leq Q^m(x)\ \forall x \in [0, x_c]$, $Q^{\tau}(x) > Q^m(x)\ \forall x \in (x_c, \overline{x}^m]$, for some $x_c \in (0, \overline{x}^m)$. In this case, it follows that $\overline{x}^{\tau} > \overline{x}^m$ (Figure B.1.b).

Case 3. $Q^{\tau}(x)$ and $Q^m(x)$ intersect two or more times.



(a) Case 1



(b) Case 2

Figure B.1: Relationship between bid rent curves.

We show that cases 1 and 3 cannot occur.

For the Case 1, we proceed by contradiction: suppose $Q^\tau(x) \leq Q^m(x)$ $\forall x \in [0, \overline{x}^\tau]$. Then, consider any $x \in [0, \overline{x}^\tau]$. We will show that $l^\tau(x) \geq l^m(x)$. Note that the income net of transportation costs and taxes/subsidies at x in the market city is $I^m(x) = y + \frac{\mu R^m}{L} - tx$, while in the city when using τ is $I^\tau(x) = y + \frac{\mu R^\tau}{L} - tx - \tau(x)$. In general, $I^\tau(x) \geq I^m(x)$ or $I^\tau(x) < I^m(x)$ may hold, since depending on the value of x, $\tau(x)$ might be a tax or a subsidy. Additionally, the excess land rents are different in both cities. Thus, we show that $l^\tau(x) \geq l^m(x)$ for both possibilities.

Case 1.a. Suppose $I^\tau(x) \geq I^m(x)$. In this case, since $Q^\tau(x) \leq Q^m(x)$ and housing is a normal good, it follows directly that the housing consumption at x does not decrease compared to the market city, i.e., $l^\tau(x) \geq l^m(x)$. This last inequality is strict if either $I^\tau(x) > I^m(x)$ or $Q^\tau(x) < Q^m(x)$.

Case 1.b. Suppose $I^\tau(x) < I^m(x)$. In this case, since $Q^\tau(x) \leq Q^m(x)$ and the disposable income decreases, the consumption of the composite good c decreases (due to the income and substitution effect). Then, since we assume that the equilibrium utility in the city when using τ is higher, the consumption of housing must increase, i.e. $l^\tau(x) > l^m(x)$.

To conclude, note that we proved that $l^\tau(x) \geq l^m(x)$ $\forall x \in [0, \overline{x}^\tau]$, with strict inequality if $Q^\tau(x) < Q^m(x)$. Then,

$$
\begin{aligned}
L &= \int_0^{\overline{x}^\tau} \frac{1}{l^\tau(x)} dx \\
&< \int_0^{\overline{x}^\tau} \frac{1}{l^m(x)} dx \\
&\leq L
\end{aligned}
$$

The first inequality is strict since $Q^\tau(0) < Q^m(0)$, and by continuity, $l^\tau(x) > l^m(x)$ in an interval to the right of $x = 0$. The last inequality follows from the fact that, in this case, the market city is more extended (Figure B.1.a). Thus, this case cannot arise in equilibrium.

For the Case 3, we once again proceed by contradiction. Suppose that Q^τ and Q^m intersect at least two times. It is easy to see that, since $Q^\tau(0) < Q^m(0)$, at the first intersection, x_1, the following inequality holds:

$$\left.\frac{\partial Q^m}{\partial x}\right|_{x=x_1} < \left.\frac{\partial Q^\tau}{\partial x}\right|_{x=x_1} \tag{B.44}$$

In words, at the first intersection, Q^m is decreasing at a faster rate than Q^τ. Naturally, at the second intersection the reverse is true:

$$\left.\frac{\partial Q^m}{\partial x}\right|_{x=x_2} > \left.\frac{\partial Q^\tau}{\partial x}\right|_{x=x_2} \tag{B.45}$$

Using the Muth-Mills condition, condition (B.45) implies

$$\frac{t}{l^m(x_2)} < \frac{t + \tau'(x_2)}{l^\tau(x_2)} \tag{B.46}$$

$$\Rightarrow l^\tau(x_2) < l^m(x_2) \tag{B.47}$$

where we used $\tau'(x_2) < 0$ for the last inequality. Then, as Q^m and Q^τ intersect at x_2, $Q^m(x_2) = Q^\tau(x_2)$ must hold. From this, again two possibilities may arise since $I^\tau(x_2) \geq I^m(x_2)$ or $I^\tau(x_2) < I^m(x_2)$ may hold:

Case 2.a $I^\tau(x_2) \geq I^m(x_2)$. Since residents living at x_2 in the city that uses the instrument τ have at least the same disposable income than the residents living at x_2 in the market city, and they face the same prices for c and q, they consume at least the same amount of housing. Thus, (B.47) cannot hold.

Case 2.b $I^\tau(x_2) < I^m(x_2)$. In this case, residents living at x_2 in the city that uses the instrument τ have less disposable income than the residents living at x_2 in the market city, and they face the same prices for c and q. Thus, it is impossible for them to attain a higher utility, contradicting one of our hypotheses.

B.3. Proof of Proposition 8

In this proof, we show that if one considers a social welfare function that is the sum of the compensating variation of residents plus the share of the absentee landlord rents, then the market equilibrium does not maximize social welfare under some mild conditions for any $\mu < 1$. Those conditions are (i) an increasing marginal utility of income with distance (ii) decreasing spending in housing with distance. We show that these conditions hold if $u(c, l)$ is strictly quasi-concave. We proceed by showing that some income redistribution from the city center to the outskirts increases welfare.

We start by considering two equilibria. First, the resulting equilibrium when using a linear revenue-neutral combination of taxes and subsidies $\tau(x) = ax$. Second, we consider the resulting equilibrium when including a linear revenue-neutral combination of taxes and subsidies $\tau(x) = (a + da)x$, with $da \to 0$. That is, the second equilibrium corresponds to a marginal deviation from the first one. In this case, the compensating variation associated with this marginal deviation is defined as:

$$CV = -\left.\frac{dy}{da}\right|_{u=\text{constant}} = \frac{du}{da} \Big/ \frac{du}{dy} \tag{B.48}$$

With this, the change in social welfare, denoted by SW, when a changes to $a + da$ is:

$$\frac{dSW}{da} = H \cdot CV + (1 - \mu)\frac{dR}{da} \tag{B.49}$$

We are interested in the sign of $\frac{dSW}{da}$ evaluated at $a = 0$. If this derivative is negative, then an income redistribution from the city center to the outskirts is welfare increasing. To find this sign, we first calculate CV, studying the initial equilibrium. The urban equilibrium condition can be stated as

$$v\left(\frac{\mu R}{H} + \frac{G}{H} + y - (t + a)x - Ql, l\right) = \overline{U} \qquad \forall x \in [0, \overline{x}] \tag{B.50}$$

with v the indirect utility function. Totally differentiating (B.50) with respect to a leads to

$$v_c \cdot \left(\frac{\mu}{H}\frac{dR}{da} + \frac{1}{H}\frac{dG}{da} - x - l\frac{dQ}{da} - Q\frac{dl}{da}\right) + v_l \cdot \frac{dl}{da} = \frac{d\overline{U}}{da} \qquad \forall x \in [0, \overline{x}] \tag{B.51}$$

while totally differentiating (B.50) with respect to y leads to:

$$v_c \cdot \left(\frac{\mu}{H}\frac{dR}{dy} + \frac{1}{H}\frac{dG}{dy} + 1 - l\frac{dQ}{dy} - Q\frac{dl}{dy}\right) + v_l \cdot \frac{dl}{dy} = \frac{d\overline{U}}{dy} \qquad \forall x \in [0, \overline{x}] \tag{B.52}$$

On the other hand, the Muth-Mills condition is:

$$\frac{\partial Q}{\partial x} = -\frac{t + a}{l(x)} \quad \forall x \in [0, \overline{x}] \tag{B.53}$$

Integrating (B.53) with respect to x we get:

$$Q(x=0) - Q_a = H(t+a) \tag{B.54}$$

and thus

$$\frac{dQ(x=0)}{da} = H \tag{B.55}$$

$$\frac{dQ(x=0)}{dy} = 0 \tag{B.56}$$

Then, evaluating (B.51) and (B.52) in $x = 0$, using (B.55), (B.56) and the utility maximization condition $\frac{v_l}{v_c} = Q$ leads to

$$\frac{d\overline{U}}{da} = v_c(x=0) \cdot \left(\frac{\mu}{H}\frac{dR}{da} + \frac{1}{H}\frac{dG}{da} - Hl(x=0) \right) \tag{B.57}$$

$$\frac{d\overline{U}}{dy} = v_c(x=0) \cdot \left(\frac{\mu}{H}\frac{dR}{dy} + \frac{1}{H}\frac{dG}{dy} + 1 \right) \tag{B.58}$$

where we denote, with some abuse of notation, $v_c\left(\frac{\mu R}{H} + \frac{G}{H} + y - Q(x=0)l(x=0), l(x=0)\right)$ by $v_c(x=0)$. It follows that

$$CV = \frac{\frac{\mu}{H}\frac{dR}{da} + \frac{1}{H}\frac{dG}{da} - Hl(x=0)}{\frac{\mu}{H}\frac{dR}{dy} + \frac{1}{H}\frac{dG}{dy} + 1} \tag{B.59}$$

To find expressions for $\frac{dG}{da}$ and $\frac{dG}{dy}$, we use the definition of G:

$$G = \int_0^{\overline{x}} \frac{ax}{l(x)} dx \tag{B.60}$$

Replacing $l(x)$ through the Muth-Mills condition (B.53)

$$G = -\frac{a}{(t+a)} \int_0^{\overline{x}} \left(\frac{\partial Q}{\partial x} \right) x \; dx \tag{B.61}$$

$$= -\frac{a}{(t+a)} \int_0^{\overline{x}} \frac{\partial (Qx)}{\partial x} - Q(x) \; dx \tag{B.62}$$

$$= -\frac{a}{(t+a)} \left(Q(\overline{x})\overline{x} - \int_0^{\overline{x}} Q(x) \; dx \right) \tag{B.63}$$

$$= -\frac{a}{(t+a)} \left(\int_0^{\overline{x}} Q_a - Q(x) dx \right) \tag{B.64}$$

$$= \frac{a}{(t+a)} R \tag{B.65}$$

Using (B.60) and (B.65), we can obtain:

$$\int_0^{\overline{x}} \frac{x}{l(x)} dx = \frac{R}{t+a} \tag{B.66}$$

while totally differentiating (B.65) with respect to a and y, we get

$$\frac{dG}{da} = \frac{t}{(t+a)^2} R + \frac{a}{(t+a)} \frac{dR}{da} \tag{B.67}$$

$$\frac{dG}{dy} = \frac{a}{(t+a)} \frac{dR}{dy} \tag{B.68}$$

Since we are interested in the sign of $\frac{dSW}{da}$ evaluated at $a = 0$, we evaluate (B.67) and (B.68) at $a = 0$:

$$\left.\frac{dG}{da}\right|_{a=0} = \frac{R}{t} \tag{B.69}$$

$$\left.\frac{dG}{dy}\right|_{a=0} = 0 \tag{B.70}$$

To obtain expressions for $\dfrac{dR}{da}$ and $\dfrac{dR}{dy}$, we return to the definition of R:

$$R = \int_0^{\overline{x}} Q(x) - Q_a \; dx \tag{B.71}$$

Totally differentiating (B.71) with respect to y gives us

$$\frac{dR}{dy} = (Q(\overline{x}) - Q_a) \frac{d\overline{x}}{dy} + \int_0^{\overline{x}} \frac{dQ}{dy} dx \tag{B.72}$$

$$= \int_0^{\overline{x}} \frac{1}{l} \frac{\mu}{H} \frac{dR}{dy} + \frac{1}{l} + \frac{1}{H \cdot l} \frac{dG}{dy} - \frac{1}{l \cdot v_c} \frac{d\overline{U}}{dy} \; dx \tag{B.73}$$

$$= H \left(\frac{\mu}{H} \frac{dR}{dy} + 1 + \frac{1}{H} \frac{dG}{dy} \right) - \frac{d\overline{U}}{dy} \int_0^{\overline{x}} \frac{1}{l \cdot v_c} \; dx \tag{B.74}$$

$$= H \left(\frac{\mu}{H} \frac{dR}{dy} + 1 + \frac{1}{H} \frac{dG}{dy} \right) - \left(\frac{\mu}{H} \frac{dR}{dy} + 1 + \frac{1}{H} \frac{dG}{dy} \right) \int_0^{\overline{x}} \frac{v_c(x=0)}{l \cdot v_c} \; dx \tag{B.75}$$

For the second equality we used $Q(\overline{x}) = Q_a$, and the characterization of $\frac{dQ}{dy}$ given by the identity of (B.52). For the fourth inequality, we used (B.58). From (B.75) we can isolate $\frac{dR}{dy}$:

$$\frac{dR}{dy} = \left(\frac{H}{H(1-\mu) + \mu \int_0^{\overline{x}} \frac{v_c(x=0)}{l \cdot v_c} \; dx} \right) \left(H - \int_0^{\overline{x}} \frac{v_c(x=0)}{l \cdot v_c} \; dx \right) \left(1 + \frac{1}{H} \frac{dG}{dy} \right) \tag{B.76}$$

Similarly, totally differentiating (B.71) with respect to a allow us to get

$$\frac{dR}{da} = (Q(\overline{x}) - Q_a)\frac{d\overline{x}}{da} + \int_0^{\overline{x}} \frac{dQ}{da}dx \tag{B.77}$$

$$= \int_0^{\overline{x}} \frac{1}{l}\frac{\mu}{H}\frac{dR}{da} - \frac{x}{l} + \frac{1}{H \cdot l}\frac{dG}{da} - \frac{1}{l \cdot v_c}\frac{d\overline{U}}{da}\ dx \tag{B.78}$$

$$= H\left(\frac{\mu}{H}\frac{dR}{da} + \frac{1}{H}\frac{dG}{da}\right) - \frac{R}{t+a} - \frac{d\overline{U}}{da}\int_0^{\overline{x}} \frac{1}{l \cdot v_c}\ dx \tag{B.79}$$

$$= H\left(\frac{\mu}{H}\frac{dR}{da} + \frac{1}{H}\frac{dG}{da}\right) - \frac{R}{t+a} - CV\left(\frac{\mu}{H}\frac{dR}{dy} + 1\right)\int_0^{\overline{x}} \frac{v_c(x=0)}{l \cdot v_c}\ dx \tag{B.80}$$

For the second equality we used $Q(\overline{x}) = Q_a$, and the characterization of $\frac{dQ}{da}$ given by the identity of (B.51). For the third equality, we used (B.66). For the fourth inequality, we used (B.59) together with (B.57). It follows from (B.80) that

$$(1-\mu)\frac{dR}{da} = \frac{dG}{da} - \frac{R}{t+a} - CV\left(\frac{\mu}{H}\frac{dR}{dy} + 1\right)\int_0^{\overline{x}} \frac{v_c(x=0)}{l \cdot v_c}\ dx \tag{B.81}$$

Once again, since we are interested in the sign of $\frac{dSW}{da}$ evaluated at $a = 0$, we evaluate (B.76) and (B.81) at $a = 0$:

$$\left.\frac{dR}{dy}\right|_{a=0} = \left(\frac{H}{H(1-\mu) + \mu\displaystyle\int_0^{\overline{x}} \frac{v_c(x=0)}{l \cdot v_c}\ dx}\right)\left(H - \int_0^{\overline{x}} \frac{v_c(x=0)}{l \cdot v_c}\ dx\right) \tag{B.82}$$

$$(1-\mu)\left.\frac{dR}{da}\right|_{a=0} = -CV\left(\frac{\mu}{H}\frac{dR}{dy} + 1\right)\int_0^{\overline{x}} \frac{v_c(x=0)}{l \cdot v_c}\ dx \tag{B.83}$$

where we used (B.69) and (B.70) to replace $\frac{dG}{da}$ and $\frac{dG}{dy}$.

Note that from (B.82), it follows that $\left.\frac{dR}{dy}\right|_{a=0} > 0$ if the marginal utility of income is increasing with distance. Indeed, the first term on the right-hand size is clearly positive, while the second term is positive if $H > \int_0^{\overline{x}} \frac{v_c(x=0)}{l \cdot v_c}\ dx$. A sufficient condition for this is precisely $\frac{\partial v_c}{\partial x} > 0$.

Now, we study the sign of $CV|_{a=0}$, starting from (B.59):

$$CV|_{a=0} = \frac{\frac{\mu}{H}\left.\frac{dR}{da}\right|_{a=0} + \frac{R}{Ht} - H \cdot l(x=0)}{\frac{\mu}{H}\left.\frac{dR}{dy}\right|_{a=0} + 1} \tag{B.84}$$

Considering the previous comment, if the marginal utility of income is increasing with distance, then the denominator of (B.84) is clearly greater than zero. On the other hand, we show that if the spending in housing is decreasing with distance, then $\frac{R}{Ht} - H \cdot l(x=0) < 0$.

Indeed:

$$\frac{R}{Ht} - H \cdot l(x=0) = \frac{R - Ht \cdot H \cdot l(x=0)}{Ht} \tag{B.85}$$

$$= \frac{\int_0^{\overline{x}} Q(x) - Q_a \; dx - H \cdot l(x=0)(Q(x=0) - Q_a)}{Ht} \tag{B.86}$$

$$= \frac{\displaystyle\int_0^{\overline{x}} \frac{Q(x)l(x) - Q(x=0)l(x=0) - Q_a(l(x) - l(x=0))}{l(x)}}{Ht} \tag{B.87}$$

For the second equality we used (B.54) evaluated at $a = 0$. For the third equality we used $H = \int_0^{\overline{x}} \frac{1}{l} dx$.

Since $l(x) - l(x=0) \geq 0$, from (B.87) it follows that a sufficient condition for $\frac{R}{Ht} - H \cdot l(x = 0) < 0$ is indeed $\frac{\partial Q(x)l(x)}{\partial x} < 0$. Finally, note that if this holds, then $\left.\frac{dR}{da}\right|_{a=0}$ cannot be less or equal than zero. We proceed by contradiction. If $\left.\frac{dR}{da}\right|_{a=0} \leq 0$, then $CV|_{a=0} < 0$ under the previous assumptions, since the denominator in (B.84) is positive, and $\frac{R}{Ht} - H \cdot l(x=0) < 0$. But, from (B.83), $\left.\frac{dR}{da}\right|_{a=0}$ and $CV|_{a=0}$ have opposite signs for any $\mu < 1$, a contradiction. Consequently, $\left.\frac{dR}{da}\right|_{a=0} > 0$, and then $CV|_{a=0} < 0$. The only exception for this is when $\mu = 1$. In this case, from (B.83), $CV|_{a=0} = 0$.

Finally, we study the sign of $\left.\frac{dSW}{da}\right|_{a=0}$:

$$\left.\frac{dSW}{da}\right|_{a=0} = H \cdot CV|_{a=0} + (1-\mu) \left.\frac{dR}{da}\right|_{a=0} \tag{B.88}$$

$$= CV|_{a=0} \left(H - \left(\frac{\mu}{H} \left.\frac{dR}{dy}\right|_{a=0} + 1 \right) \int_0^{\overline{x}} \frac{v_c(x=0)}{l \cdot v_c} \right) \tag{B.89}$$

where we used (B.83) for the last equality. Clearly, $\left.\frac{dSW}{da}\right|_{a=0} = 0$ for $\mu = 1$, since $CV|_{a=0} = 0$. This is consistent with the well-known fact that the market equilibrium maximizes welfare under a public-ownership setting. We focus now on the case $\mu \neq 1$, where we have $CV|_{a=0} < 0$. Note that if the marginal utility of income is increasing with distance, then $H > \int_0^{\overline{x}} \frac{v_c(x=0)}{l \cdot v_c} \; dx$ and the following holds from (B.82):

$$\left.\frac{dR}{dy}\right|_{a=0} = \left(\frac{H}{H(1-\mu) + \mu \displaystyle\int_0^{\overline{x}} \frac{v_c(x=0)}{l \cdot v_c} \; dx} \right) \left(H - \int_0^{\overline{x}} \frac{v_c(x=0)}{l \cdot v_c} \; dx \right) \tag{B.90}$$

$$< \frac{H}{\displaystyle\int_0^{\overline{x}} \frac{v_c(x=0)}{l \cdot v_c} \; dx} \left(H - \int_0^{\overline{x}} \frac{v_c(x=0)}{l \cdot v_c} \; dx \right) \tag{B.91}$$

Using (B.91) in (B.89)

$$\left.\frac{dSW}{da}\right|_{a=0} \leq CV|_{a=0}\left[H - \left\{\frac{\mu}{\int_0^{\overline{x}} \frac{v_c(x=0)}{l\cdot v_c}\ dx}\left(H - \int_0^{\overline{x}} \frac{v_c(x=0)}{l\cdot v_c}\ dx\right) + 1\right\}\int_0^{\overline{x}} \frac{v_c(x=0)}{l\cdot v_c}\right] \tag{B.92}$$

$$= CV|_{a=0}\,(1-\mu)\left(H - \int_0^{\overline{x}} \frac{v_c(x=0)}{l\cdot v_c}dx\right) \tag{B.93}$$

$$< 0 \tag{B.94}$$

(B.92) holds with equality for $\mu = 0$. For the last inequality, we used $CV|_{a=0} < 0$ and $H > \int_0^{\overline{x}} \frac{v_c(x=0)}{l\cdot v_c}\ dx$ under our assumptions. Thus, a marginal resdistribution of income from the city center to the outskirts is welfare improving for any $\mu \in [0,1)$, provided two assumptions used in this proof: (i) $\frac{\partial v_c}{\partial x} > 0$ (ii) $\frac{\partial (Ql)}{\partial x} < 0$. We now show that these conditions hold if $u(c,l)$ is strictly quasi-concave and $\sigma > -1$:

(i)

$$\frac{\partial v_c}{\partial x} = v_{cc}\frac{\partial c}{\partial x} + v_{cl}\frac{\partial l}{x} \tag{B.95}$$

Then, since the utility is the equilibrium utility is constant throughout the city, it is easy to obtain $v_c\dfrac{\partial c}{\partial x} = -v_l\dfrac{\partial l}{\partial x}$. Using this in (B.95), we obtain:

$$\frac{\partial v_c}{\partial x} = \frac{1}{v_c}\frac{\partial l}{\partial x}\left(v_c v_{cl} - v_l v_{cc}\right) \tag{B.96}$$

We conclude using two well-known results: if $u(c,l)$ is strictly quasi-concave, then $\frac{\partial l}{\partial x} > 0$ (e.g., Brueckner, 1987) and $v_c v_{cl} - v_l v_{cc} > 0$ (e.g., Section 2.1 of Kanemoto, 1980).

(ii) As Brueckner (1987) shows

$$\frac{\partial (Ql)}{\partial x} = (1+\sigma)l\frac{\partial Q}{\partial x} \tag{B.97}$$

with σ the income-compensated price elasticity of demand for housing. Using the fact that $\frac{\partial Q}{\partial x} < 0$, it follows directly that $\frac{\partial (Ql)}{\partial x}$ if $\sigma > -1$.